Probability of Miss Analysis for Packet CDMA Acquisition on Rician-Fading Channels

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***Abstract*— We analyze the probability of miss in the acquisition of direct sequence spread-spectrum packets transmitted on Rician-fading channels. The results are useful to compute the probability of a random-access packet success, and the expected number of packet transmissions needed to achieve success, on LEO satellite code-division multiple-access channels.**

***Index Terms*—Acquisition, packet CDMA, Rician fading.**

I. Introduction

IN DIRECT-SEQUENCE code-division multiple-access (DS-CDMA) cellular systems, access packets are sent by the mobiles on the reverse link access channel, using a random-access protocol, either autonomously for purposes like registration, or in response to a base station command [1]. These access packets have a preamble at the beginning, in order to enable the base station to acquire and track a packet transmission with adequate time and frequency accuracy, so that the following message payload can be demodulated. The base station searches the entire search space defined by the maximum time and frequency uncertainties involved due to propagation delays and Doppler effects. The time and frequency uncertainties on LEO satellite channels can be quite large (in the order of several tens of milliseconds of time uncertainty and several tens of kilohertz of frequency uncertainty). The probability of miss to acquire the access packets is often used to evaluate the overall packet success rate and the expected number of transmissions needed to achieve success. In [2]–[4], the probabilities of miss and false alarm in parallel matched filter acquisition schemes with and without reference filtering are investigated for Rayleigh- and Rician-fading channels. In this letter, we provide a simplified analysis to compute the probability of miss to acquire a DS-CDMA packet in the presence of frequency nonselective Rician fading, which is applicable to LEO satellite channels. These channels are often characterized by the presence of a direct line-of-sight (LOS) component in addition to the scattered component due to multipath Rayleigh fading. The Rice factor, K, defines the ratio between the powers in the LOS component and the scattered component.

II. Analysis

We define the probability of a miss as the probability for which an access packet that is sent in a slot goes unacquired at the base station. The search space is partitioned into several frequency bins and discrete time offsets. Let L_c be the number of frequency bins and M_c be the number of discrete time offsets. The total number of hypotheses searched is L_cM_c. In each hypothesis, the searcher at the base station performs K_c chip coherent integrations of the obtained energy, and N_c noncoherent accumulations of such coherent integrations. To simplify the analysis, a packet is considered to be acquired if the correct hypothesis happens to be the one with the largest energy among all possible hypotheses.

For the additive white Gaussian noise (AWGN) case, the total energy E resulting from the coherent and noncoherent accumulations is given by

$$E = \sum_{j=0}^{N_c-1} \left(\left(\sum_{k=0}^{K_c-1} c_{jk}^{(i)} \left[u_{jk}^{(i)} + n_{jk}^{(i)} \right] \right)^2 + \left(\sum_{k=0}^{K_c-1} c_{jk}^{(q)} \left[u_{jk}^{(q)} + n_{jk}^{(q)} \right] \right)^2 \right) \quad (1)$$

where $u_{jk}^{(i)}$ ($+1$ or -1) is the signal component of the kth in-phase chip, $n_{jk}^{(i)}$ is the noise component of the kth in-phase chip ($n_{jk}^{(i)} \sim N(0, \sigma^2)$), and $c_{jk}^{(i)}$ ($+1$ or -1) is the kth in-phase correlator reference chip. Likewise, $u_{jk}^{(q)}$, $n_{jk}^{(q)}$, and $c_{jk}^{(q)}$ correspond to the signal, noise, and reference for the kth quadrature chip. The noise components $n_{jk}^{(i)}$ and $n_{jk}^{(q)}$ are independent identically distributed (i.i.d.).

Let E_s be the total energy when signal is present, and E_N be the noise energy when signal is not present. When there is no signal present in the hypothesis, the total noise energy E_N has a χ^2 distribution with $2N_c$ degrees of freedom, so that

$$P(E_N < x) = 1 - e^{-(x/2K_c\sigma^2)} \sum_{n=0}^{N_c-1} \frac{1}{n!} \left(\frac{x}{2K_c\sigma^2} \right)^n. \quad (2)$$

When there is a signal present in the hypothesis, the total energy, E_s, has a noncentral χ^2 distribution with $2N_c$ degrees of freedom. Thus,

$$P(E_s < x) = 1 - Q_{N_c}\left(\sqrt{2N_cK_c\frac{E_c}{I_t}},\ \sqrt{\frac{x}{K_c\sigma^2}} \right) \quad (3)$$

where $E_c/I_t = (u^{(i)^2} + u^{(q)^2})/2\sigma^2$, $Q_m(a, b)$ represents the Marcum Q-function, and the probability density function (pdf)

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is given by

$$f_s(x) = \frac{1}{2K_c\sigma^2}\left(\frac{x}{2N_cK_c^2\sigma^2(E_c/I_t)}\right)^{(N_c-1)/2} \cdot \exp\left[-\left(\frac{x+2N_cK_c^2\sigma^2\left(\frac{E_c}{I_t}\right)}{2K_c\sigma^2}\right)\right] \cdot I_{N_c-1}\left(\frac{1}{\sigma}\sqrt{2N_c(E_c/I_t)x}\right). \quad (4)$$

where $I_{N_c-1}(\cdot)$ is the modified Bessel function of the first kind and (N_c-1)th order. Let $\{E_{N,k}\}$ be the energies obtained from the noise hypotheses $(1 \le k \le L_cM_c - 1)$. The probability of a miss is then given by

$$P(\text{miss}) = P\left(E_s < \max_k E_{N,k}\right) = 1 - \int_0^\infty P(E_{N,k} < x)^{L_cM_c-1} f_s(x)\,dx. \quad (5)$$

While considering the Rician-fading case, in order to make the analysis tractable, we assume that the Rician fading process is constant over each coherent integration, and independent over successive integrations. From (4), a single coherent integration without fading has a noncentral χ^2 distribution with 2 degrees of freedom with density function given by

$$f_s(x) = \frac{1}{2K_c\sigma^2}\exp\left[-\left(\frac{x+2K_c^2\sigma^2\left(\frac{E_c}{I_t}\right)}{2K_c\sigma^2}\right)\right] \cdot I_0\left(\frac{1}{\sigma}\sqrt{2(E_c/I_t)x}\right). \quad (6)$$

The Rician-fading effect can be introduced by multiplying E_c with a factor λ^2, where λ is a Rician random variable with density function

$$f_\lambda(\lambda) = \frac{\lambda}{\sigma_f^2}\cdot\exp\left[-\left(\frac{\lambda^2+A^2}{2\sigma_f^2}\right)\right] I_0\left(\lambda\frac{A}{\sigma_f^2}\right). \quad (7)$$

Because of the assumption that fading is constant over a coherent integration period, and independent over successive noncoherent accumulations, the pdf of the faded integration Y is given by

$$f_Y(y) = \int_0^\infty f_{Y|\Lambda}(y|\lambda) f_\Lambda(\lambda)\,d\lambda. \quad (8)$$

We normalize the average power in the fading process $(A^2 + 2\sigma_f^2)$, and the total received power $(K_c(E_c + I_t)/2)$ to 1. From [5], the integral in (8) can be written as

$$f_Y(x) = \frac{1}{2\sigma_a^2}\cdot\exp\left[-\left(\frac{x+a^2}{2\sigma_a^2}\right)\right] I_0\left(\frac{a}{\sigma_a^2}\sqrt{x}\right) \quad (9)$$

where

$$a^2 = \frac{K}{1+K}\left(\frac{2K_c(E_c/I_t)}{1+(E_c/I_t)}\right) \quad (10)$$

$$\sigma_a^2 = \frac{1}{1+(E_c/I_t)} + \frac{1}{1+K}\left(\frac{K_c(E_c/I_t)}{1+(E_c/I_t)}\right) \quad (11)$$

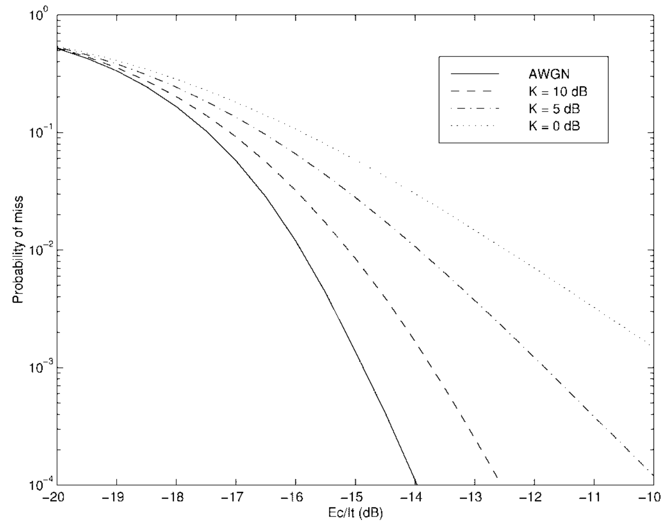

Fig. 1. Probability of miss versus E_c/I_t at different Rice factors, $K = 0, 5, 10$ dB. $L_c = 8$, $M_c = 128$, $K_c = 256$, and $N_c = 4$.

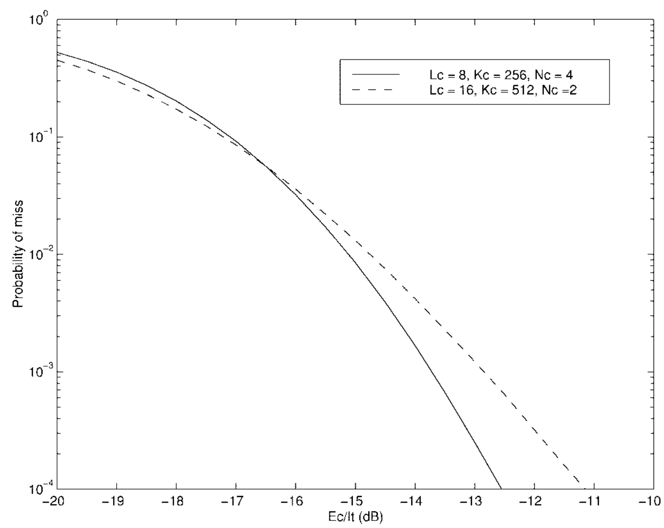

Fig. 2. Probability of miss versus E_c/I_t at $K = 10$ dB. Case (i) $L_c = 8$, $M_c = 128$, $K_c = 256$, $N_c = 4$. Case (ii) $L_c = 16$, $M_c = 128$, $K_c = 512$, $N_c = 2$.

and $K = A^2/2\sigma_f^2$ is the LOS-to-scatter power ratio (also known as the Rice factor). From (9), it can be seen that each coherent integration with Rician fading is still a noncentral χ^2 distribution with 2 degrees of freedom, but with a different mean and variance given by (10) and (11). Thus, the total energy in N_c independent integrations is also noncentral χ^2 distributed with $2N_c$ degrees of freedom with pdf given by

$$f_s(x) = \frac{1}{2\sigma_a^2}\left(\frac{x}{N_ca^2}\right)^{(N_c-1)/2}\cdot\exp\left[-\left(\frac{x+N_ca^2}{2\sigma_a^2}\right)\right] \cdot I_{N_c-1}\left[\frac{a}{\sigma_a^2}\sqrt{x}\right]. \quad (12)$$

The probability of miss can then be calculated by applying (12) in (5).

III. Numerical Results

In the following numerical examples, a processing gain of 64, frequency uncertainty of 10 kHz, and a time offset resolution of half a chip are used. Fig. 1 shows the probability of miss as a function of E_c/I_t in decibels for different values of Rice factors, $K = 0, 5,$ and 10 dB. The performance under AWGN is also shown for comparison. The following parameters are used for generating the results in Fig. 1: $L_c = 8$, and $M_c = 128$, $K_c = 256$, $N_c = 4$. As expected, increasing the value of Rice factor brings the performance curves closer to the AWGN case. In Fig. 2, we plot the probability of miss results for two different combination of acquisition parameters at a given Rice factor of 10 dB, viz. (i) $L_c = 8$, $M_c = 128$, $K_c = 256$, $N_c = 4$ and (ii) $L_c = 16$, $M_c = 128$, $K_c = 512$, $N_c = 2$. The frequency bin size in case (ii) is half of that in case (i). Note that the number of frequency bins and the number of coherent accumulations in case (ii) are both double ($L_c = 16$, $K_c = 512$) that in case (i) ($L_c = 8$, $K_c = 256$). The energy measurement time for each hypothesis is held constant in both cases (i.e., $K_c N_c = 1024$ in both cases). The performance crossover seen in Fig. 2 can be explained as follows. For large values of E_c/I_t, case (ii) performs worse because the number of frequency bins and hence the number of noise hypotheses are doubled, thereby increasing the chances of a miss. Whereas at low values of E_c/I_t, case (ii) performs better because under high noise conditions the energy estimation is better with increased coherent accumulation periods [case (ii) has twice the coherent accumulation period compared to case (i)].

In summary, for a given number of coherent and noncoherent accumulations, the analytical expressions derived in this letter can be used to obtain the probability of a miss which, in turn, can be used to evaluate the overall packet success rate in packet DS-CDMA systems taking acquisition performance into account. Alternately, to achieve a desired probability of miss, the required number of coherent and noncoherent accumulations, and hence the required preamble length can be determined.

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