Optimum Selection Combining of Binary NCFSK Signals on Independent Rayleigh Fading Channels

A. Ramesh*, A. Chockalingam† and L. B. Milstein‡

* Wireless and Broadband Communications, Synopsys (India) Pvt. Ltd., Bangalore 560095, INDIA

† Department of Electrical Communication Engineering, Indian Institute of Science, Bangalore 560012, INDIA

‡ Department of Electrical and Computer Engineering, University of California, San Diego, La Jolla, CA 92093, U.S.A

Abstract*— In this paper, we derive a new selection combining (SC) scheme for noncoherent binary FSK signals on independent (but not necessarily identically distributed) Rayleigh fading channels with L-antenna diversity reception. With this combining scheme, we choose the diversity branch having the largest magnitude of the logarithm of the ratio of the a posteriori probabilities (log-APP ratio – LAPPR) of the transmitted information bit. We show that this scheme minimizes the probability of bit error, thus proving the optimality. We also show that a) the traditional square-law combining of all L diversity branches is equivalent to combining the LAPPRs of all the L diversity branches, and b) the SC scheme proposed by Neasmith and Beaulieu is a special case of the proposed optimum SC scheme for independent and identically distributed (i.i.d) Rayleigh fading. Bit error probability results show that, at 10^{-4} BER, a) for i.i.d Rayleigh fading, the proposed optimum SC combining scheme performs better than the existing SC schemes by 0.5 dB for $L = 3$ and 1.5 dB for $L = 5$, and performs within 0.5 dB of the scheme which square-law combines all the L diversity branches, and b) for independent Rayleigh fading, the proposed optimum SC scheme gives an additional gain of 2.0 dB over the SC schemes of Pierce and Chyi *et al.

I. Introduction

Diversity reception is a well known technique for mitigating the effects of fading in wireless communication systems [1]. For binary noncoherent FSK (NCFSK) modulation on Rayleigh fading channels, Pierce, in [2], derived the bit error probability of a *selection combining* (SC) scheme in which the diversity branch that has the largest instantaneous signal power (which is proportional to the square of the fading amplitude on that branch) is chosen for the subsequent signal detection. In [3], Chyi *et al* analyzed a SC scheme, known as the maximum output selection combining scheme, for M-ary modulated signals. In this scheme, the diversity branch with the largest square-law detector output is chosen. However, neither of the SC schemes in [2] and [3] are optimal. Recently, Kim and Kim, in [4], derived the optimum SC scheme for binary phase shift keying (BPSK) signals in Rayleigh fading.

In this paper, we propose a new SC scheme for binary NCFSK signals in which the diversity branch having the largest magnitude of the logarithm of the a posteriori probabilities ratio (LAPPR) of the transmitted information bit is chosen. We show that our scheme minimizes the probability of bit error, thus proving its optimality. We derive the bit error performance of the proposed optimum SC scheme for binary NCFSK signals on independent (but not necessarily identically distributed) Rayleigh fading channels with L-antenna receive diversity. For the case of i.i.d fading, we obtain a closed-form expression for the probability of bit error. We compare the performance of the proposed optimum SC scheme with that of the schemes proposed by Pierce in [2] and Chyi *et al* in [3].

We show that a) the traditional square-law combining all the L diversity branches is equivalent to combining the LAPPRs of all the L diversity branches, and b) the SC scheme proposed by Neasmith and Beaulieu in [5] is a special case of the proposed optimum SC scheme for i.i.d Rayleigh fading. Our bit error probability results show that, at a BER of 10^{-4}, a) for i.i.d Rayleigh fading, the proposed optimum SC combining scheme performs better than the existing SC schemes by 0.5 dB for $L = 3$ and 1.5 dB for $L = 5$, and performs within 0.5 dB of the scheme which square-law combines all the L diversity branches, and b) for independent Rayleigh fading, the proposed SC scheme gives an additional gain of 2.0 dB over the SC schemes of Pierce and Chyi *et al.*

The rest of the paper is organized as follows. In Section II, we introduce the system model and derive the LAPPR for bit detection. In Section III, the bit error performance of the proposed optimum SC scheme is derived. In Section IV, the independent fading extensions to the i.i.d fading analyses of other SC schemes are given. Section V gives the comparative performance of the proposed optimum SC scheme versus other existing schemes, and conclusions are given in Section VI.

II. System Model

We assume that the transmitted symbols are BFSK modulated with $\underline{s}_0 = [1,0]^T$ and $\underline{s}_1 = [0,1]^T$ denoting the BFSK symbols, associated with the messages m_0 and m_1, respectively. The complex orthonormal basis functions $\phi_1(t) = exp(j2\pi f_1 t)$ and $\phi_2(t) = exp(j2\pi f_2 t)$ represent the trans-

This work was supported in part by the Office of Naval Research under grant N00014-02-1-0001, and by the TRW foundation.

mitted information symbol $\underline{s}_m = [s_{m,x}, s_{m,y}]$. That is, $s_m(t) = s_{m,x}\phi_1(t) + s_{m,y}\phi_2(t)$.

The transmitted symbols are passed through a fading channel and noise is added to them at the receiver front end. We assume that the fading process is slow, frequency non-selective and remains constant over one symbol interval. Assuming perfect symbol timing at the receiver, the equivalent lowpass representation of the received symbols, after the noncoherent demodulation as shown in Fig. 1, is given by [6]

$$\underline{r}_c^{(l)} = \alpha^{(l)}\underline{s}_i \cos\theta^{(l)} + \underline{n}_c^{(l)}, \tag{1}$$

$$\underline{r}_s^{(l)} = \alpha^{(l)}\underline{s}_i \sin\theta^{(l)} + \underline{n}_s^{(l)}, \qquad l = 1, 2, \ldots, L, \tag{2}$$

where $\underline{s}_i$ is the transmitted BFSK signal point corresponding to the message m_i, $i \in \{0, 1\}$, and $\underline{r}_c^{(l)}$ and $\underline{r}_s^{(l)}$ denote the vector valued outputs of the quadrature demodulators at the l^{th} antenna. The random phase on the l^{th} antenna path, $\theta^{(l)}$, is distributed uniformly over $[0, 2\pi]$, and $\alpha^{(l)}$ is the random fade experienced by the transmitted symbol $\underline{s}_i$ on the l^{th} antenna path. We assume that the $\alpha^{(l)}$'s, are independent, but *not necessarily identically distributed,* Rayleigh random variables. The second moment of $\alpha^{(l)}$ is set to Ω_l (i.e., $E([\alpha^{(l)}]^2) = \Omega_l$), and $\underline{n}_c^{(l)}$ and $\underline{n}_s^{(l)}$ denote the in phase and quadrature phase noise vectors on the l^{th} antenna path whose components have zero mean and variance $\sigma^2 = N_0/2E_b$, where E_b/N_0 is the SNR per bit. It is noted that the $\underline{r}_c^{(l)}$, $\underline{r}_s^{(l)}$, $\underline{s}_i$, $\underline{n}_c^{(l)}$, $\underline{n}_s^{(l)}$ are vectors, each having two dimensions. With the assumption of signal point $\underline{s}_1$ being transmitted corresponding to the data bit '1', we have

$$\underline{r}_c^{(l)} = [r_{c,x}^{(l)}, r_{c,y}^{(l)}] = [n_{c,x}^{(l)}, \alpha^{(l)}\cos\theta^{(l)} + n_{c,y}^{(l)}], \tag{3}$$

$$\underline{r}_s^{(l)} = [r_{s,x}^{(l)}, r_{s,y}^{(l)}] = [n_{s,x}^{(l)}, \alpha^{(l)}\sin\theta^{(l)} + n_{s,y}^{(l)}]. \tag{4}$$

We propose to choose the diversity branch whose magnitude of the LAPPR (i.e., $|LAPPR|$) of the transmitted information symbol is the largest. The LAPPR of the transmitted information symbol $\underline{s}_i$ on the l^{th} antenna path is given by

$$LAPPR^{(l)} \triangleq \Lambda^{(l)} = \log\left(\frac{\text{Prob}(\underline{s}_i = \underline{s}_1|\underline{r}_c^{(l)}, \underline{r}_s^{(l)})}{\text{Prob}(\underline{s}_i = \underline{s}_0|\underline{r}_c^{(l)}, \underline{r}_s^{(l)})}\right). \tag{5}$$

For equally probable message signals,

$$\Lambda^{(l)} = \log\left(\frac{f(\underline{r}_c^{(l)}, \underline{r}_s^{(l)}|\underline{s}_i = \underline{s}_1)}{f(\underline{r}_c^{(l)}, \underline{r}_s^{(l)}|\underline{s}_i = \underline{s}_0)}\right). \tag{6}$$

The quantity $f(\underline{r}_c^{(l)}, \underline{r}_s^{(l)}|\underline{s}_i = \underline{s}_m)$, $m \in \{0, 1\}$, can be calculated as follows [7]:

$$f_{\underline{r}_c^{(l)}, \underline{r}_s^{(l)}}(\underline{x}, \underline{y}|\underline{s}_i = \underline{s}_m) = E_{\alpha^{(l)}}\left\{E_{\theta^{(l)}}\left[f_{\underline{r}_c^{(l)}\underline{r}_s^{(l)}}(\underline{x}, \underline{y}|\underline{s}_m, \alpha^{(l)}, \theta^{(l)})\right]\right\}, \tag{7}$$

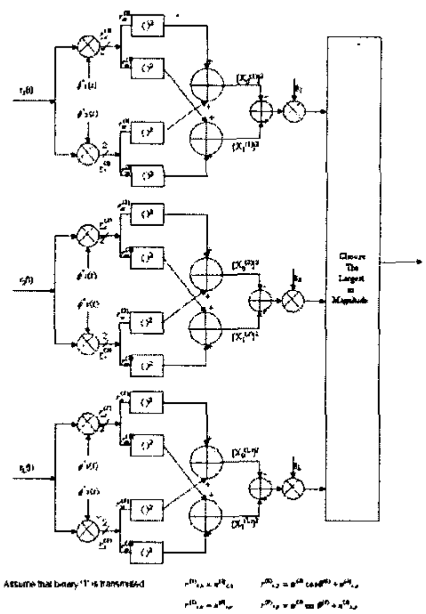

Fig. 1. Proposed Optimum SC Receiver for Binary NCFSK Signals.

where $E_{\alpha^{(l)}}[\cdot]$ and $E_{\theta^{(l)}}[\cdot]$ denote the expectation operations with respect to $\alpha^{(l)}$ and $\theta^{(l)}$, respectively. The quantity $f_{\underline{r}_c^{(l)}, \underline{r}_s^{(l)}}(\underline{x}, \underline{y}|\underline{s}_m, \alpha^{(l)}, \theta^{(l)})$ can be calculated as

$$\begin{aligned} &= f_{\underline{n}_c^{(l)}}(\underline{x} - \alpha^{(l)}\underline{s}_m\cos\theta^{(l)}) f_{\underline{n}_s^{(l)}}(\underline{y} - \alpha^{(l)}\underline{s}_m\sin\theta^{(l)}) \\ &\sim e^{-[\alpha^{(l)}]^2\frac{E_b}{N_0}} e^{\frac{2E_b}{N_0}(\underline{x}\cdot\underline{s}_m\alpha^{(l)}\cos\theta^{(l)} + \underline{y}\cdot\underline{s}_m\alpha^{(l)}\sin\theta^{(l)})} \\ &\sim e^{-[\alpha^{(l)}]^2\frac{E_b}{N_0}} e^{\frac{2E_b X_m^{(l)}}{N_0}\alpha^{(l)}\cos\left(\theta^{(l)} - \phi_m^{(l)}\right)}, \end{aligned} \tag{8}$$

where $X_m^{(l)} = \sqrt{(\underline{x}\cdot\underline{s}_m)^2 + (\underline{y}\cdot\underline{s}_m)^2}$ and $\phi_m^{(l)} = \tan^{-1}\left(\frac{\underline{y}\cdot\underline{s}_m}{\underline{x}\cdot\underline{s}_m}\right)$. From (8), $f_{\underline{r}_c^{(l)}, \underline{r}_s^{(l)}}(\underline{x}, \underline{y}|\underline{s}_m, \alpha^{(l)})$ is equal to

$$\begin{aligned} &= E_{\theta^{(l)}}\left(\frac{E_b}{\pi N_0} e^{-(2+[\alpha^{(l)}]^2)\frac{E_b}{N_0}} e^{\frac{2X_m^{(l)}E_b}{N_0}\alpha^{(l)}\cos(\theta^{(l)} - \phi_m^{(l)})}\right) \\ &\sim e^{-[\alpha^{(l)}]^2\frac{E_b}{N_0}} I_0\left(2\alpha^{(l)}\frac{X_m^{(l)}E_b}{N_0}\right), \end{aligned} \tag{9}$$

where $I_0(\cdot)$ is the modified Bessel function of the zeroth order and first kind [6]. Finally, we can obtain $f_{\underline{r}_c^{(l)}, \underline{r}_s^{(l)}}(\underline{x}, \underline{y}|\underline{s}_m)$, from (9), as

$$\begin{aligned} f_{\underline{r}_c^{(l)}, \underline{r}_s^{(l)}}(\underline{x}, \underline{y}|\underline{s}_m) &\sim E_{\alpha^{(l)}}\left(e^{-[\alpha^{(l)}]^2\frac{E_b}{N_0}} I_0\left(2\alpha^{(l)}\frac{X_m^{(l)}E_b}{N_0}\right)\right) \\ &\sim \frac{1}{1+\Omega_l\gamma} e^{\frac{[X_m^{(l)}]^2\Omega_l\gamma^2}{1+\Omega_l\gamma}}, \end{aligned} \tag{10}$$

where[1] $\gamma = E_b/N_0$. Finally, substituting (10) in (6) and scaling by γ^2, we obtain

$$\Lambda^{(l)} = g_l \left([X_1^{(l)}]^2 - [X_0^{(l)}]^2\right), \quad (11)$$

where g_l is the weighting factor on l^{th} antenna path and is given by

$$g_l = \frac{\Omega_l}{1+\Omega_l\gamma}. \quad (12)$$

We choose the diversity branch whose magnitude of the LAPPR in (6) is the largest. In Appendix-A1, we show that this proposed combining scheme minimizes the error probability of reception, and hence is optimum. The traditional square-law combining of all the L diversity branches can be obtained by combining the LAPPRs of all the L available diversity branches. That is, $\sum_{l=1}^{L} \Lambda^{(l)}$ is the decision statistic for the L-branch square-law combining [1],[6].

III. Bit Error Probability Analysis

In this section, we analyze the bit error probability performance of the proposed optimum SC combiner for both the i.i.d as well as the independent fading cases. First, we assume that the transmitted bit is a '1'. With this, the pdfs of $X_1^{(l)}$ and $X_0^{(l)}$ are given by [7]

$$\begin{aligned} f_{X_1^{(l)}}(x) &= 2\lambda_1^{(l)} x e^{-\lambda_1^{(l)} x^2} \\ f_{X_0^{(l)}}(x) &= 2\lambda_2^{(l)} x e^{-\lambda_2^{(l)} x^2}, \end{aligned} \quad (13)$$

where $\lambda_1^{(l)} = \frac{\gamma}{1+\Omega_l\gamma}$ and $\lambda_2^{(l)} = \gamma$. The pdf of $\Lambda^{(l)}$ is given by

$$f_{\Lambda^{(l)}}(x) = \begin{cases} \frac{a_l}{g_l} e^{-\frac{\lambda_1^{(l)} x}{g_l}} & x \geq 0 \\ \frac{a_l}{g_l} e^{\frac{\lambda_2^{(l)} x}{g_l}} & x < 0, \end{cases} \quad (14)$$

where $a_l = \frac{\lambda_1^{(l)}\lambda_2^{(l)}}{\lambda_1^{(l)}+\lambda_2^{(l)}}$. It is not difficult to show that the pdf of $|\Lambda^{(l)}|$ is

$$f_{|\Lambda^{(l)}|}(x) = \frac{a_l}{g_l}\left(e^{-\frac{\lambda_1^{(l)} x}{g_l}} + e^{-\frac{\lambda_2^{(l)} x}{g_l}}\right), \quad x \geq 0. \quad (15)$$

A. Independent Fading Case

In this subsection, we assume that the fading process is independent (but not identically distributed) on all the L antennas. With the assumption of '1' being transmitted, bit detection error occurs if, of the L statistics $\Lambda^{(l)}$, the one with the largest magnitude is negative. Accordingly, the error probability of the proposed optimum SC (OSC) scheme is given by the following expression:

$$\begin{aligned} P_e^{OSC,indep} &= \sum_{l=1}^{L} \text{Prob}\left(\max\left(|\Lambda^{(j),j\neq l}|\right) < |\Lambda^{(l)}|, \Lambda^{(l)} < 0\right) \\ &= \sum_{l=1}^{L} \text{Prob}\left(\max\left(|\Lambda^{(j),j\neq l}|\right) + \Lambda^{(l)} < 0\right) = \sum_{l=1}^{L} P_e(l), \end{aligned} \quad (16)$$

where

$$\begin{aligned} P_e(l) &= \text{Prob}\left(\max\left(|\Lambda^{(j),j\neq l}|\right) + \Lambda^{(l)} < 0\right) \\ &= \text{Prob}\left(Z_l + \Lambda^{(l)} < 0\right) = \int_{z=0}^{\infty}\int_{x=-\infty}^{-z} f_{Z_l}(z) f_{\Lambda^{(l)}}(x)\, dz\, dx \\ &= \frac{a_l}{g_l}\int_{z=0}^{\infty} f_{Z_l}(z) e^{-\frac{\lambda_2^{(l)} z}{g_l}} dz = \frac{1+\overline{\gamma}_l}{2+\overline{\gamma}_l}\sum_{j=1, j\neq l}^{L} \Phi(l,j). \end{aligned} \quad (17)$$

In the above, $Z_l = \max\left(|\Lambda^{(j),j\neq l}|\right)$, $\overline{\gamma}_l = \Omega_l\gamma$, $l = 1,\ldots,L$, and

$$\Phi(l,j) = \int_{\psi=0}^{\frac{\pi}{2}} \prod_{k=1, k\neq\{j,l\}}^{L} \left(1 - \frac{(1+\overline{\gamma}_k)e^{-\frac{\overline{\gamma}_l \tan\psi}{\overline{\gamma}_k}} + e^{-\frac{\overline{\gamma}_l \tan\psi}{\overline{\gamma}_k}}}{2+\overline{\gamma}_k}\right) \cdot e^{-(1+\overline{\gamma}_l)\tan\psi} \sec^2\psi\, d\psi. \quad (18)$$

The derivation of (18) is given in Appendix-A2. Unfortunately, (18) cannot be expressed in a closed-form. Nevertheless, the integral $\Phi(l,j)$ can be evaluated with minimal effort as the integration limits are finite and the integrand involves a simple-to-compute expression. The performance for the i.i.d fading case can be derived as a special case of this independent fading result, as given in the following subsection.

B. I.I.D. Fading Case

With the assumption of i.i.d fading on each antenna path, we have $\Omega_l = \Omega$, $\lambda_1^{(l)} = \lambda_1$, $\lambda_2^{(l)} = \lambda_2$ and $a_l = a$, $\forall\, l = 1, 2, \cdots, L$. Without loss of generality, we can assume $\Omega = 1$ and drop the g_l's. With this, the probability of bit detection error, $P_e^{OSC,iid}$, is given by

$$\begin{aligned} P_e^{OSC,iid} &= \sum_{l=1}^{L} \text{Prob}\left(\max\left(|\Lambda^{(j),j\neq l}|\right) < |\Lambda^{(l)}|, \Lambda^{(l)} < 0\right) \\ &= L\cdot\text{Prob}\left(\max\left(|\Lambda^{(j),j\neq l}|\right) + \Lambda^{(l)} < 0\right) \\ &= L(L-1)\sum_{k=0}^{L-2}\sum_{j=0}^{k}\binom{L-2}{k}\binom{k}{j}\frac{(-1)^k a^2}{(\lambda_1+\lambda_2)^k}\lambda_1^{k-j}\lambda_2^{j} \\ &\cdot\left(\frac{1}{\lambda_2(\lambda_1(j+1)+\lambda_2(k-j+1))} + \frac{1}{\lambda_2(\lambda_1 j+\lambda_2(k-j+2))}\right). \end{aligned} \quad (19)$$

The derivation of (19) is given in Appendix-A3. It is to be noted that, unlike (17) and (18), the expression in (19) is in closed-form and can be evaluated very easily.

[1] In deriving (10) we have used the result $\int_{t=0}^{\infty} te^{-at^2}J_0(bt)dt = \frac{1}{2a}e^{-\frac{b^2}{4a}}$, with $J_0(it) = I_0(t)$, $i = \sqrt{-1}$.

IV. Extensions to Existing Analyses

We would like to compare the error probability performance of the proposed optimum SC scheme with other existing SC schemes, as well as with the L-branch square-law combining scheme. We consider the three SC schemes proposed in [2], [3], and [5] for our comparison. In [2],[3],[5], only i.i.d Rayleigh fading is considered. In order to enable the comparison of these SC schemes with our scheme, we extend the analyses of these schemes to the independent fading case.

We have derived the bit error probability expressions, in closed-form, for Pierce SC scheme and Chyi *et al* SC scheme, for independent Rayleigh fading, as [8]

$$P_e^{Pierce,indep} = \sum_{l=1}^{L} \left(\frac{1}{1+\overline{\gamma}_l} - \sum_{i=1,i\neq l}^{L} \frac{1}{1+\overline{\gamma}_l + \frac{2\overline{\gamma}_l}{\overline{\gamma}_i}} + \sum_{i=1,j=1,i\neq j\neq l}^{L} \frac{1}{\frac{2\overline{\gamma}_l}{\overline{\gamma}_i} + \frac{2\overline{\gamma}_l}{\overline{\gamma}_j}} + \cdots + \frac{(-1)^{L-1}}{1+\overline{\gamma}_l + \sum_{i=1,i\neq l}^{L} \frac{2\overline{\gamma}_l}{\overline{\gamma}_i}} \right), \quad (20)$$

and

$$P_e^{Chyi,indep} = 1 - \sum_{l=1}^{L}\sum_{m=0}^{L} \left[\frac{(-1)^m \binom{L}{m}}{1+m(1+\overline{\gamma}_l)} - \sum_{i=1,i\neq l}^{L} \frac{(-1)^m \binom{L}{m}}{1+(1+\overline{\gamma}_l)(m+\frac{\overline{\gamma}_i}{1+\overline{\gamma}_i})} + \cdots + \frac{(-1)^{L-1+m}\binom{L}{m}}{1+(1+\overline{\gamma}_l)(m+\sum_{i=1,i\neq l}^{L}\frac{\overline{\gamma}_i}{1+\overline{\gamma}_i})} \right]. \quad (21)$$

In the SC scheme by Neasmith and Beaulieu [5], the branch with largest energy difference, $[X_1^{(l)}]^2 - [X_0^{(l)}]^2$, is chosen. Clearly, for the case of i.i.d Rayleigh fading, the error probability of the Neasmith SC scheme is the same as that of the proposed optimum SC scheme. For the case of independent Rayleigh fading, the error probability expression for the Neasmith scheme can be obtained by substituting $g_l = 1$ in (17) and (18).

Finally, combining the LAPPRs of all the diversity branches gives the performance of the L-branch square-law combining. The error probability of the L-branch square-law combining scheme on i.i.d Rayleigh fading is given by [6]

$$P_e^{Sq.Law,iid} = \frac{\lambda_2^L}{\Gamma(L)} \int_{y=0}^{\infty} e^{-\lambda_2 y} y^{L-1} \Gamma_c(\lambda_1 y, L)\, dy, \quad (22)$$

where $\Gamma_c(x,n) = \frac{1}{\Gamma(n)} \int_{u=0}^{x} e^{-u} u^{n-1} du$.

V. Results and Discussion

Fig. 2 shows the comparative performance of the proposed optimum SC scheme, the SC scheme of Pierce, and the SC scheme of Chyi *et al*, for i.i.d Rayleigh fading when $L = 3$, 4, and 5. The performance of the square-law combiner, which combines all the L available LAPPRs, is also plotted for comparison. With three antenna diversity reception ($L = 3$), at a bit error rate of 10^{-4}, the optimum SC scheme performs 0.3 dB poorer compared to the L-branch square-law combiner, but performs better than the SC schemes of Pierce and Chyi *et al* by 0.9 dB and 0.4 dB, respectively. As the order of diversity increases from $L = 3$ to 5, the diversity gain of the proposed optimum SC increases over Pierce and Chyi *et al* schemes. For example, for $L = 5$, the diversity gain of the optimum SC is 1.6 dB over the Pierce SC scheme and 0.8 dB over Chyi *et al* SC scheme. The Chyi *et al* scheme chooses the branch with the maximum demodulator output. This scheme gives a decision error when one demodulator output with noise-only exceeds the demodulator output with signal-plus-noise. The proposed optimum SC scheme, on the other hand, performs subtraction of the outputs of the demodulators (signal-plus-noise energy output due to correct hypothesis, and noise-only energy output due to incorrect hypothesis). This subtraction basically eliminates the dominance of the noise-only demodulator output and this results in enhanced performance of the optimum SC scheme over the other schemes.

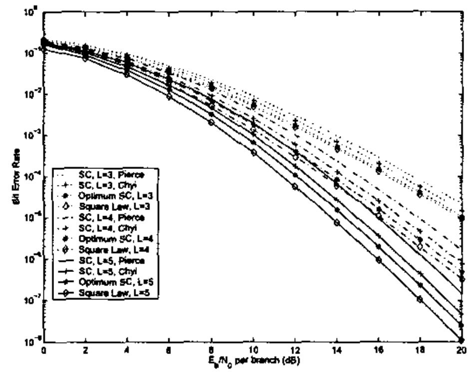

Fig. 2. Comparison of the bit error performance of various SC schemes on i.i.d Rayleigh fading for $L = 3$, 4, and 5.

It is noted that, for i.i.d Rayleigh fading, the performance of the Neasmith SC scheme is the same as that of the optimum SC scheme. However, as we will see next, in the case of independent fading, the optimum SC scheme performs better than the Neasmith SC scheme. In fact, the Neasmith scheme performs poorer than Pierce SC scheme at low SNRs and large multipath intensity profile (MIP) decay factors.

Fig. 3 shows the comparative performance of the various SC schemes for the case of independent Rayleigh fading. The second moment of the fading, Ω_l, is decreased linearly (in dB scale) in steps of 2 dB. That is, $\Omega_l = -2(l-1)$ dB, $l = 1, 2, \cdots, L$. From Fig. 3, we observe that at high SNRs, the

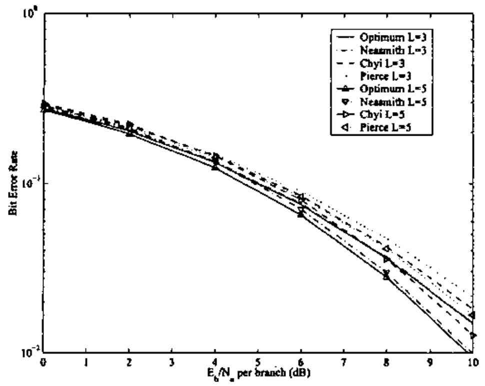

Fig. 3. Comparison of the bit error performance of various SC schemes on independent Rayleigh fading. MIP is [0dB, −2dB, −4dB] for $L = 3$, and [0dB, −2dB, −4dB, −6dB, −8dB] for $L = 5$.

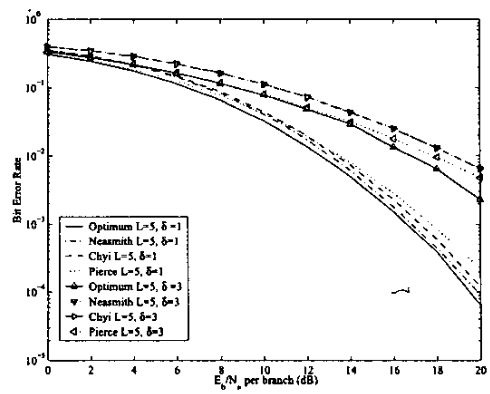

Fig. 4. Comparison of the bit error performance of various SC schemes on multipath Rayleigh fading channels with exponentially decaying MIP for $L = 5$. Decay factor, $\delta = 1.0, 3.0$.

Neasmith SC scheme performs the same as that of the optimum SC, whereas at low SNRs the optimum SC performs better. This behavior is more clearly observed in Fig. 4. The reason for this is as follows: At high SNRs, with the approximation $1 + \gamma\Omega_l = \gamma\Omega_l$ in (12), the weighting factors g_l become independent of the second moment Ω of the Rayleigh fades. At low SNRs, with the approximation $1 + \gamma\Omega_l = 1$, we have $g_l = \Omega_l$. The Neasmith SC scheme does not take this gain factor into account, whereas the proposed optimum SC scheme approximately weights the energy differences before the selection is made, which improves the performance.

Fig. 4 shows the comparative performance of the various SC schemes for $L = 5$, for exponentially decaying L antenna independent Rayleigh fading with $\Omega_l = \Omega_1 e^{-\delta(l-1)}$, $l = 1, 2, \cdots, L$ and $\Omega_1 = 1$. The values of decay factor, δ, considered are 1 and 3. From Fig. 4, we observe that for small values of the decay factor, δ, and high values of SNR, Neasmith SC performs close to optimum SC. However, at large values of δ and at low SNRs, the Neasmith scheme performs worse than optimum SC. In fact, at low SNR and high MIP decay factors, the Neasmith scheme performs poorer than the scheme which chooses the branch with largest signal power (i.e., the Pierce SC scheme). This illustrates an important conclusion of this paper. That is, while the Neasmith SC scheme is optimum for i.i.d Rayleigh fading, it performs poorer than the Pierce SC scheme for independent Rayleigh fading at low SNRs and large MIP decay factors, whereas our proposed SC scheme provides the optimum performance for arbitrary independent Rayleigh fading.

VI. Conclusions

We proposed a new selection combining scheme for binary NCFSK signals on independent (but not necessarily identically distributed) Rayleigh fading channels with L-antenna diversity reception. With this scheme, the diversity branch having the largest magnitude of the LAPPR of the transmitted information bit is chosen. We showed that this scheme minimized the probability of bit error, thus proving its optimality. At 10^{-4} BER, a) on i.i.d Rayleigh fading, the proposed optimum SC combining scheme performed better than the existing SC schemes by 0.5 dB for $L = 3$ and 1.5 dB for $L = 5$, and performed within 0.5 dB of the scheme which square-law combines all the L diversity branches, and b) on independent Rayleigh fading, the proposed optimum SC scheme provided an additional gain of 2.0 dB over the SC schemes of Pierce and Chyi *et al.*

Appendices

A1. Optimality of the Proposed SC Scheme

In this appendix, we show that choosing the largest LAPPR magnitude among the available L LAPPR magnitudes minimizes the probability of error, thus proving the optimality of the proposed selection combining scheme. The general expression for the error probability is given by

$$P_b = \int_{\mathbf{X}} \text{Prob}(\hat{\underline{s}}_m \neq \underline{s}_m|\mathbf{X}) f_{\mathbf{X}}(\mathbf{X})\, d\mathbf{X} = 1 - \int_{\mathbf{X}} \text{Prob}(\hat{\underline{s}}_m = \underline{s}_m|\mathbf{X}) f_{\mathbf{X}}(\mathbf{X})\, d\mathbf{X}, \qquad (23)$$

where $\mathbf{X} = (X_0^{(1)}, X_1^{(1)}, X_0^{(2)}, X_1^{(2)}, \cdots, X_0^{(L)}, X_1^{(L)})$ and $\hat{\underline{s}}_m$ is the detected symbol when symbol $\underline{s}_m$ is transmitted. The $\text{Prob}(\hat{\underline{s}}_m = \underline{s}_m|\mathbf{X})$ can be derived as follows:

$$\text{Prob}(\hat{\underline{s}}_m = \underline{s}_m|\mathbf{X}) = \sum_{l=1}^{L} \text{Prob}(\hat{\underline{s}}_m = \underline{s}_m|\mathbf{X},\ l^{th}\text{antenna is chosen}) \cdot \text{Prob}(l^{th}\text{antenna is chosen}|\mathbf{X})$$

$$= \sum_{l=1}^{L} \text{Prob}(\widehat{\underline{s}}_m = \underline{s}_m | X_0^{(l)}, X_1^{(l)}) \cdot \text{Prob}(l^{th} \text{antenna is chosen}|\mathbf{X})$$

$$\leq \max\left(\text{Prob}(\widehat{\underline{s}}_m = \underline{s}_m | X_0^{(l)}, X_1^{(l)})\right). \quad (24)$$

The last step in the above equation is due to the fact that $\sum_{l=1}^{L} \text{Prob}(l^{th} \text{antenna is chosen}|\mathbf{X}) = 1$ and $0 \leq \text{Prob}(l^{th} \text{antenna is chosen}|\mathbf{X}) \leq 1$. It is to be noted that equality in (24) is achieved, and hence P_b is minimized, by selecting the branch providing the maximum $\text{Prob}(\widehat{\underline{s}}_m = \underline{s}_m | X_0^{(l)}, X_1^{(l)})$. With this, we proceed to derive $\text{Prob}(\widehat{\underline{s}}_m = \underline{s}_m | X_0^{(l)}, X_1^{(l)})$ and show that it is a monotonically increasing function of $|\Lambda^{(l)}|$. The $\text{Prob}(\widehat{\underline{s}}_m = \underline{s}_m | X_0^{(l)}, X_1^{(l)})$ can be calculated as

$$\begin{aligned} \text{Prob}(\widehat{\underline{s}}_m = \underline{s}_m | X_0^{(l)}, X_1^{(l)}) &= \text{Prob}(\widehat{\underline{s}}_m = \underline{s}_0,\ \underline{s}_m = \underline{s}_0 | X_0^{(l)}, X_1^{(l)}) \\ &\quad + \text{Prob}(\widehat{\underline{s}}_m = \underline{s}_1,\ \underline{s}_m = \underline{s}_1 | X_0^{(l)}, X_1^{(l)}). \end{aligned} \quad (25)$$

From (5) and (11), we obtain

$$\begin{aligned} \text{Prob}(\widehat{\underline{s}}_m = \underline{s}_1 | X_0^{(l)}, X_1^{(l)}) &= \frac{1}{1 + e^{-\Lambda^{(l)}}}, \quad \text{and} \\ \text{Prob}(\widehat{\underline{s}}_m = \underline{s}_0 | X_0^{(l)}, X_1^{(l)}) &= \frac{1}{1 + e^{\Lambda^{(l)}}}. \end{aligned} \quad (26)$$

Noting that $\Lambda^{(l)} \geq 0$ when $\widehat{\underline{s}}_m = \underline{s}_1$, $\underline{s}_m = \underline{s}_1$ and $\Lambda^{(l)} < 0$ when $\widehat{\underline{s}}_m = \underline{s}_0$, $\underline{s}_m = \underline{s}_0$, (26) can be simplified as

$$\text{Prob}(\widehat{\underline{s}}_m = \underline{s}_m | X_0^{(l)}, X_1^{(l)}) = \frac{1}{1 + e^{-|\Lambda^{(l)}|}}. \quad (27)$$

The above expression is clearly a monotonic increasing function of $|\Lambda^{(l)}|$. Therefore, the error probability is minimized by choosing the branch with the largest magnitude of LAPPR.

A2. Derivation of $P_e^{OSC,indep}$

We first find the pdf of $Z_l = \max(|\Lambda^{(j),j\neq l}|)$ and use this in step 2 of (17) to obtain (18).

$$\begin{aligned} F_{Z_l}(z) &= \text{Prob}\left(\max\left(|\Lambda^{(j),j\neq l}|\right) \leq z\right) \\ &= \prod_{j=1,j\neq l}^{L} F_{|\Lambda^{(j)}|}(z). \end{aligned} \quad (28)$$

Taking logarithms on both sides and differentiating with respect to z and then upon rearranging, we arrive at

$$f_{Z_l}(z) = \sum_{j=1,j\neq l}^{L} f_{|\Lambda^{(j)}|}(z) \prod_{k=1,k\neq\{j,l\}}^{L} F_{|\Lambda^{(k)}|}(z). \quad (29)$$

The expression $P_e(l)$ in (16) is derived as follows:

$$P_e(l) = \text{Prob}(Z_l + \Lambda^{(l)} < 0)$$

$$\begin{aligned} &= \int_{z=0}^{\infty} \int_{x=-\infty}^{-z} f_{Z_l}(z) f_{\Lambda^{(l)}}(x)\, dx\, dz = \frac{a_l}{g_l} \int_{z=0}^{\infty} f_{Z_l}(z) e^{-\frac{\lambda_2^{(l)} z}{g_l}} dz \\ &= \frac{a_l}{g_l} \sum_{j=1,j\neq l}^{L} \int_{z=0}^{\infty} \prod_{k=1,k\neq\{j,l\}}^{L} \left(1 - \frac{\lambda_2^{(k)} e^{-\frac{\lambda_1^{(k)} z}{g_k}} + \lambda_1^{(k)} e^{-\frac{\lambda_2^{(k)} z}{g_k}}}{\lambda_1^{(k)} + \lambda_2^{(k)}} \right) \\ &\quad \cdot e^{-\frac{\lambda_2^{(l)} z}{g_l}} dz. \end{aligned} \quad (30)$$

In deriving (30), we made use of the expression for $f_{Z_l}(z)$ of (29). Upon substituting $z = \tan\psi$, we arrive at (18).

A3. Derivation of $P_e^{OSC,iid}$

$$\begin{aligned} P_e^{OSC,iid} &= \sum_{l=1}^{L} \text{Prob}\left(\max\left(|\Lambda^{(j),j\neq l}|\right) < |\Lambda^{(l)}|, \Lambda^{(l)} < 0\right) \\ &= L\ \text{Prob}(Z_l + \Lambda^{(l)} < 0) \\ &= L \int_{y=-\infty}^{0} \int_{z=0}^{-y} f_{Z_l}(z) f_{\Lambda^{(l)}}(y)\, dz\, dy. \end{aligned} \quad (31)$$

The density function of the quantity $Z_l = \max\left(|\Lambda^{(j),j\neq l}|\right)$ can be derived as follows:

$$\begin{aligned} f_{Z_l}(z) &= \frac{d}{dz}\left[\text{Prob}(Z_l \leq z)\right] \\ &= (L-1)\left(F_{|\Lambda|}(z)\right)^{L-2} f_{|\Lambda|}(z) \\ &= a(L-1)\left(1 - \frac{\lambda_2 e^{-\lambda_1 z} + \lambda_1 e^{-\lambda_2 z}}{\lambda_1 + \lambda_2}\right)^{L-2} \left(e^{-\lambda_1 z} + e^{-\lambda_2 z}\right) \\ &= a(L-1) \sum_{k=0}^{L-2} \sum_{j=0}^{k} \frac{(-1)^k \lambda_1^{k-j} \lambda_2^j}{(\lambda_1 + \lambda_2)^k} \binom{L-2}{k} \binom{k}{j} \\ &\quad \cdot \left(e^{-(\lambda_1(j+1) + \lambda_2(k-j))} + e^{-(\lambda_1(j) + \lambda_2(k-j+1))}\right). \end{aligned} \quad (32)$$

Substituting (32), and the expression for $f_{\Lambda^{(l)}}(y)$ from (14), in (31), and performing the integration, we obtain (19).

References

[1] M. K. Simon and M. S. Alouini, *Digital Communications over Generalized Fading Channels: A Unified Approach to the Performance Analysis*, Wiley & Sons, Inc., 2000.

[2] J. N. Pierce, "Theoretical Diversity Improvement in Frequency-Shift Keying," *Proc. IRE*, vol. 46, pp. 903-910, May 1958.

[3] G. Chyi, J. G. Proakis, and C. M. Keller, "On the Symbol Error Probability of Maximum-Selection Diversity Reception Schemes over a Rayleigh Fading Channel," *IEEE Trans. Commun.*, vol. 37, pp. 79-83, January 1989.

[4] Y. G. Kim and S. W. Kim, "Optimum Selection Diversity for BPSK Signals in Rayleigh Fading Channels," *Proc. ISSTA'2000*, pp. 50-52, 2000. Also in *IEEE Trans. Commun.*, October 2001.

[5] E. A. Neasmith and N. C. Beaulieu, "New Results on Selection Diversity," *IEEE Trans. Commun.*, vol. 46, pp. 695-704, May 1998.

[6] J. M. Wozencraft and I. M. Jacobs, *Principles of Communication Engineering*, New York: Wiley, 1965.

[7] A. Ramesh, A. Chockalingam, and L. B. Milstein, "Performance of Noncoherent Turbo Detection on Rayleigh Fading Channels," *Proc. IEEE GLOBECOM'2001*, November 2001.

[8] A. Ramesh, A. Chockalingam, and L. B. Milstein, "Further Results on Selection Combining of NCFSK Signals in Rayleigh Fading Channels," submitted to *IEEE Trans. Commun.*, March 2002.