Performance of Media-Based Modulation in Two-Way Full-Duplex Relaying Networks

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***Abstract*—Media-based modulation (MBM) is a recently proposed modulation scheme, which conveys information using digitally controllable parasitic elements (a.k.a radio frequency mirrors) placed near the transmit antenna. In this paper, we investigate the performance of MBM in a *two-way relaying network* where two nodes exchange information with the help of a relay node. We consider full-duplex (FD) operation and MBM based transmission at all the nodes, and decode-and-forward protocol for relaying. We refer to the considered system as *two-way FD relaying with MBM* (TW-FDR-MBM) system. We introduce the TW-FDR-MBM system model and investigate its bit error performance. In particular, we carry out an average bit error probability (BEP) analysis of this system. Our performance results show that TW-FDR-MBM scheme achieves better performance compared to TW-FDR scheme that uses conventional modulation schemes like PSK/QAM.**

Keywords – Two-way relaying, media-based modulation, full-duplex, average bit error probability.

I. Introduction

Future wireless networks are expected to provide higher spectral efficiencies and coverage with lower power consumption compared to the current existing networks [1]. Full-duplex (FD) is a promising technique that can achieve double the spectral efficiency compared to half-duplex (HD) systems, by simultaneous transmission and reception on the same frequency [2]. However, the self-interference (SI) caused by the simultaneous transmission and reception is a main bottleneck in FD systems. Several cancellation techniques (classified as passive and active) have been proposed to alleviate the SI problem [3]. The residual SI after cancellation can be modeled as either Rayleigh or Rician random variable [4]. The performance of FD with the residual SI has been extensively studied in the literature [2]. Relaying is an appealing approach to improve the network coverage and throughput [5]. Decode-and-forward (DF) and amplify-and-forward (AF) protocols are widely studied relaying protocols. Various studies on relaying with FD operation have been summarized in [6].

Media-based modulation (MBM) is a potential modulation scheme that offers multiple-input multiple-output (MIMO) benefits by placing digitally controllable parasitic elements (called as radio frequency mirrors) around the transmit antenna [7]. An unit comprising of a transmit antenna and radio frequency (RF) mirrors is called as the 'MBM transmit unit (MBM-TU)'. Each RF mirror can be switched either to an ON state or to an OFF state by an information bit. A mirror allows the incident wave to pass through it when it is ON, and reflects back the incident wave when it is OFF. The ON/OFF status of all the mirrors is called as a 'mirror activation pattern (MAP)'. The number of possible MAPs is $2^{m_{rf}}$, where m_{rf} is the number of mirrors. Different MAPs result in different channel fade realizations. The collection of $2^{m_{rf}}$ channel fades corresponding to $2^{m_{rf}}$ possible MAPs (referred to as MBM channel alphabet) can convey m_{rf} information bits. Further, the transmit antenna transmits a conventional modulation symbol (e.g., PSK/QAM) to convey additional information bits. Studies on MBM have shown that it performs significantly better than conventional modulation schemes [7]-[11]. The performance of FD with MBM has been studied in a point-to-point scenario in [10]. The performance of MBM in a one-way FD relaying network has been investigated in [11].

In this paper, we investigate the bit error performance of MBM in a *two-way relaying network* with FD operation, which has not been reported before. In this system, two FD nodes exchange information with the help of a FD relay node, where every node uses MBM for transmission, and the relay node uses DF protocol for forwarding the received signal. We refer to this system as two-way FD relaying system with MBM (TW-FDR-MBM). Our contributions in this paper can be outlined as follows. First, we introduce the two-way FD relaying scheme using MBM. We then analyze the end-to-end average bit error probability (BEP) performance of TW-FDR-MBM system and compare it with that of a TW-FDR system which uses conventional modulation (CM) schemes like PSK/QAM. We refer to the TW-FDR system with PSK/QAM as TW-FDR-CM system. Our numerical results show that, for a given spectral efficiency, TW-FDR-MBM scheme performs better than TW-FDR-CM scheme, highlighting the performance benefit of using MBM in two-way relaying systems with FD operation.

The rest of this paper is organized as follows. The TW-FDR-MBM system model is introduced in Section II. The end-to-end average BEP analysis is presented in Section III. Performance results and discussions are presented in Section IV. Finally, conclusions are presented in Section V.

II. System model

We consider a two-way relaying network, where two FD nodes (denoted by node 1 and node 2) exchange information with the help of a FD relay node R as shown in Fig. 1. The transmitter at the node p, $p = 1, 2, R$ consisits of $n_{tu}^{(p)}$ MBM-TUs, $m_{rf}^{(p)}$ RF mirrors in each MBM-TU, and $n_{rf}^{(p)}$ transmit RF chains, $n_{rf}^{(p)} \leq n_{tu}^{(p)}$. The node p's receiver is equipped with $n_r^{(p)}$ receive antennas. We assume that the node 1 and node 2 are separated by a large distance such that they can

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We have used 'r' in the superscript and subscript notations to represent the relay node R.

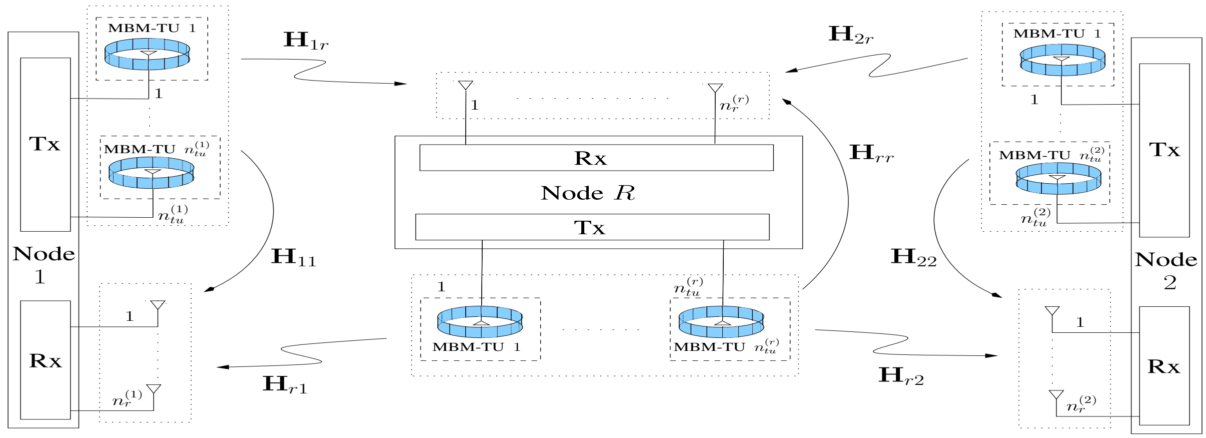

Fig. 1. Two-way full-duplex relaying with MBM.

communicate only through the relay, i.e., no direct link exists between them. We consider that all the nodes transmit using generalized spatial modulation MBM (GSM-MBM) scheme, and the relay node forwards using DF protocol. We refer to this system as two-way FD relaying with MBM (TW-FDR-MBM) system.

A. *Transmitter at node p*

The transmitter with GSM-MBM at node p is shown in Fig. 2. In every channel use, GSM-MBM conveys information as follows: $i)$ $\lfloor \log_2 \binom{n_{tu}^{(p)}}{n_{rf}^{(p)}} \rfloor$ bits are conveyed by selecting $n_{rf}^{(p)}$ out of $n_{tu}^{(p)}$ MBM-TUs; $ii)$ $n_{rf}^{(p)} \log_2 M^{(p)}$ bits are conveyed by transmitting $n_{rf}^{(p)}$ $M^{(p)}$-ary QAM/PSK symbols on the chosen $n_{rf}^{(p)}$ MBM-TUs (one symbol is transmitted on one selected MBM-TU); and $iii)$ $n_{rf}^{(p)} m_{rf}^{(p)}$ bits are conveyed by controlling (making ON/OFF) the $n_{rf}^{(p)} m_{rf}^{(p)}$ mirrors in the selected $n_{rf}^{(p)}$ MBM-TUs ($m_{rf}^{(p)}$ mirrors in each of the selected MBM-TUs). Therefore, spectral efficiency of node p (in bpcu) is given by

$$\eta^{(p)} = \underbrace{\left\lfloor \log_2 \binom{n_{tu}^{(p)}}{n_{rf}^{(p)}} \right\rfloor}_{\text{MBM-TU index bits}} + \underbrace{n_{rf}^{(p)} m_{rf}^{(p)}}_{\text{mirror index bits}} + \underbrace{n_{rf}^{(p)} \log_2 M^{(p)}}_{\text{QAM/PSK symbol bits}} \text{ bpcu.} \quad (1)$$

In a given channel use, every MBM-TU is made either ON or OFF such that only $n_{rf}^{(p)}$ out of $n_{tu}^{(p)}$ MBM-TUs are made ON. A realization (which is an $n_{tu}^{(p)} \times 1$ vector consists of 1's and 0's) of the ON/OFF status of the $n_{tu}^{(p)}$ MBM-TUs (1 and 0 correspond to the ON and OFF status of MBM-TU, respectively) is called as the 'MBM-TU activation pattern'. Let $\mathbb{S}_t^{(p)}$ denote the set of $2^{\lfloor \log_2 \binom{n_{tu}^{(p)}}{n_{rf}^{(p)}} \rfloor}$ MBM-TU activation patterns (only these many are required for signaling) chosen from the set of $\binom{n_{tu}^{(p)}}{n_{rf}^{(p)}}$ possible MBM-TU activation patterns. For instance, when $n_{tu}^{(p)} = 4$ and $n_{rf}^{(p)} = 2$, a possible MBM-TU activation patterns set $\mathbb{S}_t^{(p)}$ is given by

$$\mathbb{S}_t^{(p)} = \{[1\ 0\ 1\ 0]^T, [0\ 1\ 1\ 0]^T, [1\ 1\ 0\ 0]^T, [0\ 0\ 1\ 1]^T\}. \quad (2)$$

GSM-MBM specializes to other MBM schemes such as MIMO-MBM and SM-MBM, and non-MBM schemes such as GSM, SM, and MIMO [11].

Each of the $n_{rf}^{(p)}$ MBM-TUs that is made ON, transmits a symbol from the $M^{(p)}$-ary QAM/PSK alphabet $\mathbb{A}^{(p)}$. An MBM-TU in OFF state can be viewed as transmitting 0. Hence, the symbol transmitted on the jth MBM-TU, denoted by $x_j^{(p)}$, belongs to the set $\mathbb{A}^{(p)} \cup \{0\}$ such that $\|\mathbf{x}^{(p)}\|_0 = n_{rf}^{(p)}$ and $\mathcal{I}(\mathbf{x}^{(p)}) \in \mathbb{S}_t^{(p)}$, where $\mathbf{x}^{(p)}$ is the $n_{tu}^{(p)} \times 1$-sized vector with $x_j^{(p)}$ as its jth element, $\|\mathbf{x}^{(p)}\|_0$ denotes the L_0-norm of $\mathbf{x}^{(p)}$, and $\mathcal{I}(\mathbf{x}^{(p)})$ is a function (which maps $\mathbf{x}^{(p)}$ to a same sized vector whose jth element takes the value 1 when $x_j^{(p)} \neq 0$, and value 0 otherwise) that gives the MBM-TU activation pattern for $\mathbf{x}^{(p)}$. For instance, when $n_{tu}^{(p)} = 4$, $n_{rf}^{(p)} = 2$, $\mathbb{A}^{(p)}$ is BPSK, and $\mathbf{x}^{(p)} = [0\ 0\ {+1}\ {-1}]^T$, then $\mathcal{I}([0\ 0\ {+1}\ {-1}]^T) = [0\ 0\ 1\ 1]^T$. Let $\mathbb{S}_{\text{gsm}}^{(p)}$ be the set of all such $\mathbf{x}^{(p)}$ vectors, i.e., $\mathbb{S}_{\text{gsm}}^{(p)} = \{\mathbf{x}^{(p)} : x_j^{(p)} \in \mathbb{A}^{(p)} \cup \{0\}, \|\mathbf{x}^{(p)}\|_0 = n_{rf}^{(p)}, \mathcal{I}(\mathbf{x}^{(p)}) \in \mathbb{S}_t^{(p)}\}$.

In every MBM-TU, an RF mirror is made either ON or OFF. Similar to the MBM-TU activation pattern, a realization of the ON/OFF status of the $m_{rf}^{(p)}$ mirrors is called as the 'mirror activation pattern (MAP)'. Let $N_m^{(p)} = 2^{m_{rf}^{(p)}}$ denote the number of possible MAPs. The MAP index on jth MBM-TU, denoted by $l_j^{(p)}$, is chosen as follows: $i)$ $l_j^{(p)}$ takes an integer value in $[1, N_m^{(p)}]$, based on $m_{rf}^{(p)}$ information bits when jth MBM-TU is ON (i.e., $s_j^{(p)} \neq 0$); $ii)$ otherwise $l_j^{(p)}$ takes some fixed value (say 1) independent of information bits.

B. *Transmission protocol*

All the nodes (1, 2, and R) operate in FD mode and the relay node R uses DF protocol. We assume that all the nodes use the same average power denoted by E, i.e., uniform power allocation among the nodes. The information exchange between the two end nodes (i.e., node 1 and node 2) takes place in two phases.

In the first phase, both the end nodes transmit their information to the relay node R. Let $\mathbf{b}_1^{(1)}$ and $\mathbf{b}_1^{(2)}$ be the information bit vectors (of size $\eta^{(1)}$ and $\eta^{(2)}$, respectively) transmitted (using GSM-MBM) by the node 1 and node 2, respectively. The relay node R detects these bit vectors in the presence of its self-interference (SI, which is a signal broadcasted by the node R) that results from the relay node's FD operation.

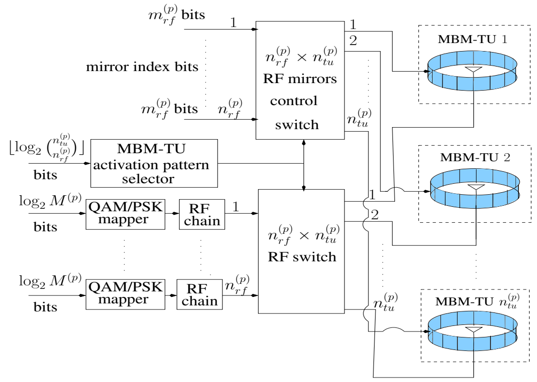

Fig. 2. Transmitter at node p, $p = 1, 2, R$.

Let $\hat{\mathbf{b}}_1^{(1,r)}$ and $\hat{\mathbf{b}}_1^{(2,r)}$ denote the estimates of $\mathbf{b}_1^{(1)}$ and $\mathbf{b}_1^{(2)}$, respectively, at the relay node R.

In the second phase, the relay node R forwards the detected data $\hat{\mathbf{b}}_1^{(1,r)}$ and $\hat{\mathbf{b}}_1^{(2,r)}$ to the node 1 and node 2 by transmitting the bit vector $\mathbf{b}_2^{(r)} = \hat{\mathbf{b}}_1^{(1,r)} \oplus \hat{\mathbf{b}}_1^{(2,r)}$ (using GSM-MBM), where $\oplus$ denotes the bit-wise XOR operation (we append the zeros to the smaller vector to make $\hat{\mathbf{b}}_1^{(1,r)}$ and $\hat{\mathbf{b}}_1^{(2,r)}$ equal sized vectors). The transmission parameters of the relay node are chosen such that the spectral efficiency $\eta^{(r)}$ is equal to the size of $\mathbf{b}_2^{(r)}$, i.e., $\eta^{(r)} = \max\{\eta^{(1)}, \eta^{(2)}\}$. The node p, $p = 1, 2$, detects the $\mathbf{b}_2^{(r)}$ in the presence of its SI (which is a new information transmitted to the relay node R). Let $\hat{\mathbf{b}}_2^{(r,p)}$ be the estimate of $\mathbf{b}_2^{(r)}$ at node p, $p = 1, 2$. Using $\mathbf{b}_1^{(p)}$ and $\hat{\mathbf{b}}_2^{(r,p)}$, the node p, $p = 1, 2$, estimates the information transmitted by node q, $q = 1, 2$, $q \neq p$, in first phase as $\hat{\mathbf{b}}_1^{(q)} = \mathbf{b}_1^{(p)} \oplus \hat{\mathbf{b}}_2^{(r,p)}$ (append zeros to the $\mathbf{b}_1^{(p)}$ vector before XOR operation to make them equal sized vectors and discard the last $(\eta^{(r)} - \eta^{(q)})$ bits after XOR operation). Therefore, the node 1 and node 2 exchange a total of $(\eta^{(1)} + \eta^{(2)})$ bits in each channel use.

C. Channel model

All the links (channels) are assumed to experience independent fading. Each link in the system can be either a desired link (which is a link between two nodes) or an undesired SI link (which is a link from a node to itself). We refer to the desired link from the node p to the node q as p to q-desired link, where (p, q) is an ordered pair belongs to the set $\{(1, R), (2, R), (R, 1), (R, 2)\}$. Similarly, we refer to the undesired SI link from the node p to itself as p to p-SI link, where p belongs to the set $\{1, 2, R\}$.

p to q-desired link: Let $h_{i,k}^{j,pq}$ be the complex channel fade at the ith receive antenna of the node q when the kth MAP is active on the jth MBM-TU of the node p, $i = 1, 2, \cdots, n_r^{(q)}$, $j = 1, 2, \cdots, n_{tu}^{(p)}$, and $k = 1, 2, \cdots, N_m^{(p)}$. The channel fades (i.e., $h_{i,k}^{j,pq}$s) are assumed to be i.i.d. and distributed as $\mathcal{CN}(0, 1)$. Let $\mathbf{h}_k^{j,pq} = [h_{1,k}^{j,pq} \ h_{2,k}^{j,pq} \ \cdots \ h_{n_r^{(q)},k}^{j,pq}]^T$ denote the $n_r^{(q)} \times 1$-sized complex channel fade vector at the receiver of node q when the kth MAP is active on the jth MBM-TU of node p. These channel fade vectors constitute the MBM channel alphabet from the jth MBM-TU of node p to the node q. Let $\mathbf{H}_{pq}^j = [\mathbf{h}_1^{j,pq} \ \mathbf{h}_2^{j,pq} \ \cdots \ \mathbf{h}_{N_m^{(p)}}^{j,pq}]$ denote the $n_r^{(q)} \times N_m^{(p)}$ channel matrix at the node q corresponding to the jth MBM-TU of the node p. Let $\mathbf{H}_{pq} = [\mathbf{H}_{pq}^1 \ \mathbf{H}_{pq}^2 \ \cdots \ \mathbf{H}_{pq}^{n_{tu}^{(p)}}]$ denote the $n_r^{(q)} \times N_m^{(p)} n_{tu}^{(p)}$ overall channel matrix from the node p to the node q.

p to p-SI link: Let $h_{i,k}^{j,pp}$ be the residual SI (after cancellation) channel fade coefficient at the ith receive antenna of node p when the kth MAP is active on the jth MBM-TU to node p, $i = 1, 2, \cdots, n_r^{(p)}$, $j = 1, 2, \cdots, n_{tu}^{(p)}$, and $k = 1, 2, \cdots, N_m^{(p)}$. Assuming the SI cancellation technique completely eliminates the line-of-sight (LOS) component [12], the $h_{i,k}^{j,pp}$s are modeled as i.i.d. and distributed as $\mathcal{CN}\big(0, (E/\sigma_p^2)^{-\lambda^{(p)}}\big)$, where σ_p^2 is the average noise power at the node p and $\lambda^{(p)}$ is a small positive constant that accounts for the quality of the SI cancellation scheme used by node p [4], [12]. For example, $\lambda^{(p)} = 1$ and $\lambda^{(p)} = 0$ refers to high and low quality SI cancellation schemes, respectively. Let $\mathbf{h}_k^{j,pp} = [h_{1,k}^{j,pp} \ h_{2,k}^{j,pp} \ \cdots \ h_{n_r^{(p)},k}^{j,pp}]^T$ denote the $n_r^{(p)} \times 1$-sized residual SI channel fade vector at the receiver of node p when the kth MAP is active on the jth MBM-TU of node p. As in the p to q-desired link, let $\mathbf{H}_{pp}^j$ and $\mathbf{H}_{pp}$ denote the $n_r^{(p)} \times N_m^{(p)}$ SI channel matrix and $n_r^{(p)} \times N_m^{(p)} n_{tu}^{(p)}$ overall SI channel matrix, respectively, from node p to itself.

D. Received signal

In the first phase, both the node 1 and node 2 transmit their information to the relay node R. Let $x_j^{(p),1}$ and $l_j^{(p),1}$ be the transmitted symbol and selected MAP index, respectively, on the jth MBM-TU of the node p, $p = 1, 2$, in the first phase. Now, the $n_r^{(r)}$-sized received signal vector $\mathbf{y}^{(r),1}$ at the relay in the first phase is given by

$$
\begin{aligned}
\mathbf{y}^{(r),1} &= \underbrace{\sum_{j=1}^{n_{tu}^{(1)}} x_j^{(1),1} \mathbf{h}_{l_j^{(1),1}}^{j,1r}}_{\text{desired signal from node 1}} + \underbrace{\sum_{j=1}^{n_{tu}^{(2)}} x_j^{(2),1} \mathbf{h}_{l_j^{(2),1}}^{j,2r}}_{\text{desired signal from node 2}} \\
&\quad + \underbrace{\sum_{j=1}^{n_{tu}^{(r)}} x_j^{(r),1} \mathbf{h}_{l_j^{(r),1}}^{j,rr}}_{\text{self-interference signal}} + \mathbf{w}^{(r),1} \\
&= \sum_{j=1}^{n_{tu}^{(1)}} x_j^{(1),1} \mathbf{H}_{1r}^j \mathbf{e}_{l_j^{(1),1}}^{(1)} + \sum_{j=1}^{n_{tu}^{(2)}} x_j^{(2),1} \mathbf{H}_{2r}^j \mathbf{e}_{l_j^{(2),1}}^{(2)} \\
&\quad + \sum_{j=1}^{n_{tu}^{(r)}} x_j^{(r),1} \mathbf{H}_{rr}^j \mathbf{e}_{l_j^{(r),1}}^{(r)} + \mathbf{w}^{(r),1} \\
&= \mathbf{H}_{1r}\mathbf{s}^{(1),1} + \mathbf{H}_{2r}\mathbf{s}^{(2),1} + \mathbf{H}_{rr}\mathbf{s}^{(r),1} + \mathbf{w}^{(r),1}, \qquad (3)
\end{aligned}
$$

where $x_j^{(r),1}$ and $l_j^{(r),1}$ denote the transmitted symbol and active MAP index, respectively, on the jth MBM-TU of the relay node R in the first phase (broadcast signal to node 1 and node 2, which results in SI), $\mathbf{e}_l^{(p)}$, $p = 1, 2, R$ is the lth column of $N_m^{(p)} \times N_m^{(p)}$ identity matrix, $\mathbf{w}^{(r),1}$ is the noise vector distributed as $\mathcal{CN}(\mathbf{0}_{n_r^{(r)}}, \sigma_r^2 \mathbf{I}_{n_r^{(r)}})$, $\mathbf{0}_d$ denotes the $d \times 1$ all zero vector, $\mathbf{I}_d$ denotes the $d \times d$ identity matrix, and

$\mathbf{s}^{(p),1},\ p=1,2,R$ is the $N_m^{(p)} n_{tu}^{(p)} \times 1$ transmit vector belongs to the GSM-MBM signal set $\mathbb{S}_{\text{gm}}^{(p)}$ (of size $|\mathbb{S}_{\text{gm}}^{(p)}| = 2^{\eta^{(p)}}$) at node p, which is given by

$$\mathbb{S}_{\text{gm}}^{(p)} = \Big\{ \mathbf{s} = [\mathbf{s}_1^T\ \mathbf{s}_2^T\ \cdots\ \mathbf{s}_{n_{tu}^{(p)}}^T]^T : \mathbf{s}_j = x_j \mathbf{e}_{l_j}^{(p)}, \\ l_j \in \{1, \cdots, N_m^{(p)}\}; \mathbf{x} = [x_1\ x_2\ \cdots\ x_{n_{tu}^{(p)}}]^T \in \mathbb{S}_{\text{gsm}}^{(p)} \Big\}. \quad (4)$$

Note that $\mathbf{s}^{(1),1}$, $\mathbf{s}^{(2),1}$, and $\mathbf{s}^{(r),1}$ are independent of each other, since the node 1 and node 2 transmit independent data and $\mathbf{s}^{(r),1}$ depends only on the previous data transmitted by the node 1 and node 2. The relay jointly detects the transmitted vectors $\mathbf{s}^{(1),1}$ and $\mathbf{s}^{(2),1}$ by employing the interference-oblivious ML detector, whose decision rule is given by

$$\{\hat{\mathbf{s}}^{(1,r),1}, \hat{\mathbf{s}}^{(2,r),1}\} = \underset{\substack{\mathbf{s}^{(p)} \in \mathbb{S}_{\text{gm}}^{(p)} \\ p=1,2}}{\text{argmin}} \|\mathbf{y}^{(r),1} - \mathbf{H}_{1r}\mathbf{s}^{(1)} - \mathbf{H}_{2r}\mathbf{s}^{(2)}\|^2, \quad (5)$$

where $\hat{\mathbf{s}}^{(1,r),1}$ and $\hat{\mathbf{s}}^{(2,r),1}$ denote the estimates of $\mathbf{s}^{(1),1}$ and $\mathbf{s}^{(2),1}$, respectively, at the relay node R. Let $\hat{\mathbf{b}}_1^{(1,r)}$ and $\hat{\mathbf{b}}_1^{(2,r)}$ denote the bit vectors corresponding to the estimates $\hat{\mathbf{s}}^{(1,r),1}$ and $\hat{\mathbf{s}}^{(2,r),1}$, respectively. Next, in the second phase, the relay node R broadcasts the XOR-ed bit vector $\mathbf{b}_2^{(r)} = \hat{\mathbf{b}}_1^{(1,r)} \oplus \hat{\mathbf{b}}_1^{(2,r)}$ to the node 1 and node 2. Let $\mathbf{s}^{(r),2} \in \mathbb{S}_{\text{gm}}^{(r)}$ denote the transmit vector corresponding to the bit vector $\mathbf{b}_2^{(r)}$. Now, the $n_r^{(p)} \times 1$ received vector $\mathbf{y}^{(p),2}$ at the node $p,\ p=1,2$, in the second phase is given by

$$\mathbf{y}^{(p),2} = \mathbf{H}_{rp}\mathbf{s}^{(r),2} + \mathbf{H}_{pp}\mathbf{s}^{(p),2} + \mathbf{w}^{(p),2}, \quad (6)$$

where $\mathbf{s}^{(p),2}$ denotes the signal transmitted by node $p,\ p = 1,2$, in the second phase (which results in SI) and $\mathbf{w}^{(p),2}$ is the noise vector distributed as $\mathcal{CN}(\mathbf{0}_{n_r^{(p)}}, \sigma_p^2 \mathbf{I}_{n_r^{(p)}})$. Note that $\mathbf{s}^{(r),2}$ and $\mathbf{s}^{(p),2}$ are independent of each other, since the $\mathbf{s}^{(r),2}$ depends on the data transmitted by node 1 and node 2 in the first phase. The interference-oblivious ML decision rule at the node $p,\ p=1,2$ is given by

$$\hat{\mathbf{s}}^{(r,p),2} = \underset{\mathbf{s} \in \mathbb{S}_{\text{gm}}^{(r)}}{\text{argmin}} \|\mathbf{y}^{(p),2} - \mathbf{H}_{rp}\mathbf{s}\|^2. \quad (7)$$

Finally, the node $p\ (=1,2)$ estimates the information transmitted by the node $q\ (=1,2),\ q \neq p$ in the first phase as $\hat{\mathbf{b}}_1^{(q)} = \mathbf{b}_1^{(p)} \oplus \hat{\mathbf{b}}_2^{(r,p)}$ (append zeros to the $\mathbf{b}_1^{(p)}$ vector before XOR operation to make them equal sized vectors and discard the last $(\eta^{(r)} - \eta^{(q)})$ bits after XOR operation), where $\mathbf{b}_1^{(p)}$ and $\hat{\mathbf{b}}_2^{(r,p)}$, $p=1,2$ denote the information bit vectors corresponding to the vectors $\mathbf{s}^{(p),1}$ and $\hat{\mathbf{s}}^{(r,p),2}$, respectively.

III. Average bit error probability analysis

In this section, we analyze the end-to-end average bit error probability (BEP) of the TW-FDR-MBM system described in Sec. II. We assume that all the possible transmit vectors are equi-probable. Let $\mathbf{b}_1^{(p)}$ denote the $\eta^{(p)}$-length information bit vector transmitted by the node $p,\ p=1,2$, and $\hat{\mathbf{b}}_1^{(p)}$ denote the $\mathbf{b}_1^{(p)}$'s estimate at the node $q\ (\neq p),\ q=1,2$. Then, the average BEP from the node p to node q is given by

$$\begin{aligned}
P_B^{pq} &= P\big(\hat{\mathbf{b}}_1^{(p)} \neq \mathbf{b}_1^{(p)}\big) \\
&= \sum_{\mathbf{s}' \in \mathbb{S}_{\text{gm}}^{(p)}} \sum_{\mathbf{s}'' \in \mathbb{S}_{\text{gm}}^{(q)}} \sum_{\hat{\mathbf{s}} \in \mathbb{S}_{\text{gm}}^{(r)}} P\Big(\substack{\hat{\mathbf{b}}_1^{(p)} \neq \mathbf{b}_1^{(p)}, \mathbf{s}^{(p),1} = \mathbf{s}', \mathbf{s}^{(q),1} = \mathbf{s}'', \\ \hat{\mathbf{s}}^{(r,q),2} = \hat{\mathbf{s}}}\Big) \\
&= \sum_{\mathbf{s}' \in \mathbb{S}_{\text{gm}}^{(p)}} \sum_{\mathbf{s}'' \in \mathbb{S}_{\text{gm}}^{(q)}} \sum_{\hat{\mathbf{s}} \in \mathbb{S}_{\text{gm}}^{(r)}} P\big(\mathbf{s}^{(p),1} = \mathbf{s}', \mathbf{s}^{(q),1} = \mathbf{s}''\big) \times \\
&\qquad P\big(\hat{\mathbf{s}}^{(r,q),2} = \hat{\mathbf{s}} | \mathbf{s}^{(p),1} = \mathbf{s}', \mathbf{s}^{(q),1} = \mathbf{s}''\big) \times \\
&\qquad P\big(\hat{\mathbf{b}}_1^{(p)} \neq \mathbf{b}_1^{(p)} | \mathbf{s}^{(p),1} = \mathbf{s}', \mathbf{s}^{(q),1} = \mathbf{s}'', \hat{\mathbf{s}}^{(r,q),2} = \hat{\mathbf{s}}\big) \\
&= \frac{1}{2^{\eta^{(p)}} 2^{\eta^{(q)}}} \sum_{\mathbf{s}' \in \mathbb{S}_{\text{gm}}^{(p)}} \sum_{\mathbf{s}'' \in \mathbb{S}_{\text{gm}}^{(q)}} \sum_{\hat{\mathbf{s}} \in \mathbb{S}_{\text{gm}}^{(r)}} \frac{\delta(\mathbf{s}', \mathbf{s}'', \hat{\mathbf{s}})}{\eta^{(p)}} \times \\
&\qquad P\big(\hat{\mathbf{s}}^{(r,q),2} = \hat{\mathbf{s}} | \mathbf{s}^{(p),1} = \mathbf{s}', \mathbf{s}^{(q),1} = \mathbf{s}''\big) \\
&= \frac{1}{2^{\eta^{(p)}} 2^{\eta^{(q)}}} \sum_{\mathbf{s}', \hat{\mathbf{s}}' \in \mathbb{S}_{\text{gm}}^{(p)}} \sum_{\mathbf{s}'', \hat{\mathbf{s}}'' \in \mathbb{S}_{\text{gm}}^{(q)}} \sum_{\hat{\mathbf{s}} \in \mathbb{S}_{\text{gm}}^{(r)}} \frac{\delta(\mathbf{s}', \mathbf{s}'', \hat{\mathbf{s}})}{\eta^{(p)}} \times \\
&\qquad \underbrace{P\big(\hat{\mathbf{s}}^{(p,r),1} = \hat{\mathbf{s}}', \hat{\mathbf{s}}^{(q,r),1} = \hat{\mathbf{s}}'' \big| \big\{\substack{\mathbf{s}^{(p),1} = \mathbf{s}', \\ \mathbf{s}^{(q),1} = \mathbf{s}''}\big\}\big)}_{\triangleq\ P_{pqr}(\hat{\mathbf{s}}', \hat{\mathbf{s}}'' | \mathbf{s}', \mathbf{s}'')} \times \\
&\qquad P\big(\hat{\mathbf{s}}^{(r,q),2} = \hat{\mathbf{s}} \big| \big\{\substack{\hat{\mathbf{s}}^{(p,r),1} = \hat{\mathbf{s}}', \hat{\mathbf{s}}^{(q,r),1} = \hat{\mathbf{s}}'', \\ \mathbf{s}^{(p),1} = \mathbf{s}', \mathbf{s}^{(q),1} = \mathbf{s}''}\big\}\big) \\
&= \frac{1}{2^{\eta^{(p)}} 2^{\eta^{(q)}}} \sum_{\mathbf{s}', \hat{\mathbf{s}}' \in \mathbb{S}_{\text{gm}}^{(p)}} \sum_{\mathbf{s}'', \hat{\mathbf{s}}'' \in \mathbb{S}_{\text{gm}}^{(q)}} \sum_{\hat{\mathbf{s}} \in \mathbb{S}_{\text{gm}}^{(r)}} \frac{\delta(\mathbf{s}', \mathbf{s}'', \hat{\mathbf{s}})}{\eta^{(p)}} \times \\
&\qquad P_{pqr}(\hat{\mathbf{s}}', \hat{\mathbf{s}}'' | \mathbf{s}', \mathbf{s}'') \underbrace{P\big(\hat{\mathbf{s}}^{(r,q),2} = \hat{\mathbf{s}} | \mathbf{s}^{(r),2} = f_r(\hat{\mathbf{s}}', \hat{\mathbf{s}}'')\big)}_{\triangleq\ P_{rq}(\hat{\mathbf{s}} | f_r(\hat{\mathbf{s}}', \hat{\mathbf{s}}''))}, \quad (8)
\end{aligned}$$

where $\delta(\mathbf{s}', \mathbf{s}'', \hat{\mathbf{s}})$ is the number of error bits, given by the Hamming distance between bit vectors $\mathbf{b}_{\mathbf{s}'}$ and $\mathbf{b}_{\hat{\mathbf{s}}} \oplus \mathbf{b}_{\mathbf{s}''}$ ($\mathbf{b}_{\mathbf{s}'}$, $\mathbf{b}_{\mathbf{s}''}$, and $\mathbf{b}_{\hat{\mathbf{s}}}$ denote the bit vectors corresponding to the signal vectors $\mathbf{s}'$, $\mathbf{s}''$, and $\hat{\mathbf{s}}$, respectively), $0 \leq \delta(\mathbf{s}', \mathbf{s}'', \hat{\mathbf{s}}) \leq \eta^{(p)}$, the equality in (8) follows from the fact that $\hat{\mathbf{s}}^{(r,q),2}$ is independent of $\mathbf{s}^{(p),1}$ and $\mathbf{s}^{(q),1}$ given $\hat{\mathbf{s}}^{(p,r),1}$ and $\hat{\mathbf{s}}^{(q,r),1}$, $f_r(\hat{\mathbf{s}}', \hat{\mathbf{s}}'')$ is the signal vector in $\mathbb{S}_{\text{gm}}^{(r)}$ corresponding to the bit vector $\mathbf{b}_{\hat{\mathbf{s}}'} \oplus \mathbf{b}_{\hat{\mathbf{s}}''}$ ($\mathbf{b}_{\hat{\mathbf{s}}'}$ and $\mathbf{b}_{\hat{\mathbf{s}}''}$ denote the bit vectors corresponding to the signal vectors $\hat{\mathbf{s}}'$ and $\hat{\mathbf{s}}''$, respectively), $P_{pqr}(\hat{\mathbf{s}}', \hat{\mathbf{s}}'' | \mathbf{s}', \mathbf{s}'')$ is the probability of decoding the transmitted vectors $\mathbf{s}^{(p),1} = \mathbf{s}'$ and $\mathbf{s}^{(q),1} = \mathbf{s}''$ as $\hat{\mathbf{s}}^{(p,r),1} = \hat{\mathbf{s}}'$ and $\hat{\mathbf{s}}^{(q,r),1} = \hat{\mathbf{s}}''$, respectively, at the relay, and $P_{rq}(\hat{\mathbf{s}} | f_r(\hat{\mathbf{s}}', \hat{\mathbf{s}}''))$ is the probability of the node q decoding the relay's transmitted vector $\mathbf{s}^{(r),2} = f_r(\hat{\mathbf{s}}', \hat{\mathbf{s}}'')$ as $\hat{\mathbf{s}}^{(r,q),2} = \hat{\mathbf{s}}$.

Derivation of $P_{pqr}(\hat{\mathbf{s}}', \hat{\mathbf{s}}'' | \mathbf{s}', \mathbf{s}'')$: The probability $P_{pqr}(\hat{\mathbf{s}}', \hat{\mathbf{s}}'' | \mathbf{s}', \mathbf{s}'')$ can be written as

$$\begin{aligned}
&P_{pqr}(\hat{\mathbf{s}}', \hat{\mathbf{s}}'' | \mathbf{s}', \mathbf{s}'') \\
&= \sum_{\tilde{\mathbf{s}} \in \mathbb{S}_{\text{gm}}^{(r)}} P\big(\big\{\substack{\hat{\mathbf{s}}^{(p,r),1} = \hat{\mathbf{s}}', \hat{\mathbf{s}}^{(q,r),1} = \hat{\mathbf{s}}'', \\ \mathbf{s}^{(r),1} = \tilde{\mathbf{s}}}\big\} \big| \big\{\substack{\mathbf{s}^{(p),1} = \mathbf{s}', \\ \mathbf{s}^{(q),1} = \mathbf{s}''}\big\}\big) \\
&= \sum_{\tilde{\mathbf{s}} \in \mathbb{S}_{\text{gm}}^{(r)}} P\big(\mathbf{s}^{(r),1} = \tilde{\mathbf{s}} | \mathbf{s}^{(p),1} = \mathbf{s}', \mathbf{s}^{(q),1} = \mathbf{s}''\big) \times \\
&\qquad \underbrace{P\big(\big\{\substack{\hat{\mathbf{s}}^{(p,r),1} = \hat{\mathbf{s}}', \\ \hat{\mathbf{s}}^{(q,r),1} = \hat{\mathbf{s}}''}\big\} \big| \big\{\substack{\mathbf{s}^{(p),1} = \mathbf{s}', \mathbf{s}^{(q),1} = \mathbf{s}'', \\ \mathbf{s}^{(r),1} = \tilde{\mathbf{s}}}\big\}\big)}_{\triangleq P_{pqr}(\hat{\mathbf{s}}', \hat{\mathbf{s}}'' | \mathbf{s}', \mathbf{s}'', \tilde{\mathbf{s}})}, \quad (9)
\end{aligned}$$

where $P_{pqr}(\hat{\mathbf{s}}', \hat{\mathbf{s}}'' | \mathbf{s}', \mathbf{s}'', \tilde{\mathbf{s}})$ is the probability of the relay decoding the transmitted vectors $\mathbf{s}^{(p),1} = \mathbf{s}'$ and $\mathbf{s}^{(q),1} = \mathbf{s}''$ as $\hat{\mathbf{s}}^{(p,r),1} = \hat{\mathbf{s}}'$ and $\hat{\mathbf{s}}^{(q,r),1} = \hat{\mathbf{s}}''$, respectively, given that the relay transmitted $\mathbf{s}^{(r),1} = \tilde{\mathbf{s}}$. Since $\mathbf{s}^{(r),1}$ is independent of

$\mathbf{s}^{(p),1}$ and $\mathbf{s}^{(q),1}$, (9) simplifies to

$$P_{pqr}(\hat{\mathbf{s}}', \hat{\mathbf{s}}''|\mathbf{s}', \mathbf{s}'') = \frac{1}{2^{\eta^{(r)}}} \sum_{\tilde{\mathbf{s}} \in \mathbb{S}_{\text{gm}}^{(r)}} P_{pqr}(\hat{\mathbf{s}}', \hat{\mathbf{s}}''|\mathbf{s}', \mathbf{s}'', \tilde{\mathbf{s}}). \quad (10)$$

The probability $P_{pqr}(\hat{\mathbf{s}}', \hat{\mathbf{s}}''|\mathbf{s}', \mathbf{s}'', \tilde{\mathbf{s}})$ can be written as

$$\begin{aligned} &P_{pqr}(\hat{\mathbf{s}}', \hat{\mathbf{s}}''|\mathbf{s}', \mathbf{s}'', \tilde{\mathbf{s}}) \\ &\quad = \mathbb{E}_{\mathbf{H}_{pr}, \mathbf{H}_{qr}}\left(P_{pqr}(\hat{\mathbf{s}}', \hat{\mathbf{s}}''|\mathbf{s}', \mathbf{s}'', \tilde{\mathbf{s}}, \mathbf{H}_{pr}, \mathbf{H}_{qr})\right), \end{aligned} \quad (11)$$

where $\mathbb{E}(\cdot)$ denotes the expectation operator. From (3) and (5), the probability $P_{pqr}(\hat{\mathbf{s}}', \hat{\mathbf{s}}''|\mathbf{s}', \mathbf{s}'', \tilde{\mathbf{s}}, \mathbf{H}_{pr}, \mathbf{H}_{qr})$ can be written as

$$\begin{aligned} &P_{pqr}(\hat{\mathbf{s}}', \hat{\mathbf{s}}''|\mathbf{s}', \mathbf{s}'', \tilde{\mathbf{s}}, \mathbf{H}_{pr}, \mathbf{H}_{qr}) \\ &= P\left(\bigcap_{\substack{(\dot{\mathbf{s}}, \ddot{\mathbf{s}}) \in \mathbb{S}_{\text{gm}}^{(p),(q)} \\ (\dot{\mathbf{s}}, \ddot{\mathbf{s}}) \neq (\hat{\mathbf{s}}', \hat{\mathbf{s}}'')}} \left\{ \begin{array}{c} \|\mathbf{H}_{pr}(\mathbf{s}' - \hat{\mathbf{s}}') + \mathbf{H}_{qr}(\mathbf{s}'' - \hat{\mathbf{s}}'') + \tilde{\mathbf{w}}^{(r),1}\|^2 \\ < \|\mathbf{H}_{pr}(\mathbf{s}' - \dot{\mathbf{s}}) + \mathbf{H}_{qr}(\mathbf{s}'' - \ddot{\mathbf{s}}) + \tilde{\mathbf{w}}^{(r),1}\|^2 \end{array} \right\} \right), \end{aligned} \quad (12)$$

where $\tilde{\mathbf{w}}^{(r),1} = \mathbf{H}_{rr}\tilde{\mathbf{s}} + \mathbf{w}^{(r),1}$ and $\mathbb{S}_{\text{gm}}^{(p),(q)} = \{(\dot{\mathbf{s}}, \ddot{\mathbf{s}}) : \dot{\mathbf{s}} \in \mathbb{S}_{\text{gm}}^{(p)}, \ddot{\mathbf{s}} \in \mathbb{S}_{\text{gm}}^{(q)}\}$. It is easy to see that the elements of $\tilde{\mathbf{w}}^{(r),1}$ are i.i.d. and distributed as $\mathcal{CN}(0, \sigma_r^2 + \|\tilde{\mathbf{s}}\|^2(E/\sigma_r^2)^{-\lambda^{(r)}})$. From the monotonicity of probability (i.e., $P(\cap_k A_k) \leq P(A_k)$), (12) can be upper bounded as

$$\begin{aligned} &P_{pqr}(\hat{\mathbf{s}}', \hat{\mathbf{s}}''|\mathbf{s}', \mathbf{s}'', \tilde{\mathbf{s}}, \mathbf{H}_{pr}, \mathbf{H}_{qr}) \\ &\leq \begin{cases} P\left(\begin{array}{c} \|\mathbf{H}_r(\mathbf{s}'_c - \hat{\mathbf{s}}'_c) + \tilde{\mathbf{w}}^{(r),1}\|^2 \\ < \|\tilde{\mathbf{w}}^{(r),1}\|^2 \end{array} \right) & \text{if } \mathbf{s}'_c \neq \hat{\mathbf{s}}'_c \\ \min\limits_{\dot{\mathbf{s}}_c \neq \mathbf{s}'_c} P\left(\begin{array}{c} \|\mathbf{H}_r(\mathbf{s}'_c - \dot{\mathbf{s}}_c) + \tilde{\mathbf{w}}^{(r),1}\|^2 \\ > \|\tilde{\mathbf{w}}^{(r),1}\|^2 \end{array} \right) & \text{if } \mathbf{s}'_c = \hat{\mathbf{s}}'_c \end{cases} \\ &= \begin{cases} Q\left(\sqrt{\frac{\|\mathbf{H}_r(\mathbf{s}'_c - \hat{\mathbf{s}}'_c)\|^2}{2(\sigma_r^2 + \|\tilde{\mathbf{s}}\|^2(E/\sigma_r^2)^{-\lambda^{(r)}})}} \right) & \text{if } \mathbf{s}'_c \neq \hat{\mathbf{s}}'_c \\ 1 - \max\limits_{\dot{\mathbf{s}}_c \neq \mathbf{s}'_c} Q\left(\sqrt{\frac{\|\mathbf{H}_r(\mathbf{s}'_c - \dot{\mathbf{s}}_c)\|^2}{2(\sigma_r^2 + \|\tilde{\mathbf{s}}\|^2(E/\sigma_r^2)^{-\lambda^{(r)}})}} \right) & \text{if } \mathbf{s}'_c = \hat{\mathbf{s}}'_c \end{cases}, \end{aligned} \quad (13)$$

where $\mathbf{H}_r = [\mathbf{H}_{pr} \ \ \mathbf{H}_{qr}]$, $\mathbf{s}'_c = [\mathbf{s}'^T \ \ \mathbf{s}''^T]^T$, $\hat{\mathbf{s}}'_c = [\hat{\mathbf{s}}'^T \ \ \hat{\mathbf{s}}''^T]^T$, $\dot{\mathbf{s}}_c = [\dot{\mathbf{s}}^T \ \ \ddot{\mathbf{s}}^T]^T$, and $Q(\cdot)$ denotes the Q-function. Substituting (13) in (11) and simplifying [13], we get

$$\begin{aligned} &P_{pqr}(\hat{\mathbf{s}}', \hat{\mathbf{s}}''|\mathbf{s}', \mathbf{s}'', \tilde{\mathbf{s}}) \\ &\leq \begin{cases} g_{n_r^{(r)}}\left(\frac{\|\mathbf{s}'_c - \hat{\mathbf{s}}'_c\|^2}{4(\sigma_r^2 + \|\tilde{\mathbf{s}}\|^2(E/\sigma_r^2)^{-\lambda^{(r)}})} \right) & \text{if } \mathbf{s}'_c \neq \hat{\mathbf{s}}'_c \\ 1 - \max\limits_{\dot{\mathbf{s}}_c \neq \mathbf{s}'_c} g_{n_r^{(r)}}\left(\frac{\|\mathbf{s}'_c - \dot{\mathbf{s}}_c\|^2}{4(\sigma_r^2 + \|\tilde{\mathbf{s}}\|^2(E/\sigma_r^2)^{-\lambda^{(r)}})} \right) & \text{if } \mathbf{s}'_c = \hat{\mathbf{s}}'_c \end{cases} \\ &= \begin{cases} g_{n_r^{(r)}}\left(\frac{\|\mathbf{s}'_c - \hat{\mathbf{s}}'_c\|^2}{4(\sigma_r^2 + \|\tilde{\mathbf{s}}\|^2(E/\sigma_r^2)^{-\lambda^{(r)}})} \right) & \text{if } \mathbf{s}'_c \neq \hat{\mathbf{s}}'_c \\ 1 - g_{n_r^{(r)}}\left(\frac{\min\limits_{\dot{\mathbf{s}}_c \neq \mathbf{s}'_c} \|\mathbf{s}'_c - \dot{\mathbf{s}}_c\|^2}{4(\sigma_r^2 + \|\tilde{\mathbf{s}}\|^2(E/\sigma_r^2)^{-\lambda^{(r)}})} \right) & \text{if } \mathbf{s}'_c = \hat{\mathbf{s}}'_c \end{cases}, \end{aligned} \quad (14)$$

where

$$g_k(\alpha) = f(\alpha)^k \sum_{j=0}^{k-1} \binom{k-1+j}{j} (1 - f(\alpha))^j,$$

$f(\alpha) = \frac{1}{2}\left(1 - \sqrt{\frac{\alpha}{1+\alpha}}\right)$, and the non-increasing monotonicity of $g_k(\alpha)$ results in (14).

Derivation of $P_{rq}(\hat{\mathbf{s}}|f_r(\hat{\mathbf{s}}', \hat{\mathbf{s}}''))$: The probability $P_{rq}(\hat{\mathbf{s}}|f_r(\hat{\mathbf{s}}', \hat{\mathbf{s}}''))$ can be written as

$$\begin{aligned} &P_{rq}(\hat{\mathbf{s}}|f_r(\hat{\mathbf{s}}', \hat{\mathbf{s}}'')) \\ &= \sum_{\bar{\mathbf{s}} \in \mathbb{S}_{\text{gm}}^{(q)}} P\left(\hat{\mathbf{s}}^{(r,q),2} = \hat{\mathbf{s}}, \mathbf{s}^{(q),2} = \bar{\mathbf{s}} | \mathbf{s}^{(r),2} = f_r(\hat{\mathbf{s}}', \hat{\mathbf{s}}'')\right) \\ &= \sum_{\bar{\mathbf{s}} \in \mathbb{S}_{\text{gm}}^{(q)}} P\left(\mathbf{s}^{(q),2} = \bar{\mathbf{s}} | \mathbf{s}^{(r),2} = f_r(\hat{\mathbf{s}}', \hat{\mathbf{s}}'')\right) \times \\ &\qquad \underbrace{P\left(\hat{\mathbf{s}}^{(r,q),2} = \hat{\mathbf{s}} | \mathbf{s}^{(r),2} = f_r(\hat{\mathbf{s}}', \hat{\mathbf{s}}''), \mathbf{s}^{(q),2} = \bar{\mathbf{s}}\right)}_{\triangleq P_{rq}(\hat{\mathbf{s}}|f_r(\hat{\mathbf{s}}', \hat{\mathbf{s}}''), \bar{\mathbf{s}})}, \end{aligned} \quad (15)$$

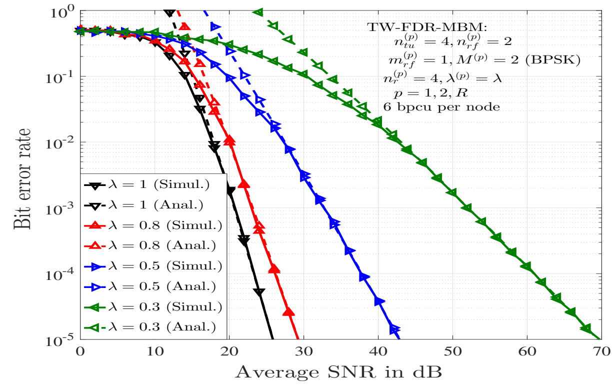

Fig. 3. Comparison between simulated BER and analytical upper bound on BER of the TW-FDR-MBM system.

where $P_{rq}(\hat{\mathbf{s}}|f_r(\hat{\mathbf{s}}', \hat{\mathbf{s}}''), \bar{\mathbf{s}})$ is the probability of the node q decoding the relay's transmitted vector $\mathbf{s}^{(r),2} = f_r(\hat{\mathbf{s}}', \hat{\mathbf{s}}'')$ as $\hat{\mathbf{s}}^{(r,q),2} = \hat{\mathbf{s}}$ given that node q transmitted $\mathbf{s}^{(q),2} = \bar{\mathbf{s}}$. Since $\mathbf{s}^{(q),2}$ is independent of $\mathbf{s}^{(r),2}$, (15) simplifies to

$$P_{rq}(\hat{\mathbf{s}}|f_r(\hat{\mathbf{s}}', \hat{\mathbf{s}}'')) = \frac{1}{2^{\eta^{(q)}}} \sum_{\bar{\mathbf{s}} \in \mathbb{S}_{\text{gm}}^{(q)}} P_{rq}(\hat{\mathbf{s}}|f_r(\hat{\mathbf{s}}', \hat{\mathbf{s}}''), \bar{\mathbf{s}}). \quad (16)$$

Following similar steps from (11)-(14), $P_{rq}(\hat{\mathbf{s}}|f_r(\hat{\mathbf{s}}', \hat{\mathbf{s}}''), \bar{\mathbf{s}})$ can be upper bounded as

$$\begin{aligned} &P_{rq}(\hat{\mathbf{s}}|f_r(\hat{\mathbf{s}}', \hat{\mathbf{s}}''), \bar{\mathbf{s}}) \leq \\ &\begin{cases} g_{n_r^{(q)}}\left(\frac{\|\hat{\mathbf{s}} - f_r(\hat{\mathbf{s}}', \hat{\mathbf{s}}'')\|^2}{4(\sigma_q^2 + \|\bar{\mathbf{s}}\|^2(E/\sigma_q^2)^{-\lambda^{(q)}})} \right) & \text{if } \hat{\mathbf{s}} \neq f_r(\hat{\mathbf{s}}', \hat{\mathbf{s}}'') \\ 1 - g_{n_r^{(q)}}\left(\frac{\min\limits_{\acute{\mathbf{s}} \neq \hat{\mathbf{s}}} \|\hat{\mathbf{s}} - \acute{\mathbf{s}}\|^2}{4(\sigma_q^2 + \|\bar{\mathbf{s}}\|^2(E/\sigma_q^2)^{-\lambda^{(q)}})} \right) & \text{if } \hat{\mathbf{s}} = f_r(\hat{\mathbf{s}}', \hat{\mathbf{s}}'') \end{cases}, \end{aligned} \quad (17)$$

where $\acute{\mathbf{s}} \in \mathbb{S}_{\text{gm}}^{(r)}$. Substituting (10), (14), (16), and (17) in (8) gives an upper bound on the average BEP from the node p to node q.

IV. Numerical results and discussions

In this section, we present the numerical results on the bit error rate (BER) performance of the TW-FDR-MBM system. We consider same transmission parameters at all the nodes, and therefore the performance is symmetric from one end node to the other end node.

In Fig. 3, we present the end-to-end average BER performance of the TW-FDR-MBM system with $n_{tu}^{(p)} = 4$, $n_{rf}^{(p)} = 2$, $m_{rf}^{(p)} = 1$, $M^{(p)} = 2$ (BPSK), $n_r^{(p)} = 4$, $p = 1, 2, R$, and 6 bpcu per node for various values of $\lambda^{(p)} = 1, 0.8, 0.5, 0.3$. It can be seen that the performance degrades as the quality of SI cancellation becomes poor (i.e., $\lambda^{(p)}$ decreases). For instance, the average SNR required to achieve 10^{-5} BER is about 26 dB, 30 dB, 43 dB, and 70 dB for $\lambda^{(p)} = 1, 0.8, 0.5$, and

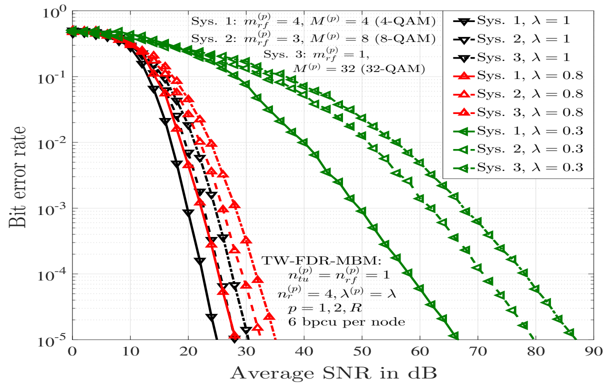

Fig. 4. Bit error performance of TW-FDR-MBM system for various combinations of $m_{rf}^{(p)}$ and $M^{(p)}$ that achieves same bpcu.

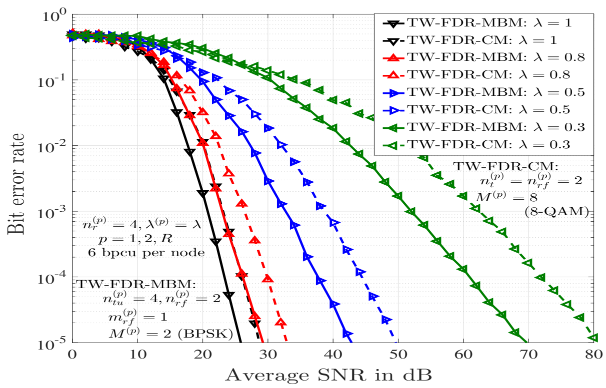

Fig. 5. Performance comparison between TW-FDR-MBM and TW-FDR-CM systems.

0.3, respectively. Fig. 3 also illustrates the tightness of the analytical upper bound on the BER. It can be seen that the bound becomes increasingly tight with increasing SNR.

In Fig. 4, we present the simulated BER performance of TW-FDR-MBM system with $n_{tu}^{(p)} = n_{rf}^{(p)} = 1$, $n_r^{(p)} = 4$, $\lambda^{(p)} = 1, 0.8, 0.3$, and 6 bpcu per node for various combinations of $m_{rf}^{(p)}$ and $M^{(p)}$ that achieves 6 bpcu. The considered combinations are: $i)$ $m_{rf}^{(p)} = 4$, $M^{(p)} = 4$ (QAM); $ii)$ $m_{rf}^{(p)} = 3$, $M^{(p)} = 8$ (QAM); $iii)$ $m_{rf}^{(p)} = 1$, $M^{(p)} = 32$ (QAM). It is noticed that the bit error performance of TW-FDR-MBM improves as $m_{rf}^{(p)}$ increases. For instance, to achieve 10^{-4} BER with $\lambda^{(p)} = 0.8$, the average SNR required is about 32 dB, 30 dB, and 20 dB for $m_{rf}^{(p)} = 1, 2$, and 4, respectively. This is because, increase in $m_{rf}^{(p)}$ allows the system to use a smaller sized PSK/QAM alphabet, which, in turn, results in SNR gain. Further, it is observed that the system with lower $m_{rf}^{(p)}$ (which needs higher $M^{(p)}$) is more sensitive to SI (i.e., $\lambda^{(p)}$). For instance, at 10^{-4} BER, the degradation in performance is about 3 dB, 4 dB, and 4.5 dB for $(m_{rf}^{(p)}, M^{(p)})$ is $(4,4)$, $(3,8)$, and $(1,32)$, respectively, when $\lambda^{(p)}$ is changed from 1 to 0.8.

Figure 5 compares the simulated BER performance of TW-FDR-MBM and TW-FDR-CM systems with $\lambda^{(p)} \in \{1, 0.8, 0.5, 0.3\}$, $n_{rf}^{(p)} = 2$, $n_r^{(p)} = 4$, and $\eta^{(p)} = 6$ bpcu. The following system parameters are considered. TW-FDR-MBM: $n_{tu}^{(p)} = 4$, $m_{rf}^{(p)} = 1$, $M^{(p)} = 2$ (BPSK); TW-FDR-CM: $n_t^{(p)} = 2$, $M^{(p)} = 8$ (QAM), where $n_t^{(p)}$ is the number of transmit antennas at node p. It is observed that the TW-FDR-MBM achieves better performance than the TW-FDR-CM. For example, to achieve 10^{-4} BER with $\lambda^{(p)} = 1$, TW-FDR-MBM system requires about 3 dB less SNR than TW-FDR-CM. This is because, to achieve the given spectral efficiency, TW-FDR-MBM system can use a smaller sized alphabet ($M^{(p)} = 2$) compared to TW-FDR-CM system ($M^{(p)} = 8$). For the same reason, TW-FDR-MBM is also more robust to SI than TW-FDR-CM. For instance, at a BER of 10^{-4}, the performance of TW-FDR-CM degrades by about 4 dB and 18 dB when $\lambda^{(p)}$ is changed from 1 to 0.8 and 0.5, respectively, whereas the degradation in TW-FDR-MBM is only about 3 dB and 15 dB, respectively.

V. Conclusions

We investigated the performance of MBM in a two-way relaying network (referred to as TW-FDR-MBM system), where two FD nodes exchange information with the help of a FD relay node using DF protocol. We carried out an average BEP analysis of the TW-FDR-MBM system. Our simulation results showed that, for a given spectral efficiency, TW-FDR-MBM system achieves better bit error performance than TW-FDR-CM system which uses conventional modulation schemes like PSK/QAM. Power allocation and the effect of spatial correlation and imperfect channel estimation in TW-FDR-MBM systems can be considered for future study.

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