Parallel Interference Cancellation in Multicarrier DS-CDMA Systems

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***Abstract*— In this paper, we present and analyze the performance of a parallel interference cancellation (PIC) scheme for multicarrier (MC) direct-sequence code-division multiple-access (DS-CDMA) systems. At each cancellation stage in the proposed PIC scheme, on each subcarrier, a weighted sum of the soft outputs of the other users in the current stage is cancelled from the soft output of the desired user (rather than making hard bit decisions of the other users and regenerating and canceling the interfering signals) to form the input to the next stage. At the last stage, the interference cancelled outputs from all the subcarriers are maximal ratio combined (MRC) to form the decision statistic. The scheme has the advantage of not requiring the amplitude estimates of the other users. We derive analytical expressions for the bit error rate (BER) at different stages in the proposed PIC scheme on Rayleigh fading channels. Analytical results are found to agree well with the simulation results. The results show that the proposed PIC scheme offers better interference suppression capability than the conventional matched filter receiver. We also obtain bounds on the coded BER of the proposed PIC scheme for a convolutionally coded multicarrier DS-CDMA system where the PIC-MRC output feeds a soft decision Viterbi decoder.**

Keywords – Parallel interference cancellation, multicarrier DS-CDMA, fading channels.

I. INTRODUCTION

Multicarrier (MC) approach offers several advantages including robustness in fading and interference, operation at lower chip rates and non-contiguous bandwidth operation. Several studies have analyzed the performance of multicarrier DS-CDMA systems [1]-[2]. In this paper, we consider parallel interference cancellation (PIC) for multicarrier DS-CDMA. PIC schemes for multicarrier DS-CDMA have been studied in [4]-[5]. These studies consider hard decision PIC receivers, where hard bit decisions are made on the output of the matched filters (MF), which are then is used to regenerate and cancel the multiple access interference (MAI) in parallel. Our contribution in this paper is that we propose and analyze a PIC scheme which directly uses the soft output of the MFs (rather than the hard bit decisions) for cancellation and does not require the estimation of the amplitudes of the users.

In the proposed PIC scheme, at each cancellation stage, on each subcarrier, a weighted sum of the soft outputs of the other users in the current stage is subtracted from the soft output of the desired user (rather than making hard bit decisions of the other users and regenerating and canceling the MAI) to form the input to the next stage. At the last stage, the interference cancelled outputs from all the subcarriers are maximal ratio combined (MRC) to form the decision statistic. We derive analytical expressions for the bit error rate (BER) for the first and second stages in the proposed PIC scheme on Rayleigh fading channels. Analytical results are found to agree well with the simulation results. The proposed PIC scheme is shown to offer better performance than the conventional MF receiver. We also obtain bounds on the coded BER of the proposed PIC scheme for a convolutionally coded multicarrier DS-CDMA system where the PIC-MRC output feeds a soft decision Viterbi decoder.

II. SYSTEM MODEL

We consider a K-user synchronous DS-CDMA system. Each user employs a multicarrier modulator, where the transmit data sequence multiplied by a spreading sequence modulates M subcarriers, as shown in Fig. 1. The transmitted signal for the kth user is given by

$$s_k(t) = \sqrt{2E_{ck}} \sum_{i=-\infty}^{\infty} b_k^i a_k(t-iT) \sum_{m=1}^{M} \cos(\omega_m t + \theta_{k,m}), \tag{1}$$

where E_{ck} is the transmitted energy-per-chip for the kth user (assumed to be same for all users), b_k^i is the transmitted bit of the kth user, T is one bit duration, M is the number of subcarriers, ω_m is the mth subcarrier frequency, $\theta_{k,m}$ is the phase of the mth subcarrier of the kth user which is uniformly distributed over $[0, 2\pi)$, and $a_k(t)$ is the signature waveform of user k, given by

$$a_k(t) = \sum_{n=0}^{N-1} c_k^{(n)} h(t - nMT_c), \tag{2}$$

where $c_k^{(n)}$ is the spreading sequence of the kth user, N is the processing gain, $h(t)$ is the impulse response of the chip wave shaping filter assumed to satisfy the constraint $\int_{-\infty}^{\infty} |H(f)|^2 df = 1$, and $T = NMT_c$ where T_c is one chip duration in a single carrier ($M = 1$) system. The channel in each subband is assumed to be slow-varying, flat Rayleigh channel with transfer function $\zeta_{k,m} = \alpha_{k,m}\exp(j\beta_{k,m})$, where $\{\alpha_{k,m}\}$ are i.i.d Rayleigh random variables with unit second moment, and $\{\beta_{k,m}\}$ are i.i.d uniform random variables over $[0, 2\pi)$, for both user k and subcarrier m. The received signal is given by

$$r(t) = \sum_{k=1}^{K} \sqrt{2E_{ck}} \sum_{i=-\infty}^{\infty} b_k^i a_k(t-iT) \sum_{m=1}^{M} \alpha_{k,m} \cos(\omega_m t + \theta'_{k,m}) + n_w(t), \quad (3)$$

where $\theta'_{k,m} = \theta_{k,m} + \beta_{k,m}$ and $n_w(t)$ is white Gaussian noise with psd of $\eta_0/2$.

A. PIC Receiver Structure

The proposed L-stage PIC receiver for multicarrier DS-CDMA is shown in Figs. 2, 3. Fig. 2 shows the multiuser multistage PIC demodulator on the mth subcarrier (there are M such demodulators, one on each subcarrier), and Fig. 3 shows the maximal ratio combining (of outputs from all subcarriers of the kth user) and bit decision for the kth user.

III. Performance Analysis

In this section, we derive analytical expressions for the BER at the output of the 2nd- and 3rd-stages of the PIC receiver. The lowpass filter output of the mth subcarrier for the kth user, $L_{k,m}(t) = \text{LPF}\{r'_m(t)\sqrt{2}\cos(\omega_m t + \theta'_{k,m})\}$, is given by

$$L_{k,m}(t) = \sum_{k'=1}^{K} \sqrt{E_{ck'}} \sum_{i=-\infty}^{\infty} b_{k'}^i \alpha_{k',m} \cos(\theta'_{k',m} - \theta'_{k,m}) g_{k'}(t-iT) + n^0_{k,m}(t) \quad (4)$$

where

$$g_{k'}(t) = \sum_{n=0}^{N-1} c_{k'}^{(n)} x(t - nMT_c), \quad (5)$$

$$n^0_{k,m}(t) = \text{LPF}\{n'_{wm}(t)\sqrt{2}\cos(\omega_m t + \theta'_{k,m})\}, \quad (6)$$

$x(t) = F^{-1}|H(f)|^2$, and $r'_m(t)$ and $n'_{wm}(t)$ are respectively, $r(t)$ and $n_w(t)$ after passing through the mth bandpass filter. The power spectral density of $n^0_{k,m}(t)$ is $\eta_0/2$.

Without any loss of generality, in the following we analyze the system decision for the ith bit of the kth user. Let $Z^{(j)}_{k,m}(i)$ denote the soft output of the ith bit of the kth user ($k = 1, 2, \cdots, K$) on the mth subcarrier ($m = 1, 2, \cdots, M$) of the jth stage ($j = 1, 2, \cdots, L$) of the PIC receiver. The soft output of the ith bit of the kth user on the mth subcarrier of the 1st stage, $Z^{(j)}_{k,m}(i)$, is given by

$$Z^{(1)}_{k,m}(i) = \frac{1}{N} \sum_{n=0}^{N-1} c_k^{(n)} L_{k,m}(iT + nMT_c). \quad (7)$$

The output $Z^{(1)}_{k,m}(i)$ consists of the desired signal component, MAI and noise components, and can be written as

$$Z^{(1)}_{k,m}(i) = S_{k,m}(i) + I_{k,m}(i), \quad (8)$$

where

$$S_{k,m}(i) = \sqrt{E_{ck}}\, b_k^i\, \alpha_{k,m} \quad (9)$$

$$I_{k,m}(i) = \frac{1}{N} \sum_{k' \neq k} V_{k'k,m}(i)\, \rho_{k'k} + n_{k,m}(i) \quad (10)$$

$$V_{k'k,m}(i) = S_{k',m}(i) \cos(\theta'_{k',m} - \theta'_{k,m}) \quad (11)$$

$$\rho_{k'k} = \sum_{n=0}^{N-1} c_k^{(n)} \sum_{n'=0}^{N-1} c_{k'}^{(n')} x((n-n')MT_c) \quad (12)$$

$$n_{k,m}(i) = \frac{1}{N} \sum_{n=0}^{N-1} c_k^{(n)} n^0_{k,m}(iT + nMT_c), \quad (13)$$

and the variance of $n_{k,m}(i)$ is $\eta_0/2N$.

A. 2nd Stage Output Statistics of the m-th Subcarrier

The soft outputs from the 1st stage, $Z^{(1)}_{k,m}(i)$, are fed as input to the 2nd stage where cancellation is performed. The objective of the interference cancellation is to estimate the MAI of the kth user in parallel and cancel the estimate from the kth user's total signal. It is noted that, from (10), we can compute the MAI as long as we have the knowledge of $V_{k'k,m}(i)$ and $\rho_{k'k}$. Given the knowledge of the spreading sequences of all the users, $\rho_{k'k}$ can be determined. Thus the problem to estimate the MAI of the kth user is to find an accurate enough estimate of $V_{k'k,m}(i)$. An estimate of $V_{k'k,m}(i)$, though imperfect, may be obtained by multiplying $Z^{(1)}_{k',m}(i)$ by a factor $G^{(1)}_{k',m}$, i.e.,

$$\widehat{V}_{k'k,m}(i) = Z^{(1)}_{k',m}(i)\, G^{(1)}_{k',m}, \quad (14)$$

where $G^{(1)}_{k',m}$ is given by

$$G^{(1)}_{k',m} = \frac{1}{N} \cos(\theta'_{k',m} - \theta'_{k,m})\, \rho_{k'k}. \quad (15)$$

Using the approximate estimate of $V_{k'k,m}(i)$ in (14), the interference cancelled output of the 2nd stage, $Z^{(2)}_{k,m}(i)$, can be obtained as

$$Z^{(2)}_{k,m}(i) = Z^{(1)}_{k,m}(i) - \sum_{k'=1,\, k' \neq k}^{K} \widehat{V}_{k'k,m}(i). \quad (16)$$

Note that in the cancellation process, the kth user signal component remains unaltered whereas the interference component is altered. The statistics of the interference and noise terms at the 2nd stage output is of interest. Eqn. (16) can be written as

$$Z^{(2)}_{k,m}(i) = \sqrt{E_{ck}}\, b_k^i\, \alpha_{k,m} + W^{(2)}_{k,m}(i), \quad (17)$$

where

$$W^{(2)}_{k,m}(i) = n_{k,m}(i) - \frac{1}{N} \sum_{k'=1, k' \neq k}^{K} \cos(\theta'_{k',m} - \theta'_{k,m})\, \rho_{k'k} I_{k,m}(i). \quad (18)$$

For large K, $I_{k,m}(i)$ can be approximated as a Gaussian r.v. with zero mean and variance

$$Var\{I_{k,m}(i)\} = \sum_{k' \neq k} \frac{E_{ck}}{2N^2}\rho_{k'k}^2 + \frac{\eta_0}{2N}. \quad (19)$$

Therefore, the variance of the interference and noise at the 2nd stage output for the kth user, $\eta_{k,m}^{(2)}$, can be obtained as

$$\eta_{k,m}^{(2)} = \frac{\eta_0}{2N} + \sum_{k'=1, k' \neq k}^{K} \frac{1}{2N^2}\rho_{k'k}^2 \, Var\{I_{k,m}(i)\}. \quad (20)$$

B. 3rd Stage Output Statistics of the m-th Subcarrier

In the 3rd stage, we essentially try to cancel the interference caused by the 'imperfect' cancellation in the 2nd stage. As we did in the 2nd stage cancellation, here also we obtain an imperfect estimate of the interference term and use it in the cancellation. Further, in the 3rd stage cancellation this estimate must be added (not subtracted) to the previous stage output in order to the cancel the negative imperfect cancellation term introduced in the 2nd stage cancellation. Accordingly, the interference cancelled output of the kth user at the 3rd stage, $Z_{k,m}^{(3)}(i)$, can be written as

$$\begin{aligned} Z_{k,m}^{(3)}(i) &= Z_{k,m}^{(2)}(i) + \sum_{k'=1}^{K} Z_{k',m}^{(2)}(i) \, G_{k',m}^{(2)}(i) \quad (21) \\ &= \sqrt{E_{ck}} \, b_k^i \, \alpha_{k,m} + W_{k,m}^{(3)}(i), \end{aligned}$$

The variance of the interference and noise at the 3rd stage output for the kth user, $\eta_{k,m}^{(3)}$, is given by $\eta_{k,m}^{(3)} = Var\{W_{k,m}^{(3)}(i)\} = E\{W_{k,m}^{(3)}(i)\}^2$ and can be obtained using the Gaussian approximation.

C. Maximal Ratio Combining and BER

The last stage (stage-L) outputs of the M subcarrier demodulators of the kth user are maximal ratio combined and a hard bit decision is made on the combined output a shown in Fig. 3. If we assume perfect knowledge of the channel fading gain of the kth user, the output of the PIC followed by the MRC is given by

$$Z_k = \sum_{m=1}^{M} g_{k,m} \, Z_{k,m}^{(L)}, \quad (22)$$

where, in order to maximize the signal-to-noise ratio [3], we set $g_{k,m}$ to be

$$g_{k,m} = \frac{E\{Z_{k,m}^{(L)} | \alpha_{k,m}\}}{Var\{Z_{k,m}^{(L)} | \alpha_{k,m}\}}. \quad (23)$$

The average bit-error probability is then given by

$$P_e = \int_0^\infty Q(\sqrt{2x}) f_{\gamma_k}(x) dx, \quad (24)$$

a closed-form solution of which can be obtained as [3]

$$P_e = \left[\frac{1}{2}(1-\mu)\right]^M \sum_{m=0}^{M-1} \binom{M-1+k}{k} \left[\frac{1}{2}(1+\mu)\right]^m, \quad (25)$$

where $\mu = \sqrt{\frac{\overline{\gamma}}{1+\overline{\gamma}}}$ and $\overline{\gamma}$ is the average SNR per subcarrier.

D. Coded PIC Scheme

Now, we extend the proposed PIC scheme to a convolutionally coded multicarrier DS-CDMA system and study its performance. We consider a convolutionally coded multicarrier DS-CDMA system, where the user data bits are input to a rate-$1/R$ convolutional encoder which generates R coded symbols for each data bit. The output code symbols are interleaved and serial-to-parallel converted such that the R parallel code symbols may be transmitted simultaneously. Then, each of the R code symbols multiplied by the user-specific spreading sequence modulates M subcarriers, which results in RM subcarriers.

The PIC receiver for the above coded multicarrier system employs a bank of RM PIC demodulators Note that in the symbol demapper, a total of RM subcarriers are remapped into R subcarrier groups, and each group consists of M subcarriers which carries the same code symbol. In the demapper, the outputs from subcarriers which carry the same code symbol are combined. The output of the demapper is parallel-to-serial converted, and deinterleaved. The output of the deinterleaver is then fed to the soft decision Viterbi decoder.

The cancellation process in the L-stage PIC demodulator is the same as that for the uncoded system shown in Figs. 2(a),(b), except that code symbols are considered here instead of uncoded data bits. Also, the MRC and hard bit decision as shown in Fig. 2(c) are not performed here. Instead, the RM soft outputs corresponding to the RM subcarriers at the last cancellation stage is fed to the demapper, where the outputs from subcarriers which carry the same code symbol are combined as follows:

$$Z_k^n(i) = \sum_{m \in \{A_n\}} \alpha_{k,m} \, Z_{k,m}^{(L)}(i), \quad (26)$$

where $Z_k^n(i)$, $n = 1, \cdots, R$, is the demapper output corresponding to the nth convolutionally coded symbol, $\{A_n\} \triangleq \{n, n+R, ..., n+(M-1)R\}$ represents the set of subcarriers which carry the nth coded symbol.

Let the data block length be P bits. The soft outputs from the demapper/deinterleaver, $Z_k^n(i)$, $n = 1, \cdots, R$, $i = 1, \cdots, P$ are fed as input to the soft decision Viterbi decoder.

Suppose the decoder chooses that path which has the maximum path metric, where the branch metrics, $\mu_i^{(r)}$, and path metrics of the rth path, $U^{(r)}$, respectively, are defined as

$$\mu_i^{(r)} = \sum_{n=1}^{R} Z_k^n(i)\, d_{n,i}^{(r)}, \quad \text{and} \quad U^{(r)} = \sum_{i=1}^{P} \mu_i^{(r)}, \qquad (27)$$

where $d_{n,i}^{(r)}$ represents the nth coded symbol of the rth trellis path at time index i, then, with sufficient interleaving of the coded symbols, an upper bound on the coded BER performance can be obtained using the transfer function of the convolutional code used, as [2]

$$P_e < \frac{dT(D_1, ..., D_M, B)}{dB}\Big|_{B=1, D_n = \prod_{m \in \{A_n\}} \frac{1}{(1+\overline{\gamma}_c)}} \qquad (28)$$

where $\overline{\gamma}_c \triangleq E_{ck}/2\eta_k^{(L)}$, and $T(D_1, D_2, ..., D_M, B)$ is the transfer function of the convolutional code used.

IV. Performance results

In this section, we present the numerical results of the bit error rate performance of the proposed PIC receiver for multicarrier DS-CDMA, both for the uncoded as well as coded schemes. We have assumed perfect channel estimation at the receiver. We compare the BER performance at various stages of the PIC (i.e., $L = 1, 2, 3$). Fig. 4 shows the BER performance of the PIC receiver as a function of the number of users (K), at various stages of the PIC ($L = 1, 2, 3$) for the uncoded system. The figure shows the plots corresponding to both the analytical results evaluated through Eqn. (25), as well as the results obtained through bit error simulations. In the simulations, random binary sequences are used for spreading. From Fig. 4, we observe that the proposed PIC receiver with 2 stages ($L = 2$) gives significant improvement in the BER performance compared to the conventional MF-MRC receiver (i.e., $L = 1$). This improvement is further enhanced when one more additional stage of cancellation ($L = 3$) is used.

Fig. 5 compares the BER performance of the uncoded system for different values of the number of subcarriers (M) and processing gains (N), at the 2nd ($L = 2$) and 3rd ($L = 3$) stages of the PIC. As expected, for a given number of PIC stages, the performance improves as the number of subcarriers is increased, because of the frequency diversity benefit offered by the multicarrier approach.

The BER performance of the PIC for the convolutionally coded system is shown in Figs. 6 and 7. In Fig. 6, upper bounds on the coded BER evaluated from Eqn. (28) are plotted for rate $1/2, 1/4$, and $1/8$ convolutional codes (i.e., $R = 2, 4, 8$) and different number of subcarriers ($M = 2, 4, 8$) . Three different systems all having same system bandwidth with a total number of 8 subcarriers (i.e, $RM = 8$), but with different combinations of R and M, namely ($R = 2, M = 4$), ($R = 4, M = 2$) and ($R = 8, M = 1$), are considered. For all these three systems, the BER performance at various stages of the PIC ($L = 1, 2, 3$) are also shown. It is observed that for all the three coded systems, the proposed PIC receiver provides significantly improved BER performance compared to the conventional MF-MRC ($L = 1$) receiver. Also, for a given number of PIC stages, the performance of the ($R = 2, M = 4$), ($R = 4, M = 2$) and ($R = 8, M = 1$) systems are similar, with the lowest rate code ($R = 8, M = 1$) system performing slightly better. A larger M implies a smaller R to keep the system bandwidth constant, where the frequency diversity benefit is more (due to large M) but the time diversity benefit due to a high rate code (i.e., small R) is less. On the other hand, a low rate code (large R) implies more time diversity benefit and less frequency gain (due to small M). Thus, to achieve a desired performance in a given system bandwidth, Fig. 6 points to a possible complexity tradeoff between systems having different values of R and M.

In Fig. 7, the theoretical upper bounds on coded BER are compared with simulation BER results for various stages of cancellation for $R = 2$, $M = 1$, $N = 32$, and $E_b/\eta_0 = 15$ dB. The analytical results are found to reasonably agree with the simulation results. Thus, the proposed PIC receiver effectively cancels the MAI and improves the BER performance both in uncoded and coded systems.

V. Conclusion

We presented and analyzed the performance of a parallel interference cancellation (PIC) scheme for multicarrier DS-CDMA systems. The scheme has the advantage of not requiring the amplitude estimates of the other users. We derived analytical expressions for the bit error rate (BER) at different stages in the proposed PIC scheme on Rayleigh fading channels, both for uncoded as well as convolutionally coded systems. Analytical results were shown to agree well with the simulation results. The proposed PIC receiver was shown to effectively cancel the MAI and significantly improve the BER performance compared to conventional MF-MRC receiver, both for uncoded as well as coded systems.

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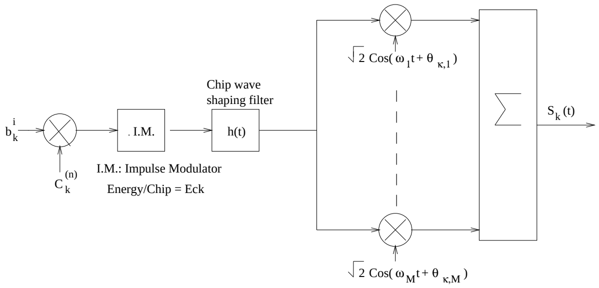

Fig. 1. Multicarrier DS-CDMA Transmitter of kth user

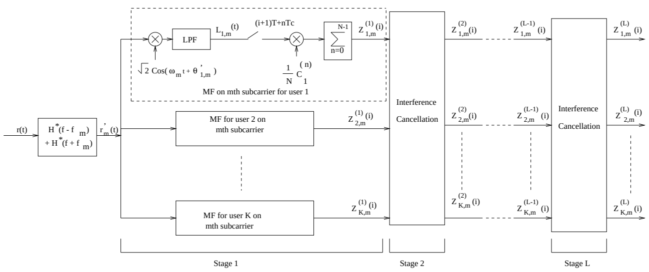

Fig. 2. L-stage PIC demodulator on the mth subcarrier

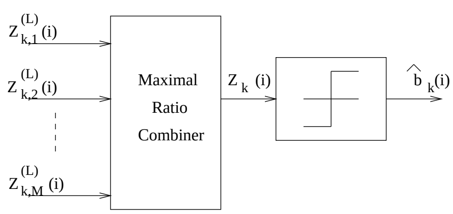

Fig. 3. MRC and bit decision for the kth user

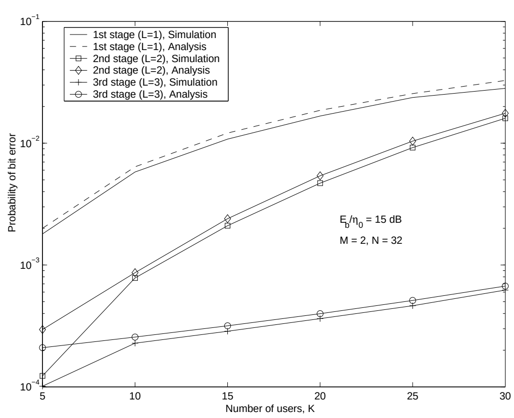

Fig. 4. Probability of bit error, P_e, versus number of users, K, at different stages of the PIC receiver for the uncoded system. Analysis and simulation plots. $M = 2$, $N = 32$, $E_b/\eta_0 = 15$ dB.

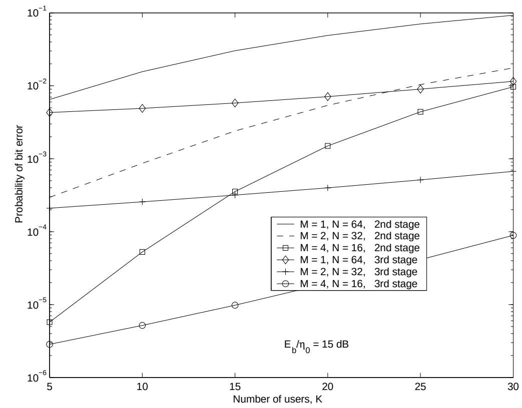

Fig. 5. Probability of bit error, P_e, versus number of users, K, at different stages of the PIC for the uncoded system with different values of M ($M = 1, N = 64; M = 2, N = 32; M = 4, N = 16$). Analysis plots only. $E_b/\eta_0 = 15$ dB.

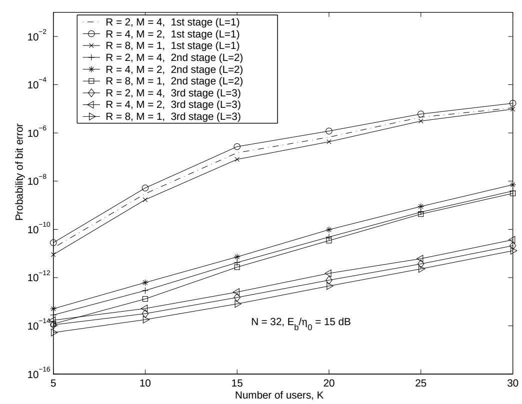

Fig. 6. Upper bound on the coded BER versus number of users, K, at different PIC stages of the convolutionally coded system with different combinations of code rate ($1/R$) and number of subcarriers (M). $N = 32$, $E_b/\eta_0 = 15$ dB.

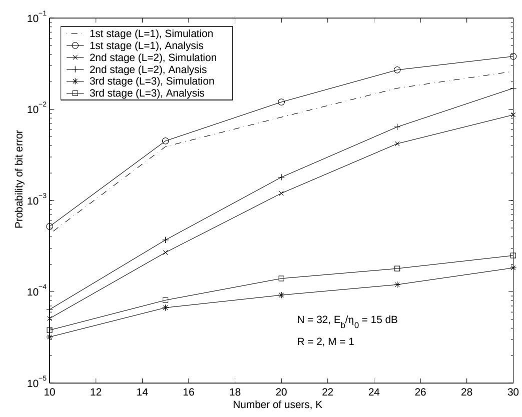

Fig. 7. Comparison of the analytical bounds on the coded BER versus the simulated BER at different PIC stages of the convolutionally coded system. $R = 2$, $M = 1$, $N = 32$, $E_b/\eta_0 = 15$ dB.