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Single-Symbol ML Decodable Distributed STBCs for Partially-Coherent Cooperative Networks

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***Abstract*—Space-time block codes (STBCs) that are single-symbol decodable (SSD) in a co-located multiple antenna setting need not be SSD in a distributed cooperative communication setting. A relay network with N relays and a single source-destination pair is called a partially-coherent relay channel (PCRC) if the destination has perfect channel state information (CSI) of all the channels and the relays have only the phase information of the source-to-relay channels. In this paper, first, a new set of necessary and sufficient conditions for a STBC to be SSD for co-located multiple antenna communication is obtained. Then, this is extended to a set of necessary and sufficient conditions for a distributed STBC (DSTBC) to be SSD for a PCRC, by identifying the additional conditions. Using this, several SSD DSTBCs for PCRC are identified among the known classes of STBCs. It is proved that even if a SSD STBC for a co-located MIMO channel does not satisfy the additional conditions for the code to be SSD for a PCRC, single-symbol decoding of it in a PCRC gives full-diversity and only coding gain is lost.**

Keywords – *Cooperative communications, amplify-and-forward protocol, distributed STBC, single-symbol decoding.*

I. Introduction

The problem of fading and the ways to combat it through spatial diversity techniques have been an active area of research. Multiple-input multiple-output (MIMO) techniques have become popular in realizing spatial diversity and high data rates through the use of multiple transmit antennas. For such co-located multiple transmit antenna systems low maximum-likelihood (ML) decoding complexity space-time block codes (STBCs) have been studied by several researchers [1]-[5], which include the well known complex orthogonal designs (CODs) and their generalizations. Recent research has shown that the advantages of spatial diversity could be realized in single-antenna user nodes through user cooperation [6],[7] via relaying. A simple wireless relay network of $N+2$ nodes consists of a single source-destination pair with N relays. For such relay channels, use of CODs [1],[2] has been studied in [8]. CODs are attractive for cooperative communications mainly because they admit very fast ML decoding (single-symbol decoding (SSD)). However, it should be noted that this property applies only to the decode-and-forward (DF) policy at the relay node. In a scenario where the relays amplify and forward (AF) the signal, it is known that the orthogonality is lost, and hence the destination has to use a complex multi-symbol ML decoding or sphere decoding [8],[9]. It should be noted that the AF policy is attractive for two reasons: i) the complexity at the relay is greatly reduced, and ii) the restrictions on the rate because the relay has to decode is avoided [10].

In order to avoid the complex ML decoding at the destination, in [11], the authors propose an alternative code design strategy and propose a SSD code for 2 and 4 relays. For arbitrary number of relays, recently in [12], distributed orthogonal STBCs (DOSTBCs) have been studied and it is shown that if the destination has the complete channel state information (CSI) of all the source-to-relay channels and the relay-to-destination channels, then the maximum possible rate is upper bounded by $\frac{2}{N}$ complex symbols per channel use for N relays. Towards improving the rate of transmission and achieving simultaneously both full-diversity as well as SSD at the destination, in this paper, we study relay channels with the assumption that the relays have the phase information of the source-to-relay channels and the destination has the CSI of all the channels.

The contributions of this paper can be summarized as follows: $1)$ First, a new set of necessary and sufficient conditions for a STBC to be SSD for co-located multiple antenna communication is obtained. $2)$ A set of necessary and sufficient conditions for a distributed STBC (DSTBC) to be SSD for a PCRC is obtained by identifying the additional conditions. Using this, several SSD DSTBCs for PCRC are identified among the known classes of STBCs for co-located multiple antenna system. $3)$ It is proved that even if a SSD STBC for a co-located MIMO channel does not satisfy the additional conditions for the code to be SSD for a PCRC, single-symbol decoding of it in a PCRC gives full-diversity and only coding gain is lost.

The rest of the paper is organized as follows: In Section II, the signal model for a PCRC is developed. Using this model, in Section III, a new set of necessary and sufficient conditions for a STBC to be SSD in a co-located MIMO is presented. Then, in Section IV, SSD DSTBCs for PCRC are characterized by identifying a set of necessary and sufficient conditions. Also, it is shown that SSD codes for co-located MIMO, under suboptimal SSD decoder for PCRC offer full diversity. Concluding remarks and discussion of simulation results constitute Section V.

II. System Model

Consider a wireless network with $N+2$ nodes consisting of a source, a destination, and N relays, as shown in Fig. 1. All nodes are half-duplex nodes, i.e., a node can either transmit or receive at a time on a specific frequency. We consider amplify-and-forward (AF) transmission at the relays. Trans-

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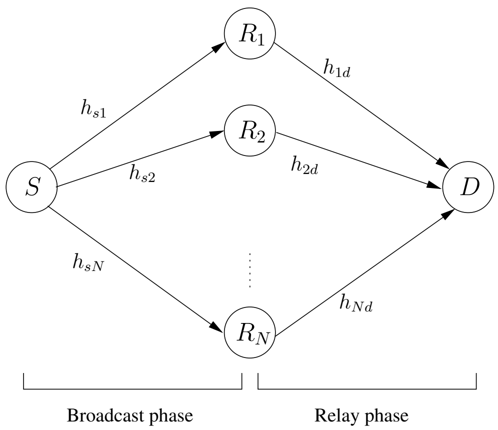

Fig. 1. A cooperative relay network with N relays.

mission from the source to the destination is carried out in two phases. In the first phase, the source transmits information symbols $x^{(i)}, 1 \leq i \leq T_1$ in T_1 time slots. All the N relays receive these T_1 symbols. This phase is called the *broadcast phase*. In the second phase (*relay phase*), all the N relays perform distributed space-time block encoding on their received vectors and transmit the resulting encoded vectors in T_2 time slots. We assume that the source-to-relay channels remain static over T_1 time slots, and the relay-to-destination channels remain static over T_2 time slots.

A. No CSI at the Relays

The received signal at the jth relay, $j = 1, \cdots, N$, in the ith time slot, $i = 1, \cdots, T_1$, denoted by $v_j^{(i)}$, can be written as[1]

$$v_j^{(i)} = \sqrt{E_1} h_{sj} x^{(i)} + z_j^{(i)}, \tag{1}$$

where h_{sj} is the complex channel gain from the source s to the jth relay, $z_j^{(i)}$ is additive white Gaussian noise at relay j with zero mean and unit variance, E_1 is the transmit energy per symbol in the broadcast phase, and $E\left[\left(x^{(i)}\right)^* x^{(i)}\right] = 1$. Under the assumption of no CSI at the relays, the amplified ith received signal at the jth relay can be written as $\widehat{v}_j^{(i)} = G\, v_j^{(i)}$, where $G = \sqrt{\frac{E_2}{E_1+1}}$ is the amplification factor at the relay that makes the total transmission energy per symbol in the relay phase to be equal to E_2 [8]. Let E_t denote the total energy per symbol in both the phases put together. Then, it is shown in [10] that the optimum energy allocation that maximizes the receive SNR at the destination is when half the energy is spent in the broadcast phase and the remaining half in the relay phase when the time allocations for the relay and broadcast phase are same, i.e., $T_1 = T_2$. We also assume that the energy is distributed equally, i.e., $E_1 = \frac{E_t}{2}$ and $E_2 = \frac{E_t}{2M}$, where M is the number of transmissions per symbol in the STBC. For the unequal-time allocation ($T_1 \neq T_2$) this distribution might not be optimal.

[1] We use the following notation: Vectors are denoted by boldface lowercase letters, and matrices are denoted by boldface uppercase letters. Superscripts T and $\mathcal{H}$ denote transpose and conjugate transpose operations, respectively and $*$ denotes conjugation operation. $\mathbf{j} = \sqrt{-1}$.

At relay j, a $2T_1 \times 1$ real vector $\widehat{\mathbf{v}}_j$ given by

$$\widehat{\mathbf{v}}_j = \left[\widehat{v}_{jI}^{(1)}, \widehat{v}_{jQ}^{(1)}, \widehat{v}_{jI}^{(2)}, \widehat{v}_{jQ}^{(2)}, \cdots, \widehat{v}_{jI}^{(T_1)}, \widehat{v}_{jQ}^{(T_1)}\right]^T, \tag{2}$$

is formed, where $\widehat{v}_{jI}^{(i)}$ and $\widehat{v}_{jQ}^{(i)}$, respectively, are the in-phase (real part) and quadrature (imaginary part) components of $\widehat{v}_j^{(i)}$. Now, (2) can be written in the form

$$\widehat{\mathbf{v}}_j = G\sqrt{E_1}\,\mathbf{H}_{sj}\,\mathbf{x} + \widehat{\mathbf{z}}_j, \tag{3}$$

where $\mathbf{x}$ is the $2T_1 \times 1$ data symbol real vector, given by $\mathbf{x} = [x_I^{(1)}, x_Q^{(1)}, x_I^{(2)}, x_Q^{(2)}, \cdots, x_I^{(T_1)}, x_Q^{(T_1)}]^T$, $\widehat{\mathbf{z}}_j$ is the $2T_1 \times 1$ noise vector, given by $\widehat{\mathbf{z}}_j = [\widehat{z}_{jI}^{(1)}, \widehat{z}_{jQ}^{(1)}, \widehat{z}_{jI}^{(2)}, \widehat{z}_{jQ}^{(2)}, \cdots, \widehat{z}_{jI}^{(T_1)}, \widehat{z}_{jQ}^{(T_1)}]^T$, where $\widehat{z}_j^{(i)} = G\, z_j^{(i)}$, and $\mathbf{H}_{sj}$ is a $2T_1 \times 2T_1$ block-diagonal matrix, given by

$$\mathbf{H}_{sj} = \begin{bmatrix} \begin{bmatrix} h_{sjI} & -h_{sjQ} \\ h_{sjQ} & h_{sjI} \end{bmatrix} & \cdots & \mathbf{0} \\ \vdots & \ddots & \vdots \\ \mathbf{0} & \cdots & \begin{bmatrix} h_{sjI} & -h_{sjQ} \\ h_{sjQ} & h_{sjI} \end{bmatrix} \end{bmatrix}. \tag{4}$$

Let $\mathbf{C} = \left[\mathbf{c}_1, \mathbf{c}_2, \cdots, \mathbf{c}_N\right]$ denote the $T_2 \times N$ distributed STBC matrix to be sent in the relay phase jointly by all N relays, where $\mathbf{c}_j$ denotes the jth column of $\mathbf{C}$. The jth column $\mathbf{c}_j$ is manufactured by the jth relay as

$$\mathbf{c}_j = \mathbf{A}_j \widehat{\mathbf{v}}_j = \underbrace{G\sqrt{E_1}\mathbf{A}_j\,\mathbf{H}_{sj}}_{\mathbf{B}_j}\,\mathbf{x} + \mathbf{A}_j\,\widehat{\mathbf{z}}_j, \tag{5}$$

where $\mathbf{A}_j$ is the complex processing matrix of size $T_2 \times 2T_1$ for the jth relay, called the *relay matrix* and $\mathbf{B}_j$ can be viewed as the column vector representation matrix [3] for the distributed STBC with the difference that in our case the vector $\mathbf{x}$ is real whereas in [3] it is complex. Let $\mathbf{y}$ denote the $T_2 \times 1$ received signal vector at the destination in T_2 time slots. Then, $\mathbf{y}$ can be written as

$$\mathbf{y} = \sum_{j=1}^{N} h_{jd}\mathbf{c}_j + \mathbf{z}_d, \tag{6}$$

where h_{jd} is the complex channel gain from the jth relay to the destination, and $\mathbf{z}_d$ is the AWGN noise vector at the destination with zero mean and $E[\mathbf{z}_d\,\mathbf{z}_d^*] = \mathbf{I}$. Substituting (5) in (6), we can write

$$\mathbf{y} = G\sqrt{E_1}\left(\sum_{j=1}^{N} h_{jd}\mathbf{H}_{sj}\mathbf{A}_j\right)\mathbf{x} + \sum_{j=1}^{N} h_{jd}\mathbf{A}_j\widehat{\mathbf{z}}_j + \mathbf{z}_d. \tag{7}$$

B. With Phase Only Information at the Relays

In this subsection, we obtain a signal model for the case of partial CSI at the relays, where we assume that each relay has the knowledge of the channel phase on the link between the source and itself in the broadcast phase. That is, defining the channel gain from source to relay j as $h_{sj} = \alpha_{sj} e^{\mathbf{j}\theta_{sj}}$, we assume that relay j has perfect knowledge of only θ_{sj} and does not have the knowledge of α_{sj}.

In the proposed scheme, we perform a phase compensation operation on the amplified received signals at the relays, and space-time encoding is done on these phase-compensated signals. That is, we multiply $\widehat{v}_j^{(i)}$ in (2) by $e^{-\mathbf{j}\theta_{sj}}$ before space-time encoding. Note that multiplication by $e^{-\mathbf{j}\theta_{sj}}$ does not change the statistics of $z_j^{(i)}$. Therefore, with this phase compensation, the $\widehat{\mathbf{v}}_j$ vector in (3) becomes

$$\begin{aligned}\widehat{\mathbf{v}}_j &= \left(G\sqrt{E_1}\,\mathbf{H}_{sj}\,\mathbf{x} + \widehat{\mathbf{z}}_j\right) e^{-\mathbf{j}\theta_{sj}} \\ &= G\sqrt{E_1}\,|h_{s_j}|\,\mathbf{x} + \widehat{\mathbf{z}}_j. \end{aligned} \quad (8)$$

Consequently, the $\mathbf{c}_j$ vector generated by relay j is given by

$$\begin{aligned}\mathbf{c}_j &= \mathbf{A}_j\widehat{\mathbf{v}}_j \\ &= \underbrace{G\sqrt{E_1}\mathbf{A}_j\ |h_{sj}|}_{\triangleq\ \mathbf{B}_j'}\ \mathbf{x} + \mathbf{A}_j\,\widehat{\mathbf{z}}_j, \end{aligned} \quad (9)$$

where $\mathbf{B}_j^{'}$ is the equivalent weight matrix with phase compensation. Now, we can write the received vector $\mathbf{y}$ as

$$\mathbf{y} = G\sqrt{E_1}\left(\sum_{j=1}^{N} h_{jd}|h_{sj}|\mathbf{A}_j\right)\mathbf{x} + \underbrace{\sum_{j=1}^{N} h_{jd}\mathbf{A}_j\widehat{\mathbf{z}}_j + \mathbf{z}_d}_{\tilde{\mathbf{z}}_d\ :\ \text{total noise}}. \quad (10)$$

Systems with phase compensation at the relays will be referred as *partially-coherent relay channels* (PCRC). A distributed STBC which is SSD for a PCRC will be referred as SSD-DSTBC-PCRC.

III. Conditions for SSD and Full-Diversity for Co-located MIMO

The received vector $\mathbf{y}$ in a co-located MIMO setup can be written as

$$\mathbf{y} = \sqrt{E_t}\left(\sum_{j=1}^{N} h_{jd}\mathbf{A}_j\right)\mathbf{x} + \mathbf{z}_d. \quad (11)$$

Theorem 1: For co-located MIMO with N transmit antennas, the linear STBC as given in (11) is SSD *iff*

$$\begin{aligned}&\mathbf{A}_{jI}^T\mathbf{A}_{jI} + \mathbf{A}_{jQ}^T\mathbf{A}_{jQ} = \mathbf{D}_{jj}^{(1)}; \forall j,\\ &\mathbf{A}_{jI}^T\mathbf{A}_{iI} + \mathbf{A}_{jQ}^T\mathbf{A}_{iQ} + \mathbf{A}_{iI}^T\mathbf{A}_{jI} + \mathbf{A}_{iQ}^T\mathbf{A}_{jQ} = \mathbf{D}_{ij}^{(2)}; \forall i,j, i\neq j,\\ &\mathbf{A}_{jI}^T\mathbf{A}_{iQ} + \mathbf{A}_{jQ}^T\mathbf{A}_{iI} - \mathbf{A}_{iI}^T\mathbf{A}_{jQ} - \mathbf{A}_{iQ}^T\mathbf{A}_{jI} = \mathbf{D}_{ij}^{(3)}; \forall i,j, i\neq j, \end{aligned} \quad (12)$$

where $\mathbf{A}_j = \mathbf{A}_{jI} + \mathbf{j}\mathbf{A}_{jQ},\ j = 1,2,\cdots,N$, where $\mathbf{A}_{jI}$ and $\mathbf{A}_{jQ}$ are real matrices, and $\mathbf{D}_{jj}^{(1)}, \mathbf{D}_{ij}^{(2)}$ and $\mathbf{D}_{ij}^{(3)}$ are block diagonal matrices of the form

$$\mathbf{D}_{ij}^{(k)} = \begin{bmatrix} \mathbf{D}_{ij,1}^{(k)} & \mathbf{0} & \cdots & \mathbf{0} \\ \mathbf{0} & \mathbf{D}_{ij,2}^{(k)} & \cdots & \mathbf{0} \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{0} & \cdots & \cdots & \mathbf{D}_{ij,T_1}^{(k)} \end{bmatrix}, \quad (13)$$

where $\mathbf{D}_{ij,l}^{(k)} = \begin{bmatrix} a_{ij,l}^{(k)} & b_{ij,l}^{(k)} \\ b_{ij,l}^{(k)} & c_{ij,l}^{(k)} \end{bmatrix}$ and it is understood that whenever the superscript is (1) as in $\mathbf{D}_{ij}^{(1)}$, then $i = j$.

Proof: Can be found in [15].

Lemma 1: For co-located MIMO, the linear STBC as given in (11) with the $\mathbf{D}_{ij}^{(k)}$ matrices in (12) satisfying $\mathbf{D}_{ij}^{(2)} = \mathbf{D}_{ij}^{(3)} = \mathbf{0}$ achieves maximum diversity for all signal constellations *iff*

$$a_{jj,l}^{(1)}c_{jj,l}^{(1)} - {b_{jj,l}^{(1)}}^2 > 0,\quad 1\le j\le N;\quad 1\le l\le T_1, \quad (14)$$

i.e., $\mathbf{D}_{jj,l}^{(1)}$ is positive definite for all j, l.

Proof: Can be found in [15].

IV. SSD Codes for PCRC

In the previous section, we saw that SSD is achieved if the relay matrices satisfy the condition (12). However, to achieve SSD in the case of distributed STBC with AF protocol, the equivalent weight matrices $\mathbf{B}_j$'s must satisfy the condition in (12). It can be seen that for any $\mathbf{A}_j$ that satisfies the condition in (12), the corresponding $\mathbf{B}_j$'s need not satisfy (12). We note that, in [11], code designs which retain the SSD feature have been obtained for no CSI at the relays, but only for $N = 2$ and 4. Our key contribution in this paper is that by using partial CSI at the relays (i.e., only the channel phase information of the source-to-relay links), the SSD feature at the destination can be restored for a large subclass of SSD codes for co-located MIMO communication.

The main result of this paper is given in the following theorem, which characterizes the class of SSD codes for PCRC.

Theorem 2: A code as given by (5) is SSD-DSTBC-PCRC *iff* the relay matrices $\mathbf{A}_j,\ j = 1,2,\cdots,N$, satisfy (12) (i.e., the code is SSD for a co-located MIMO set up), and, in addition,

$$\begin{aligned}&{\mathbf{A}_{j_1I}}^T\left(\mathbf{A}_{j_2I}{\mathbf{A}_{j_2I}}^T + \mathbf{A}_{j_2Q}{\mathbf{A}_{j_2Q}}^T\right)\mathbf{A}_{j_3I} + \\ &{\mathbf{A}_{j_3I}}^T\left(\mathbf{A}_{j_2I}{\mathbf{A}_{j_2I}}^T + \mathbf{A}_{j_2Q}{\mathbf{A}_{j_2Q}}^T\right)\mathbf{A}_{j_1I} + \\ &{\mathbf{A}_{j_1Q}}^T\left(\mathbf{A}_{j_2I}{\mathbf{A}_{j_2I}}^T + \mathbf{A}_{j_2Q}{\mathbf{A}_{j_2Q}}^T\right)\mathbf{A}_{j_3Q} + \\ &{\mathbf{A}_{j_3Q}}^T\left(\mathbf{A}_{j_2I}{\mathbf{A}_{j_2I}}^T + \mathbf{A}_{j_2Q}{\mathbf{A}_{j_2Q}}^T\right)\mathbf{A}_{j_1Q} = \mathbf{D}_{j_1,j_2,j_3}^{'}, \\ &\qquad \forall j_1,j_2,j_3, \end{aligned} \quad (15)$$

$$\begin{aligned}&{\mathbf{A}_{j_1I}}^T\left(\mathbf{A}_{j_2I}{\mathbf{A}_{j_2Q}}^T + \mathbf{A}_{j_2Q}{\mathbf{A}_{j_2I}}^T\right)\mathbf{A}_{j_3Q} + \\ &{\mathbf{A}_{j_3Q}}^T\left(\mathbf{A}_{j_2I}{\mathbf{A}_{j_2Q}}^T + \mathbf{A}_{j_2Q}{\mathbf{A}_{j_2I}}^T\right)\mathbf{A}_{j_1I} + \\ &{\mathbf{A}_{j_1Q}}^T\left(\mathbf{A}_{j_2I}{\mathbf{A}_{j_2Q}}^T + \mathbf{A}_{j_2Q}{\mathbf{A}_{j_2I}}^T\right)\mathbf{A}_{j_3I} + \\ &{\mathbf{A}_{j_3I}}^T\left(\mathbf{A}_{j_2I}{\mathbf{A}_{j_2Q}}^T + \mathbf{A}_{j_2Q}{\mathbf{A}_{j_2I}}^T\right)\mathbf{A}_{j_1Q} = \mathbf{D}_{j_1,j_2,j_3}^{''}, \\ &\qquad \forall j_1,j_2,j_3, \end{aligned} \quad (16)$$

where $\mathbf{D}_{j_1,j_2,j_3}^{'}$ and $\mathbf{D}_{j_1,j_2,j_3}^{''}$, $1\le j_1,j_2,j_3\le N$, are block diagonal matrices of the form in (13).

Proof: First we show the sufficiency part. It can be easily verified that the matrices $\mathbf{B}_j' = G\sqrt{E_1}\mathbf{A}_j\ |h_{sj}|$ satisfy the condition (12) in spite of the fact that $|h_{sj}|$ are random variables. Let $\mathbf{H}_{eq}^{(pc)} = G\sqrt{E_1}\sum_{j=1}^{N}|h_{sj}|h_{jd}\mathbf{A}_j$. It can be readily seen that $\Re\left({\mathbf{H}_{eq}^{(pc)}}^{\mathcal{H}}\mathbf{H}_{eq}^{(pc)}\right)$ is block diagonal of the form in (13). Hence, the term $\Re\left({\mathbf{H}_{eq}^{(pc)}}^{\mathcal{H}}\mathbf{y}\right)$ decomposes $\mathbf{y}$ into terms containing the information of each symbol. Hence, for SSD, it suffices to show that noise in each of these terms are uncorrelated, i.e., the DSTBC is SSD *iff* $E\left[\Re\left({\mathbf{H}_{eq}^{(pc)}}^{\mathcal{H}}\tilde{\mathbf{z}}_d\right)\Re\left({\mathbf{H}_{eq}^{(pc)}}^{\mathcal{H}}\tilde{\mathbf{z}}_d\right)^T\right]$ is a block diagonal matrix of the form (13). We have

$$
\begin{aligned}
&E\left[\Re\left(\mathbf{H}_{eq}^{\mathcal{H}}\tilde{\mathbf{z}}_d\right)\Re\left(\mathbf{H}_{eq}^{\mathcal{H}}\tilde{\mathbf{z}}_d\right)^T\right] = \\
&\sum_{j_1=1}^{N}\sum_{j_2=1}^{N}\sum_{j_3=1}^{N}|h_{sj_1}||h_{rj_2}|^2|h_{sj_3}|\left(h_{rj_1I}h_{rj_3I}+h_{rj_1Q}h_{rj_3Q}\right)\\
&\quad\cdot\Big({\mathbf{A}_{j_1I}}^T\left(\mathbf{A}_{j_2I}{\mathbf{A}_{j_2I}}^T+\mathbf{A}_{j_2Q}{\mathbf{A}_{j_2Q}}^T\right)\mathbf{A}_{j_3I}\\
&\quad+{\mathbf{A}_{j_3I}}^T\left(\mathbf{A}_{j_2I}{\mathbf{A}_{j_2I}}^T+\mathbf{A}_{j_2Q}{\mathbf{A}_{j_2Q}}^T\right)\mathbf{A}_{j_1I}\\
&\quad+{\mathbf{A}_{j_1Q}}^T\left(\mathbf{A}_{j_2I}{\mathbf{A}_{j_2I}}^T+\mathbf{A}_{j_2Q}{\mathbf{A}_{j_2Q}}^T\right)\mathbf{A}_{j_3Q}\\
&\quad+{\mathbf{A}_{j_3Q}}^T\left(\mathbf{A}_{j_2I}{\mathbf{A}_{j_2I}}^T+\mathbf{A}_{j_2Q}{\mathbf{A}_{j_2Q}}^T\right)\mathbf{A}_{j_1Q}\Big)\\
&+\sum_{j_1=1}^{N}\sum_{j_2=1}^{N}\sum_{j_3=1}^{N}|h_{sj_1}||h_{rj_2}|^2|h_{sj_3}|\left(h_{rj_1I}h_{rj_3Q}+h_{rj_1Q}h_{rj_3I}\right)\\
&\quad\cdot\Big({\mathbf{A}_{j_1I}}^T\left(\mathbf{A}_{j_2I}{\mathbf{A}_{j_2Q}}^T+\mathbf{A}_{j_2Q}{\mathbf{A}_{j_2I}}^T\right)\mathbf{A}_{j_3Q}\\
&\quad+{\mathbf{A}_{j_3Q}}^T\left(\mathbf{A}_{j_2I}{\mathbf{A}_{j_2Q}}^T+\mathbf{A}_{j_2Q}{\mathbf{A}_{j_2I}}^T\right)\mathbf{A}_{j_1I}\\
&\quad+{\mathbf{A}_{j_1Q}}^T\left(\mathbf{A}_{j_2I}{\mathbf{A}_{j_2Q}}^T+\mathbf{A}_{j_2Q}{\mathbf{A}_{j_2I}}^T\right)\mathbf{A}_{j_3I}\\
&\quad+{\mathbf{A}_{j_3I}}^T\left(\mathbf{A}_{j_2I}{\mathbf{A}_{j_2Q}}^T+\mathbf{A}_{j_2Q}{\mathbf{A}_{j_2I}}^T\right)\mathbf{A}_{j_1Q}\Big). \quad (17)
\end{aligned}
$$

From (17) it can be seen that if the conditions in (15) and (16) are met, the covariance matrix is of the form (13). Hence, along with (12) the conditions in (15) and (16) constitute a set of sufficient conditions.

To show the "necessary part," since the terms $h_{sj_1}||h_{rj_2}|^2|h_{sj_3}|$ $(h_{rj_1I}h_{rj_3I}+h_{rj_1Q}h_{rj_3Q})$ and $h_{sj_1}||h_{rj_2}|^2|h_{sj_3}|(h_{rj_1I}h_{rj_3Q}+h_{rj_1Q}h_{rj_3I})$ are independent and if the co-variance matrix has to be block diagonal for all the realizations of h_{sj} and h_{rj}, then the conditions in (15) and (16) have to be necessarily satisfied. Also, in the similar lines of the proof for *Theorem 1*, it can be deduced that $\mathbf{B}'_j$ satisfying condition (12) is necessary for the term $\Re\left({\mathbf{H}_{eq}^{(pc)}}^{\mathcal{H}}\mathbf{y}\right)$ to decompose $\mathbf{y}$ into terms containing information of each symbol. □

A. Full-diversity, Single-Symbol Non-ML Detection

Theorem 3: The PCRC system given by (10) achieves full diversity irrespective of whether the total noise ($\tilde{\mathbf{z}}_d$) is correlated or not, if the STBC achieves full diversity in the co-located case and condition (12) is satisfied.

Proof: Since the noise $\tilde{\mathbf{z}}_d$ is not assumed to be uncorrelated, the optimal detection of $\mathbf{x}$ in the maximum likelihood sense is given by

$$\widehat{\mathbf{x}} = \arg\min\ (\mathbf{y}-\mathbf{H}_{eq}^{(pc)}\mathbf{x})^{\mathcal{H}}\mathbf{\Omega}^{-1}(\mathbf{y}-\mathbf{H}_{eq}^{(pc)}\mathbf{x}), \quad (18)$$

where $\mathbf{\Omega}$ is co-variance matrix of the noise, given by $\mathbf{\Omega} = E\{\tilde{\mathbf{z}}_d\tilde{\mathbf{z}}_d^{\mathcal{H}}\}$. We consider the sub-optimal metric (ignoring $\mathbf{\Omega}^{-1}$)

$$\widehat{\mathbf{x}} = \arg\min\ (\mathbf{y}-\mathbf{H}_{eq}^{(pc)}\mathbf{x})^{\mathcal{H}}(\mathbf{y}-\mathbf{H}_{eq}^{(pc)}\mathbf{x}), \quad (19)$$

and show that this decision metric achieves full diversity. By Chernoff bound, the pair-wise error probability is upper bounded by

$$P\left(\mathbf{x}_1\rightarrow\mathbf{x}_2\right)\le E\left\{e^{-d^2(\mathbf{x}_1,\mathbf{x}_2)E_t/4}\right\}, \quad (20)$$

where the Euclidean distance in (20) can be written as

$$d^2(\mathbf{x}_1,\mathbf{x}_2) = (\mathbf{x}_2-\mathbf{x}_1)^T\Re\left({\mathbf{H}_{eq}^{(pc)}}^{\mathcal{H}}\mathbf{H}_{eq}^{(pc)}\right)(\mathbf{x}_2-\mathbf{x}_1). \quad (21)$$

Define $\Delta\mathbf{x}^{(i)} = [\Delta x_I^{(i)}\Delta x_Q^{(i)}]^T = [(x_{2I}^{(i)}-x_{1I}^{(j)}),(x_{2Q}^{(i)}-x_{1Q}^{(i)})]^T$. Given that the conditions (12) are satisfied, the distance metric can be written as sum of T_1 terms as

$$
\begin{aligned}
d^2(\mathbf{x}_1,\mathbf{x}_2) &= \sum_{i=1}^{T_1}{\Delta\mathbf{x}^{(i)}}^T\left(\sum_{j=1}^{N}|h_{sj}|^2|h_{jd}|^2\mathbf{D}_{j,i}^{(1)}\right)\Delta\mathbf{x}^{(i)} \quad (22)\\
&= \sum_{j=1}^{N}|h_{sj}|^2|h_{jd}|^2\left(\sum_{i=1}^{T_1}{\Delta\mathbf{x}^{(i)}}^T\mathbf{D}_{j,i}^{(1)}\Delta\mathbf{x}^{(i)}\right). \quad (23)
\end{aligned}
$$

Substituting (23) in (20) and evaluating the expectation with respect to $|h_{jd}|^2$, we get

$$
\begin{aligned}
&P\left(\mathbf{x}_1\rightarrow\mathbf{x}_2|\,h_{sj}\right) \le \\
&\prod_{j=1}^{N}\left(\frac{1}{1+|h_{sj}|^2\sum_{i=1}^{T_1}{\Delta\mathbf{x}^{(i)}}^T\mathbf{D}_{j,i}^{(1)}\Delta\mathbf{x}^{(i)}E_t/4,}\right), \quad (24)
\end{aligned}
$$

which, for high SNRs, could be approximated as

$$
\begin{aligned}
&P\left(\mathbf{x}_1\rightarrow\mathbf{x}_2|h_{sj}\right) \le \\
&\prod_{j=1}^{N}\left(\frac{1}{\sum_{i=1}^{T_1}{\Delta\mathbf{x}^{(i)}}^T\mathbf{D}_{j,i}^{(1)}\Delta\mathbf{x}^{(i)}E_t/4}\right)\prod_{j=1}^{N}\left(\frac{1}{|h_{sj}|^2}\right). \quad (25)
\end{aligned}
$$

Now, evaluating the expectation with respect to $|h_{sj}|$, we get

$$P\left(\mathbf{x}_1\rightarrow\mathbf{x}_2\right)\le\prod_{j=1}^{N}\left(\frac{1}{\sum_{i=1}^{T_1}{\Delta\mathbf{x}^{(i)}}^T\mathbf{D}_{j,i}^{(1)}\Delta\mathbf{x}^{(i)}E_t/4}\right)(\mathbf{Ei}(0))^N, \quad (26)$$

where $\mathbf{Ei}(x)$ is the exponential integral $\int_x^\infty\frac{e^{-t}}{t}dt$. From (26), it is clear that the condition for achieving maximum diversity is identical to that of co-located MIMO (14). □

Theorem 3 means that by using any STBC which satisfies the conditions (12) and achieves full diversity in co-located MIMO system, it is possible to do decoding of one symbol at a time and achieve full diversity, though not optimal in the ML sense, in a distributed setup with phase compensation done at the relay, even if *(15) and (16) are not satisfied* .

V. Discussion and Simulation Results

We consider the following codes in our discussions: $i)$ Complex Orthogonal Designs [1], COD_2, COD_4 and COD_8 for 2, 4, and 8 transmit antennas, $ii)$ Coordinate Interleaved Orthogonal Designs [4], $CIOD_4$ and $CIOD_8$ for 4 and 8 transmit antennas, $iii)$ Clifford Unitary Weight Code [5] CUW_4 for 4 transmit antennas, and $iv)$ the eight antenna code RR_8 proposed in [13].

The results of our necessary and sufficient conditions (12), (15) and (16) as well as the sufficient condition in [13], evaluated for various classes of codes for PCRC are shown in Table I. As can be seen from the last column of Table I, the sufficient condition in [13] identifies only COD_2 (Alamouti) and CUW_4 as SSDs for PCRC. However, our conditions (12), (15)

Code	Number of Relays	Rate	Necessary and sufficient Conditions (12), (15) & (16)	Sufficient Condition in [13]
COD_2 (Alamouti)	$N = 2$	1	True	True
COD_4	$N = 4$	3/4	False	False
$CIOD_4$	$N = 4$	1	True	False
CUW_4	$N = 4$	1	True	True
COD_8	$N = 8$	1/2	False	False
$CIOD_8$	$N = 8$	3/4	False	False
RR_8	$N = 8$	1	True	False

TABLE I

TEST FOR NECESSARY AND SUFFICIENT CONDITIONS FOR VARIOUS CLASSES OF CODES FOR PCRC.

and (16) identify $CIOD_4$ and RR_8, in addition to COD_2 and CUW_4, as SSDs for PCRC (4th column of Table I).

Next, we present the bit error rate (BER) performance of various classes of codes without and with phase compensation at the relays (i.e., PCRC). For the purposes of the simulation results and discussions in this section, we classify the decoding of codes for PCRC into two categories: $i)$ codes for which single symbol decoding is ML-optimal; we refer to this decoding as ML-SSD; we consider ML-SSD of COD_2 , $ii)$ codes which when decoded using single symbol decoding are not ML-optimal, but achieve full diversity; we refer to this decoding as non-ML-SSD; we consider non-ML-SSD of COD_4 and COD_8. When no phase compensation is done at the relays, we consider ML decoding.

In Fig. 2, we plot the BER performance for COD_2, COD_4, and COD_8 without and with phase compensation at the relays (i.e., PCRC) for 16-QAM. Note that COD_2 is SSD for PCRC whereas COD_4 and COD_8 are not SSD for PCRC. So decoding of COD_2 with PCRC is ML-SSD, whereas decoding of COD_4 and COD_8 with PCRC is non-ML-SSD. When no phase compensation is done at the relays, we do ML decoding for all COD_2, COD_4, and COD_8. The following observations can be made from Fig. 2: $i)$ COD_2 without and with phase compensation at the relays (PCRC) achieve the full diversity order of 2, $ii)$ COD_2 with PCRC and ML-SSD achieves better performance by about 3 dB at a BER of 10^{-2} compared to ML decoding of COD_2 without phase compensation, and $iii)$ even the non-ML-SSD of COD_4 and COD_8 with PCRC achieves full diversity of 4 and 8, respectively (but not the ML performance corresponding to PCRC), and even with this suboptimum decoding, PCRC achieves about 1 dB and 0.5 dB better performance at a BER of 10^{-2}, respectively, compared to ML decoding of COD_4 and COD_8 without phase compensation at the relays. More simulation results can be found in [15].

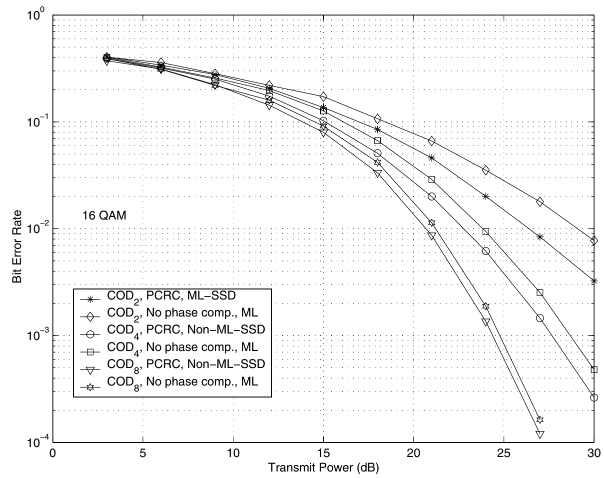

Fig. 2. Comparison of BER performance of COD_2, COD_4, and COD_8 without and with phase compensation at the relays. 16-QAM.

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