Generalized Spatial Modulation for Large-Scale MIMO Systems: Analysis and Detection

T. Lakshmi Narasimhan, P. Raviteja, and A. Chockalingam
Department of Electrical and Communication Engineering
Indian Institute of Science, Bangalore, India

***Abstract*—Generalized Spatial modulation (GSM) uses n_t antenna elements but fewer radio frequency chains (n_{rf}) at the transmitter. Spatial modulation and spatial multiplexing are special cases of GSM with $n_{rf} = 1$ and $n_{rf} = n_t$, respectively. In GSM, apart from conveying information bits through n_{rf} modulation symbols, information bits are also conveyed through the indices of the active n_{rf} transmit antennas. In this paper, we derive analytical bounds on the code-word and bit error probabilities of maximum likelihood detection in GSM. The bounds are shown to be tight at medium to high signal-to-noise ratios (SNR). We also present a low-complexity detection algorithm based on reactive tabu search (RTS) for GSM in large-scale MIMO systems. Simulation results show that the proposed algorithm performs well and scales well in complexity.**

Keywords *– Large-scale MIMO systems, generalized spatial modulation, performance analysis, detection, reactive tabu search.*

I. Introduction

Multiple-input multiple-output (MIMO) systems with a large number of antennas (tens to hundreds) can provide several advantages like increased spectral and power efficiencies, which are key requirements in next generation of wireless communication systems. Key technological issues that need to be addressed in the practical realization of such large-scale MIMO systems include design and placement of compact antenna arrays, multiple radio frequency (RF) chains, and large-dimension transmit/receive signal processing techniques and algorithms [1],[2]. Spatial modulation [3], a relatively new modulation scheme for multi-antenna systems, can alleviate the need to have a large number of RF chains in large-scale MIMO systems.

In spatial modulation (SM), the transmitter has multiple transmit antennas but only one transmit RF chain. This means that only one antenna can be active at a time and the remaining antennas have to remain silent. The choice of the active antenna at a given time is made based on information bits. If $n_t = 2^m$ is the number of transmit antennas, then the index of the active antenna is chosen using $\log_2 n_t = m$ information bits. A conventional modulation (e.g., QAM) symbol is sent on the chosen antenna. If $\mathbb{A}$ is the modulation alphabet used, then the number of bits conveyed in one channel use in SM is $m + \log_2 |\mathbb{A}|$. It has been shown that SM outperforms conventional modulation in multiuser MIMO systems on the uplink [4],[5],[6]. This is because, for a given spectral efficiency, a reduced modulation alphabet size can be used in SM compared to that in conventional modulation.

The advantages of SM can be further enhanced through generalized spatial modulation (GSM) [1],[7]. In GSM, the transmitter is allowed to have more than one transmit RF chain. Let n_{rf} denote the number of RF chains at the transmitter. In GSM, $1 \leq n_{rf} \leq n_t$. Spatial modulation and spatial multiplexing are special cases of GSM with $n_{rf} = 1$ and $n_{rf} = n_t$, respectively. In GSM, in each channel use, n_{rf} modulation symbols are transmitted from n_{rf} antennas out of the n_t available antennas. The choice of n_{rf} out of n_t antennas conveys $\lfloor \log_2 \binom{n_{rf}}{n_t} \rfloor$ information bits. This is in addition to the information bits conveyed by the n_{rf} modulation symbols. It has been shown that for a given modulation alphabet and n_t, there exits an optimum n_{rf} that maximizes the spectral efficiency, and that this optimum n_{rf} can be less than n_t [7].

In this paper, we are interested in the performance analysis of GSM and detection of GSM signals in large-scale MIMO systems. Our new contributions in this paper can be summarized as follows.

- We first analytically characterize the code-word error probability (CEP) and the bit error probability (BEP) of the GSM system and derive closed-form expressions for the upper bounds on CEP and BEP for maximum likelihood (ML) detection. The obtained bounds are tight in the moderate-to-high SNR regime. The analytical bounds and simulation results show that, for a given spectral efficiency, GSM can outperform SM and spatial multiplexing.
- We then propose a algorithm for the detection of GSM signals in large-scale MIMO systems. The algorithm is based on reactive tabu search (RTS). An interesting aspect here is a neighborhood definition appropriate for GSM signal set. Simulation results show that the algorithm performs well in large-scale GSM-MIMO systems.

The rest of the paper is organized as follows. The system model for GSM-MIMO is presented in Section II. The analysis of CEP and BEP of GSM-MIMO is presented in Section III. In Section IV, we present the detection algorithm for large-scale GSM-MIMO. Section V concludes the paper.

II. GSM-MIMO system model

Consider a GSM-MIMO system with n_t antennas and n_{rf} RF chains at the transmitter, and n_r antennas at the receiver. The transmitter uses GSM. The GSM transmitter is shown in Fig. 1. In each channel use, the transmitter selects n_{rf} out of n_t antennas to transmit n_{rf} modulation symbols from

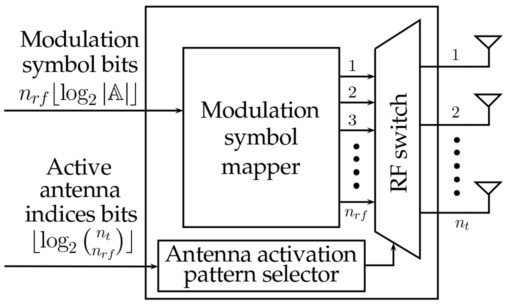

Fig. 1. The GSM transmitter.

a modulation alphabet $\mathbb{A}$. This choice of n_{rf} antennas can be any one of the possible $\binom{n_t}{n_{rf}}$ combinations. Thus, the number of information bits conveyed through indices of the chosen antennas is $\lfloor \log_2 \binom{n_t}{n_{rf}} \rfloor$. In addition to this, the number of information bits conveyed through the n_{rf} modulation symbols is $n_{rf}\lfloor \log_2 \mathbb{A} \rfloor$. Therefore, the total number of bits conveyed in a channel use in GSM is given by

$$\eta = \left\lfloor \log_2 \binom{n_t}{n_{rf}} \right\rfloor + n_{rf} \lfloor \log_2 \mathbb{A} \rfloor \quad \text{bpcu}. \tag{1}$$

Let $\mathbb{S}^{n_{rf}}_{n_t,\mathbb{A}}$ denote the GSM signal set, which is the set of all possible GSM signal vectors that can be transmitted. Out of the $\binom{n_t}{n_{rf}}$ possible antenna activation patterns[1], only $2^{\lfloor \log_2 \binom{n_t}{n_{rf}} \rfloor}$ activation patterns are needed for signaling. Let $\mathcal{S}$ denote this set of selected antenna activation patterns, where $|\mathcal{S}| = 2^{\lfloor \log_2 \binom{n_t}{n_{rf}} \rfloor}$. Then, $\mathbb{S}^{n_{rf}}_{n_t,\mathbb{A}}$ is given by

$$\mathbb{S}^{n_{rf}}_{n_t,\mathbb{A}} = \big\{ \mathbf{x} : x_i \in \mathbb{A} \cup \{0\}, \, \|\mathbf{x}\|_0 = n_{rf}, \, \mathcal{I}(\mathbf{x}) \in \mathcal{S} \big\}, \tag{2}$$

where $\mathbf{x}$ is the $n_t \times 1$ transmit vector, x_i is the ith entry of $\mathbf{x}$, $i = 1, \cdots, n_t$, $\|\mathbf{x}\|_0$ is the l_0-norm of the vector $\mathbf{x}$, and $\mathcal{I}(\mathbf{x})$ is a function that gives the activation pattern for $\mathbf{x}$; for e.g., $\mathcal{I}(\mathbf{x} = [+1 \ -1 \ -1 \ 0]^T) = [1 \ 1 \ 1 \ 0]^T$.

Let us give an example of the GSM signal set. Let $n_t = 4$, $n_{rf} = 2$, BPSK modulation, and $\mathcal{S} = \{[1\ 1\ 0\ 0]^T, [1\ 0\ 1\ 0]^T, [1\ 0\ 0\ 1]^T, [0\ 1\ 1\ 0]^T\}$. The GSM signal set for this example is given by

$$\mathbb{S}^2_{4,\text{BPSK}} = \left\{ \begin{bmatrix} +1 \\ +1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} +1 \\ -1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} -1 \\ -1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} -1 \\ +1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} +1 \\ 0 \\ +1 \\ 0 \end{bmatrix}, \begin{bmatrix} +1 \\ 0 \\ -1 \\ 0 \end{bmatrix}, \begin{bmatrix} -1 \\ 0 \\ -1 \\ 0 \end{bmatrix}, \begin{bmatrix} -1 \\ 0 \\ +1 \\ 0 \end{bmatrix}, \right.$$
$$\left. \begin{bmatrix} +1 \\ 0 \\ 0 \\ +1 \end{bmatrix}, \begin{bmatrix} +1 \\ 0 \\ 0 \\ -1 \end{bmatrix}, \begin{bmatrix} -1 \\ 0 \\ 0 \\ -1 \end{bmatrix}, \begin{bmatrix} -1 \\ 0 \\ 0 \\ +1 \end{bmatrix}, \begin{bmatrix} 0 \\ +1 \\ +1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ +1 \\ -1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ -1 \\ -1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ -1 \\ +1 \\ 0 \end{bmatrix} \right\}.$$

The $n_r \times 1$ received signal vector $\mathbf{y} = [y_1 \ y_2 \cdots \ y_{n_r}]^T$ at the receiver can be written as

$$\mathbf{y} \quad = \quad \mathbf{H}\mathbf{x} + \mathbf{w}, \tag{3}$$

where $\mathbf{x} \in \mathbb{S}^{n_{rf}}_{n_t,\mathbb{A}}$ is the transmit vector, $\mathbf{H} \in \mathbb{C}^{n_r \times n_t}$ is the channel gain matrix, whose (i,j)th entry $h_{i,j} \sim \mathcal{CN}(0,1)$ denotes the complex channel gain from the jth transmit antenna to the ith receive antenna, and $\mathbf{w} = [w_1 \ w_2 \cdots \ w_{n_r}]^T$ is the noise vector whose entries are modeled as complex Gaussian with zero mean and variance σ^2. For this system model, the ML detection rule is given by

$$\hat{\mathbf{x}} = \underset{\mathbf{x} \in \mathbb{S}^{n_{rf}}_{n_t,\mathbb{A}}}{\text{argmin}} \ \|\mathbf{y} - \mathbf{H}\mathbf{x}\|^2, \tag{4}$$

where $\|\mathbf{y} - \mathbf{H}\mathbf{x}\|^2$ is the ML cost.

III. CEP and BEP Analysis of GSM-MIMO

In this section, we analyze the CEP and BEP performance of ML detection in GSM-MIMO. Assume that all the transmit GSM signal vectors are equally likely. The ML detection rule in (4) can be written as

$$\hat{\mathbf{x}} = \underset{\mathbf{x} \in \mathbb{S}^{n_{rf}}_{n_t,\mathbb{A}}}{\text{argmin}} \ \|\mathbf{y} - \sum_{k=1}^{n_t} x_k \mathbf{h}_k\|^2, \tag{5}$$

where x_k is the kth element of $\mathbf{x}$, and $\mathbf{h}_k$ is the kth column of $\mathbf{H}$. The pairwise error probability (PEP) that $\mathbf{x}$ can be decoded as $\tilde{\mathbf{x}} \in \mathbb{S}^{n_{rf}}_{n_t,\mathbb{A}}$ can be written as

$$\begin{aligned} P(\mathbf{x} \to \tilde{\mathbf{x}}|\mathbf{H}) &= P\Big(\|\mathbf{y} - \sum_{k=1}^{n_t} x_k \mathbf{h}_k\|^2 > \|\mathbf{y} - \sum_{k=1}^{n_t} \tilde{x}_k \mathbf{h}_k\|^2 \Big| \mathbf{H}\Big) \\ &= P\Big(\sum_{r=1}^{n_r} |y_r - \sum_{k=1}^{n_t} x_k h_{r,k}|^2 > \sum_{r=1}^{n_r} |y_r - \sum_{k=1}^{n_t} \tilde{x}_k h_{r,k}|^2 \Big| \mathbf{H}\Big), \end{aligned} \tag{6}$$

where $h_{r,k}$ is the (r,k)th element of $\mathbf{H}$. Let $A_r = \sum_{k=1}^{n_t} x_k h_{r,k}$ and $\tilde{A}_r = \sum_{k=1}^{n_t} \tilde{x}_k h_{r,k}$. Since $\mathbf{x}$ is the transmitted vector, $y_r = A_r + w_r$, $r = 1, \cdots, n_r$. Now, we can write

$$\begin{aligned} P(\mathbf{x} \to \tilde{\mathbf{x}}|\mathbf{H}) &= P\Big(\sum_{r=1}^{n_r} |y_r - A_r|^2 > \sum_{r=1}^{n_r} |y_r - \tilde{A}_r|^2 \Big| \mathbf{H}\Big) \\ &= P\Big(\sum_{r=1}^{n_r} |w_r|^2 > \sum_{r=1}^{n_r} |A_r + w_r - \tilde{A}_r|^2 \Big| \mathbf{H}\Big) \\ &= P\Big(\sum_{r=1}^{n_r} 2\Re((\tilde{A}_r - A_r) w_r^*) > \sum_{r=1}^{n_r} |A_r - \tilde{A}_r|^2 \Big| \mathbf{H}\Big), \end{aligned} \tag{7}$$

where $\sum_{r=1}^{n_r} 2\Re((\tilde{A}_r - A_r) w_r^*)$ is a Gaussian random variable with mean zero and variance $2\sigma^2 \sum_{r=1}^{n_r} |A_r - \tilde{A}_r|^2$. Therefore,

$$\begin{aligned} P(\mathbf{x} \to \tilde{\mathbf{x}}|\mathbf{H}) &= Q\left(\sqrt{\sum_{r=1}^{n_r} |A_r - \tilde{A}_r|^2 / 2\sigma^2}\right) \\ &= Q\left(\sqrt{\Big\|\sum_{k=1}^{n_t} (x_k - \tilde{x}_k)\mathbf{h}_k\Big\|^2 / 2\sigma^2}\right). \end{aligned} \tag{8}$$

The argument in (8) is a central χ^2-distribution with $2n_r$ degrees of freedom. Computation of the unconditional PEP requires the expectation of the $Q(.)$ function in (8) w.r.t. $\mathbf{H}$, which can be obtained as follows [8]:

$$\begin{aligned} \text{PEP}(\mathbf{x} \to \tilde{\mathbf{x}}) &= \mathbb{E}_{\mathbf{H}}\{P(\mathbf{x} \to \tilde{\mathbf{x}}|\mathbf{H})\} \\ &= f(\gamma)^{n_r} \sum_{r=0}^{n_r-1} \binom{n_r - 1 + r}{r} (1 - f(\gamma))^r, \end{aligned} \tag{9}$$

[1] An antenna activation pattern is a $n_t \times 1$ vector consisting of 1's and 0's, where a 1 in a coordinate indicates that the antenna corresponding to that coordinate is active and a 0 indicates that the corresponding antenna is silent.

where $f(\gamma) \triangleq \frac{1}{2}\Big(1 - \sqrt{\frac{\gamma}{1+\gamma}}\Big)$, $\gamma \triangleq \frac{\alpha}{4\sigma^2}$, $\alpha = \sum_{k=1}^{n_t} \theta_k$, and $\theta_k \triangleq |x_k - \tilde{x}_k|^2$. Now, an upper bound on the average CEP can be obtained as

$$P_{\text{CEP}} \leq \frac{1}{2^\eta} \sum_{\mathbf{x}\in\mathbb{S}^{n_{rf}}_{n_t,\mathbb{A}}} \sum_{\tilde{\mathbf{x}}\in\mathbb{S}^{n_{rf}}_{n_t,\mathbb{A}}\backslash\mathbf{x}} \text{PEP}(\mathbf{x}\to\tilde{\mathbf{x}}). \tag{10}$$

From (10), the average BEP can be upper bounded as

$$P_{\text{BEP}} \leq \frac{1}{2^\eta} \sum_{\mathbf{x}\in\mathbb{S}^{n_{rf}}_{n_t,\mathbb{A}}} \sum_{\tilde{\mathbf{x}}\in\mathbb{S}^{n_{rf}}_{n_t,\mathbb{A}}\backslash\mathbf{x}} \text{PEP}(\mathbf{x}\to\tilde{\mathbf{x}}) \frac{d(\mathbf{x},\tilde{\mathbf{x}})}{\eta}, \tag{11}$$

where $d(\mathbf{x},\tilde{\mathbf{x}})$ is the number of bits in which $\mathbf{x}$ differs from $\tilde{\mathbf{x}}$. The total number of PEPs that are to be calculated is $(2^\eta)\times(2^\eta-1)$. Therefore, the complexity of the computation of the above bounds on CEP and BEP will increase exponentially for large values of n_t, n_{rf}. In the following subsection, we propose a simplification that reduces this computational complexity.

A. Computation of the upper bounds for large n_t, n_{rf}

The CEP expression in (10) can be written in the form:

$$P_{\text{CEP}} \leq \frac{1}{2^\eta} \sum_{i=1}^{|\mathcal{S}|}\sum_{j=1}^{|\mathcal{S}|} \sum_{\mathbf{x}:\mathcal{I}(\mathbf{x})=\mathbf{s}_i\in\mathcal{S}} \sum_{\substack{\tilde{\mathbf{x}}:\mathcal{I}(\tilde{\mathbf{x}})=\mathbf{s}_j\in\mathcal{S},\\ \tilde{\mathbf{x}}\neq\mathbf{x}}} \text{PEP}(\mathbf{x}\to\tilde{\mathbf{x}}). \tag{12}$$

For a given pair of activation patterns $\mathbf{s}_i$ and $\mathbf{s}_j$, $i,j \in \{1,\cdots,|\mathcal{S}|\}$, the total number of PEPs are $|\mathbb{A}|^{2n_{rf}}$ when $i\neq j$, and $|\mathbb{A}|^{n_{rf}}(|\mathbb{A}|^{n_{rf}}-1)$ when $i=j$.

Complexity reduction 1: For a pair of activation patterns $\mathbf{s}_i$ and $\mathbf{s}_j$, let $\mathcal{A}_{ij}$ denote the set of active antennas that are common to both $\mathbf{s}_i$ and $\mathbf{s}_j$. Define $\beta_{ij} = n_{rf} - |\mathcal{A}_{ij}|$. Note that $\beta_{ij} \in \{0,1,\cdots,\min(n_{rf}, n_t-n_{rf})\}$. Also, note that for any i,j for which $\beta_{ij}=q$, the value of the summation $\sum\limits_{\mathbf{x}:\mathcal{I}(\mathbf{x})=\mathbf{s}_i} \sum\limits_{\tilde{\mathbf{x}}:\mathcal{I}(\tilde{\mathbf{x}})=\mathbf{s}_j,\,\tilde{\mathbf{x}}\neq\mathbf{x}} \text{PEP}(\mathbf{x}\to\tilde{\mathbf{x}})$ in (12) will be the same, and so it is enough to compute this summation only once for each q. With this simplification, (12) can be written as

$$P_{\text{CEP}} \leq \frac{1}{2^\eta} \sum_{q=0}^{\min(n_{rf},n_t-n_{rf})} \phi(q) \sum_{\mathbf{x}:\mathcal{I}(\mathbf{x})=\mathbf{s}_i} \sum_{\substack{\tilde{\mathbf{x}}:\mathcal{I}(\tilde{\mathbf{x}})=\mathbf{s}_j\\ \beta_{ij}=q}} \text{PEP}(\mathbf{x}\to\tilde{\mathbf{x}}), \tag{13}$$

where $\phi(q)$ is the number of $(\mathbf{s}_i,\mathbf{s}_j)$ pairs for which $\beta_{ij}=q$, which can be computed easily.

Complexity reduction 2: For each value of q, we need to compute $|\mathbb{A}|^{2n_{rf}}$ PEPs. We propose to reduce this complexity as follows. The parameter α in (9) is the summation of n_t terms. Out of these n_t terms, $n_t-(n_{rf}+q)$ terms will be zero for a given value of q. Of the $(n_{rf}+q)$ non-zero terms, $2q$ terms will take values from $\mathbb{J} \triangleq \{|c|^2 : c\in\mathbb{A}\}$, and $n_{rf}-q$ terms will take values from $\mathbb{L} \triangleq \{|c-\tilde{c}|^2 : c,\tilde{c}\in\mathbb{A}\}$. Let $\mathbb{J}=\{j_1,j_2,\cdots,j_m\}$ and $\mathbb{L}=\{l_1,l_2,\cdots,l_n\}$, where $j_1<j_2<\cdots<j_m$, $l_1<l_2<\cdots<l_n$, $m=|\mathbb{J}|$, and $n=|\mathbb{L}|$. We write α as $\alpha=\alpha_1+\alpha_2$, where α_1 is the sum of $2q$ terms from $\mathbb{J}$ and α_2 is the sum of $n_{rf}-q$ terms from $\mathbb{L}$. Note that α_1 can take values in the range $2qj_1$ to $2qj_m$. For a given value of α_1, the following equations must be satisfied:

$$\sum_{i=1}^{m} j_i v_i = \alpha_1, \qquad \sum_{i=1}^{m} v_i = 2q, \tag{14}$$

where v_i is an integer such that $v_i \in \{0,1,\cdots,\lfloor(\alpha_1 - \sum_{k=i+1}^{m} j_k v_k)/j_i\rfloor\}$. Similarly, α_2 can take values in the range $(n_{rf}-q)l_1$ to $(n_{rf}-q)l_n$, and, for a given value of α_2, the following equations must be satisfied:

$$\sum_{i=1}^{n} l_i u_i = \alpha_2, \qquad \sum_{i=1}^{n} u_i = n_{rf}-q, \tag{15}$$

where u_i is an integer such that $u_i \in \{0,1,\cdots,\lfloor(\alpha_2 - \sum_{k=i+1}^{n} l_k u_k)/l_i\rfloor\}$.

Since $\alpha=\alpha_1+\alpha_2$, α lies in the range $2qj_1+(n_{rf}-q)l_1$ to $2qj_m+(n_{rf}-q)l_n$. The choice of v_i's and u_i's to attain a particular α is not unique, i.e., there exist multiple pairs of $\mathbf{x}$ and $\tilde{\mathbf{x}}$ that correspond to different values of v_i's and u_i's but the same value of α. Thus, we need to evaluate (9) only once for a given value of α and count the number of possible combinations of v_i's and u_i's that correspond to that α.

Remark: The above complexity reduction schemes significantly simplify the computation of (12), because without these simplifications the sum $\sum\limits_{\mathbf{x}:\mathcal{I}(\mathbf{x})=\mathbf{s}_i} \sum\limits_{\tilde{\mathbf{x}}:\mathcal{I}(\tilde{\mathbf{x}})=\mathbf{s}_j,\,\tilde{\mathbf{x}}\neq\mathbf{x}} \text{PEP}(\mathbf{x}\to\tilde{\mathbf{x}})$ needs to be computed for all i,j, which is prohibitive for large n_t, n_{rf}. The following examples illustrate the achieved complexity reduction.

Example 1: For $n_t=22$, $n_{rf}=16$, we have $|\mathcal{S}|=2^{16}$. A direct computation of (12) which involves a double summation from 1 to $|\mathcal{S}|$ is prohibitive. Whereas for these parameters, $q\in\{0,1,\cdots,6\}$. Hence (13) can be easily computed in much fewer computations. This illustrates complexity reduction 1.

Example 2: For $n_t=4$, $n_{rf}=3$, $\mathbb{A}=\{-1-j,-1+j,1-j,1+j\}$, we have $\mathbb{J}=\{2\}$, $\mathbb{L}=\{0,4,8\}$. For a particular value of q, say $q=1$, the summation in (13) requires computation of the PEPs for 64 different pairs of GSM signal vectors. But since α lies in the range 4 to 20, we need to compute only 17 PEPs. This illustrates complexity reduction 2.

B. Results and discussion

In this subsection, we present numerical results of the CEP and BEP performance of GSM-MIMO. We compare the analytical upper bounds with the simulation results. We use the notation '(n_t,n_{rf})-GSM' to refer to a GSM-MIMO system with n_t transmit antennas and n_{rf} transmit RF chains.

In Fig. 2, we compare the simulated CEP and BEP with the analytical upper bounds for the (4,3)-GSM system with $n_r=4$ and 4-QAM, at a spectral efficiency of 8 bits per channel use (bpcu). From Fig. 2, we see that the analytical upper bound is quite tight in the medium-to-high SNR regime.

In Figs. 3 and 4, we present a comparison between the performance of GSM-MIMO with those of SM-MIMO and V-BLAST (spatial multiplexing) MIMO. Recall that SM-MIMO and V-BLAST MIMO are special cases of GSM-MIMO with $n_{rf}=1$ and $n_{rf}=n_t$, respectively. Figure 3 shows the CEP

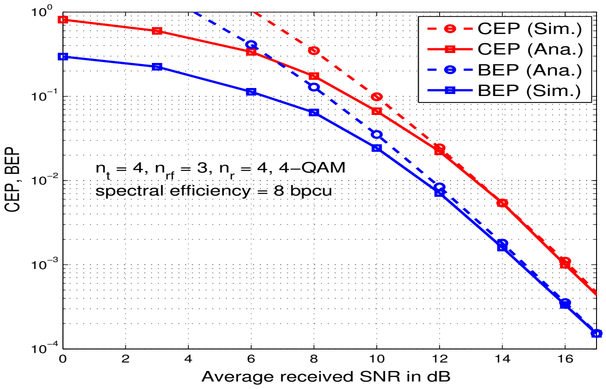

Fig. 2. BEP and CEP performance of (4,3)-GSM system with n_r = 4, 4-QAM, 8 bpcu.

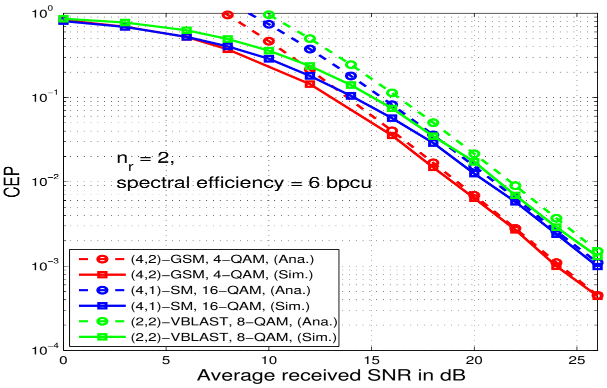

Fig. 3. CEP comparison between i) (4,2)-GSM system with 4-QAM, ii) (4,1)-GSM system (i.e., SM-MIMO system) with 16-QAM, and iii) (2,2)-GSM system (i.e., V-BLAST system) with 8-QAM. n_r = 2, 6 bpcu.

comparison's between i) (4,2)-GSM system with 4-QAM, ii) (4,1)-GSM system (i.e., SM system) with 16-QAM, and iii) (2,2)-GSM system (i.e., V-BLAST system) with 8-QAM, with $n_r = 2$. Note that all the three systems have the same spectral efficiency of 6 bpcu. Figure 4 shows the corresponding BEP plots. From Figs. 3 and 4, we can observe that i) the upper bounds are tight at medium-to-high SNRs, and ii) the GSM-MIMO system outperforms both SM-MIMO and V-BLAST MIMO systems.

IV. Detection in large-scale GSM-MIMO

The complexity of ML detection in GSM-MIMO increases exponentially with increase in n_t, n_{rf}. In this section, we propose a low complexity algorithm for GSM-MIMO signal detection. The algorithm is based on reactive tabu search with random restarts (R3TS). Details of the R3TS algorithm for detection in V-BLAST MIMO systems are available in [9],[10]. For adapting this algorithm for detection of GSM-MIMO signals, we need to define appropriate neighborhood for the GSM signal set. We define the neighborhood as follows.

Neighborhood definition for GSM signal set: We define the neighborhood $\mathcal{N}(\mathbf{x})$ for a GSM signal vector $\mathbf{x} \in \mathbb{S}^{n_{rf}}_{n_t,\mathbb{A}}$ as the set of all possible signal vectors which differ from $\mathbf{x}$ in

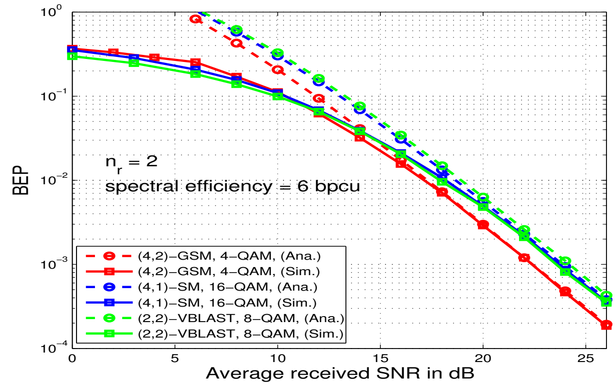

Fig. 4. BEP comparison between i) (4,2)-GSM system with 4-QAM, ii) (4,1)-GSM system (i.e., SM-MIMO system) with 16-QAM, and iii) (2,2)-GSM system (i.e., V-BLAST system) with 8-QAM. n_r = 2, 6 bpcu.

either one modulation symbol or in one active antenna index. That is, $\mathcal{N}(\mathbf{x}) = \mathcal{N}_1(\mathbf{x}) \cup \mathcal{N}_2(\mathbf{x})$, where

$$\begin{aligned}\mathcal{N}_1(\mathbf{x}) = \{\mathbf{z} : z_k = x_k, \forall k \text{ except for some } k_1; \\ \mathcal{I}(\mathbf{z}) = \mathcal{I}(\mathbf{x}),\, z_{k_1} \in \mathbb{A} \setminus x_{k_1}\}, \end{aligned} \quad (16)$$

$$\begin{aligned}\mathcal{N}_2(\mathbf{x}) = \{\mathbf{z} : \beta_{ij} = 1, \text{where } \mathcal{I}(\mathbf{x}) = \mathbf{s}_i, \mathcal{I}(\mathbf{z}) = \mathbf{s}_j; \\ z_k = x_k, \forall k \text{ except for some } k_1, k_2 \\ \text{s.t. } x_{k_1} = 0, z_{k_1} \in \mathbb{A}, z_{k_2} = 0\}. \end{aligned} \quad (17)$$

So, a transmitted vector $\mathbf{x}$ will have $(|\mathbb{A}|-1)n_{rf} + (n_t - n_{rf})n_{rf}|\mathbb{A}|$ neighbors. For e.g., for n_t = 3, n_{rf} = 2 and BPSK,

$$\mathcal{N}_1\left(\begin{bmatrix}+1\\+1\\0\end{bmatrix}\right) = \left\{\begin{bmatrix}-1\\+1\\0\end{bmatrix}, \begin{bmatrix}+1\\-1\\0\end{bmatrix}\right\}. \quad (18)$$

$$\mathcal{N}_2\left(\begin{bmatrix}+1\\+1\\0\end{bmatrix}\right) = \left\{\begin{bmatrix}0\\+1\\+1\end{bmatrix}, \begin{bmatrix}0\\+1\\-1\end{bmatrix}, \begin{bmatrix}+1\\0\\+1\end{bmatrix}, \begin{bmatrix}+1\\0\\-1\end{bmatrix}\right\}. \quad (19)$$

Tabu matrix: For the above neighborhood definition, the tabu matrix $\mathbf{T}$ is of size $(n_t + \binom{n_t}{2})|\mathbb{A}| \times |\mathbb{A}|$, where the first $n_t|\mathbb{A}|$ rows correspond to $\mathcal{N}_1(\mathbf{x})$ and the next $\binom{n_t}{2}|\mathbb{A}|$ rows correspond to $\mathcal{N}_2(\mathbf{x})$. For a solution vector $\mathbf{x}$, if $\mathbf{z} \in \mathcal{N}_1(\mathbf{x})$ then it corresponds to $((k_1-1)|\mathbb{A}| + t, t')$th position in the tabu matrix, where $x_{k_1} = a_t$, $z_{k_1} = a_{t'}$, and $a_t, a_{t'} \in \mathbb{A}$. If $\mathbf{z} \in \mathcal{N}_2(\mathbf{x})$, then it corresponds to $\big(n_t|\mathbb{A}| + \frac{(\tilde{k_1}-1)(2n_t-\tilde{k_1})}{2}|\mathbb{A}| + (\tilde{k_2}-\tilde{k_1}-1)|\mathbb{A}| + t, t'\big)$th position in the tabu matrix $\mathbf{T}$, where $\tilde{k_1} = \min(k_1,k_2)$, $\tilde{k_2} = \max(k_1,k_2)$, $x_{k_1} = 0, z_{k_1} = a_{t'}$, $x_{k_2} = a_t$, and $a_t, a_{t'} \in \mathbb{A}$.

R3TS-GSM detection algorithm: The algorithm starts with an initial solution vector $\mathbf{x}^{(0)}$ as the current solution. For example, $\mathbf{x}^{(0)}$ can be the MMSE solution vector $\mathbf{x}_{MMSE}$. All the entries of the tabu matrix are initially set to zero. Let m denote the iteration index, P the tabu period, $\mathbf{g}^{(m)}$ the vector with the least ML cost till the mth iteration, l_{rep} the average number of iterations between two successive occurrences of the same solution vector. Initialize $P = P_0$, $\mathbf{g}^{(0)} = \mathbf{x}^{(0)}$, and $l_{rep} = 0$. In each iteration (e.g., mth iteration), perform the following steps.

Step 1: Find $\mathbf{z}^{best_1} = \underset{\mathbf{z}\in\mathcal{N}(\mathbf{x}^m)}{\text{argmin}} \ \|\mathbf{y}-\mathbf{H}\mathbf{z}\|^2$. The move from $\mathbf{x}^m$ to $\mathbf{z}^{best_1}$ is accepted if any one of the following conditions is satisfied:
(i) $\|\mathbf{y}-\mathbf{H}\mathbf{z}^{best_1}\|^2 < \|\mathbf{y}-\mathbf{H}\mathbf{x}^m\|^2$,
(ii) If $\mathbf{z}^{best_1} \in \mathcal{N}_1(\mathbf{x}^m)$, then $\mathbf{T}((k_1-1)|\mathbb{A}|+t,t')) = 0$. If $\mathbf{z}^{best_1} \in \mathcal{N}_2(\mathbf{x}^m)$, then $\mathbf{T}\big(n_t|\mathbb{A}| + \frac{(\tilde{k_1}-1)(2n_t-\tilde{k_1})}{2}|\mathbb{A}| + (\tilde{k_2}-\tilde{k_1}-1)|\mathbb{A}|+t,t'\big) = 0$. If a move is accepted, then $\mathbf{x}^{(m+1)} = \mathbf{z}^{best_1}$. If a move is not accepted, then find $\mathbf{z}^{best_2} = \underset{\mathbf{z}\in\mathcal{N}(\mathbf{x}^m)\backslash\mathbf{z}^{best_1}}{\text{argmin}} \ \|\mathbf{y}-\mathbf{H}\mathbf{z}\|^2$, and check the above conditions for $\mathbf{z}^{best_2}$. If this is also not accepted, then repeat the procedure for $\mathbf{z}^{best_3}$, and so on. If all the neighbors are tabu, then all the entries in the $\mathbf{T}$ are decremented by the minimum value in $\mathbf{T}$. Then repeat the procedure from $\mathbf{z}^{best_1}$ to find $\mathbf{x}^{m+1}$.

Step 2: After step 1, the new solution $\mathbf{x}^{m+1}$ is checked for repetition. Repetition can be checked by comparing ML costs of all the solutions in the previous iterations. If there is a repetition, then l_{rep} is updated and $P \leftarrow P+1$. If $\|\mathbf{y}-\mathbf{H}\mathbf{x}^{m+1}\|^2 < \|\mathbf{y}-\mathbf{H}\mathbf{x}^m\|^2$, then do

- if $\mathbf{x}^{m+1} \in \mathcal{N}_1(\mathbf{x}^m)$ then $\mathbf{T}((k_1-1)|\mathbb{A}|+t,t')) = 0$,
- if $\mathbf{x}^{m+1} \in \mathcal{N}_2(\mathbf{x}^m)$ then $\mathbf{T}\big(n_t|\mathbb{A}| + \frac{(\tilde{k_1}-1)(2n_t-\tilde{k_1})}{2}|\mathbb{A}| + (\tilde{k_2}-\tilde{k_1}-1)|\mathbb{A}|+t,t'\big) = 0$,
- if $\mathcal{I}(\mathbf{x}^{m+1}) \in \mathcal{S}$, then $\mathbf{g}^{(m+1)} = \mathbf{x}^{m+1}$

else

- if $\mathbf{x}^{m+1} \in \mathcal{N}_1(\mathbf{x}^m)$, then $\mathbf{T}((k_1-1)|\mathbb{A}|+t,t')) = P+1$,
- if $\mathbf{x}^{m+1} \in \mathcal{N}_2(\mathbf{x}^m)$, then $\mathbf{T}\big(n_t|\mathbb{A}| + \frac{(\tilde{k_1}-1)(2n_t-\tilde{k_1})}{2}|\mathbb{A}| + (\tilde{k_2}-\tilde{k_1}-1)|\mathbb{A}|+t,t'\big) = P+1$,
- $\mathbf{g}^{(m+1)} = \mathbf{g}^{(m)}$.

Step 3: Update the entries of the tabu matrix as $\mathbf{T}(r,s) = \max(\mathbf{T}(r,s)-1,0)$.

The algorithm can be stopped after a maximum number of iterations max_iter or when the l_{rep} value exceeds a threshold max_rep. The performance of the algorithm can be improved by using multiple restarts, where, in each restart, we start with a different initial solution. The algorithm is stopped after a particular number of maximum restarts max_rest or if the ML cost of the solution vector obtained so far is below the $n_r\sigma^2 + \sqrt{n_r\sigma^4}$. The best solution with least ML cost is declared as the final output solution.

A. Results and discussions

In Fig. 5, we show the performance (22,16)-GSM system with 4-QAM and (16,16)-GSM system (i.e., V-BLAST system) with 8-QAM, both at 48 bpcu and $n_r = 16$. For V-BLAST detection, we used sphere decoding (i.e., ML detection). For GSM detection, we used the R3TS algorithm with the following parameters: $max\_iter = 1000$, $max\_rep = 300$, $max\_rest = 50$. We have plotted CEP upper bounds as well as simulated CEP and BEP. The following observations can be made from Fig. 5: $i)$ the proposed complexity reduction techniques allow us to compute the CEP bounds for ML detection for large n_t (=16,22) and n_{rf} (=16), and these bounds are tight at moderate-to-high SNRs (e.g., for $n_t = n_r = 16$), $ii)$ at high SNRs, the simulated CEP of R3TS detection (which a low complexity suboptimum detection) is close to the CEP upper bound of ML detection for (22,16)-GSM system, and $iii)$ at moderate-to-high SNRs, (22,16)-GSM system with R3TS detection outperforms V-BLAST system with sphere decoding.

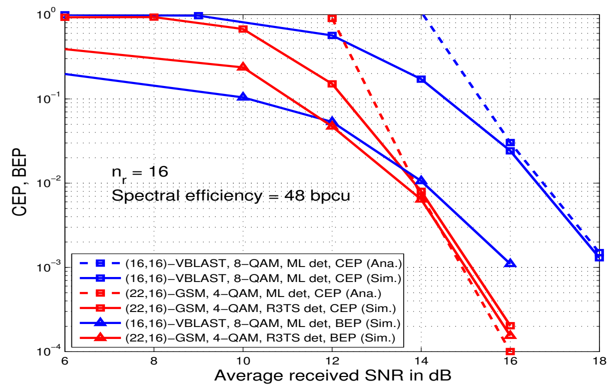

Fig. 5. CEP and BEP of (22,16)-GSM system with 4-QAM and (16,16)-GSM system (i.e., V-BLAST system) with 8-QAM. $n_r = 16$, 48 bpcu.

V. Conclusions

We studied large-scale generalized spatial modulation MIMO (GSM-MIMO) systems. We first derived analytical upper bounds on the CEP and BEP performance of GSM-MIMO and showed that the bounds are tight at medium to high SNRs. We also proposed complexity reduction schemes that allowed the computation of the bounds for large n_t, n_{rf}. We then presented a reactive tabu search (RTS) based algorithm for the detection of large-scale GSM-MIMO signals. Our analytical and simulation results show that, for a given spectral efficiency, GSM-MIMO system can achieve better performance compared to SM-MIMO and V-BLAST systems.

References

[1] A. Chockalingam and B. Sundar Rajan, *Large MIMO Systems*, Cambridge Univ. Press, Feb. 2014.

[2] F. Rusek, D. Persson, B. K. Lau, E. G. Larsson, T. L.Marzetta, O. Edfors, F. Tufvesson, "Scaling up MIMO: opportunities and challenges with very large arrays," *IEEE Sig. Proc. Mag.*, vol. 30, no. 1, pp. 40-60, Jan. 2013.

[3] M. Di Renzo, H. Haas, A. Ghrayeb, S. Sugiura, and L. Hanzo, "Spatial modulation for generalized MIMO: challenges, opportunities and implementation," *Proceedings of the IEEE*, vol. 102, no. 1, Jan. 2014.

[4] N. Serafimovski1, S. Sinanovic, M. Di Renzo, and H. Haas, "Multiple access spatial modulation," *EURASIP J. Wireless Commun. and Networking 2012*, 2012:299.

[5] T. Lakshmi Narasimhan, P. Raviteja, and A. Chockalingam, "Large-scale multiuser SM-MIMO versus massive MIMO," *Proc. ITA'2014*, Feb. 2014.

[6] P. Raviteja, T Lakshmi Narasimhan and A. Chockalingam, "Detection in large-scale multiuser SM-MIMO systems: algorithms and performance," *accepted in IEEE VTC'2014-Spring*, May 2014.

[7] T. Datta and A. Chockalingam, "On generalized spatial modulation," *Proc. IEEE WCNC'2013*, Apr. 2013.

[8] M. S Alouini and A.Goldsmith, "A unified approach for calculating error rates of linearly modulated signals over generalized fading channels," *IEEE Trans. on Commun.*, vol. 47, no. 9, pp. 13241334, Sep. 1999.

[9] N. Srinidhi, T. Datta, A. Chockalingam, and B. S. Rajan, "Layered tabu search algorithm for large-MIMO detection and a lower bound on ML performance," *IEEE Trans. Commun.*, pp. 2955-2963, Nov. 2011.

[10] T. Datta, N. Srinidhi, A. Chockalingam, and B. S. Rajan, "Random-restart reactive tabu search algorithm for detection in large-MIMO systems," *IEEE Commun. Lett.*, vol. 14, no. 12, pp. 1107-1109, Dec. 2010.