Capacity of DS-CDMA Networks on Frequency Selective Fading Channels with Open-Loop Power Control*

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Abstract

In this paper, we present the coded bit error performance for the reverse link of a direct sequence code division multiple access (DS-CDMA) network employing an open-loop power control scheme over a frequency selective Rayleigh fading channel. A quasi-analytical approach has been adopted, wherein the uncoded bit error performance is evaluated through simulation and the coded performance is arrived at through analytical bounds. A rate-1/3 convolutional code of constraint length 9, with hard decision Viterbi decoding is considered. The system capacity degradation due to open-loop power control error, which is approximated by a log-normally distributed random variable, is estimated. It is found that, for typical voice applications, the capacity degradation compared to perfect power control remains less than 3% as long as the standard deviation of the power control error (σ_δ) is less than 1 dB, and increases to 17% when σ_δ is 2 dB.

1 Introduction

A lot of recent research has been focussed on the application of direct sequence code division multiple access (DS-CDMA) to improve capacity of cellular mobile radio communications [1]-[2]. Apart from their application in the commercial arena, high capacity CDMA systems have been proposed for military communications as well, owing to the increasing amount of information (including voice, data, video) to be communicated securely in battlefield environments [3]. In a typical cellular CDMA network, the mobile-to-base station link (reverse link) uses asynchronous CDMA access, and the base-to-mobile link (forward link) employs synchronous CDMA access. The reverse and the forward links use different frequency bands. On the reverse link, multiple mobile users in a cell can access the base station simultaneously, over the same radio bandwidth, each using a different spreading code. At the base station, correlation receivers tuned to various codes are used to demodulate each individual user's data.

Many results have been published on the capacity of asynchronous DS-CDMA systems over both frequency nonselective and frequency selective Rayleigh/Rician fading channels [4]-[6]. In [7] and [8], the capacity of DS-CDMA systems employing both coherent (BPSK) and non-coherent (DPSK) modulation formats with a RAKE receiver, and operating over Nakagami fading channels, is considered. However, all the above works assume *perfect power control*, i.e., all the users' transmissions arrive with the same power at the base station receiver. In a practical mobile radio environment, this assumption is not true. In particular, an adaptive power control (APC) scheme is always essential to compensate for the shadowing, distance losses, and fading effects. Such a scheme attempts to maintain a constant average performance among the users, and reduce the multiple-access interference effect. Closed-loop APC, though very effective at low vehicle speeds, may find it difficult to track the relatively fast variations associated with rapid channel fading, resulting from increased mobile velocity, increased carrier frequency, or a combination of both [2]. An alternate, and simpler, form of power control is an open-loop APC, in which the mobile estimates the channel state on the forward link and and uses this to derive an estimate of the channel state on the reverse link [9]-[10]. The open-loop APC, in compensating for the large scale variations on the channel, which are the same on both links (such as shadowing), attempts to minimize the effect of the multipath fading component by averaging it out. This results in a randomly varying power control error (PCE) that causes performance degradation. Evaluation of the system capacity degradation due to open-loop PCE is the main focus of this paper. The statistics of the PCE of an open-loop APC scheme over flat fading and frequency selective Rayleigh fading channels have been studied in [9] and [10], respectively. It has been shown that the distribution of the PCE can be well-approximated by a *log-normal* random variable, and that the standard de-

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viation of the PCE (σ_δ) typically varies in the range 1 - 4 dB, depending on the measurement time used in the open-loop power control algorithm, and the Doppler frequency.

We consider the capacity of a DS-CDMA network over frequency selective Rayleigh fading channels, in the presence of an open-loop power control scheme. Using a quasi-analytic approach, we estimate the coded bit error performance of a DS-CDMA system employing coherent RAKE reception. The system capacity degradation as a function of the standard deviation of the PCE (σ_δ in dB) is estimated. Section 2 describes the system model, including the transmitter, channel, and the receiver. In Section 3, the simulation model for estimating the uncoded bit error performance of the system, the capacity estimation based on an analytical upper bound to the coded bit error performance, and the capacity degradation as a function of the standard deviation of PCE, are discussed. Section 4 highlights the conclusions and the areas of further study.

2 System Model

We consider a DS-CDMA system consisting of $(J+1)$ simultaneously transmitting mobile users, J being the number of interfering users. Each user is assigned a unique CDMA code sequence, and the code sequences have a common chip rate of $\frac{1}{T_c}$, where $T_c = \frac{T_b}{N_c}$. T_b and T_c are the bit and chip durations, repectively, and N_c is the number of chips/bit (i.e., the processing gain). Let $c_k(t)$ denote the code sequence waveform of the k^{th} user, and let $\{c_i{}^{(k)}\}$ be the corresponding sequence of elements of {+1, -1}. Then

$$c_k(t) = \sum_{i=-\infty}^{\infty} c_j{}^{(k)} P_c(t - iT_c),$$

where

$$P_a(t) \triangleq \begin{cases} 1 & 0 < t < T_a \\ 0 & \text{otherwise.} \end{cases}$$

Similarly, the data waveform can be written as

$$b_k(t) = \sum_{i=-\infty}^{\infty} b_i{}^{(k)} P_b(t - iT_b).$$

It follows that the transmitted signal for the k^{th} user is given by

$$s_k(t) = \mathrm{Re}[A\lambda_k b_k(t) c_k(t) e^{j(\omega_o t + \theta_k)}],$$

where $A = \sqrt{\frac{2E_b}{T_b}}$ is common to all users, ω_o is the common carrier frequency, θ_k is the carrier phase of the k^{th} user, and λ_k is the power control error which is a random variable due to imperfect open-loop power control. We consider λ_k to be log-normally distributed with standard deviation σ_δ dB. In otherwords, $\lambda_k = 10^{(\frac{x}{20})}$, where the variable x follows a normal distribution.

Assuming asynchronous operation, the signals from all the users (other than the user of interest) are misaligned with respect to the signal from the user of interest by an amount τ_k, $k = 1, 2, .., J$, such that τ_k is uniformly distributed in $[0, T_b)$. Thus, the composite signal at the input to the channel is given by

$$S(t) = \mathrm{Re}[\sum_{k=0}^{J} A\lambda_k b_k(t - \tau_k) c_k(t - \tau_k) e^{j(\omega_o t + \phi_k)}], \qquad (1)$$

where $\phi_k = \theta_k - \omega_o \tau_k$, and $\theta_0 = \tau_0 = 0$. Note that θ_0 and τ_0 are the carrier phase and the time delay, respectively, of the user of interest. Further, $\{\phi_k\}$, $k = 1, 2, .., J$, are independent identically distributed random variables uniformly distributed in $[0, 2\pi)$.

Frequency Selective Channel

We consider a frequency selective Rayleigh fading channel with an exponential decaying multipath intensity profile (MIP). The time-variant frequency selective channel is modelled as a tapped delay line with tap spacing T_c, and tap coefficients $\{z_i(t)\}$, which are zero-mean, complex-valued, stationary, mutually independent Gaussian random processes. Thus, the complex low pass equivalent channel impulse response is given by

$$h(\Delta; t) = \sum_{i=0}^{L_p - 1} z_i(t) \delta(\Delta - iT_c),$$

where L_p is the number of resolvable paths, each spaced T_c apart. Figure 1 shows the channel model for the k^{th} user. Using the standard wide sense stationary uncorrelated scattering (WSSUS) assumption implies that the $\{z_i(t)\}$ are mutually uncorrelated and thus statistically independent. We can write $z_i(t) = \alpha_i(t) e^{j\psi_i(t)}$, where $\{\alpha_i(t)\}$ are Rayleigh distributed and the phases $\{\psi_i(t)\}$ are uniformly distributed in $[0, 2\pi)$. The average path strength Ω_i is the second moment of α_i (i.e., $\Omega_i = \mathrm{E}[\alpha_i^2]$), and is assumed to be related to the second moment of the initial path strength by

$$\Omega_i = \Omega_0 e^{-di}, \qquad d \geq 0. \qquad (2)$$

Equation (2) describes the decay of the average path strength as a function of path delay; the parameter d reflects the rate at which this decay occurs. Actual measurements indicate that the above exponential decaying MIP assumption is fairly accurate for congested urban areas [11]. If the multipath spread is T_m, then the number of resolvable paths is $L_p = \lfloor \frac{T_m}{T_c} \rfloor + 1$, and L_p is assumed to be less than N (equivalent to asssuming $T_m < T_b$). Finally, in Fig. 1, $n_w(t)$ is the complex valued low pass equivalent AWGN with two-sided power spectral density η_o.

RAKE Receiver

We adopt the coherent RAKE receiver model used in [7]. Figure 2 illustrates the RAKE receiver for the user of interest, with L_r independent fading paths coherently combined. The correlator is matched to the CDMA code of the user of

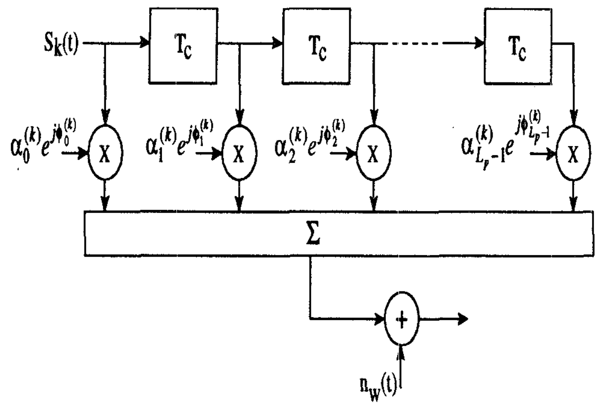

Figure 1: Frequency selective channel model

interest, and the correlator is assumed to have achieved time synchronization with the initial path. The tap weights and phases are assumed to be perfect estimates, which in practice can be estimated through dedicated circuits [12]. For the receiver considered here, to ensure combined multipath demodulation, the sampling instants are $nT_b + (L_r - 1)T_c$, where $n = 0, 1, 2, 3, \ldots$ During the sampling instants, the threshold detector outputs +1 if the corresponding input is greater than 0, and outputs -1 otherwise.

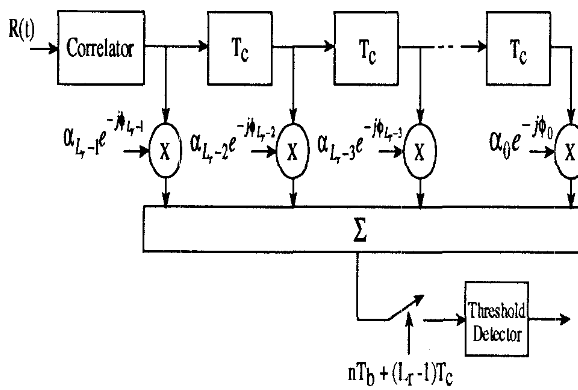

Figure 2: Rake receiver model

3 Capacity Estimation

We take a quasi-analytic approach [13] to estimate the bit error performance, and thus the capacity on the reverse link of the DS-CDMA system model described above. We first estimate the uncoded bit error performance of the system at different system parameter settings through large scale simulations. The occurrence of bit errors in such simulation experiments would be *bursty* due to sudden and deep fades appearing on the channel. In practice, the bursty nature of the errors due to the memory on the channel can be manipulated to appear as independent *random errors* by interleaving the coded data over sufficient depth before transmission, and deinterleaving the data before decoding at the receiver. We assume *perfect interleaving*, and evaluate an upper bound on the coded bit error performance of the system using convolutional codes with hard decision Viterbi decoding. From the coded bit error performance, we then estimate the system capacity, which is defined as the number of simultaneous users that can be supported while maintaining an acceptable BER performance needed by the specific application (e.g., 10^{-3} for voice).

Uncoded BER performance

The reverse link of the DS-CDMA system has been simulated to estimate the uncoded bit error performance of the system. A set of CDMA simulation tools developed in *C* language has been used to synthesize the simulation program. Random binary sequences of length 127 are used as the spreading codes for different users. All the users transmit asynchronously with different time delays τ_k with respect to the user of interest, such that τ_k is chosen randomly in the set $\{0, T_s, 2T_s, \ldots, (N_c K - 1)T_s\}$, where T_s is the sampling interval, and K is the number of samples per chip. A sampling rate corresponding to 4 samples per chip ($K = 4$) is employed. System parameters such as the number of simultaneous users (J), the number of independent fading paths (L_p), the multipath intensity profile exponent (d), the number of paths combined at the RAKE receiver (L_r), the signal-to-noise ratio per bit, and the standard deviation of the power control error (σ_δ), can be varied in the simulation program to study their effect on the system performance.

Figure 3 shows the simulated bit error perofrmance of the system as a function of average E_b/η_o, over a frequency selective Rayleigh fading channel ($L_p = 3$), using a RAKE receiver with maximal ratio combining of all the resolvable paths ($L_p = L_r$). A MIP exponent value of $d = 0.2$ is considered. The BER curves for a 30 user system at different values of σ_δ (0 - 4 dB) are plotted. It is seen that, when $\sigma_\delta = 0$ dB (i.e., perfect power control), a BER of 10^{-2} is achieved at an average E_b/η_o of 16 dB. The simulation results obtained for the perfect power control case are found to closely match the analytical results derived by Eng in [7]. It is noted that the bit error performance degrades with increased σ_δ values, as expected. It is interesting to note that the degradation in performance is marginal when $\sigma_\delta = 1$ dB. However, for $\sigma_\delta > 2$ dB, significant degradation is observed. A target BER of 10^{-2} can be achieved asymptotically when $\sigma_\delta = 2$ dB, whereas such performance is unachievable if the σ_δ value increases beyond 3 dB.

Coded BER performance

For convolutional codes with hard decision Viterbi decoding, the BER transfer characteristic can be upper-bounded

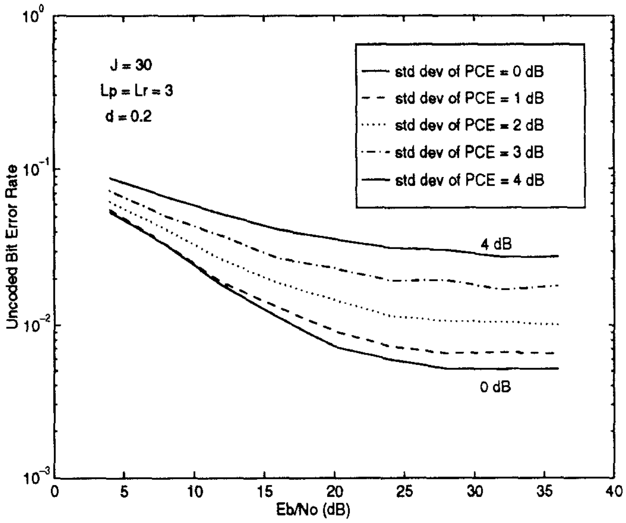

Figure 3: Uncoded BER vs E_b/η_o for different vaules of PCE standard deviation (σ_δ); $J = 30$; $L_p = L_r = 3$; $d = 0.2$.

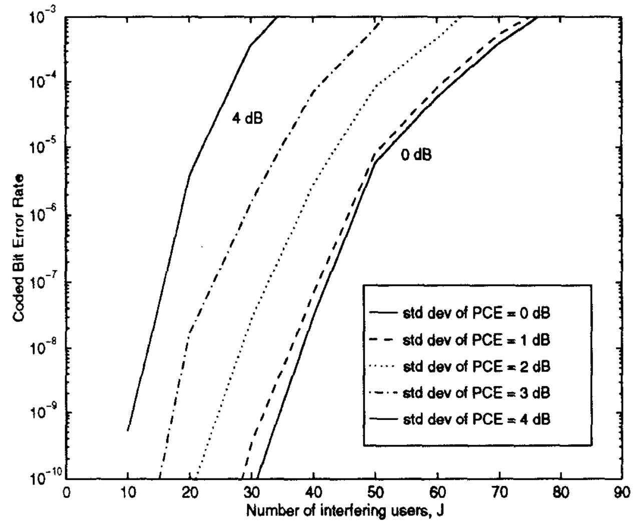

Figure 4: Upper bound on the coded BER vs number of interfering users (J) for different vaules of PCE standard deviation (σ_δ); No AWGN; $L_p = L_r = 3$; $d = 0.2$.

by the well known transfer function bound [12]

$$p_o < \sum_{x=d_f}^{\infty} \beta_x P(x), \tag{3}$$

where d_f is the free distance of the code, and $\{\beta_x\}$ are the coefficients in the expansion of the derivative of $T(D, N)$, the transfer function (or generating function) of the code evaluated at $N = 1$ [13]. $P(x)$ is the probabilty of selecting an incorrect path, which can be bounded by the expression

$$P(x) < [4p(1-p)]^{d/2},$$

where p is the uncoded BER.

We consider the use of a rate-1/3 convolutional code of constraint length 9 on the reverse link. The $\{\beta_x\}$ coefficients for the corresponding code are taken from [13]. The upper bound on the coded BER performance of the system as a function of the number of interfering users (J), when $L_p = L_r = 3$, $d = 0.2$, and $E_b/\eta_o = \infty$ (i.e., no AWGN) is plotted in Figure 4. The curves are parameterized by different σ_δ values. The system capacity values as a function of σ_δ in dB for different bit error rates (10^{-3} for voice, and 10^{-6} or 10^{-10} for data) are tabulated in Table 1. It is found that with no power control error, 77 and 47 simultaneous voice and data circuits, respectively, can be supported by the system considered. These capacity figures degrade by less than 3% when the standard deviation of PCE is 1 dB, and by 17% - 19% when σ_δ is 2 dB. The capacity degradation is very high when σ_δ increases beyond 2 dB (33% for 3 dB and 56% for 4 dB for voice). In this study, we have considered just a single cell system, whereas in practical cellular systems, the effect of the dynamic adjacent cell power control variations must be considered in the system capacity estimation. Thus, our current estimates of capacity are optimistic.

Figure 5 shows the effect of having a larger number of resolvable paths and coherently combining all of them at the RAKE receiver. The graph shows the upper bound on the coded BER *vs* L_p (= L_r) as a function of σ_δ for a 50 user system when $d = 0.2$. The figure illustrates the potential improvement in performance that can be achieved due to increased frequency selectivity of the channel (i.e, when the number of resolvable paths combined at the receiver is high), which is realized at the expense of increased receiver complexity and bandwidth.

4 Conclusions

We presented the coded bit error performance on the reverse link of a direct sequence code division multiple access network which employs an open-loop power control scheme over a frequency selective Rayleigh fading channel. A quasi-analytical approach was adopted, wherein the uncoded bit error performance was evaluated through simulation and the coded performance was arrived at through analytical bounds. The system capacity degradation due to open-loop power control error, which was approximated by a log-normally distributed random variable, was estimated. It was shown that, for typical voice applications, the capacity degradation compared to perfect power control remained less than 3% as long as the standard deviation of PCE (σ_δ) was less than 1 dB, and increased to 17% when σ_δ was 2 dB. The area that needs further study is the effect of adjacent cell power control.

BER	System Capacity, J (% degradation)				
	Standard deviation of PCE (σ_δ)				
	0 dB	1 dB	2 dB	3 dB	4 dB
10^{-3} Voice	77 (0%)	75 (2.6%)	64 (16.9%)	52 (32.5%)	34 (55.8%)
10^{-6} Data	47 (0%)	46 (2.1%)	38 (19.2%)	29 (38.3%)	18 (61.7%)
10^{-10}	31 (0%)	29 (6.5%)	21 (32.3%)	15 (51.2%)	< 10 (77.4%)

Table 1: CDMA system capacity with power control error for N = 127, $L_p = L_r = 3$, $d = 0.2$, No AWGN, rate-1/3 convolutional code (K=9) with perfect interleaving.

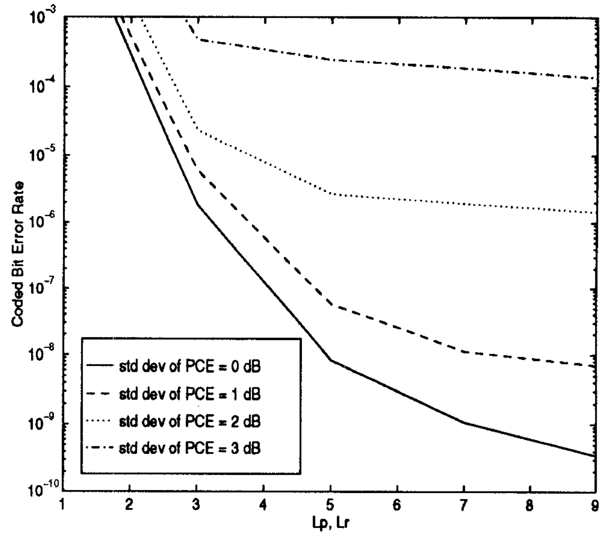

Figure 5: Upper bound on the coded BER vs L_p, L_r for different values of PCE standard deviation (σ_δ); d = 0.2, No AWGN; J = 50.

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