Performance of a Wireless Media Access Protocol on a Markovian Fading Channel*

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Abstract

The throughput performance of a wireless media access protocol taking into account the effect of channel fading is analyzed. For efficient access on the uplink (mobile-to-base), the protocol makes use of the uplink channel status information which is conveyed to the mobile through a *busy/idle flag* on the downlink (base-to-mobile link). A first-order Markov model is used to describe the packet successes and failures on a Rayleigh fading channel. A closed-form expression for the throughput is derived by modeling the system as a finite-state, discrete-time Markov chain. The analytical results obtained through the first-order Markov approximation of the channel are compared to those obtained from an i.i.d channel model. The Markovian fading channel model is shown to provide better performance results than the i.i.d channel model. Results obtained by direct simulation of the fading process and the protocol operation show that a first-order Markov approximation of the fading process is quite accurate.

1 Introduction

Wireless networks using radio transmission are becoming increasingly popular for personal, multimedia communications [1]. The media access protocols play a vital role in determining the network efficiency when multiple users share a common transmission medium. Classical random access protocols, for which extensive analyses have been carried out, are random ALOHA, slotted ALOHA, and CSMA. In this paper, we study the performance of a media access protocol that uses the uplink (mobile-to-base station link) channel status information, which is conveyed to the mobile through the downlink (base station-to-mobile link) broadcast from the base station by periodically inserting a busy/idle flag [2]. One key issue that is to be addressed in a mobile radio environment is the effect of channel fading on the protocol performance. In the past literature, most models for data block transmission (e.g., in data-link protocols) have assumed that the block transmissions were independent and identically distributed (i.i.d). Also, many protocols were designed for an i.i.d channel, and techniques were developed to eliminate channel memory (e.g., by interleaving). A newer approach is to take advantage of the channel memory (e.g., exploiting some prediction techniques) to obtain better performance, instead of destroying it. In the latter context, the study of channels with memory becomes important. Some real-world channels do not lend themselves to an analytical study, and the development of simplified models is desirable. A natural approach is to approximate a channel with memory by means of a Markov model. Recently, in [3], Wang investigated the accuracy of a first-order Markov process in modeling data transmission on a Rayleigh fading channel, finding that such an approximation is, in fact, satisfactory. The process he refers to is the sequence of values of the envelope of the complex Gaussian process used to model the multiplicative effect of the channel. On the other hand, more often it is not the value of the channel envelope (or of the SNR, which is proportional to the square of it) which is of direct interest, but rather some nonlinear function of it, which depends on the transmission techniques, modulation, coding, and so on. In particular, the binary process which describes the successes and the failures of the packet transmissions is of primary importance, and very often is the only information which is available. In [4], Zorzi *et al*, using mutual information, investigated the accuracy of a first-order binary Markov model for the success/failure process of data packets. For an ARQ protocol, the results were shown to be relatively insensitive to the memory of the Markov model, meaning that a first-order model is adequate and the consideration of higher-order models is not needed.

In this paper, we apply the Markovian channel model developed in [4] to investigate the effect of Rayleigh fading on the performance of the proposed media access protocol. We derive a closed-form expression for the channel utilization by modeling the system as a finite-state, discrete-time Markov chain. An enhancement to the basic protocol that takes advantage of the memory in the fading channel behavior is proposed and analyzed. The proposed media access protocol is described in Section 2. The fading channel model, packet success/failure process, and the parameters of the Markov approximation of the channel are described in Section 3. In Section 4, the throughput analysis for both the basic and the enhanced protocol are provided. Numerical and simulation results are discussed in Section 5. Section 6 provides the conclusions.

2 Protocol Description

In the following description of the protocol, it is assumed that the uplink channel is slotted to one packet duration, transmission attempts are made only at the slot boundaries, and each message consists of a random number of data packets. The mobile, once it receives a message to be delivered to the base station, first checks the status of the busy/idle flag which is received periodically on the downlink from the base station. If the flag is set to busy, the mobile

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refrains from making a transmission attempt, and reschedules the attempt to a later time. If the flag is set to idle, then the mobile makes a transmission attempt by sending a header packet (which could contain the number of packets in the message, and other control information like source and destination addresses, etc.) in the immediately following slot on the uplink. The mobile expects the base station to respond to this header in the form of the busy/idle flag being set to busy, in the event of successful header packet reception. If this happens, the mobile transmits the packets continuously on the uplink until all the packets in the message are sent. The base station sets the flag back to idle once the entire massage transmission is complete. In the event of the header packet loss (either due to collision or channel errors), the base station makes no response. Thus, the continued idle status of the flag prompts the mobiles to reschedule their transmission attempts to a later time.

3 Fading Channel Model

In the literature, the flat fading channel is modeled as a multiplicative complex function, $\alpha(t)$, which is adequately described as a random process. A popular model considers a Gaussian random process with a given mean and covariance function defined as [5]

$$K(\tau) = E[(\alpha(t+\tau)-\mu)^*(\alpha(t)-\mu)]. \qquad (1)$$

Note that if $\mu = 0$, the envelope of $\alpha(t)$ is Rayleigh distributed for any t, and the envelope squared has an exponential distribution. When $\mu > 0$, the resulting fading envelope is *Rician* distributed which accounts for the presence of a line-of-sight (LOS) component, and is often more accurate in micro- and picocells. When the LOS component is absent, or has negligible power, the Rician model degenerates into the Rayleigh one.

In a widely accepted model, the Gaussian process is assumed to have a bandlimited non-rational spectrum given by

$$S(f) = S(0)\left[1-\left(\frac{f}{f_D}\right)^2\right]^{-1/2}, \quad \text{for } |f| < f_D, \qquad (2)$$

and zero otherwise [5]. This spectrum corresponds to the covariance function

$$K(\tau) = J_0(2\pi f_D|\tau|), \qquad (3)$$

whose physical meaning has been investigated in [5]. $J_0(\cdot)$ is the modified Bessel function of the first kind and of zeroth order. The correlation properties of the fading process depend only on $f_D|\tau|$. When it is small (< 0.1), the process is very correlated ("slow" fading); on the other hand, for larger values of $f_D|\tau|$ (> 0.2), the samples of the channel are almost independent ("fast" fading). For high data rates (small τ), the fading process can always be considered to be slowly varying, at least for the usual values of the carrier frequency (900-1800 MHz) and for typical vehicular speeds. Therefore, the dependence between transmissions of consecutive packets of data cannot be neglected. In particular, the assumption that the successes/failures of data packets constitute an i.i.d process is far from reality, and may lead to incorrect results when used to evaluate the performance of a transmission scheme or protocol. A more general model for the success/failure process, which accounts for dependence, is described as follows.

3.1 A model for packet success/failure process

We model the packet success/failure process as the outcome of a comparison of the instantaneous signal-to-noise ratio to the threshold value, SNR_t: if the received power is above the threshold, the packet is successfully decoded with probability 1; otherwise, it is lost with probability 1. If F is the value of the *fading margin*, the instantaneous signal-to-noise ratio (taking into account the effect of fading) is given by $SNR_tF|\alpha^2(t)|$. Hence, the binary process that describes packet successes/failures on the channel, β_j, can be obtained by quantization of the squared magnitude of the complex Gaussian description with the threshold $1/F$, i.e.,

$$\beta_j = \begin{cases} 0 & \text{if } v_j^2 > 1/F, \\ 1 & \text{if } v_j^2 \le 1/F, \end{cases} \qquad (4)$$

where $v_j = |\alpha(jT)|$ is the amplitude of the fading envelope, and "1" stands for a packet failure. As in [4], we describe the above success/failure process on a mobile radio channel by a first-order Markov model. The parameters of the Markov model can be determined based on the fading model and the characteristics of the communications scheme. The transition probability matrix that describes the channel is given by

$$M_c = \begin{pmatrix} p & 1-p \\ 1-q & q \end{pmatrix}, \qquad (5)$$

where p and $1-q$ are the probabilities that the transmission in slot i is successful, given that the transmission in slot $i-1$ was successful or unsuccessful, respectively. Given the matrix M_c, the channel properties are completely characterized. In particular, it is possible to find the steady state distribution of the chain. Based on this, the steady-state probability that an error occurs, P_E, is

$$P_E = \frac{1-p}{2-p-q}. \qquad (6)$$

Also, note that $(1-q)^{-1}$ represents the average length of a burst of errors , which is described by a geometric r.v.

3.2 Parameters of the Markov model

In order to carry out the performance analysis, the parameters of the above Markov model are to be found. There are essentially two independent parameters to be computed, e.g., p and q. We will derive P_E and q, from which p can be obtained from Eqn.(6). Let $A(v)$ be the event of an unsuccessful reception, with probability $P_w(v)$, conditioned on the value of the fading envelope, v. The average probability of a packet error is therefore

$$P_E = P[1] = E[P[A(v)]] = \int_0^\infty P_w(a)f_v(a)da. \qquad (7)$$

Also,

$$q = P[1|1] = \frac{E[P[A(v_1)]P[A(v_2)]]}{P_E}. \qquad (8)$$

If

$$P[A(v)] = \begin{cases} 0 & v^2 > 1/F \\ 1 & v^2 \le 1/F \end{cases}, \qquad (9)$$

we have

$$P_E = F_v(\sqrt{1/F}), \;\; P[1,1] = F_{v_1 v_2}(\sqrt{1/F}, \sqrt{1/F}), \quad (10)$$

where F_v and $F_{v_1 v_2}$ are the marginal and joint cdfs of the fading envelope. For the Rayleigh fading case, P_E and q can be found as [4]:

$$P_E = 1 - e^{-1/F}, \quad (11)$$

and

$$q = 1 - \frac{Q(\theta, \rho\theta) - Q(\rho\theta, \theta)}{e^{1/F} - 1}, \quad (12)$$

where

$$\theta = \sqrt{\frac{2/F}{1-\rho^2}}, \quad (13)$$

$\rho^2 = J_0(2\pi f_D T)$ is the correlation coefficient of the two successive samples T seconds apart, and $Q(\cdot,\cdot)$ is the Marcum Q function.

4 Throughput Analysis

To analyze the system throughput, we assume that the busy/idle flag on the downlink is received instantaneously and error-free by all the mobiles. Further, we assume perfect power control and neglect the effect of capture. With the above assumptions, we model the system as a discrete-time Markov chain with a finite number of states. During a slot, the system can be in any one of the following states, namely, 1) *idle state* (i) — state in which there is no activity on the uplink; 2) *single header transmission states* (s_s, s_f) — states in which there is only one header packet being transmitted in a slot on the uplink (implying that there is no packet loss due to collision). State s_s corresponds to the "good" channel condition, where the header packet is received error-free. State s_f corresponds to the "bad" channel condition in which the header packet is lost due to channel-introduced errors; 3) *header collision state* (c) — state in which a header packet is lost due to more than one simultaneous header packet transmissions in a slot; 4) *busy channel states* (b_s, b_f) — states in which a data packet is being transmitted in a slot. Again, this corresponds to two distinct states, b_s and b_f, similar to the single header transmission states.

The expressions for various state transition probabilities are first derived, which are then used to solve for the steady state probability vector, π, to estimate the throughput. The different states are ordered according to the vector

$$(\; i \quad c \quad s_s \quad s_f \quad b_s \quad b_f \;), \quad (14)$$

and the corresponding steady state probabilities are ordered as

$$(\; \pi_1 \quad \pi_2 \quad \pi_3 \quad \pi_4 \quad \pi_5 \quad \pi_6 \;). \quad (15)$$

Let p_{ij}, $1 \le i, j \le 6$, be the state transition probabilites according to the above ordering. In deriving these transition probabilities, the following assumptions are made on the message arrival process and message length distributions: 1) the message arrival process is Poisson with G message arrivals per time slot, and 2) the message lengths (measured in integer number of packets) follow a geometric distribution with parameter g_m and pdf

$$P[\text{message length} = x] = \begin{cases} g_m(1-g_m)^{x-1} & x = 1, 2, 3, \ldots \\ 0 & \text{otherwise.} \end{cases}$$

Let p_0 be the probability that there are no message arrivals in a slot, p_1 be the probability of a single message arrival in a slot, and P_E be the steady-state packet error probability given by Eqn.(11). The state transition probability matrix, M, can then be derived as

$$M = \begin{bmatrix} X_0 & X_1 & X_2 & X_3 & 0 & 0 \\ X_0 & X_1 & X_2 & X_3 & 0 & 0 \\ 0 & 0 & 0 & 0 & p & 1-p \\ X_0 & X_1 & X_2 & X_3 & 0 & 0 \\ g_m X_0 & g_m X_1 & g_m X_2 & g_m X_3 & X_4 & X_5 \\ g_m X_0 & g_m X_1 & g_m X_2 & g_m X_3 & X_6 & X_7 \end{bmatrix}, \quad (16)$$

where $X_0 = p_0$, $X_1 = 1 - p_0 - p_1$, $X_2 = p_1(1-P_E)$, $X_3 = p_1 P_E$, $X_4 = (1-g_m)p$, $X_5 = (1-g_m)(1-p)$, $X_6 = (1-g_m)(1-q)$, and $X_7 = (1-g_m)q$. Note that for a Poisson arrival process, $p_0 = e^{-G}$ and $p_1 = Ge^{-G}$. The steady state probability vector, π, is obtained by solving the equation

$$\pi = \pi M, \quad (17)$$

and using the additional conservation relationship

$$\sum_j \pi_j = 1. \quad (18)$$

Working through the steady state analysis of the system, we obtain the expression, in closed-form, for the steady state channel throughput of the proposed protocol, as

$$\eta = \frac{g_m X_2}{g_m + X_2}\left[1 + \frac{g_m p + (1-g_m)(1-q)}{g_m\{1 + (1-g_m)(1-p-q)\}}\right]. \quad (19)$$

Note that, since $g_m X_2/(g_m + X_2)$ a monotonically increasing function of X_2, and since X_2 is the only quantity in Eqn.(19) which depends on G, the value of G which maximizes X_2 also maximizes the throughput. This optimal value of the offered load can be easily found to be $G = 1$, regardless of the values of p, q and g_m. In the above, successful header packet transmissions are considered to be useful in the throughput computation, so that $\eta = \pi_3 + \pi_5$, where π_3 and π_5 are the steady-state probabilities of states s_s and b_s, respectively. The protocol throughput for an i.i.d channel model can be derived by setting $1 - p = q = P_E$ in Eqn.(19), to obtain

$$\eta_{iid} = \frac{X_2[g_m + (1-P_E)]}{g_m + X_2}. \quad (20)$$

4.1 Enhanced Protocol with Error-Detect Feature

In the protocol described and analyzed above (call it the *basic protocol*), we allow the mobile to continuously transmit all the packets in a message even when a packet or a series of packets in the busy state are lost due to fading. However, the memory in the fading process can be exploited by using the knowledge about the channel status to modify the data transmission strategy. For example, under slow fading conditions (where events in successive slots are expected to be highly correlated), the fact that the packet in the current slot is in error implies that the packet in the subsequent slot will also be in error with high probability. Therefore, when a mobile detects a bad channel condition while transmitting, it chould decide to release the channel, avoiding the occurrence of many errors and allowing other users (which may, on the other hand, experience good channel conditions) to transmit.

To investigate how this idea can be used to enhance the protocol performance, we analyze a simple, modified version of the basic protocol. We refer to the modified scheme as the protocol with an error-detect (ED) feature. According to the *ED protocol*, if a packet is received in error during the busy state, the mobile terminates the ongoing data transmission, thereby allowing the other mobiles to access the channel. Note that the transition probability matrix for the ED protocol, M', will be the same as that of the basic protocol (matrix M, given by (Eqn.16)), except for the transition probabilities from state b_f. In fact, the transition probabilities from state b_f will be same as those from the idle state. Accordingly, the transition probability matrix for the ED protocol is given by

$$\mathrm{M}' = \begin{bmatrix} X_0 & X_1 & X_2 & X_3 & 0 & 0 \\ X_0 & X_1 & X_2 & X_3 & 0 & 0 \\ 0 & 0 & 0 & 0 & p & 1-p \\ X_0 & X_1 & X_2 & X_3 & 0 & 0 \\ g_m X_0 & g_m X_1 & g_m X_2 & g_m X_3 & X_4 & X_5 \\ X_0 & X_1 & X_2 & X_3 & 0 & 0 \end{bmatrix}. \quad (21)$$

The throughput expression for the ED protocol is derived as

$$\eta_{ed} = \frac{X_2(1 + g_m p)}{1 - (1 - g_m)p + X_2}, \quad (22)$$

which, again, is maximum for $G = 1$.

5 Results and Discussion

The throughput performance of the protocol obtained from Eqns. (19), (20), and (22) is plotted vs. the message arrival rate, G, in Figure 1. A g_m value of 0.1, corresponding to an average message length of 10 packets per message (not including the header) is used. Plots are shown for fading margins, F, of 5 dB and 10 dB. For the basic and ED protocols, a normalized Doppler bandwidth, $f_D T$, of 0.02 (representing slow fading) is chosen. The extreme case of i.i.d packet errors with the same marginal error rate is also reported for comparison. The effect of varying normalized Doppler bandwidths, fading margins, and average message lengths are illustrated in the subsequent graphs (Figs. 2, 3, 4). The solid lines represent the analytical results obtained from the throughput expressions, whereas the markers represent the simulation points. The simulation results are obtained by explicit simulation of the protocol on a flat Rayleigh fading channel, which is simulated using the method proposed by Jakes [5].

From Figure 1, we observe the following. As was analytically computed, for both protocol schemes (basic/ED) and both channel models (slow fading/i.i.d), the maximum achievable throughput occurs when $G = 1$, and this gives good robustness against either possible variations or wrong estimates of the channel parameters. Even with a small fading margin of 5 dB, the basic protocol offers a maximum throughput of about 0.69. This is very good, considering that for that value of F, the marginal packet error rate is $P_E \simeq 0.27$, and therefore the throughput cannot be larger than 0.73. The ED protocol is found to perform even better than the basic protocol (e.g., maximum throughput of 0.75 for the ED protocol against 0.69 for the basic protocol). This was expected, because the fade rate considered is small (i.e., $f_D T = 0.02$), and the basic protocol allows the mobile to transmit all the packets in

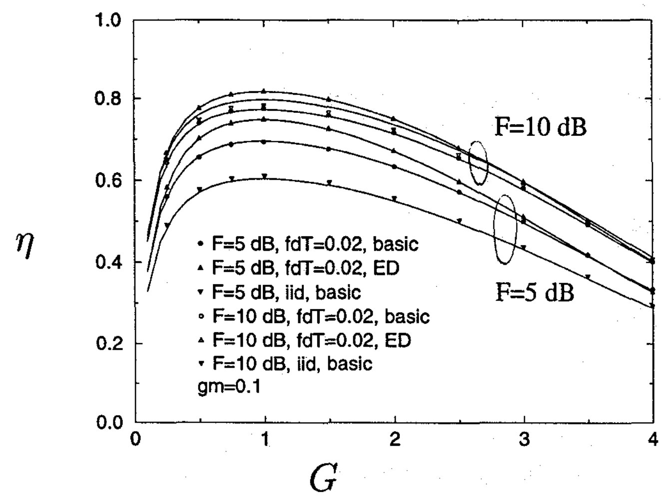

Figure 1: Throughput *vs* message arrival rate (G). $g_m = 0.1$.

a message without any break, even under deep fade situations. On the other hand, if a mobile is in a deep fade during the busy state, the ED protocol releases the channel from that mobile, thereby allowing other mobiles to access the channel. This also explains why the throughput of the ED protocol can be larger than the steady-state packet error rate. When the fade margin is 10 dB, the maximum throughput for the basic and the ED protocols are 0.8 and 0.82, respectively, showing a tendency of diminishing performance gain. For very high fade margins, the basic and ED protocols perform almost the same. However, under slow fading conditions, over a practical range of fade margins (5 dB - 10 dB), the ED protocol shows a noticeable improvement over the basic protocol.

A comparison between the performance predictions of an i.i.d channel model and the Markov fading model is made from Figure 1. It is seen that the i.i.d model provides a conservative estimate of the throughput performance compared to the Markov model under slow fading conditions ($f_D T = 0.02$). For fast fading conditions ($f_D T = 2$), however, both the i.i.d and Markov models tend to perform similarly. It is further noted that the first-order Markov approximation of the fading process is quite accurate, since the results obtained through both analysis (using the Markov approximation) and simulation (performed without the approximation) closely agree.

Figure 2 illustrates the effect of varying $f_D T$ on the maximum achievable throughput for both the basic and ED protocols at $g_m = 0.1$ and $G = 1$. As stated earlier, the ED protocol performs better than the basic protocol for small values of $f_D T$ (e.g., < 0.04 for $F = 10$ dB), whereas, at high values of $f_D T$ (i.e., for weaker correlation between packet errors) the basic protocol performs better than the ED protocol. This performance crossover is due to the fact that at high values of $f_D T$, each data packet during the busy state would experience a nearly i.i.d success/failure process, and terminating the data transmission in the event of a single failure event (as is done in ED protocol) reduces the average number of success events during the busy period. This observation suggests that an adaptive use of both the basic and the ED protocols, given some knowledge about the fade rate and margin, would be advantageous. Also, note that in the ED protocol, a decision to terminate an ongoing data transmission is triggered by a single

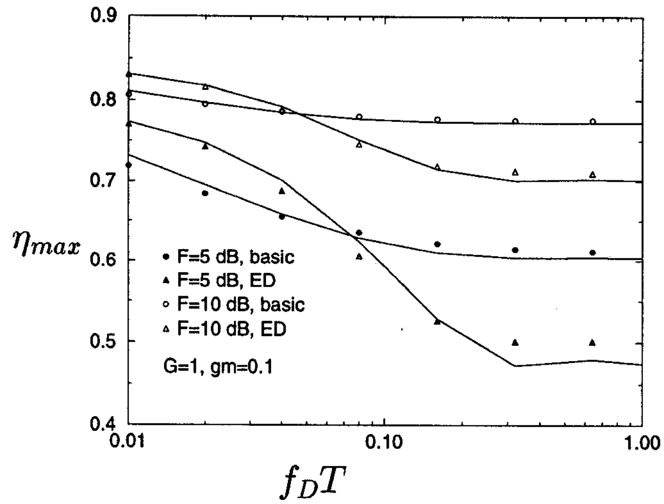

Figure 2: Maximum throughput *vs* normalized Doppler bandwidth, $f_D T$. $G = 1$. $g_m = 0.1$.

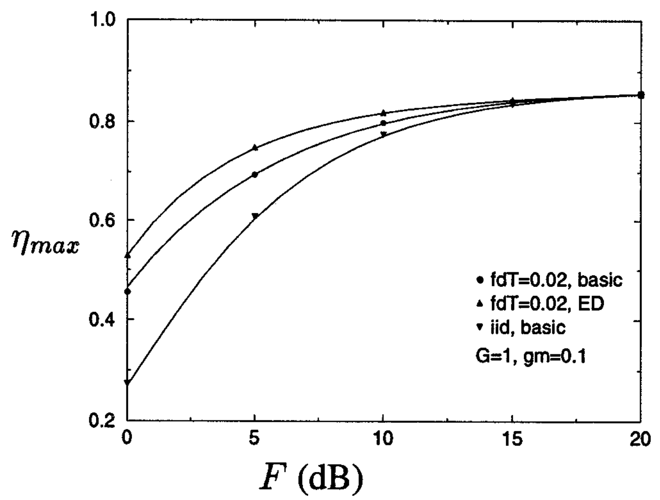

Figure 3: Maximum throughput *vs* fading margin, F, in dB. $G = 1$. $g_m = 0.1$.

failure event in the busy state. However, it is possible to consider more than one success/failure event in a row to make a decision as to whether to continue or terminate the transmission. Such a scheme is expected to perform better under fast fading conditions.

Figure 3 shows the effect of varying the fade margin on the throughput perfomance. At low fade margins, there is marked difference between the performance of the different protocols. However, for fade margins greater than 15 dB, the performance of all the schemes tends to be the same. Figure 4 shows the effect of increasing the average message length ($1/g_m$) on the throughput. It can be seen that the channel utilization improves for smaller values of g_m, i.e., for longer message lengths. This suggests that the protocol is suitable for messaging applications like file transfers, etc. However, the increased throughput for large message sizes comes at the expense of increased delay performance of the protocol, which needs to be studied.

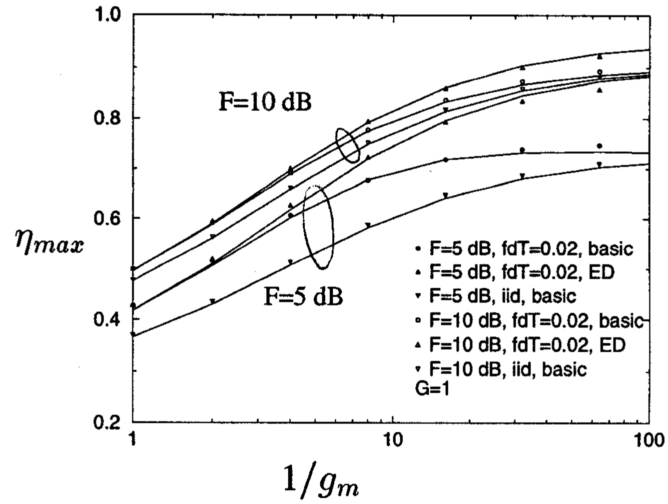

Figure 4: Maximum throughput *vs* average message length, $1/g_m$. $G = 1$. $g_m = 0.1$.

6 Conclusions

We analyzed the throughput performance of a wireless media access protocol, taking into account the effect of channel fading. A first-order Markov model was used to describe the Rayleigh fading process. A closed-form expression for the throughput was derived by modeling the system as a finite-state, discrete-time, Markov chain. The analytical results obtained through the first-order Markov approximation of the channel were compared to those obtained from an i.i.d channel model. The Markovian fading channel model was shown to provide better performance results than the i.i.d channel model. Simulations also showed that a first-order Markov approximation of the fading process is quite accurate. For slow fading channels, an enhanced scheme, which took advantage of the channel memory, was proposed and analyzed. Further directions of research include a delay analysis, the consideration of other versions of the protocol with a larger degree of memory, and the effect of feedback errors and capture.

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