# Throughput-Delay Analysis of a Multi-Channel Packet CDMA Scheme in a Fading Environment*

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*Abstract:* **In this paper, we analyze the throughput and delay performance of a multichannel wireless access protocol using CDMA in the presence of Rayleigh fading. $M$ equal-capacity traffic channels, each of which is assigned a unique orthogonal spreading code, are shared by $N$ mobile users ($N \geq M$) on the uplink (mobile-to-base station link). Transmission attempts on the uplink are made based on the *busy/idle status* of the $M$ receivers, which is broadcast by the base station, every slot, on the downlink (base station-to-mobile link). An i.i.d. fading channel model is used to describe the packet success/failure process. Analytical expressions for the average per channel throughput and delay are derived. Simulation results are shown to verify the analysis.**

## I. INTRODUCTION

Next generation wireless networks are envisaged to support high data rates, packet oriented transport, and multimedia traffic handling. The design of efficient and robust wireless media access protocols, and the evaluation of their performance in the presence of channel fading, are key technical issues [1]-[4]. High capacity wireless networks can be realized either by assigning a single wideband channel or by using multiple narrow band channels. The latter approach, which we will consider in this paper, is particularly attractive when contiguous wide bandwidth spectrum is not available. In a multichannel system, there are several independent channels, and a user can transmit on any of these channels based on a suitable access protocol. The performance of multichannel slotted ALOHA systems, where multiple equal-capacity channels are shared by many users, has been analyzed in [5], [6]. However, these studies assumed a deterministic channel model that did not consider the effect of multipath fading on the system performance. In fact, mobile radio channels are severely affected by time-varying losses due to distance, shadowing (blockage due to buildings, trees, etc.), and multipath fading.

Media access control schemes using code division multiple access (CDMA) are becoming popular for packet communication in wireless environments because of their robustness to interference, ability to perform well under multipath conditions, and need for less regulatory and frequency coordination efforts. Earlier studies on both random and slotted CDMA protocols include [7], [8]. In [9], [10], we presented a media access protocol that makes use of the uplink (mobile-to-base station link) channel status information, which is conveyed to the mobiles through a *busy/idle* flag on the downlink (base station-to-mobile link) broadcast. The throughput performance of a system that uses a single traffic channel on the uplink was analyzed in a correlated Rayleigh fading environment. In [11], we extend this protocol to utilize multiple traffic channels on the uplink by assigning a unique, orthogonal spreading code to each channel, and analyze the throughput performance. In this paper, we further analyze the delay performance of the proposed multichannel protocol in the presence of fading.

The detailed description of the proposed multichannel access protocol is presented in Section 2. In Section 3, we analyze the throughput and delay performance of the protocol in the presence of fading. Numerical and simulation results are presented in Section 4. Conclusions and areas of future work are provided in Section 5.

## II. MULTICHANNEL PACKET CDMA PROTOCOL

Consider a packet communication wireless network where $N$ mobile users share $M$ equal-capacity traffic channels ($N \geq M$) on the uplink to communicate with the base station. All the uplink channels are synchronized and slotted to one packet duration. Each uplink channel is assigned a unique, orthogonal spreading code such that the packets transmitted on a given channel are spread using its assigned spreading code. In other words, all the users in the network can use any one of $M$ different spreading codes following the access rules. The base station is provided with $M$ receivers, each tuned to a different spreading code, to demodulate all the uplink channels' traffic. Based on the busy/idle status of the $M$ receivers, the base station broadcasts an $M$-bit *busy/idle word*, every slot, on the downlink. The busy/idle flag corresponding to each channel is set or reset depending on whether or not the data packets are being transmitted on that channel.

Each message generated at the mobiles consists of two segments, namely the *header segment* and the *data segment.* The header segment is of one packet length. It carries a preamble for spreading code acquisition, and control information like the destination address, the number of packets in the data segment, etc. The data segment, which represents the actual traffic, consists of a random number of packets. Transmission attempts are made by the mobiles only at the slot boundaries by sending the header packet.

The mobile, once it receives a message to be sent to the base station, first checks the status of the busy/idle word which it periodically receives from the base station on the downlink. If all the

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$M$ busy/idle flags indicate busy status, the mobile refrains from making a transmission attempt, and reschedules the attempt to a later time. If one or more flags indicate an idle status, then the mobile *randomly* picks one of these idle channels, and makes a transmission attempt by sending a header packet on the uplink slot of the chosen channel. If the header packet is received successfully (without packet loss due to fading or collision), the base station sets the corresponding channel's flag *busy* for $X$ subsequent slots, where $X$ is the number of packets in the data segment of that message. This permits the mobile to send all the data packets in those $X$ slots. During these $X$ slots, other mobiles would receive a busy status flag for this channel and so they would not make transmission attempts on it. The base station resets the flag back to idle status after $X$ slots. If the header packet is lost (due to collision or fading), then the base station will not respond with a busy status flag, but will continue to send the corresponding channel's flag as *idle*. This is an indication to the mobile that the header packet was lost, and so it has to reschedule its transmission attempt to a later time.

Note that, in case of error-free busy/idle word reception at all the mobiles, collisions are possible only during the header packet transmissions and not during the transmission of packets in the data segment.

## III. PERFORMANCE ANALYSIS

The analysis of the system delay performance is obtained by expanding the approach employed in [11]. As in [11], a single cell system with no inter-cell interference is considered. Busy/idle word is assumed to be received instantaneously, and error-free by all the mobiles. The effect of capture is ignored. All the mobiles' transmissions are assumed to be power controlled such that the slowly varying distance and shadow losses are perfectly compensated, whereas the rapidly varying multipath fading remain uncompensated. The multipath fading is considered to be frequency non-selective, the effect of which is described as a multiplicative complex function, $\alpha(t)$. The envelope of $\alpha(t)$ has an i.i.d. Rayleigh distribution. Based on a model of The packet success/failure process in the presence of fading in [9, 10], the average packet error rate, $P_E$, is given by

$$P_E = 1 - e^{-1/F}. \tag{1}$$

where $F$ is the fading margin of the link.

New messages arrive at the mobiles with probability $\lambda$ in each slot. Each mobile is assumed to have buffers to hold one message. Newly arriving messages are rejected until the stored message is successfully sent to the base station. The length of the data segment of each message, $X$, measured in integer number of packets, is assumed to follow a geometric distribution with parameter $g_m$, and probability mass function

$$P[X = i] = \begin{cases} g_m(1-g_m)^{i-1} & i = 1, 2, 3, ... \\ 0 & \text{otherwise.} \end{cases} \tag{2}$$

In any given slot, each mobile can be in any one of the three states, namely, *idle/header* state, *data_tx* state, and *backlogged* state. Refer to Figure 1. In the *idle/header* state, the mobile remains idle or generates a new message with a probability $\lambda$, randomly chooses an idle uplink channel (if available), and transmits the header packet in the uplink slot. If the header packet transmission is successful, the mobile moves from the *idle/header* state to the *data_tx* state, in which it transmits the data packets continuously until all the packets in the message are sent, and then moves back to the *idle/header* state. The mobile moves from the *idle/header* state to a *backlogged* state if all the uplink channels are found busy upon arrival of a message. Similarly, if the header packet is lost due to collision or bad channel conditions, the mobile moves from the *idle/header* state to the *backlogged* state. In the *backlogged* state, the mobile rechecks the status of the uplink channels after a random number of slots. This retransmission attempt delay is assumed to be geometrically distributed with parameter $g_r$. If a mobile in the *backlogged* state fails to retransmit its header packet successfully, it stays in this state until its header packet transmission is successful, after which it moves to the *data_tx* state.

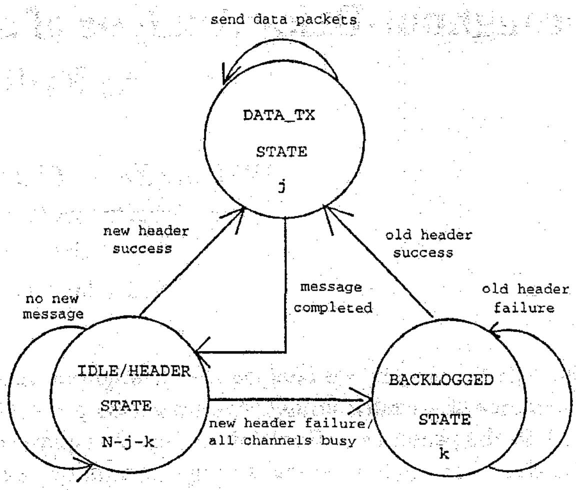

Fig. 1. Mobile state transition diagram

### A. *Throughput Performance*

Let $x_t$ be the number of mobiles in the *data_tx* state and $y_t$ be the number of mobiles in the *backlogged* state at the end of slot $t$. The two dimensional random process $\{x_t, y_t\}$ is a finite state Markov chain. Based on the conditional probability that $n$ mobiles simultaneously transmit header packets and $c_s$ of those packets are successfully received at the base station, the one step transition probability that the system moves from $(x_t = j, y_t = k)$ at time slot $t$ to $(x_{t+1} = l, y_{t+1} = m)$ at time slot $t+1$ is given by

$$\begin{aligned} P_{jk,lm} &= \sum_{n=0}^{N-j} \sum_{c_s=0}^{min(M-j,n)} \binom{N-j-k}{a} \lambda^a (1-\lambda)^{N-j-k-a} \\ &\quad \cdot \binom{k}{n-a} g_r^{n-a} (1-g_r)^{k-n+a} \binom{j}{l-c_s} g_m^{c_s+j-l} \\ &\quad \cdot (1-g_m)^{l-c_s} f(c_s|n, M-j), \end{aligned} \tag{3}$$

where $M$ is the total number of spreading codes (i.e., the number of uplink channels), $N$ is the total number of mobiles, $N \geq M$, $0 \leq j, l \leq M$, $0 \leq k, m \leq N-j$, $a = m - k + c_s$, and the conditional probability $f(c_s|n, M-j)$, which is the probability that $c_s$ header packets are successfully received, conditioned on $n$ mobiles out of $N-j-k$ mobiles in idle/header state and $k$ mobiles in backlogged state transmit header packets over $M-j$ channels, can be evaluated by a recursive expression as

$$\begin{aligned} f(c_s|n, M-j) &= \sum_{\substack{i=0 \\ i\neq 1}}^{n} p_n^{(i)} f(c_s|n-i, M-j-1) \\ &\quad + p_n^{(1)} f(c_s-1|n-1, M-j-1)(1-P_E) \\ &\quad + p_n^{(1)} f(c_s|n-1, M-j-1)P_E, \end{aligned} \qquad (4)$$

where $p_n^{(i)} \triangleq \binom{n}{i}(1-\frac{1}{M-j})^{n-i}(\frac{1}{M-j})^i$ is the probability that $i$ packets out of $n$ packets are transmitted over the an arbitrarily chosen channel. The initial conditions for $f(c_s|n, M-j)$ can be found in [11].

Let $\boldsymbol{P} = (P_{jk,lm})$ be the probability transition matrix and $\boldsymbol{\Pi} = \{\pi_{jk}\}$, $0 \leq j \leq M$, $0 \leq k \leq N-j$, denote the steady-state probability vector. $\boldsymbol{\Pi}$ can be calculated by solving the set of linear equations

$$\boldsymbol{\Pi} = \boldsymbol{\Pi P}, \qquad (5)$$

and using the conservation relationship

$$\sum_{j=0}^{M}\sum_{k=0}^{N-j} \pi_{jk} = 1. \qquad (6)$$

The network throughput, $\theta$, which is defined as the average number of packets successfully received per time slot, is obtained from the average number of successful data packets, $E\{S_d\}$, and the average number of successful header packets, $E\{S_h\}$, in the steady state as

$$\theta = E\{S_d\} + E\{S_h\}, \qquad (7)$$

where

$$\begin{aligned} E\{S_d\} &= \sum_{j=1}^{M}\sum_{k=0}^{N-j} \pi_{jk} \sum_{n=1}^{j} n \binom{j}{n} (1-P_E)^n P_E^{j-n} \qquad (8) \\ &= \sum_{j=1}^{M}\sum_{k=0}^{N-j} j\pi_{jk}(1-P_E) \equiv E\{S_n\}(1-P_E). \end{aligned}$$

$E\{S_n\} = \sum_{j=1}^{M}\sum_{k=0}^{N-j} j\pi_{jk}$ is the average number of users in the *data_tx* state. The average number of successful header packets, $E\{S_h\}$, is obtained as

$$\begin{aligned} E\{S_h\} &= \sum_{j=0}^{M}\sum_{k=0}^{N-j}\sum_{l=0}^{M}\sum_{m=0}^{N-l} \pi_{jk} \sum_{n=1}^{N-j} \sum_{c_s=0}^{min(M-j,n)} c_s \binom{N-j-k}{a} \\ &\quad \cdot \lambda^a (1-\lambda)^{N-j-k-a} \binom{k}{n-a} g_r^{n-a}(1-g_r)^{k-n+a} \\ &\quad \cdot \binom{j}{l-c_s} g_m^{c_s+j-l}(1-g_m)^{l-c_s} f(c_s|n, M-j). \end{aligned} \qquad (9)$$

Finally, we obtain the average per channel throughput, i.e., the average number of packets successfully received per slot per channel, as

$$\theta_c = \frac{\theta}{M}. \qquad (10)$$

### B. Delay Performance

The message transfer delay is the time elapsed between the arrival of a message at the mobile and the successful completion of its transmission to the base station. This delay consists of two components, namely, 1) the number of slots elapsed to achieve successful header packet transmission, and 2) the number of slots elapsed during the data packets transmission. The average message transfer delay, $E\{D\}$, is given by

$$E\{D\} = E\{D_h\} + E\{D_d\}, \qquad (11)$$

where $E\{D_h\}$ is the expected number of slots it takes to achieve header packet success, and $E\{D_d\}$ is the expected number of slots spent for data transmission. $E\{D_d\}$ is nothing but the expected length of the message. For a geometric distribution of message length with parameter $g_m$, $E\{D_d\}$ is given by

$$E\{D_d\} = \frac{1}{g_m}. \qquad (12)$$

$E\{D_h\}$ can be obtained by evaluating the expected number of header packets lost, $E\{F_h\}$, and the expected number of waiting slots (excluding the lost header packets) elapsed until the success of the header packet, $E\{W_r\}$. The expressions for $E\{W_r\}$ and $E\{F_h\}$ are given by

$$\begin{aligned} E\{W_r\} &= \sum_{j=1}^{M}\sum_{k=0}^{N-j} \pi_{jk} \sum_{n=1}^{k} n \binom{k}{n} (1-g_r)^n g_r^{k-n} \qquad (13) \\ &= (1-g_r)\sum_{j=1}^{M}\sum_{k=0}^{N-j} k\pi_{jk}, \end{aligned}$$

and

$$\begin{aligned} E\{F_h\} &= \sum_{j=0}^{M}\sum_{k=0}^{N-j}\sum_{l=0}^{M}\sum_{m=0}^{N-l} \pi_{jk} \sum_{n=1}^{N-j} \sum_{c_s=0}^{min(M-j,n)} (n-c_s) \\ &\quad \cdot \binom{N-j-k}{a} \lambda^a (1-\lambda)^{N-j-k-a} \\ &\quad \cdot \binom{k}{n-a} g_r^{n-a}(1-g_r)^{k-n+a} \\ &\quad \cdot \binom{j}{l-c_s} g_m^{c_s+j-l}(1-g_m)^{l-c_s} f(c_s|n, M-j), \end{aligned} \qquad (14)$$

The expression for $E\{D_h\}$ is then given by

$$E\{D_h\} = 1 + \frac{E\{F_h\} + E\{W_r\}}{E\{S_h\}} \qquad (15)$$

Note that the 1 in (15) is to account for the successful header packet slot in the delay expression.

## IV. RESULTS AND DISCUSSION

Numerical results for the average per channel throughput of the proposed multichannel protocol obtained from (7) and (10) for $M = 3$, $N = 5$, $g_m = 0.1$, and $g_r = 0.2$ are plotted in Figure 2. The various values of fading margins considered are 5, 10, and 20 dB. Results for no fading case (i.e., $F \to \infty$) are also plotted. From (12), the $g_m$ value of 0.1 corresponds to an average message length of 10 data packets. Likewise, the parameter $g_r =$

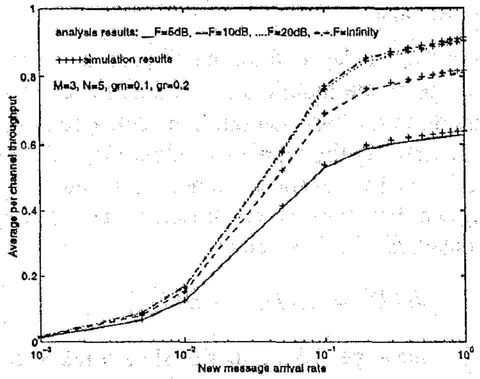

Fig. 2. Average per channel throughput, $\theta_c$, *vs* new message arrival rate, $\lambda$.

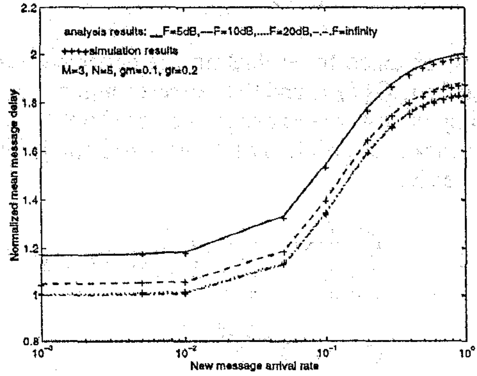

Fig. 3. Normalized mean message delay *vs* new message arrival rate, $\lambda$.

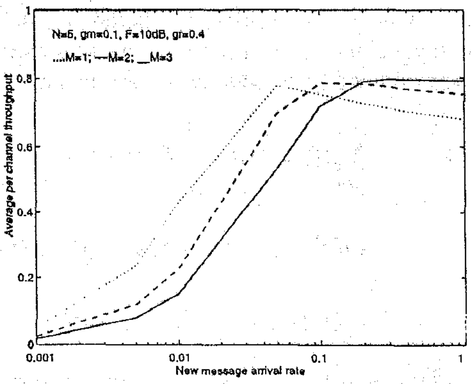

Fig. 4. Effect of number of channels, $M$, on the average per channel throughput.

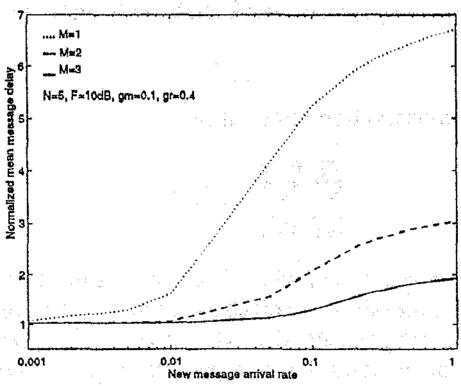

Fig. 5. Effect of number of channels, $M$, on the normalized mean message delay.

0.2 means that the average time between retransmission attempts is 5 slots.

In addition to the analysis, the proposed protocol was simulated as well, and the throughput performance collected over a million slots of simulation is also shown in Figure 2. The lines represent the analytical results and the markers represent the simulation points. When there is no fading (that is, packet losses are only due to header packet collisions), the protocol is found to offer a maximum throughput on the order of 0.9. When the channels are fading, maximum throughputs as high as the no fading case can still be achieved, provided the fading margin is maintained better than 20 dB. With practical values of fading margin, say 10 dB, the resulting maximum throughput is in excess of 0.8. When the fading margin decreases to 5 dB, the maximum throughput degrades to 0.63.

For the above set of system parameters, the message transfer delay performance of the proposed protocol is shown in Figure 3. The curves represent the average delay values obtained from (11), and normalized to the average length of the message, including the header packet (i.e., $1 + 1/g_m$). A normalized mean message delay of unity at low arrival rates implies that the messages get transmitted immediately on arrival without any waiting/retransmission delays. Both increased message arrival rates and lower fading margins are seen to increase the normalized delay beyond unity. Even when there is no fading, the normalized delay increases beyond unity at high arrival rates (e.g., a normalized delay of 1.8 at $\lambda = 1$), which is mainly due to header packet collisions and subsequent rescheduling of transmission attempts. Like the throughput, the delay performance at a fading margin of $F = 20$ dB is very close to the no fading case. Compared to the no fading case, the delay performance worsens by 5% and 20%, respectively, at fading margins of 10 dB and 5 dB. Note that the analytical and simulation results in Figures 2 and 3 closely agree, thus verifying the analysis.

The effect of the number of channels, $M$, on the throughput and delay characteristics of the proposed protocol is shown in Figures 4 and 5. The plots are parameterized by different values of $M = 1, 2, 3$ at $N = 5$, $F = 10$ dB, $g_m = 0.1$, and $g_r = 0.4$. At low arrival rates, a single channel system ($M = 1$) offers higher throughput than a multichannel system ($M = 2, 3$). This is because, in the absence of enough traffic at the mobiles, all the channels go underutilized. Also, this better throughput performance of single channel system has an associated increase in delay performance compared to a multichannel system, as seen from Figure 5. At high arrival rates, the throughput performance behavior reverses (i.e., the single channel system offers less throughput than a mutlichannel system). Here, a single channel system loses more

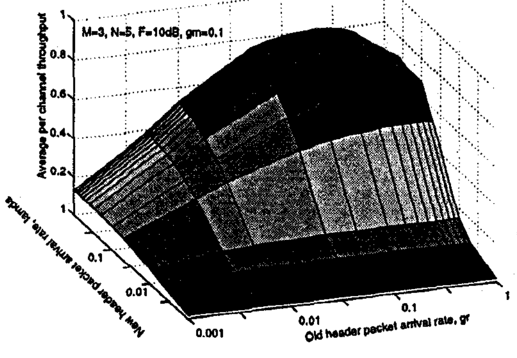

Fig. 6. Effect of $g_r$ and $\lambda$ on the average per channel throughput, $\theta_c$.

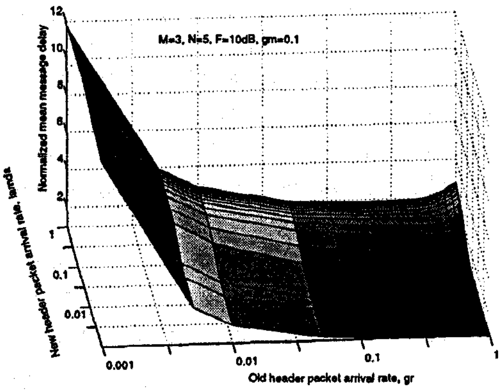

Fig. 7. Effect of $g_r$ and $\lambda$ on the normalized mean message delay.

capacity through higher collision rates than does a multichannel system. Further, the delay performance at high arrival rates is significantly better in a multichannel system than in a single channel system (e.g., normalized delay of 1.9 and 6.7, respectively, for $M = 3$ and $M = 1$ when $\lambda = 1$). In Figures 6 and 7, the effect of the parameter, $g_r$, on the throughput and delay characteristics of the proposed protocol at various values of new message arrival rate, $\lambda$, is shown as 3-D contour plots for $N = 5$, $M = 3$, $F = 10$ dB, and $g_m = 0.1$. The reason for the bell-shaped curve is that if $g_r$ is too small (i.e., more waiting slots in the *backlogged* state), more slots will go unutilized during the waiting period, and if $g_r$ is high, more slots will witness a collision resulting in decreased throughput and increased delay performance.

## V. CONCLUSIONS

We analyzed the throughput and delay performance of a multichannel wireless packet CDMA scheme in the presence of Rayleigh fading. The proposed scheme made use of multiple slotted channels, each of which employed a unique spreading sequence. The efficiency of the protocol is high because the transmission attempts on the uplink channels are made based on the busy/idle status of the receivers, which is broadcast by the base station, every slot, on the downlink. The resulting architecture is highly flexible in the sense that the system can be scaled up or down simply by adding or removing channels based on the bandwidth needs of the network. In addition, the concept of a header packet for each message resulted in significantly low overhead time employed for the code acquisition. Further ongoing investigations include the effect of capture and retransmission of erroneous data packets on the system performance. It must be noted that the Rayleigh fading is assumed to be i.i.d. in this paper, which is a good model for fast moving users in a multipath environment. An extension of the current multichannel protocol analysis to adopt a correlated fading process, which is suit for slowly moving users, will be useful future work.

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