Outage and Capacity Analysis of Cellular CDMA With Admission Control

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***Abstract*— We analyze the outage and capacity performance of an interference based admission control strategy in cellular CDMA systems. Most approaches to estimate the outage probability and the system capacity of CDMA systems in the literature do not take interference based admission control into account. In this paper, we present an analytical model to evaluate the outage probability and the capacity on the reverse link of cellular CDMA systems *with interference based admission control*. We make two main approximations in the outage analysis – one based on the central limit theorem (CLT) and the other based on the Chernoff bound (CB). We also obtain an improved approximation to the outage probability using Edgeworth expansion. It is shown that the considered admission control policy results in increased system capacity compared to that with no admission control, by about 30% for an outage probability of 0.01.**

I. Introduction

Capacity of CDMA systems without taking interference based admission control into account had been evaluated by Evans *et al* in [1], where they had obtained lower and upper bounds on CDMA system capacity. Chan *et al* in [2], and Karmani *et al* in [3], applied Edgeworth expansion to obtain better approximations to the outage probability as compared to the bounds given in [1]. It is noted that admission control strategies based on signal-to-interference ratio measurements can reduce outage probability. In this paper, we are concerned with the analytical means to evaluate the outage and capacity in cellular CDMA systems *with admission control based on interference-to-signal* (I/S) *measurements*.

We consider an admission control policy which allows only those incoming calls whose (I/S) levels at their respective base stations are below a specified threshold. Such admitted calls in a given cell can cause outage to other ongoing calls in the system, i.e., may cause the I/S at the base stations corresponding to any of the ongoing calls cross the threshold. Without admission control, new calls would be admitted irrespective of the I/S levels, provided spreading codes are available at the respective base stations. However, with admission control, by allowing only those calls that see an acceptable I/S level at their respective base stations, the outage probability can get reduced.

We develop an analytical model to evaluate the outage probability and the capacity on the reverse link of cellular CDMA systems which employ the above admission control policy. We model the system as an $M/G/\infty$ queue, and make two main approximations in the outage analysis – one based on the central limit theorem (CLT) and the other based on the Chernoff bound (CB). We further obtain an improved approximation to the outage probability using Edgeworth expansion. It is shown that the considered admission control policy results in increased system capacity compared to that with no admission control, by about 30% for an outage probability of 0.01.

II. Problem Definition

Consider a CDMA cellular system with circular cells. The problem is to devise an analytical model for computing the outage probability for this system with admission control, and evaluating the system capacity for a given outage performance. The admission control strategy considered is as follows. When a new call arrives in a cell, the corresponding base station measures the I/S ratio and compares this with a specified threshold, ϵ^o. The call is admitted if spreading codes are available for allocation and if $I/S \leq \epsilon^o$. The call is blocked otherwise. We consider the following system model for our analysis.

- There are N cells in the system (N=61) and each cell has a maximum of n spreading codes available for allocation. All the cells are assumed to be of equal radius R with the base stations located at the center of each cell.
- Mobiles are uniformly distributed over the area of each cell and all the mobiles are assumed to have either very low mobility or no mobility.
- The call arrival process in each cell is Poisson with mean arrival rate λ and the call holding time is exponentially distributed with mean μ^{-1} seconds.
- The signal undergoes distance attenuation and shadow loss. The distance loss exponent ν is taken to be 4 and the shadow loss is assumed to be log-normally distributed with zero mean and variance σ^2.
- We assume perfect power control, i.e., each base station receives unit power from the mobiles attached to it irrespective of the position of the mobiles.

III. Performance Analysis

In cellular CDMA, because of universal frequency reuse, mobiles in all the cells in the system contribute to the interference seen by any given base station. Here, we assume that the interference seen by a base station is due to the mobiles in its first tier of neighboring cells (i.e., we ignore the interference due to cells other than the first tier neighboring cells as negligible[1]). In an N-cell CDMA system, for a given cell $k, 1 \leq k \leq N$, let S_k denote the set of cells that includes cell k and its neighboring cells.

Let Δ_{ik} denote the number of active mobiles in i^{th} neighboring cell to the k^{th} cell, $1 \leq i \leq 6, 1 \leq k \leq N$. The total number of interferers Δ'_k seen by cell k is then given by $\Delta'_k = \Delta_{kk} + \Delta_k$, where Δ_k is the number of interfering mobiles to cell k from all neighbors of cell k, given by $\Delta_k = \sum_{\substack{i \in S_k \\ i \neq k}} \Delta_{ik}$, and Δ_{kk} is the number of active mobiles in cell k when a new call arrives. A cell with a maximum of n spreading codes can be modeled as an $M/G/n/n$ loss system with Δ_{kk} as the queue length process. Accordingly, the probability mass function of Δ_{kk} is given by [7]

$$\Pr\{\Delta_{kk} = m\} = \begin{cases} \frac{\frac{\rho^m}{m!}}{\sum_{l=0}^{n} \frac{\rho^l}{l!}} & 0 \leq m \leq n, \\ 0 & \text{otherwise}, \end{cases} \tag{1}$$

where $\rho = \lambda/\mu$, λ is the mean call arrival rate in a cell and μ^{-1} is the mean call holding time.

Let $M_{ij} = (r_{m_{ij}}, \theta_{m_{ij}})$ denote the co-ordinates of the j^{th} mobile in the i^{th} cell, and let $B_i = (r_{b_i}, \theta_{b_i})$ denote the co-ordinates of the base station of cell i. Let $D(M_{ij}, B_i)$ denote the distance between mobile m_{ij} and base station b_i. Hence,

$$D(M_{ij}, B_i) = \sqrt{r_{b_i}^2 + r_{m_{ij}}^2 - 2 r_{b_i} r_{m_{ij}} \cos\left(\theta_{m_{ij}} - \theta_{b_i}\right)}. \tag{2}$$

Let $I'_k(\Delta'_k)$ be the I/S experienced at the base station of cell k. The interference I is due to the mobiles present in cell k itself as well as the mobiles present in the neighboring cells to cell k. If we denote the I/S at the base station of cell k due to those Δ_k mobiles not present in cell k (or present in the neighboring cells to cell k) as $I_k(\Delta_k)$, then, with perfect power control, $I'_k(\Delta'_k)$ is given by

$$I'_k(\Delta'_k) = \Delta_{kk} + I_k(\Delta_k), \tag{3}$$

$$I_k(\Delta_k) = \sum_{\substack{i \in S_k \\ i \neq k}} \sum_{j=1}^{\Delta_{ik}} \frac{D^4(M_{ij}, B_i)\, 10^{-\frac{\psi_{ji}}{10}}}{D^4(M_{ij}, B_k)\, 10^{-\frac{\psi_{jk}}{10}}}, \tag{4}$$

where ψ_{ji} denotes a normally distributed random variable with zero mean and variance σ^2, which corresponds to the shadow loss from mobile j to the base station of cell i. Note that $I_k(\Delta_k)$ is conditioned on the number of interferers in the neighboring cells, Δ_k, and the location of the interferers in the neighboring cells. Therefore, $I_k(\Delta_k)$ needs to be averaged over the number of interferers and their locations (as will be done later in Eqns. (18), (25) and (34)).

A. Outage without Admission Control

The outage probability in a CDMA system without the I/S based admission control is given by

$$p_{out}(\epsilon^o, \Delta'_k) = \Pr\{I'_k(\Delta'_k) > \epsilon^o\}. \tag{5}$$

[1]Henceforth, we use the term neighboring cells to mean the first tier of cells around the cell-of-interest.

We first compute the outage probability conditioned on Δ_{kk} and then average over Δ_{kk}. We define $\epsilon = \epsilon^o - \Delta_{kk}$. Hence, the outage probability conditioned on Δ_{kk} when there is no I/S based admission control, $p_{out}(\epsilon)$, is given by

$$p_{out}(\epsilon) = \sum_T \Pr\{I_k(T) > \epsilon\} \Pr\{\Delta_k = T\}. \tag{6}$$

Thus, for CDMA systems with no admission control, the outage probability averaged over Δ_{kk}, is given by

$$p_{out}(\epsilon^o) = \sum_{T_k=0}^{n} p_{out}(\epsilon) \Pr\{\Delta_{kk} = T_k\}. \tag{7}$$

B. Outage with Admission Control

With I/S based admission control, we are interested in the probability that a new call arriving in cell i causes an outage in cell k. The outage probability conditioned on Δ_i, Δ_k and Δ_{kk}, denoted as $p^*_{out}(\Delta_i, \Delta_k, \epsilon)$, can be written as

$$p^*_{out}(\Delta_i, \Delta_k, \epsilon) = \Pr\{I_k(\Delta_k) > \epsilon | I_i(\Delta_i) \leq \epsilon, I_k(\Delta_k - 1) \leq \epsilon\}. \tag{8}$$

The above equation gives the probability that a newly admitted call in cell i (because of $I_i(\Delta_i) \leq \epsilon$) causes an outage in cell k ($I_k(\Delta_k) > \epsilon$) given that there was no outage in cell k before admitting the new call in cell i ($I_k(\Delta_k - 1) \leq \epsilon$). We assume that $I_i(\Delta_i)$ is statistically independent of $I_k(\Delta_k)$ to simplify the analysis. Hence, $p^*_{out}(\Delta_i, \Delta_k, \epsilon)$ becomes independent of the cell i in which the call arrives, and hence can be denoted by $p^*_{out}(\Delta_k, \epsilon)$, which can be written as

$$p^*_{out}(\Delta_k, \epsilon) = \Pr\{I_k(\Delta_k) > \epsilon | I_k(\Delta_k - 1) \leq \epsilon\}. \tag{9}$$

Averaging over Δ_k, the outage probability conditioned on Δ_{kk}, $p^*_{out}(\epsilon)$, is given by

$$p^*_{out}(\epsilon) = 1 - \prod_{\substack{i \in S_k \\ i \neq k}} \left[1 - \sum_T p^*_{out}(T, \epsilon) \Pr\{\Delta_k = T\}\right]. \tag{10}$$

To compute $\Pr\{\Delta_k = T\}$ in the above equation, we model the system as an $M/G/\infty$ queue. Note that a more accurate model for the system is an $M/G/c/c$ loss model where c denotes the maximum possible interferers from the neighboring cells to cell k. However, since c is typically large in a CDMA system, we can approximate the system by an $M/G/\infty$ queue. The number of interferers Δ_k is then the queue length process of an $M/G/\infty$ queue and hence, is a Poisson random variable [7]. Therefore, defining $\hat{\rho} = N_k \rho$, where N_k is the number of neighboring cells to cell k ($N_k = 6$),

$$\Pr\{\Delta_k = T\} = \frac{e^{-\hat{\rho}} \hat{\rho}^T}{T!}. \tag{11}$$

Finally, the outage probability in Eqn. (10) is averaged over Δ_{kk} (whose probability mass function is given in Eqn. (1)) to obtain

$$p^*_{out}(\epsilon^o) = \sum_{T_k=0}^{n} p^*_{out}(\epsilon) \Pr\{\Delta_{kk} = T_k\}. \tag{12}$$

It is noted that the key step in the outage probability computation in the above is the evaluation of Eqn. (9). In order to evaluate Eqn. (9), we need to compute the joint probability

$$P_{joint} = \Pr\{I_k(\Delta_k) > \epsilon, I_k(\Delta_k - 1) \leq \epsilon\}, \tag{13}$$

and the marginal probability

$$P_{marginal} = \Pr\{I_k(\Delta_k - 1) \leq \epsilon\}. \tag{14}$$

To evaluate the marginal probability in Eqn. (14), we use approximations based on central limit theorem (CLT) and Chernoff bound (CB). To evaluate the joint probability in Eqn. (13), one can model the random process $I_k(\Delta_k)$ as a continuous state space Markov chain, which can be solved by the theory of random walks given in [4]. However, this approach tends to be complex. Hence, we adopt a simpler method which uses the approximation made in the evaluation of the marginal probability in Eqn. (14). The CLT and CB approximations are illustrated in the following subsections.

1) Approximation using CLT: In order to evaluate the cdf of $I_k(\Delta_k - 1)$ in Eqn. (14), we approximate the summation in Eqn. (4) by a normal random variable with mean $\mu_{\Delta_k - 1}$ and variance $\sigma^2_{\Delta_k - 1}$, given by

$$\mu_{\Delta_k - 1} = \sum_{\substack{i \in S_k \\ i \neq k}} \sum_{j=1}^{\Delta_{ik}} e^{\left(-a\mu_{ik}^{(j)} + 2a^2\sigma^2\right)}, \tag{15}$$

$$\sigma^2_{\Delta_k - 1} = \sum_{\substack{i \in S_k \\ i \neq k}} \sum_{j=1}^{\Delta_{ik}} e^{\left(-2a\mu_{ik}^{(j)} + 2a^2\sigma^2\right)} (e^{2a^2\sigma^2} - 1), \tag{16}$$

where $a = \frac{\ln(10)}{10}$, $\sum_{\substack{i \in S_k \\ i \neq k}} \Delta_{ik} = \Delta_k - 1$, and

$$\mu_{ik}^{(j)} = 10 \log_{10} \frac{D^4(M_{ij}, B_i)}{D^4(M_{ij}, B_k)}. \tag{17}$$

Averaging over the location of the interferers, $\Pr\{I_k(\Delta_k - 1) > \epsilon\}$ is given by,

$$\Pr\{I_k(\Delta_k - 1) > \epsilon\} = \int \cdots \int \left(\frac{1}{\pi R^2}\right)^6 \cdot Q\left(\frac{\epsilon - \mu_{\Delta_k - 1}}{\sigma_{\Delta_k - 1}}\right) r_1 \cdots r_6 dr_1 d\theta_1 \cdots dr_6 d\theta_6, \tag{18}$$

where $Q(x) = \frac{1}{\sqrt{2\pi}} \int_x^\infty e^{-\frac{y^2}{2}} dy$. Since the mobile locations are independent of each other, the averaging over the location of the interferers can be obtained by replacing the mean of the normal random variable in Eqn. (15) by

$$\mu_{\Delta_k - 1} = (\Delta_k - 1) e^{-a\mu_{ik}^{(j)} + 2a^2\sigma^2}, \tag{19}$$

and the variance in Eqn. (16) by

$$\sigma^2_{\Delta_k - 1} = (\Delta_k - 1) e^{\left(-2a\mu_{ik}^{(j)} + 2a^2\sigma^2\right)} (e^{2a^2\sigma^2} - 1). \tag{20}$$

We can evaluate the marginal probability $\Pr\{I_k(\Delta_k) > \epsilon\}$ in a similar manner. This completes the evaluation of the marginal probability in Eqn. (14) using CLT.

Next, we proceed to obtain the joint probability in Eqn. (13) using CLT as follows. Here, $I_k(\Delta_k - 1)$ and $I_k(\Delta_k)$ are jointly normal with a correlation coefficient r. If $f_{\Delta_k}(x, y)$ is the joint pdf of $I_k(\Delta_k - 1)$ and $I_k(\Delta_k)$, then

$$f_{\Delta_k}(x, y) = \frac{1}{2\pi\sigma_1\sigma_2\sqrt{(1 - r^2)}} e^{-\frac{1}{2(1-r^2)}\left[\frac{\hat{x}^2}{\sigma_1^2} - \frac{2r\hat{x}\hat{y}}{\sigma_1\sigma_2} + \frac{\hat{y}^2}{\sigma_2^2}\right]}, \tag{21}$$

where $\hat{x} = x - \mu_{\Delta_k}, \hat{y} = y - \mu_{\Delta_k - 1}, \sigma_1 = \sigma_{\Delta_k}$ and $\sigma_2 = \sigma_{\Delta_k - 1}$. The correlation coefficient, r, is defined as

$$r = \frac{E\left[I_k(\Delta_k - 1) I_k(\Delta_k)\right] - \mu_{\Delta_k}\mu_{\Delta_k - 1}}{\sigma_{\Delta_k}\sigma_{\Delta_k - 1}}. \tag{22}$$

We can rewrite $I_k(\Delta_k)$ in Eqn. (4) as

$$I_k(\Delta_k) = I_k(\Delta_k - 1) + X_{ik}^{(j)}, \tag{23}$$

where

$$X_{ik}^{(j)} = \frac{D^4(M_{ij}, B_i) 10^{-\frac{\psi_{ji}}{10}}}{D^4(M_{ij}, B_k) 10^{-\frac{\psi_{jk}}{10}}} \tag{24}$$

denotes the I/S brought in by the newly arriving call which we say, without loss of generality, as the j^{th} mobile in cell i. From Eqns. (23) and (24), we can evaluate $E\left[I_k(\Delta_k) I_k(\Delta_k - 1)\right]$ in Eqn. (22) by using the fact that $X_{ik}^{(j)}$ and $I_k(\Delta_k - 1)$ are statistically independent. Conditioned on the location of the newly arriving call, $X_{ik}^{(j)}$ is a log-normally distributed random variable of the form $X_{ik}^{(j)} = 10^{-\frac{\Omega_{ik}^{(j)}}{10}}$, where $\Omega_{ik}^{(j)}$ is normally distributed with mean $10 \log_{10} \frac{D^4(M_{ij}, B_i) 10^{-\frac{\psi_{ji}}{10}}}{D^4(M_{ij}, B_k) 10^{-\frac{\psi_{jk}}{10}}}$ and variance $2\sigma^2$. Hence, by using the CLT approximation on $I_k(\Delta_k)$ and $I_k(\Delta_k - 1)$, we can evaluate Eqn. (13) from Eqns. (21), (22), (23) and (24), as

$$\Pr\{I_k(\Delta_k) > \epsilon, I_k(\Delta_k - 1) \leq \epsilon\} = \left(\frac{1}{\pi R^2}\right)^6 \int \cdots \int \int_\epsilon^\infty \int_{-\infty}^\epsilon f_{\Delta_k}(x, y) r_1 \cdots r_6 dy dx dr_1 d\theta_1 \cdots dr_6 d\theta_6. \tag{25}$$

Finally, we substitute the values obtained from Eqns. (18) and (25) in Eqns. (9), (10) and (12) and compute the outage probability.

2) Approximation using Chernoff Bound: In this subsection, we present another approximation to evaluate the marginal $\Pr\{I_k(\Delta_k - 1) > \epsilon\}$, by applying the Chernoff bound in Eqn. (4). For a set of m i.i.d random variables $\{Z_i\}$, $1 \leq i \leq m$, the Chernoff bound is given by [5],

$$\Pr\left\{\sum_{i=1}^m Z_i > \Gamma\right\} \leq e^{-ml\left(\frac{\Gamma}{m}\right)}, \tag{26}$$

where $l\left(\frac{\Gamma}{m}\right) = \sup_s \left[s\Gamma - \chi(s)\right]$ and $\chi(s) = \ln\left(E\left\{e^{sZ_1}\right\}\right)$ is known as the logarithm of the moment generating function (LMGF). Here, we use the equality condition in Eqn. (26) as an approximation to obtain $\Pr\{I_k(\Delta_k - 1) > \epsilon\}$. To obtain this approximation, we need to evaluate the pdf of $I_k(\Delta_k)$ in Eqn. (4). Each term in the summation in Eqn. (4) is a log-normal random variable of the form $10^{\frac{-\xi}{10}}$ where ξ is a normal random variable with mean $\mu_{ik}^{(j)}$ given by Eqn. (17) and variance $2\sigma^2$. Hence, by assuming I_j and I_k to be independent for $j \neq k$, $I_k(\Delta_{k-1})$ can be approximated to follow a log-normal distribution of the form $10^{-\frac{\Omega}{10}}$ where Ω is normally distributed with mean $\hat{\mu}_{\Delta_k-1}$ dB and standard deviation $\hat{\sigma}_{\Delta_k-1}$ dB. We apply the Fenton's method [6] (i.e., approximate the sum of independent log-normal r.v's by a log-normal r.v) to obtain the expressions for $\hat{\mu}_{\Delta_k-1}$ and $\hat{\sigma}^2_{\Delta_k-1}$, as

$$\hat{\sigma}^2_{\Delta_k-1} = \frac{1}{a^2}\ln\left[\frac{1+e^{2a^2\sigma^2}(e^{2a^2\sigma^2}-1)\eta}{e^{2a^2\sigma^2}\zeta^2}\right], \quad (27)$$

$$\hat{\mu}_{\Delta_k-1} = \frac{1}{2a}\frac{e^{a^2(\hat{\sigma}^2_{\Delta_k-1}-2\sigma^2)}}{\zeta^2}, \quad (28)$$

where $\eta = \sum_{\substack{i\in S_k \\ i\neq k}} \sum_{j=1}^{\Delta_{ik}} e^{-2a\mu_{ik}^{(j)}}$, and $\zeta = \sum_{\substack{i\in S_k \\ i\neq k}} \sum_{j=1}^{\Delta_{ik}} e^{-a\mu_{ik}^{(j)}}$.

Following a similar argument as in Section III-B.1, we can use the approximations $\sum_{\substack{i\in S_k \\ i\neq k}} \sum_{j=1}^{\Delta_{ik}} e^{-2a\mu_{ik}^{(j)}} = (\Delta_k - 1)e^{-2a\mu_{ik}^{(j)}}$, and $\sum_{\substack{i\in S_k \\ i\neq k}} \sum_{j=1}^{\Delta_{ik}} e^{-a\mu_{ik}^{(j)}} = (\Delta_k - 1)e^{-a\mu_{ik}^{(j)}}$ to compute $\hat{\mu}_{\Delta_k-1}$ and $\hat{\sigma}^2_{\Delta_k-1}$ in Eqns. (27) and (28). From these values we obtain the LMGF $\chi(s)$, using which the marginal in Eqn. (14) can be evaluated.

Next, to evaluate the joint probability in Eqn. (13), we use the Fenton's method of approximating sum of log-normals by a log-normal [6]. We rewrite Eqn. (13) as

$$\Pr\left\{\tilde{I}_k(\Delta_k) > \epsilon_{\text{dB}}, \tilde{I}_k(\Delta_k - 1) \leq \epsilon_{\text{dB}}\right\}, \quad (29)$$

where $\epsilon_{dB} = 10\log_{10}(\epsilon)$, $\tilde{I}_k(\Delta_k) = 10\log_{10} I_k(\Delta_k)$ and $\tilde{I}_k(\Delta_k - 1) = 10\log_{10} I_k(\Delta_k - 1)$. Since $I_k(\Delta_k)$ and $I_k(\Delta_k - 1)$ are log-normal, $\tilde{I}_k(\Delta_k - 1)$ and $\tilde{I}_k(\Delta_k)$ are jointly normal with correlation coefficient $\hat{r}$, given by

$$\hat{r} = \frac{E\left[\tilde{I}(\Delta_k - 1)\tilde{I}(\Delta_K)\right] - \mu_{\Delta_k-1}\mu_{\Delta_k}}{\sigma_{\Delta_k-1}\sigma_{\Delta_k}}. \quad (30)$$

To evaluate the expectation $E\left[\tilde{I}(\Delta_k - 1)\tilde{I}(\Delta_K)\right]$, we again use Eqn. (23) and use the approximation $\ln(1+x) = x$ for $|x| < 1$, to obtain

$$\tilde{I}(\Delta_k) = \tilde{I}(\Delta_k - 1) + \frac{X_{ik}^{(j)}}{I_k(\Delta_k - 1)}. \quad (31)$$

From Eqn. (31), we obtain

$$E\left[\tilde{I}_k(\Delta_k - 1)\tilde{I}_k(\Delta_k)\right] = E\left[\tilde{I}_k^2(\Delta_k - 1)\right] + E\left[\tilde{I}(\Delta_k - 1)\frac{X_{ik}^{(j)}}{I_k(\Delta_k - 1)}\right]. \quad (32)$$

Using the approximation $\ln(1+x) = x$ for $|x| < 1$ again, we have

$$E\left[\tilde{I}_k(\Delta_k - 1)\tilde{I}_k(\Delta_k)\right] = E\left[\tilde{I}_k^2(\Delta_k - 1)\right] + \frac{1}{a^2}E\left[\frac{X_{ik}^{(j)}}{I_k(\Delta_k - 1)}\right] - \frac{1}{a^2}E\left[\frac{X_{ik}^{(j)}}{I_k^2(\Delta_k - 1)}\right]. \quad (33)$$

The ratio $\frac{X_{ik}^{(j)}}{I_k(\Delta_k)-1}$ and $\frac{X_{ik}^{(j)}}{I_k^2(\Delta_k)-1}$ are log-normal variables of the form $10^{-\frac{\hat{\Omega}}{10}}$ and $10^{-\frac{\tilde{\Omega}}{10}}$, respectively, where $\hat{\Omega}$ is a normally distributed random variable with mean $\mu_{ik}^{(j)} - \mu_{\Delta_k-1}$ and variance $\sigma^2 + \sigma^2_{\Delta_k-1}$, and $\tilde{\Omega}$ is normally distributed with mean $\mu_{ik}^{(j)} - 2\mu_{\Delta_k-1}$ and variance $\sigma^2 + 2\sigma^2_{\Delta_k-1}$. From Eqns. (30) and (33), we evaluate $\hat{r}$ and hence the joint probability in Eqn. (29) as

$$\Pr\left\{\tilde{I}_k(\Delta_k) > \epsilon_{\text{dB}}, \tilde{I}_k(\Delta_k - 1) \leq \epsilon_{\text{dB}}\right\} = \left(\frac{1}{\pi R^2}\right)^6 \int_{\epsilon_{\text{dB}}}^{\infty}\int_{-\infty}^{\epsilon_{\text{dB}}} \int\cdots\int \tilde{f}_{\Delta_k}(x,y) r_1\cdots r_6\, dy\, dx\, dr_1 d\theta_1 \cdots dr_6 d\theta_6, \quad (34)$$

where if $\hat{x} = x - \mu_{\Delta_k}$ and $\hat{y} = y - \mu_{\Delta_k-1}$,

$$\tilde{f}_{\Delta_k}(x,y) = \frac{1}{2\pi\sigma_1\sigma_2\sqrt{(1-\hat{r}^2)}} e^{-\frac{1}{2(1-\hat{r}^2)}\left[\frac{\hat{x}^2}{\sigma_1^2} - \frac{2\hat{r}\hat{x}\hat{y}}{\sigma_1\sigma_2} + \frac{\hat{y}^2}{\sigma_2^2}\right]}. \quad (35)$$

From Eqns. (30), (33), (34) and (35), we can evaluate Eqn. (29) and hence the Eqn. (13). We substitute this value and the CB approximation for the marginal, to compute the conditional outage probability in Eqn. (10). Finally, we average p^*_{out} over Δ_{kk} as in Eqn. (12) to obtain the outage probability.

3) Approximation using Edgeworth Expansion: In the following, we obtain an improved approximation to the outage probability by using Edgeworth correction to the CLT approximation of the marginal derived in Section III-B.1. We define $Y = \frac{I_k(\Delta_k - 1) - \mu_{\Delta_k-1}}{\sigma_{\Delta_k-1}}$. Then, by Edgeworth expansion, the cdf of Y, $F(y) = \Pr\{Y \leq y\}$, is given by [3]

$$F(y) = P(y) - C_1(y) + C_2(y) - C_3(y) + C_4(y), \quad (36)$$

where $C_1(y), C_2(y), C_3(y), C_4(y)$ are correction terms of the Edgeworth expansion. The expressions for $C_1(y), C_2(y), C_3(y), C_4(y)$ are given by $C_1(y) = \frac{\gamma_1}{6}Z^{(2)}(y)$, $C_2(y) = \frac{\gamma_2}{24}Z^{(3)}(y) + \frac{\gamma_1^2}{72}Z^{(5)}(y)$, $C_3(y) = \frac{\gamma_3}{120}Z^{(4)}(y) + \frac{\gamma_1\gamma_2}{144}Z^{(6)}(y) + \frac{\gamma_1^3}{1296}Z^{(8)}(y)$ and $C_4(y) = \frac{\gamma_4}{720}Z^{(5)}(y) + \frac{\gamma_2^2}{1152}Z^{(7)}(y) + \frac{\gamma_1^2\gamma_2}{1728}Z^{(9)}(y) + \frac{\gamma_1^4}{31104}Z^{(11)}(y)$, where $P(y) = \int_{-\infty}^{y}\frac{1}{\sqrt{2\pi}}e^{-\frac{t^2}{2}}dt$, $Z^{(m)}(y) = \frac{d^m P(y)}{dy}$, $\gamma_{i-2} = \frac{1}{\Delta_{k-1}^{\frac{i}{2}-1}}\left(\frac{\kappa_i}{\sigma^i_{\Delta_k-1}}\right)$, and κ_i is the i^{th} cumulant of Y. Here, to compute the P_{joint}, we use the Fenton's method as described in Section III-B.2.

Note that the admission control policy will block new incoming calls if either $I/S > \epsilon^o$ or all the n spreading codes are already allocated to ongoing calls. The probability of blocking is then given by

$$p_b^* = 1 - \left[1 - p_o'(\epsilon^o, \rho)\right]\left[1 - p_b(\rho, n)\right], \tag{37}$$

where $p_b(\rho, n)$ is given by the Erlang-B formula [7] as $p_b(\rho, n) = \frac{\frac{\rho^n}{n!}}{\sum_{k=0}^{n} \frac{\rho^k}{k!}}$.

IV. Results and Discussion

In this Section, we present the numerical results for the outage probability and the capacity of a CDMA system with and without I/S based admission control. The system capacity is defined as the load up to which a desired outage probability can be maintained. We use the following values in our numerical computations: $N = 61$ cells, $n = 64$ spreading codes, cell radius $R = 1$, mean call arrival rate per cell, λ, in the range 0.1 to 0.2 in steps of 0.01, mean call holding time, $\mu^{-1} = 100$ seconds, and $\sigma = 8$ dB. As in [2],[3], we use a call admission I/S threshold value of $\epsilon^o = 14$ dB.

Figure 1 shows the outage probability as a function of Erlang load per cell with and without admission control. Both simulation as well as analytical results using CB approximation are shown. It is observed that without admission control an outage probability of 0.01 occurs at a load of about 15 Erlangs/cell, whereas with admission control the same outage performance is achieved at a load of 20 Erlangs/cell. This is due to admitting new calls only when the I/S criterion is not violated. The increase in the load essentially translates into an increase in the number of active mobiles in the system, as explained below. Each cell can be modeled as an $M/G/n/n$ loss system with a blocking probability p_b^*, as given in Eqn. (37). The average number of users in a cell for an Erlang load of ρ is given by $\rho(1 - p_b^*)$ [7]. Therefore, for an outage of 0.01, a load of 15 (or 20) Erlangs per cell translates to 14.9 (or 19.8) users per cell. This is an increase of 33% of additional users in the system due to I/S based admission control.

From Fig. 1, it is also observed that using the CB approximation to compute the $P_{marginal}$ overestimates the outage probability (i.e., underestimates capacity). The accuracy of the outage performance predicted by the CLT and CB approximations are illustrated in detail in Fig. 2. From Fig. 2, we note that while the CB approximation overestimates the outage probability, the CLT approximation underestimates it. From Fig. 1, we further observe that the CB approximation is more accurate (closer to the simulation curve) than the CLT approximation. In addition, the Edgeworth correction to the $P_{marginal}$ results in a performance that is closer to that predicted by simulation, as compared to the CLT approximation.

V. Conclusions

We presented an analytical model to evaluate the outage probability and the capacity on the reverse link of cellular CDMA systems with I/S based admission control. We used two main approximations in the outage analysis – one based on the central limit theorem and the other based on the Chernoff bound. It was shown that the CLT approximation underestimated the outage performance, whereas the CB approximation overestimated the outage. Also, the CB approximation resulted in a more accurate estimate of the outage than the CLT approximation. We further obtained an improved approximation to the outage probability using Edgeworth expansion. It was shown that the considered admission control policy results in increased system capacity compared to that with no admission control, by about 30% for an outage probability of 0.01.

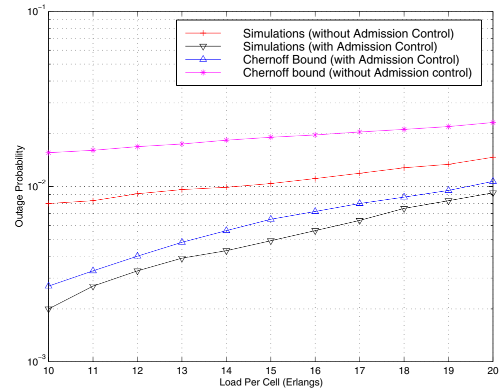

Fig. 1. Outage performance with and without admission control. $N = 61$ cells. $n = 64$ spreading codes.

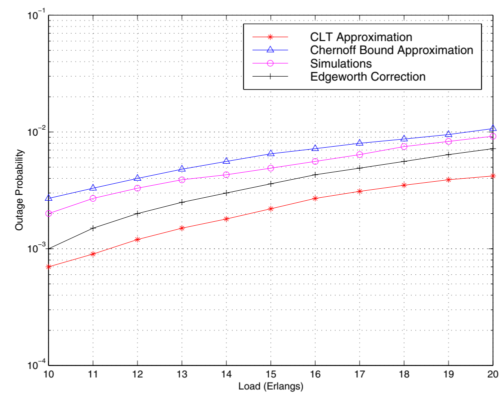

Fig. 2. Outage performance with admission control predicted by analysis and simulations. $N = 61$ cells. $n = 64$ spreading codes.

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