# A Log-Linear Closed Loop Power Control Model *

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We introduce a new model for a closed loop power control system that transforms the standard closed loop model into an analytically tractable log-linear model. From it, we obtain all first and second order received power statistics. The correlation of the received power offers insight into the burst error nature of the closed loop power control channel and can be used to specify interleaver and coding requirements. The impact of simplification of the model is studied by comparing the analytical results with a detailed simulation of the actual model. We find that the linearized model is more effective in predicting the shape of the correlation function than in predicting the actual magnitudes.

## 1 Introduction

In designing coding, interleaving and link-layer protocols, knowledge of the error characteristics of the power-controlled channel is necessary. This information can be used to adjust interleaver length, packet length, retransmission strategies, and overall link-level design.

Traditionally, the bit error rate is used as a measure of "channel quality". However, the dynamic behavior of packet based communications requires a more thorough understanding of the error characteristics. When considering the trade-offs involved in wireless link/physical layer design, it is important to study more dynamic measures of physical layer communications performance such as burst error length and channel error correlation.

In this paper, we analyze a simplified model of a closed-loop power controlled CDMA system. From the analysis, we derive system performance measures including the correlation in the received power. Although this correlation does not directly specify channel burst error characteristics, it provides insight into the bursty nature of a power controlled CDMA channel. A detailed simulation of the closed loop power control system is used to validate the results of the analysis.

## 2 Log-Linear Model

Consider the log-linear power control model shown in Figure 1. All sequences marked represent power in decibels at a particular point. The index, $n$, in all the expressions indexes bits. The boxes represent linear filters, marked by either a fixed delay or filter impulse response. This figure represents a linear system model for a single mobile/base station link. The model captures only the instantaneous powers at various points in the system, and not actual signals.

The $n^{th}$ bit is transmitted with power $P_n^t$. Not all energy transmitted in bit $n$ is received. Power is lost due to fading

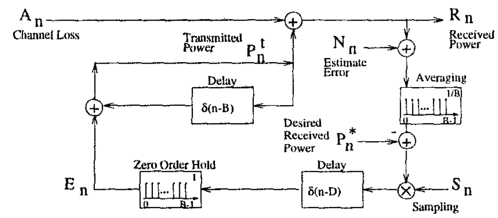


Figure 1: Log-Linear Power Control Model

represented by adding a channel loss $A_n$, which is constant over the bit. $A_n$ has a value equal to $10\log_{10}(a_n^2)$, where $a_n$ has Rayleigh distribution. Implicit in this description is the assumption that the underlying fading process $a(t)$ does not vary rapidly with respect to a single bit time. Additional loss, due to propagation, distance, and shadowing are relatively slow compared to the fading process. It is assume that they are "tracked out" perfectly by the power control algorithm. If more complex models including shadowing, fading and mobility, are desired, additional channel loss processes could be added.

The $n^{th}$ bit is received at the base station with power $R_n$. $R_n$ represents the actual power in the received signal of interest. This is not typically a measurable quantity as it does not include error involved in estimating this power. Analytically, however, the quantity $R_n$ is of primary interest, since it represents the true power of the desired signal. The base station is assumed to measure power by examining the square of the test statistic for each bit. Because of thermal noise in the receiver and received power from other mobiles, the estimate for received power contains some error. The error (in dB) is modeled by the process $N_n$. Because the square of the test statistic is used as an estimate for power, the estimate is biased and thus, the process $N_n$ has a non-zero mean. The sequence $N_n$ must not be confused with the thermal noise process at the receiver. $N_n$ is strictly a power estimate error. It is not immediately evident what the characteristics of such a process are. The modeling of the process $N_n$ is presented in Section 6.

The base station averages the received power estimates over $B$ bits, as shown by the averaging filter. The base station will use this estimate of the power to enforce power level changes at the mobile. The correction applied to the mobile's power is obtained by subtracting this estimate from the desired received power. The sampling waveform samples this average power correction once every $B$ bits. This is the value that is used by the mobile for updating its transmit power.

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Due to round-trip propagation, processing, and frame delay, it is assumed that this correction can be used by the mobile $D$ seconds after it is computed. The mobile, using a zero-order hold, reconstructs the desired change in power suggested by the base station, and adds it to its transmitted power from $B$ bits previous (i.e. adding the correction to the old transmit power yields a new transmit power $P_n^t$ that, one hopes, is more accurate).

This model captures the time evolution of the power control update process. By constructing suitable input process (i.e., $A_n$, $N_n$, $P_n^*$), one can determine statistics of the received power level $R_n$.

# 3 Assumptions

- The closed loop power control algorithm implements the perfect inverse. With each update, the base station signals the mobile with specific transmit power adjustment to equalize the mobile's received power to $P^*$. Rapidly changing channel conditions and power estimate error cause the actual received power to vary about $P^*$.
- For simplicity of analysis, we focus on a single user and assume that the interfering users are seen at the receiver with perfect power control, i.e., that their received power is fixed at $P^*$. These interfering users still contribute to the estimate error of the user of interest.
- The algorithm is assumed to track distance loss and shadowing perfectly, since their effect is typically much slower than the update rate. The channel model, therefore, does not include these losses. If these quantities can be modeled mathematically, they are easily incorporated into the model.
- The signal is assumed to undergo flat Rayleigh fading. The form of the Doppler spectrum is

$$S(f) = 1/\left(2\pi f_d\sqrt{1-f^2/f_d^2}\right), \tag{1}$$

so that the underlying Gaussian processes have normalized correlation functions given by $J_0(2\pi f_d \tau)$, where $J_0$ is the Bessel function of order 0, $f_d$ is the maximum Doppler frequency, and $\tau$ is the time delay between the specified correlated samples.

# 4 Log-Linear Model Analysis

The linear system model shown in Figure 1 contains a sampling waveform. For simplicity of results, we consider a modified linear system model, shown in Figure 2, that captures the essence of this sampled system. Note that the sampling has been removed, and consequently, so has the zero-order hold. Because the mobile has better knowledge of its received power, this approximation is likely to yield a system that performs better than the actual sampled system.

By assuming this simple linear model, we can use superposition to solve the system. To proceed, we find the transfer function for each of the inputs, $A_n$, $P_n^*$, and $N_n$. For example, setting $P_n^*$ and $N_n$ to 0, we can solve for the transfer function $H_{RA}(\Omega)$. Assuming a deterministic signal for $A_n$, and denoting the discrete Fourier transform of

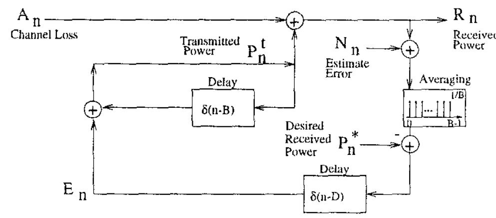


Figure 2: Simplified Log-Linear Power Control Model

$A_n$ and $R_n$ by $A(\Omega)$ and $R(\Omega)$, respectively, we can write

$$\begin{aligned} R(\Omega) &= P^t(\Omega) - A(\Omega) \\ P^t(\Omega) &= P^t(\Omega)e^{-j\Omega B} - E(\Omega) \\ E(\Omega) &= H_f(\Omega)R(\Omega)e^{-j\Omega D} \end{aligned}$$

where $H_f(\Omega)$ is the Fourier transfer function of the $B$-bit averaging filter. Solving for $R(\Omega)$ in terms of $A(\Omega)$ yields

$$H_{RA}(\Omega) = \frac{(1-e^{-j\Omega B})}{1-e^{-j\Omega B}+H_f(\Omega)e^{-j\Omega D}}, \tag{2}$$

where

$$H_f(\Omega) = \frac{1}{B}\frac{\sin(\Omega B/2)}{\sin(\Omega/2)}e^{-j\Omega\frac{B-1}{2}}. \tag{3}$$

Similarly, we can find $H_{RN}(\Omega)$ and $H_{RP^*}(\Omega)$. respectively.

We are interested in the correlation of $R_n$, the power at the output of the system. As a measure of received power correlation, we consider the auto-covariance function of the sequence $R_n$.

This system is modeled as linear. We can use superposition and represent the sequence $R_n$ as a sum of three sequences due to the three inputs $P_n^*$, $A_n$, and $N_n$, which we shall denote as $R_n^P$, $R_n^A$, and $R_n^N$ respectively. The sequence $P_n^*$ is a constant, and thus the sequence $R^P$ is deterministic and independent of $R^A$ and $R^N$. If $P_n^*$ varies, possibly to control the inter-cellular interference, the correlation of $P_n^*$ must be appropriately modeled. The sequences $R^A$ and $R^N$ are not independent. The power of the noise interference term to be presented in Section 6 has been shown to depend on the received power of the user of interest. This is clearly a function of $A_n$ and also, indirectly a function of $N_i, i \leq n$. A simple approximation that removes this correlation is presented in Sec. 6.

Based on the assumed independence of $R^A$, $R^N$ and $R^P$, the autocovariance of $R$ is the sum of the autocovariances of $R^A$, $R^N$, and $R^P$. If we can find the correlation functions, and consequently the power spectra, of the "inputs" to the linear system, we can find an expression for the received power correlation at the base station. Since, the auto-covariance function of $P_n^*$ is equal to zero, the power spectrum of $R$ reduces to

$$S_R(\Omega) = |H_{RA}(\Omega)|^2 S_A(\Omega) + |H_{RN}(\Omega)|^2 S_N(\Omega) \tag{4}$$

where $S_A(\Omega)$, $S_N(\Omega)$, and $S_P(\Omega)$ are the power spectra of the sequences $A_n$, $N_n$, and $P_n^*$, respectively. A derivation of the auto-covariance functions of $A_n$ and $N_n$ will follow in the next two sections.

# 5 The Autocorrelation of $A_n$

We assume, from [8], that the flat-fading amplitude follows a Rayleigh distribution. However, we need to consider the power (in dB) of the input process. This form of the input process is needed to force linearity of the system model.

Following the traditional analysis, we assume that the fading process is generated from two independent Gaussian processes, $X_n$ and $Y_n$. The power in the fading process, $Z$, is exponentially distributed or chi-squared distributed with 2 degrees of freedom. The $\nu^{th}$ moment is [9] $(2\sigma^2)^\nu\Gamma(1+\nu)$.

We are interested in determining the statistics of

$$A_n = 10\log_{10} Z_n$$

To find the correlation between $A_i$ and $A_j$ requires significant effort, but is central to finding the system received power correlation. To begin with, note a relationship between the moments of $Z$ and the moment generating function of $A$.

$$M_{A_i,A_j}(t_i,t_j) = E[Z_i^{\frac{10}{\ln 10}t_i} Z_j^{\frac{10}{\ln 10}t_j}]. \tag{5}$$

Using a result from [9], we know that the product moments of correlated Raleigh random variables, $a_i, a_j$, generated from a Gaussian process, with inverse correlation matrix $W$, is

$$\begin{aligned} E[a_i^\mu a_j^\nu] &= \frac{2^{\frac{1}{2}(\mu+\nu)}|W|^{\frac{1}{2}n}}{w_{11}^{\frac{1}{2}(\mu+n)} w_{22}^{\frac{1}{2}(\nu+n)}} \frac{\Gamma(\frac{1}{2}(n+\mu))\Gamma(\frac{1}{2}(n+\nu))}{\Gamma^2(\frac{1}{2}n)} \\ &\times \; {}_2F_1\left(\frac{1}{2}(n+\mu), \frac{1}{2}(n+\nu), \frac{1}{2}n, \frac{w_{12}^2}{w_{11}w_{22}}\right) \end{aligned} \tag{6}$$

where ${}_2F_1$ is the hypergeometric function and $w_{ij}$ is an element of the matrix $W$.

In our model, $Z_n = a_n^2$ and $n = 2$ degrees of freedom. The underlying Gaussians, $X$ and $Y$, have identical variances $\sigma^2$ and correlation coefficient $\rho_{i,j}$. The correlation coefficient $\rho_{i,j}$ is a function of the Doppler spectrum of the radio channel. Given this, (6) reduces to

$$\begin{aligned} E[Z_i^\mu Z_j^\nu] &= 2^{(\mu+\nu)} \left((1-\rho_{i,j}^2)\sigma^2\right)^\mu \left((1-\rho_{i,j}^2)\sigma^2\right)^\nu \\ &\times \; \Gamma(1+\mu)\Gamma(1+\nu){}_2F_1\left(1+\mu, 1+\nu, 1, \rho_{i,j}^2\right). \end{aligned} \tag{7}$$

To find the correlation $E[A_i, A_j]$, we need only the first partial derivative of (7), namely

$$\begin{aligned} E[A_iA_j] &= \left.\frac{\partial}{\partial t_i \partial t_j} M_{A_i,A_j}(t_i,t_j)\right|_{t_i=0,t_j=0} \\ &= \left.\frac{100}{\ln^2 10}\frac{\partial}{\partial\mu\partial\nu} E[Z_i^\mu Z_j^\nu]\right|_{\mu=0,\nu=0} \end{aligned}$$

Through extensive manipulations and simplifications,

$$E[A_iA_j] = \frac{100}{\ln^2 10}\left(\rho_{i,j}^2\Phi(\rho_{i,j}^2, 2, 1) + \left(\ln 2\sigma^2 - \boldsymbol{C}\right)^2\right) \tag{8}$$

where $\boldsymbol{C}$ is the Euler Gamma constant and is approximately equal to 0.577216 and $\Phi(z,s,v)$ is defined in [3] (p. 1103) as

$$z\Phi(z,2,1) = \sum_{n=1}^{\infty}\frac{z^n}{n^2} \tag{9}$$

$\Phi(z,s,v)$ should not be confused with the Gaussian probability integral. This completes the derivation of the covariance of the logarithm of the fading power $A_N$ for an arbitrary correlation $\rho_{i,j}$.

For the standard Doppler spectrum ([8]), the correlation coefficient for the process $X$ or $Y$ can be expressed as

$$\rho_{i,j} \triangleq \frac{E[X_i, X_j]}{\sigma_x\sigma_y} = J_o(2\pi f_d(i-j)T), \tag{10}$$

where $f_d$ is the maximum Doppler frequency and $T$ is bit duration. The resulting expression for $Cov[A_i, A_j]$ becomes

$$\begin{aligned} Cov[A_iA_j] &= \frac{100}{\ln^2 10} J_o(2\pi f_d(i-j)T)^2 \\ &\times \; \left(\Phi\left(J_o(2\pi f_d(i-j)T)^2, 2, 1\right)\right). \end{aligned} \tag{11}$$

# 6 The Autocorrelation of $N_n$

To characterize the noise input to the system, $N_n$, we must formulate an expression for the error in the power estimate (in dB). The square of the test statistic will be used to estimate the signal power. Consider the test statistic, without loss of generality, for user zero. The test statistic for the $i^{th}$ bit can be written as, defining the contribution of the interfering users and the thermal noise on the test statistic as $I(iT_b)$ and $N(iT_b)$, respectively,

$$g_0(iT_b) = T_b d_0^i\sqrt{2P_0} + I(iT_b) + N(iT_b)$$

where $T_b$ is the bit duration. The parameters $P_i$ and $d_i$ are the $i^{th}$ user's received power and data sequence. Consider as an estimate for the received signal power the square of $g_0(iT_b)$ appropriately normalized, namely

$$\begin{aligned} \hat{P}_0 &\triangleq g_0(iT_b)^2/(2T_b^2) \\ &= P_0 + \left(I(iT_b) + N(iT_b)\right)\sqrt{2P_0}/T_b \\ &+ \left(I(iT_b) + N(iT_b)\right)^2/(2T_b^2). \end{aligned} \tag{12}$$

Since we are interested in the error in the received power estimate in decibels, we take the ten times the logarithm of (12). This yields an equation which is difficult to simplify due to a sum term in the logarithm.

Consider the following bounds:

$$\log P_0 + \frac{x}{x+P_0} \leq \log(P_0+x)\log P_0 + \frac{x}{P_0}.$$

If $P_0$ is large compared to $x$, both curves are close to the actual values, and either curve could be used as a good approximation. As $x$ grows larger and approaches the magnitude of $P_0$, the bounds grow looser. To investigate this, we examined $10\log_{10}(x+y)$ and the above bounds for several values of $x$ with $y$ varying in each case from 0 to $x$. Results indicate the bounds are tight if $y \leq x/2$. As $y$ grows larger, the bounds grow loose. We expect the signal power $P_0$ to be significantly larger than the interference terms $x$ and therefore, we assume both bounds form close approximations and use only the upper bound here as an approximation. A closer approximation can be obtained by considering a longer expansion for the log function. We do not address the issue of lower bounds or tighter approximations.

From applying this approximation to (12) and expanding surfaces an appropriate definition for the $N_i$ process in the log-linear model. With some simplification, we can formally write

$$
\begin{aligned}
N_i &\triangleq 10/(\ln 10 P_0 T_b^2)\left(T_b\sqrt{2P_0}\,(I(iT_b)+N(iT_b))\right. \\
&+ \left.(I(iT_b)^2+N(iT_b)^2)/2+I(iT_b)N(iT_b)\right) \quad (13)
\end{aligned}
$$

Conditioned on the value of $P_0$, since received powers of interfering users will be almost the same, the Gaussian approximation is valid for the interference term. Note that even conditioned on $P_0$, $N_i$ is not a Gaussian random variable. The Gaussian assumption on $I(iT_b)$ is required to write the centralized fourth moment of $I(iT_b)$ as three times its variance. Inserting the variances, $\sigma_I^2$ and $\sigma_N^2$, and simplifying, we obtain the conditional mean and variance, $E[N_i|P_0]$ and $Var[N_i|P_0]$.

To facilitate a tractable analysis, we assume that the random term $P_0$ above is replaced by its intended quantity $P^*$. This simplifies the expressions to

$$
E[N_i] = \frac{10}{\ln 10}\left(\frac{N_0}{2E_b^*}+\frac{1}{3G}\sum_{l=1}^{S-1}\frac{P_l}{P^*}\right) \quad (14)
$$

$$
\begin{aligned}
Var[N_i] &= \frac{100}{\ln^2 10}\left(2\left(\frac{N_0}{2E_b^*}+\frac{1}{3G}\sum_{l=1}^{S-1}\frac{P_l}{P^*}\right)^2\right. \\
&+ \left.4\left(\frac{N_0}{2E_b^*}+\frac{1}{3G}\sum_{l=1}^{S-1}\frac{P_l}{P^*}\right)\right). \quad (15)
\end{aligned}
$$

where $G$ is the processing gain, $N_0$ is the unilateral received noise power, and $E_b$ is the energy per bit.

# 7 Simulation

The closed-loop power control scheme has been simulated using a set of CDMA simulation tools developed in C language. Central to the closed-loop power control simulation is the generation of the correlated Rayleigh fading process whose Doppler spectrum is as defined in [1]. We have adopted the simulation approach proposed by Jakes [7]. In principle, this method uses the sum of a number $N_0$ of randomly phased and properly weighted sinusoids to generate the Rayleigh samples with the desired Doppler spectrum. The number of sine wave oscillators, $N_0$, used in the simulations is 8. The power control loop shown in Figure 1 is simulated using the inverse algorithm. The simulation assumes perfect carrier phase and time synchronization at the receiver. For the single user case, the desired threshold $P^*$ is set to equal to the SNR value. No limit is set to the dynamic range of the power control algorithm. The time delay between the received power measurement instant at the base station receiver and the corresponding power control update at the mobile transmitter is incorporated in the simulation to a resolution of 1 bit time. The power control loop dynamics have been simulated and the statistics of the received power at different parameter settings, including Doppler frequency $f_d$, update interval $B$ and delay $D$, are evaluated.

# 8 Results

The analysis results are compared to a simulation of a realistic power controlled CDMA system described in Section

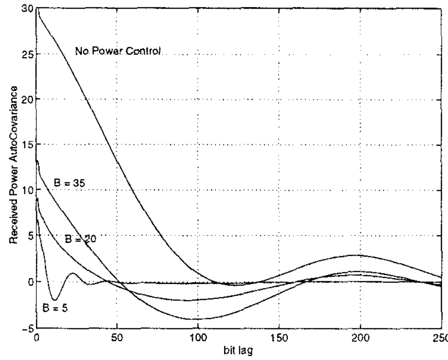


Figure 3: Effect of averaging interval, $B$, on the received power auto-covariance for $f_d = 25$, $d = 5$.

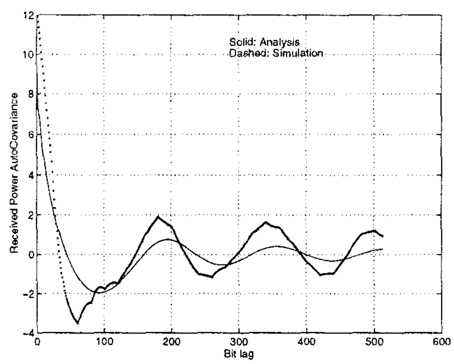


Figure 4: Comparison of analysis and simulation for $f_d = 25$, $B = 20$, $SNR = 10dB$, $d = 5$, and $T_b = 1/8000$.

7. A complete description of the simulation tool is presented in [1].

The effect of the averaging interval, $B$, on the received power correlation is shown in Figure 3 as predicted by the analytical results. For reference, the received power auto-covariance is shown for the flat-Rayleigh fading channel without power control. As the power is updated less frequently, the correlation in received power increases in magnitude and duration. In the limit as $B$ grows large, the received signal power approaches that of the flat Rayleigh fading channel. As power control update can require significant bandwidth, it is desired to keep $B$ as large as possible without sacrificing performance. These correlation curves quantify the increase in correlation as well as the increase in variance due to lengthening the update interval. Even in this ideal power control model, for reasonable $B$, the closed-loop power control does not remove all of the 'burstiness' or received power correlation present in the channel. The extent that it is reduced is quantified by these results.

Figure 4 compares the auto-covariance as predicted by the analysis and simulation. The curves, although similar in shape, differ in the magnitudes of their correlations. By examining the simulated correlation function, we can see the effect of averaging over $B$ bits and using a zero order hold at the mobile. This effect clearly manifests itself in the results as a non-smooth shaping of the correlation

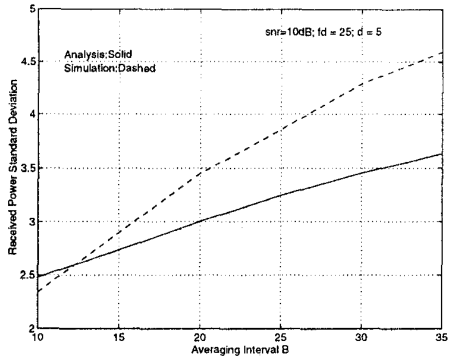


Figure 5: Comparison of Received Power standard deviation as predicted by analysis and simulation for $f_d = 25$, $SNR = 10dB$, $d = 5$, and $T_b = 1/8000$.

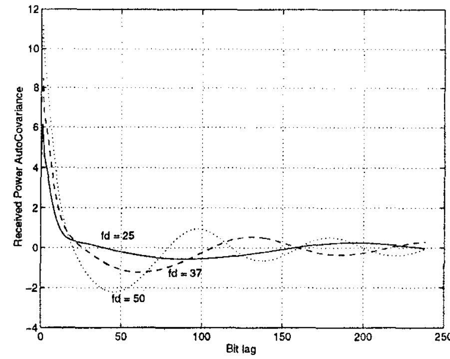


Figure 6: The effect on vehicle speed ($f_d$) on the received power auto-covariance.

function (Fig. 4 at around 100 bits of lag). Because the linear model presented here removes this sample-and-hold, the analysis curves fail to reflect this phenomenon. It is presumed that this modeling assumption not only effects the shape, but also has an impact on the magnitude of the correlation function. The variance of the received power is shown in Figure 5. This graph shown the increasing error in the magnitude of the correlation function derived from the analysis. Over this entire range, however, the shape of the correlation function, and thus, the duration of the time correlation is accurately predicted by the analysis. From this, we would expect that the bursty nature of the power-control channel is captured by this simple model.

Figure 6 shows the effect of increasing the Doppler frequency, $f_d$, (vehicle speed) on the covariance function. As the Doppler frequency increases, both the magnitude and duration of the correlation increase. In a Rayleigh fading channel with no power control, an increase in the Doppler frequency or vehicle speed decreases the correlation duration in the received power (but not the magnitude (11)). However, in the power controlled channel, increased vehicle speed will first cause an increase in the channel correlation. This is due to poor tracking of the received power by the $B$-bit averaging filter. As $f_d$ increases until $B$ is far too large to provide any tracking of the received power, the correlation begins to resemble the correlation of a Rayleigh fading channel. Again, although the correlation function shape is accurately predicted by the analysis, the error in the magnitude of the correlation increases for increasing $f_d$.

# 9 Conclusions

We have derived a linear model for a closed loop power control system for CDMA cellular communications. Analysis of the model indicates that while it accurately predicts correlation times, it significantly underestimates the magnitude of the correlation (including the received power variance).

This is primarily due to two of the modeling assumptions. To form a linear time-invariant system, we remove the sample-and-hold portion of the system model ( there was no justification for this except to form a tractable analysis). Removing this portion in the model will produce optimistic results, because the mobile has better knowledge of the fading process than with the realistic model. Secondly, we make an assumption in the derivation of the noise input to the linear system. In the noise model, we replace the term $P_0$, the received power in the signal, by its desired value $P_0^*$. This approximation will also tend to yield optimistic results. Removing the variation in this term removes some of the variance of the noise process. The variance of the noise process $N_i$ should grow in proportion to the variance of the received power. However, due to this simplification, it does not.

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