Performance Analysis of Space Diversity S-ALOHA with Steerable Beam Smart Antenna Arrays

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Abstract—**We analyze the throughput, delay and energy efficiency performance of space diversity slotted ALOHA when multibeam smart antenna arrays, capable of steering the beams selectively on smaller sectors, are used at the base station receiver. Analytical expressions to establish the effect of different beamwidths, number of beams, beam steering patterns, and beam service times on the achieved performance of S-ALOHA are derived. We show that under high load conditions *steered* beams with long beam service times offer better performance, whereas under light loads *static* coverage patterns are better. We propose and analyze an energy efficient version of S-ALOHA which uses a *beam sensed backoff* algorithm at the user terminal. We show that the beam sensed backoff algorithm results in significant energy savings that could potentially increase battery life in wireless user terminals.**

Keywords – *Space diversity, smart antennas, S-ALOHA, energy efficiency.*

I. INTRODUCTION

Capacities of wireless systems can be substantially increased by the use of *antenna diversity* [1]. Different spatial signatures of different mobile users can be exploited to separate multiple co-channel signals even if they share the same time slot or frequency band [2]. Adoption of smart antenna techniques at the base station is being provisioned for in the emerging third generation wireless systems, by way of allowing for auxiliary pilot signals, in addition to normal pilots in each cell [3]. Several investigations on smart antennas have focussed primarily on issues related to signal processing needed to realize spatial diversity by direction-of-arrival estimation and beam forming, optimum diversity combining and equalization, and evaluation of outage probability and average probability of error [2],[4]. However, the impact of employing smart antenna techniques at the physical layer on the performance of higher layer protocols in packet mode communications, is an important area which has not been adequately analyzed so far [6]-[8]. In fact, given the ability to steer beams in specified directions with specified angular resolutions, smart protocols with parameterized controls can be designed to enhance performance at higher layers, while exploiting spatial diversity advantage at the physical layer. The focus of this paper is the performance analysis of a space diversity slotted ALOHA protocol and the proposal of an energy efficient version of the protocol, when smart antenna arrays are used at the base station receiver.

We analyze the benefit of providing simultaneous space diversity using multiple narrow beams, i.e., when the base station uses n beams ($n \geq 1$), each with a beamwidth of θ radians. Specifically, the effect of different beamwidths, number of beams, beam steering patterns, and beam service times on the achieved throughput-delay performance of space diversity S-ALOHA is evaluated. The difference between our system model and the model in [6] is that, instead of considering packet acquisition from lone mobiles using PN preamble and random time offset as in [6], we consider a simple beam steering pattern which *periodically shifts the direction of beam(s) by an angular amount equal to the beamwidth.* The advantage of this model is that the resulting beam steering pattern is easy to realize, and by properly choosing the system/protocol parameters like beam service time (number of slots over which a given coverage area is illuminated by a beam), retransmission probability, etc., the system performance can be optimized for a range of system loads.

Another contribution in this paper is the analytical evaluation of energy efficiency of space diversity S-ALOHA. Design of energy efficient protocols are essential to increase battery life in wireless user terminals [9]. Here, we propose and analyze an energy efficient version of space diversity S-ALOHA which uses a *beam sensed backoff* algorithm at the user terminal. We show that the beam sensed backoff algorithm results in significant energy savings that could potentially increase battery life in wireless user terminals.

The rest of the paper is organized as follows. In Section II, a brief description of multibeam antenna arrays is presented. Section III describes the system model considered. Section IV provides the detailed mathematical analysis of the throughput, delay, and energy efficiency of the basic as well as the energy efficient version of space diversity S-ALOHA. Numerical results are discussed in Section V. Conclusions and the potential topics for future research are given in Section VI.

II. MULTIBEAM ADAPTIVE ANTENNA ARRAYS

In an adaptive antenna array, the radiation pattern, frequency response and other parameters are modified based on adaptive control, with the aim of reducing the sensitivity in the direction of unwanted signals (interference) [2],[5]. A typical antenna array consists of a number of antenna elements coupled together via some form of amplitude and phase shifting network to form a single output. Multiple output signals can be obtained from the same set of antenna elements by applying multiple sets of weights. Each set of weights yields a different array output signal, representing a different beam. It has been shown that an m

element array has $m - 1$ degrees of freedom providing up to $m - 1$ independent pattern nulls [5].

In principle, multiple beam antenna arrays can be used to track, in azimuth, each mobile (or a group of mobiles) using directive narrow beams. The directive nature of the beams ensures that the interference levels seen by any given user will be far less than when conventional wide coverage base station antennas are employed. In the context of random access protocols, this would imply reduced probability of collision among packets transmitted from geographically separated mobile users. The focus of this paper would be the evaluation of the throughput-delay performance at the media access control layer when steerable multibeam antenna arrays (with idealized beam patterns) are used at the base station receiver, rather than the issues surrounding antenna array design themselves.

III. System Model

We consider a circular cell with its base station located at the center of the cell. Mobile users are assumed to be uniformly distributed over the cell area. A slotted channel is shared by all the mobiles on the reverse link (mobile-to-base station link) for sending data packets to the base station. The base station receives packet transmissions from the mobiles through n, $n \geq 1$, different but spatially separated beams, each having a width of θ radians. The beams are assumed to have idealized, non-overlapping patterns, focusing on perfect 2-dim cones of angle θ in given directions on the two dimensional plane as shown in Figure 1. The beamwidth θ is chosen such that $\theta \leq \frac{2\pi}{n}$.

Case (a): If $\theta = \frac{2\pi}{n}$, the entire cell area is illuminated without any "hole" in coverage at any given time; for example, $n = 1$ corresponds to an omni-directional beam pattern covering the full azimuth range from 0 to 2π radians, and $n = 3$ corresponds to the classical 3-sector scheme with a beamwidth of $\frac{2\pi}{3}$ radians (i.e., 120°). We refer to this scenario as the *static beam* scenario, where steering of beams over time is not performed (i.e., the amplitude and phase weights of the array can be fixed).

Case (b): On the other hand, if $\theta < \frac{2\pi}{n}$, the cell area is partially illuminated at any given time, leaving "holes" in coverage. The angular width of the un-illuminated holes will be $(\frac{2\pi}{n} - \theta)$. In this case, to achieve full coverage over the cell area, the direction of the beams are to be steered over time, by using appropriate array weights. Users in a coverage hole at a given time will eventually be serviced because of periodic steering of the beams. We refer to this scenario as the *steered beam* scenario. A simple beam steering pattern, in such a case, would be to periodically shift the direction of beam(s) by an angular amount equal to the beamwidth θ. By doing so, the entire cell area can be swept once in $\lceil \frac{2\pi}{n\theta} \rceil$ angular shifts. The time allowed between the angular shifts will then be the service time available to the corresponding illuminated segments in the cell area. The considered beam patterns for $n = 1, 2, 3$ are illustrated in Figure 1.

Because of spatial diversity, multiple packets sent on the same slot, but from mobiles illuminated by different beams, can be successfully received by the base station. Here, we assume that packet losses occur only due to collision among simultaneous packet transmissions on the same beam. Packet losses due to physical layer characteristics like multipath fading and capture will be considered in a future investigation.

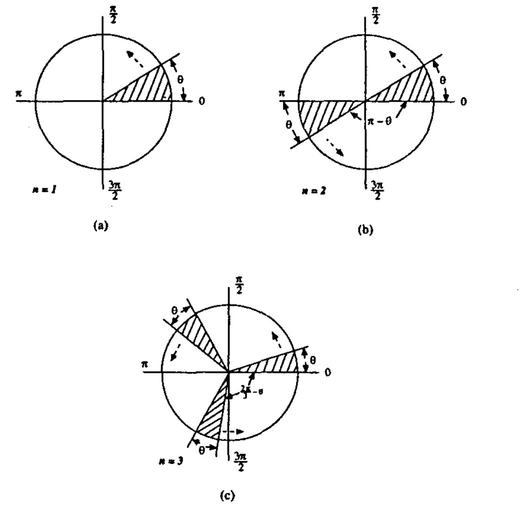

Fig. 1. Antenna Beam Steering Patterns. a) $n = 1$, b) $n = 2$, c) $n = 3$. $\theta < \frac{2\pi}{n}$.

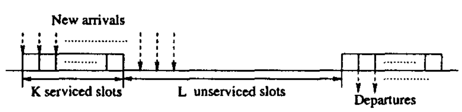

Fig. 2. A cycle of serviced-unserviced slots.

IV. Analysis

The users in the *cell* are assumed to be uniformly distributed. A new packet arrival occurs at each user node with probability p, independently in every slot. If a user node fails to transmit its newly arrived packet successfully in a slot (either due to collision or due to non-availability of beam service), it enters into a backlogged mode. We call a slot, with respect to a user, *serviced slot* if a beam is present illuminating that user's location during that slot, and *unserviced slot* if a beam is not present in that user's direction. A collision occurs if more than one user attempt packet transmission in the same serviced slot. A transmission attempt can be either due to a new packet arrival or due to retransmission of a backlogged packet. Backlogged users attempt transmission in subsequent slots with probability p_r, and reject new packet arrivals until the backlogged packet is successfully sent to the base station.

We consider $r = \left(\frac{2\pi}{n\theta}\right)$ to be greater than 1, so that there will be coverage holes present in the cell at any time. Consider a sector (an illuminating beam area) having N users. We do the following analysis for r being an integer ≥ 2, so that the serviced-unserviced periods as seen by a user in the sector under consideration show a cyclic behavior, as shown in Figure 2.

Define K and L as the number of serviced slots and unserviced slots, respectively, in a cycle. Then, $L = (r - 1)K$. Let B_i^j

be the random variable representing the number of backlogged users in the beginning of slot $i, 1 \leq i \leq (K+L)$ of the j^{th} cycle, $j \geq 1$. Define $X_i = B_l^m$, where $i = m(K+L) + l$. Observe that $\{X_i, i \geq 1\}$ is a Markov chain. For $m \geq 0$, let $\mathbf{C}$ represent the transition probability matrix of X_i (to X_{i+1}) for $i = m(K+L)+l, 1 \leq l \leq K$ and $\mathbf{H}$ represent the transition probability matrix for $i = m(K+L)+l, K+1 \leq l \leq L$. Or, in words, $C(l,m)$ is the probability that a slot has m backlogs given the previous slot during the serviced period had l backlogs, and $H(l,m)$ is the probability that a slot has m backlogs given the previous slot during the unserviced period had l backlogs. Defining $1-p = \hat{p}$ and $1-p_r = \hat{p_r}$, the expression for $C(l,m)$ can be written as, $C(l,m) =$

$$\begin{array}{l} \hat{p}^{N-l}\left[1 - lp_r\hat{p_r}^{l-1}\right] + \\ (N-l)p\hat{p}^{N-l-1}\hat{p_r}^{l} \;;\ \text{for}\ m=l,\ 0 \leq l \leq N-1, \\ 1 - Np_r\hat{p_r}^{N-1} \;;\ \text{for}\ m = l = N, \\ \hat{p}^{N-l}lp_r\hat{p_r}^{l-1} \;;\ \text{for}\ m = l-1,\ 1 \leq l \leq N, \\ 0 \;;\ \text{for}\ 0 \leq m \leq l-2,\ 2 \leq l \leq N, \\ 0 \;;\ \text{for}\ m=1,\ l=0, \\ (N-l)p\hat{p}^{N-l-1}\left[1-\hat{p_r}^{l}\right];\ \text{for}\ m=l+1,\ 1 \leq l \leq N-1, \\ \binom{N-l}{m-l}p^{m-l}\hat{p}^{N-m};\ \text{for}\ l+2 \leq m \leq N, 0 \leq l \leq N-2. \end{array}$$

The expression for $H(l,m)$ can be written as, $H(l,m) =$

$$\begin{array}{l} 0 \;;\ \text{for}\ 0 \leq l \leq N,\ 0 \leq m \leq l-1, \\ \binom{N-l}{m-l}p^{m-l}\hat{p}^{N-m} \;;\ \text{for}\ 0 \leq l \leq N,\ l \leq m \leq N. \end{array}$$

Now, sample B_i^j at the first slot of every cycle to obtain the process $\{B_1^j, j \geq 1\}$. This process is also a Markov chain with a transition probability matrix $\mathbf{P}$, given by $\mathbf{P} = \mathbf{C}^K\mathbf{H}^L$. Let B_i denote the stationary random variable corresponding to B_i^j, the stationarity considered over j. Let π_l^i represent the distribution of B_i, $1 \leq i \leq K+L$ and $0 \leq l \leq N$, i.e., $\pi_l^i = Prob[B_i = l]$. Solving $\underline{\pi}^1 = \underline{\pi}^1\mathbf{P}$ gives the stationary distribution of B_1. Then, $\underline{\pi}^i$, $2 \leq i \leq K+L$ could be obtained from $\underline{\pi}^1$, $\mathbf{C}$, and $\mathbf{H}$ using the relationships

$$\pi_l^{i+1} = \sum_{k=0}^{N} \pi_k^i C(k,l);\ 1 \leq i \leq K, \tag{1}$$

and

$$\pi_l^{i+1} = \sum_{k=0}^{N} \pi_k^i H(k,l);\ K+1 \leq i \leq K+L-1. \tag{2}$$

Next, define s_l as the probability that one user succeeds in a slot which begins with l number of backlogged users. We can write s_l, for a serviced slot, as

$$s_l = \begin{cases} lp_r\hat{p_r}^{(l-1)}\hat{p}^{(N-l)} + \\ (N-l)p\hat{p}^{(N-l-1)}\hat{p_r}^{l};\ \text{for}\ 0 \leq l \leq N-1, \\ Np_r\hat{p_r}^{(N-1)};\ \text{for}\ l = N. \end{cases} \tag{3}$$

For an unserviced slot $s_l = 0$.

A. Throughput

Define η_N as the throughput of a sector having N users. This can be written as,

$$\eta_N = \frac{1}{K+L}\sum_{i=1}^{K}\sum_{l=0}^{N}\pi_l^i s_l \tag{4}$$

We have totally $M = rn$ number of sectors in the cell. Let S_m, $1 \leq m \leq M$, be the random variable representing the number of users in the m^{th} sector. Let $\mathcal{N}$ be the total number of users in the system. Then, $0 \leq S_m \leq \mathcal{N}$ and $\sum_{m=1}^{M} S_m = \mathcal{N}$. Let $\eta(N_1, N_2, \cdots, N_M)$ be the system throughput when $S_m = N_m, 1 \leq m \leq M$, so that

$$\eta(N_1, N_2, \cdots, N_M) = \eta_{N_1} + \eta_{N_2} + \cdots + \eta_{N_M}. \tag{5}$$

Let $\beta(N_1, N_2, \cdots, N_M) = Prob[S_1 = N_1, \cdots, S_M = N_M]$, which can be derived as

$$\beta(N_1, N_2, \cdots, N_M) = \binom{\mathcal{N}}{N_1}\left(\frac{1}{M}\right)^{N_1}\left(1-\frac{1}{M}\right)^{\mathcal{N}-N_1} \cdot \binom{\mathcal{N}-N_1}{N_2}\left(\frac{1}{M-1}\right)^{N_2}\left(1-\frac{1}{M-1}\right)^{\mathcal{N}-N_1-N_2} \cdots$$

Then, the system throughput, η, can be written as,

$$\eta = \sum_{N_1=0}^{\mathcal{N}}\sum_{N_2=0}^{\mathcal{N}-N_1}\cdots\sum_{N_M=0}^{\mathcal{N}-\sum_{i=1}^{M-1}N_i}\beta(N_1, N_2, \cdots, N_M) \times \eta(N_1, N_2, \cdots, N_M). \tag{6}$$

In the case of static beams, all slots are serviced slots, and so, the stationarity is taken over the slots. We can obtain the expressions for throughput in the static beams case by putting $K = 1$ and $L = 0$ in the above Equations.

B. Delay

Let δ_N^a and δ_N^d denote the expected number of new arrivals and sum of expected delays of all new arrivals, respectively, in a period of a cycle in a sector having N users (δ_N^a and δ_N^d are derived in Appendix -A). The expected delay of a user in the system, δ, is given by,

$$\delta = \frac{\delta^d}{\delta^a}, \tag{7}$$

where,

$$\delta^a = \sum_{N_1=0}^{\mathcal{N}}\sum_{N_2=0}^{\mathcal{N}-N_1}\cdots\sum_{N_M=0}^{\mathcal{N}-\sum_{i=1}^{M-1}N_i}\beta(N_1, N_2, \cdots, N_M) \times (\delta_{N_1}^a + \delta_{N_2}^a + \cdots + \delta_{N_M}^a), \tag{8}$$

$$\delta^d = \sum_{N_1=0}^{\mathcal{N}}\sum_{N_2=0}^{\mathcal{N}-N_1}\cdots\sum_{N_M=0}^{\mathcal{N}-\sum_{i=1}^{M-1}N_i}\beta(N_1, N_2, \cdots, N_M) \times (\delta_{N_1}^d + \delta_{N_2}^d + \cdots + \delta_{N_M}^d). \tag{9}$$

C. Energy Efficiency

We define energy efficiency [9] as the reciprocal of the amount of energy consumed per successful transmission attempt, by considering one unit of energy as corresponding to the amount of energy spent in transmitting one packet. In other words, the normalized energy efficiency is unity if every transmission attempt is a successful attempt. Failed transmission attempts lead to drain in the battery power without adding to throughput, and hence will result in reduced energy efficiency.

The approach to derive the energy efficiency is very similar to the throughput analysis. Let ϕ_N be the mean number of unsuccessful transmissions per slot in a sector having N users. Similar to $\eta(N_1, N_2, \cdots, N_M)$, define $\phi(N_1, N_2, \cdots, N_M)$ to be the mean number of unsuccessful transmissions per slot of the system when $S_m = N_m$, $1 \le m \le M$. Then, ϕ, the mean number of unsuccessful transmissions per slot of the system is given by,

$$\phi = \sum_{N_1=0}^{\mathcal{N}} \sum_{N_2=0}^{\mathcal{N}-N_1} \cdots \sum_{N_M=0}^{\mathcal{N}-\sum_{i=1}^{M-1} N_i} \beta(N_1, N_2, \cdots, N_M) \times \phi(N_1, N_2, \cdots, N_M). \quad (10)$$

The derivation of ϕ_N and $\phi(N_1, N_2, \cdots, N_M)$, are given in the Appendix -B. The energy efficiency, ν, is given by $\nu = \eta/(\eta + \phi)$, where η is given by Eqn. (6).

D. Beam Sensed Backoff

In the basic version of the space diversity S-ALOHA protocol described and analyzed above, mobile users could attempt transmission during unserviced slots as well. Although leaving intentional holes in coverage (unserviced slots) may result in improved system throughput under certain system load conditions (as will be illustrated in the next Section), transmission attempts during unserviced slots will merely add to the drain on the battery power in the mobile terminal, without contributing to system throughput. The basic protocol can be made more energy efficient by constraining the mobiles not to attempt transmission during the unserviced slots. This could be achieved, for example, by the base station informing the spatially distributed mobiles individually of their beam coverage information (start and length of service period corresponding to individual mobile's location). The mobiles can then use this information to ensure that they attempt only during the serviced slots. We call this energy efficient procedure as the *beam sensed backoff* algorithm. That is, the mobile transmits a packet only if it is illuminated by a beam, and keeps idle and reschedules transmission otherwise.

The throughput and delay performance with beam sensed backoff is the same as that without backoff, because regardless of whether attempts are made during unserviced slots or not, it is only the successful transmissions during serviced slots that contribute to throughput and delay. However, the energy consumption performance with and without backoff will be different, which can be quantified analytically. The energy efficiency performance with backoff can be derived similar to without backoff, except for minor modifications in the derivation of ϕ_N as described in Appendix -B.

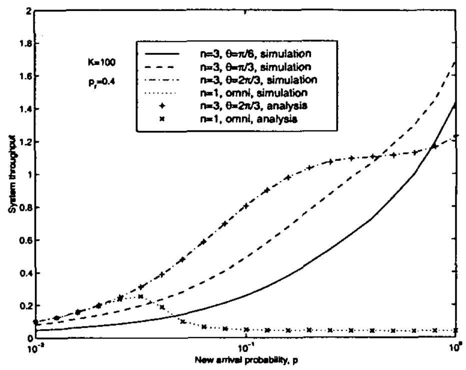

Fig. 3. System throughput, η, versus new packet arrival probability, p, for $n = 3$ at different values of θ. $\mathcal{N} = 10, p_r = 0.4, K = 100$ slots.

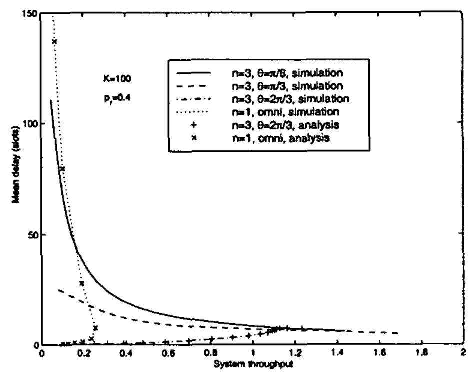

Fig. 4. Throughput-delay performance for $n = 3$ at different values of θ. $\mathcal{N} = 10, p_r = 0.4, K = 100$ slots.

V. Results

The system is studied for $\mathcal{N} = 10$ and for different values of K, p, n, θ and p_r. Figure 3 shows the system throughput (η) versus new packet arrival probability (p), for different n and θ, when $K = 100$ slots and $p_r = 0.4$. The mean delay versus system throughput for the same set of parameters is shown in Figure 4. Both analytical and simulation points are shown. Simulation results closely match with analytical results thus validating the analysis.

From Figures 3 and 4, it is seen that at light loads (low values of p), an omni-directional antenna pattern is shown to perform as good as multiple static antenna pattern, and better than steerable antenna pattern. This is because when p is low, the probability of collision is low. This makes steerable antenna pattern to perform poorer, because some possible successes are lost due to the holes in coverage. For medium loads, multiple static antenna patterns give better throughput than both omni as well as multiple steered beam patterns. This implies that there is no performance benefit in steering the beams leaving intentional holes at light to moderate loads. However, at heavy loads, steerable antenna pattern gives better throughput than static pattern. For example, for $p > 0.4$, steered beam pattern

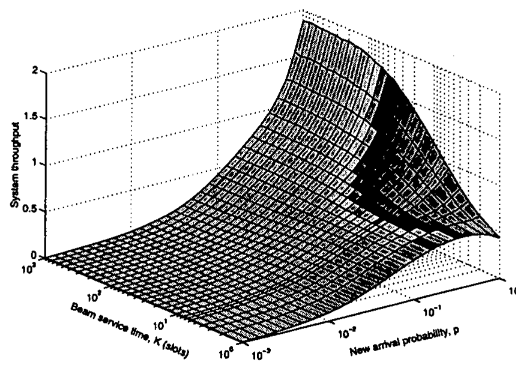

Fig. 5. System throughput, η, vs new arrival probability, p, and beam service time, K, for $n = 3$ at $\theta = \pi/3$. $\mathcal{N} = 10, p_r = 0.1$.

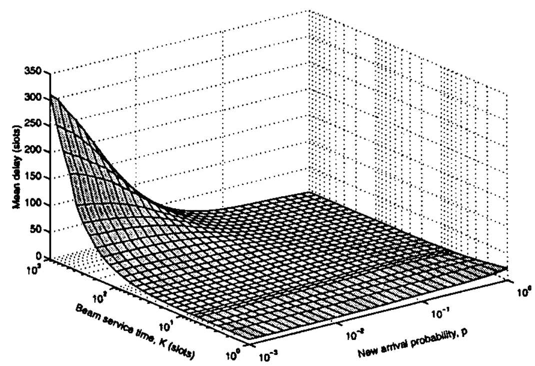

Fig. 6. Mean delay vs new arrival probability, p, and beam service time, K, for $n = 3$ at $\theta = \pi/3$. $\mathcal{N} = 10, p_r = 0.1$.

with $\theta = \frac{\pi}{3}$ performs better than static pattern with $\theta = \frac{2\pi}{3}$. This is because, the presence of coverage holes reduces the effective probability of collision during serviced slots when p is high (i.e., transmission from users in holes do not interfere with transmissions in serviced slots), thereby increasing the system throughput. The performance crossover between static versus steered beams occur at different loads, depending on the values of system parameters (e.g., θ, p_r, K).

Figures 5 and 6 show the 3-D plots of system throughput and mean delay versus p and K, for $\mathcal{N} = 10$, $n = 3$, $\theta = \frac{\pi}{3}$ and $p_r = 0.1$. At light loads ($p < 10^{-2}$), the beam service time does not significantly affect the throughput because the achieved throughputs in any case are low (< 0.1), due to low arrival and high idle rates. However, large values of K significantly increases the mean delay performance. Thus, small beam service times are desired at light loads. On the other hand, at high loads ($0.1 < p < 1$), the mean delay performance is not very much affected, but the achieved throughput increases significantly with increasing values of K. This implies that serving an illuminated sector longer (i.e., large K) is desired at high loads. Therefore, choosing K based on the system load, (i.e., a large K at high load and a small K at low load), will give better performance in terms of throughput and delay.

Finally, Figure 7 shows the energy efficiency (in dB) versus system throughput plots for both with and without the beam sensed backoff (BSBO), when $n = 3$, $\mathcal{N} = 10$, $K = 100$, and $p_r = 0.1$. The 0 dB point on the y-axis is the best possible energy efficiency corresponding to all transmission attempts being successful. Failed transmissions will result in less than unity (i.e., -ve dB) energy efficiency as shown. Each point on these energy efficiency curves correspond to a different new arrival probability p, with p increasing from 0 to 1 moving from left to right side of the x-axis. The system with beam sensed backoff is seen to achieve achieve much better energy efficiency compared to the one without backoff. For example, to achieve a throughput of 0.6 in a system with beamwidth $\theta = \pi/6$, the system without backoff needs about 3.5 dB more (i.e., more than twice) power than the one with backoff. Similarly, for $\theta = \pi/3$ the improvement in energy efficiency due to BSBO is about 2 dB. For an achieved throughput of 1, the energy savings are 2.3 dB and 1.5 dB, respectively, for $\theta = \frac{\pi}{6}$ and $\frac{\pi}{3}$. For the maximum achieved throughput of 1.8 at $\theta = \frac{\pi}{3}$, the energy savings is about 1 dB. It is noted that about 18% of the power consumed in mobile computers (laptops with wireless interface) is due to the wireless interface cards [10]. This consumption percentage could be even higher in palmtops and other such wireless terminals, where the power consumed by display, memory, etc., can be much less compared to the power consumed by RF power amplifier during packet transmissions. In the light of the above, a 3 dB energy savings at the packet transmission layer can appreciably increase the battery life – about 10% increase in battery life in the mobile computer example. Thus, we show that smart antenna techniques coupled with energy efficient protocol rules can enhance throughput-delay performance while increasing the 'talk-time' of wireless user terminals and mobile computers.

VI. Conclusions

We analyzed the throughput-delay performance of S-ALOHA when steerable multibeam antenna arrays are used at the base station receiver. Performances achieved using multiple static beams versus multiple steered beams were compared. A deterministic beam steering pattern, where the direction of beam(s) are periodically shifted by an angular amount equal to the the beamwidth, was considered. Through numerical results we showed that, under high load conditions *steered* beams with long beam service times offer better performance, whereas under light loads *static* coverage patterns are better. We also proposed and analyzed an energy efficient version of S-ALOHA which used a beam sensed backoff algorithm at the user terminal. This algorithm was shown to result in significant energy savings (about 3 dB) that could translate into extended battery life in wireless user terminals. Further, the performance of *stochastic beam steering* patterns (beam steering based on knowledge/prediction of dynamic system behavior) are expected to be better, and is worth investigating as future work.

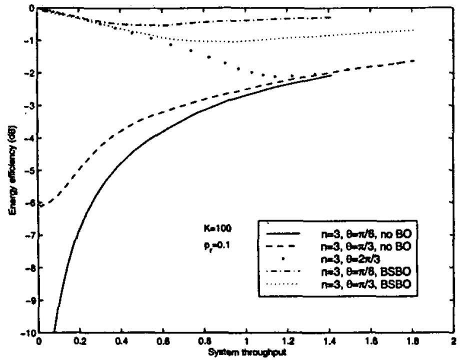

Fig. 7. Energy efficiency versus system throughput for $n = 3$ at $\theta = \pi/6$ and $\pi/3$ with and without beam sensed backoff (BSBO). $\mathcal{N} = 10$, $p_r = 0.1$, $K = 100$ slots.

APPENDIX

A. Derivation of δ_N^a and δ_N^d

Let $\mathbf{E}$ be a matrix, whose $(l,m)^{th}$ element is the probability that a *particular* user out of the l backlogged users in a serviced slot does not leave the system and goes to the next slot with m backlogged users. Observe that the elements of this matrix corresponding to no success (i.e., the transitions except $(l, l-1)$, $1 \leq l \leq N$) are same as that of $\mathbf{C}$ (Sec. IV) and $E(l, l-1) = (l - 1/l)(1-p)^{(\mathcal{N}-l)}p_r(1-p_r)^{(l-1)}$. Also define the following:

$d(l, l-1)$, $1 \leq l \leq N$, as the probability that a particular user out of l backlogged users leave the system in a serviced slot.

$g(l,m)$, $0 \leq l, m \leq N$, as the probability that a new arrival, which occurs in a serviced slot when there are l backlogged users, does not leave the system immediately and enters into backlogged mode with m backlogged users in the next slot. These are the elements of the matrix $\mathbf{G}$.

$q(l,m)$, $0 \leq l, m \leq N$, as the probability that a new arrival, which occurs in an unserviced slot when there are l backlogged users, does not leave the system immediately and enters into backlogged mode with m backlogged users in the next slot. These are the elements of the matrix $\mathbf{Q}$.

D_l^s, $1 \leq s \leq K+L$, as the random variable representing the delay of a user arriving at the s^{th} slot in a cycle, when there are l backlogged users in the system.

Figure 2 shows the departure of a user arriving in a serviced slot or unserviced slot. An arrival in a slot is shown at the beginning of the slot and the departure at the end of the slot. Let γ_l^s be the expected value of D_l^s and A_l be the expected number of new arrivals in a slot having l backlogged users. Then δ_N^d and δ_N^a are given by,

$$\delta_N^d = \sum_{s=1}^{K+L}\sum_{l=0}^{N-1} \pi_l^s \gamma_l^s A_l, \tag{11}$$

$$\delta_N^a = \sum_{s=1}^{K+L}\sum_{l=0}^{N-1} \pi_l^s A_l. \tag{12}$$

A_l is given by $\sum_{k=l+1}^{N-1} \binom{N-l}{k-l} p^{(k-l)}(1-p)^{(\mathcal{N}-k)}(k-l)$.

Let $F_s^c(l,m)$ denote the probability that a new user arriving in the s^{th} slot when there are l backlogged users has a delay greater than c number of slots. Then $\gamma_l^s = \sum_{c=0}^{\infty}(c+1)F_s^c(l,m)d(m,m-1)$. If the user departs the system in the same serviced period as it arrived, $F_s^c(l,m)$ is the $(l,m)^{th}$ element of $GE^{(c-1)}$. Observe that, c in this case is less than or equal to $K-s$. In general, if the user departs in the $\bar{c}^{th}$ slot of d^{th}, $d \geq 1$ serviced period after it arrived the system (i.e., the delay, $c = (K-s)+(d-1)(K+L)+\bar{c}$), $F_s^c(l,m)$ is the $(l,m)^{th}$ element of $GE^{(K-s)}H^L\left(\prod_{j=1}^{d-1} E^K H^L\right)E^{(\bar{c}-1)}$. Similarly, for an arrival in s^{th} unserviced slot and leaving the system in the $\bar{c}^{th}$ slot of d^{th}, $d \geq 1$ serviced period after it arrived (i.e., delay $c = (L-s)+(d-1)(K+L)+\bar{c}$), $F_s^c(l,m)$ is the $(l,m)^{th}$ element of $H^{(L-s)}\left(\prod_{j=1}^{d-1} E^K H^L\right)E^{(\bar{c}-1)}$.

B. Derivation of ϕ_N and $\phi(N_1, N_2, \cdots, N_M)$

Define f_l as the mean number of failed attempts in a slot which begins with l number of backlogged users. The mean number of attempts in the slot is given by $lp_r + (N-l)p$. In Sec. IV-A, s_l is already defined to be the mean number of successes in the slot. This gives, $f_l = lp_r + (N-l)p - s_l$. Now, ϕ_N, for no beam sensed backoff, is given by,

$$\phi_N = \frac{1}{K+L}\sum_{i=1}^{K+L}\sum_{l=0}^{N} \pi_l^i f_l \tag{13}$$

For the terms $K+1 \leq i \leq K+L$, $f_l = lp_r + (N-l)p$, since $s_l = 0$ for unserviced slots. Finally, $\phi(N_1, N_2, \cdots, N_M) = \phi_{N_1} + \phi_{N_2} + \cdots + \phi_{N_M}$.

When there is beam sensed backoff, no transmission attempt is done during the unserviced slots. Therefore, ϕ_N for system with beam sensed backoff is given by the same expression as above with the summation running over $1 \leq i \leq K$.

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