Capacities of FDMA/CDMA Systems in the Presence of Phase Noise and Multipath Rayleigh Fading

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***Abstract*—The effect of phase noise on the capacities of coherently demodulated hybrid frequency-division multiple access/code division multiple-access (FDMA/CDMA) systems operating over a multipath Rayleigh-fading channel is investigated. Using an approximate upper bound on the bit-error rate performance, which has been derived and presented in a previous paper, the capacities of the FDMA/CDMA systems are estimated for several combinations of channel and system parameters. Simulation results are also included to show the effect of the bounding error.**

***Index Terms*—Hybrid FDMA/CDMA, multipath fading, phase noise.**

I. INTRODUCTION

IN PREVIOUS studies [1], [2] the capacities of coherently demodulated hybrid frequency-division multiple access/code division multiple-access (FDMA/CDMA) systems were calculated for Rayleigh- and Rician-fading channels, assuming perfect coherence, and compared with those of noncoherently demodulated systems. The results of those studies suggest that wide-band systems provide greater capacities than narrow-band systems when coherent demodulation is used. The purpose of this letter is to determine if that same conclusion holds when phase noise is present to distort the phase reference. Two methods of capacity estimation are used in this study: 1) estimation based on an approximate upper bound on the bit-error rate (BER) [3], and 2) estimation based on Monte Carlo simulation. Estimates of the former indicate that the capacities begin to behave like those expected of a noncoherent system, i.e., FDMA/CDMA having greater capacity than wide-band CDMA in certain cases, when the signal-to-noise ratio (SNR) within the phase-locked loop (PLL) is not at least 10 dB above the system SNR E_b/η_0. However, since the estimation is based on an upper bound, the observed behavior may be partially due to bounding errors. This is confirmed by the results of the computer simulation, which indicate that wide-band CDMA provides greater capacity than FDMA/CDMA, even for PLL SNR's as low as 0 dB above E_b/η_0.

To minimize redundancy, descriptions of the system, channel, and receiver models that follow are brief and meant to serve only as a general overview. Detailed descriptions may be found in the references.

II. SYSTEM, CHANNEL, AND RECEIVER MODELS

The idea behind a hybrid FDMA/CDMA system is the division of the total available spectrum into two or more subspectra, so that separate and independent CDMA systems, each with processing gain smaller than that possible without the division, may operate within each subspectrum. The capacity of the hybrid system is defined as the sum of the capacities of the individual "narrow-band" CDMA systems. The CDMA systems each operate asynchronously, with equal processing gain, and utilize spreading sequences with periods much larger than the processing gain.

The multipath fading channel is modeled as a tapped delay line with tap spacings equal to the chip period T_c. The tap weights are independent and have Rayleigh distributed magnitudes $\{\alpha_i\}$ and uniformly distributed phases $\{\theta_i\}$. Furthermore, the weights have exponentially decaying second moments relative to the first tap weight; this is characterized by the exponent of the decay factor δ, i.e., the set of second moments is a geometric series with ratio $e^{-\delta}$

$$E[\alpha_i^2] = E[\alpha_0^2]e^{-i\delta}.$$

The receiver, like the channel, is also modeled as a tapped delay line with the same number of taps and the same tap spacings. A filter matched to the desired user's spreading sequence precedes the tapped delay line. The tap weights are arranged in reverse order of the channel tap weights, with the magnitudes having the exact values as the channel's, but the phases may contain random errors. The phase errors are assumed to fluctuate slowly, relative to the bit rate, and are modeled as being independent and Tikhonov distributed, with probability density functions (pdf's) given by

$$p(\theta) = \frac{\exp(\alpha \cos\theta)}{2\pi I_0(\alpha)}, \qquad -\pi \le \theta < \pi$$

where α is defined as the instantaneous SNR within the bandwidth of the PLL used for carrier recovery.

III. BER PERFORMANCE AND CDMA CAPACITY

Based on the channel and receiver models briefly described above, an approximate upper bound on the BER of a direct-sequence spread-spectrum (DS-SS) signal in additive white Gaussian noise (AWGN) has been derived and has been shown to be tight for many cases of interest [3]. This result may be applied to estimate CDMA performance in the presence of phase noise. In a CDMA system a large number of independent

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users are transmitting simultaneously on the same frequency; using a central limit theorem argument, the aggregate effect of all nonintended users' signals at any moment is approximately the same as that from a Gaussian random variable. When this condition holds, it has been shown, e.g., [4] and [5], that the multiple-access interference may be treated as additional Gaussian noise for the purpose of evaluating bit-error probabilty. Therefore, the results obtained in [3] for AWGN are at least good first-order approximations when applied to such a CDMA system.

From a previous study [1], CDMA capacity was calculated assuming perfect phase references. In that study the BER was found to be a function of the average received SNR E_b/η_0, the number of resolvable multipaths M, the multipath intensity profile (MIP) decay factor δ, the processing gain N, and the number of active users K. Defining the capacity as the largest K such that the BER is below the required bit-error probability, the capacity can be expressed as $C(E_b/\eta_0, M, \delta, N) = \max\{K\colon P_b(E_b/\eta_0, M, \delta, N, K) < P_{\text{breq}}\}$.

In [1] a model which relates CDMA bandwidth with the other system parameters was described. These relationships may be summarized as follows: let the parameters associated with the FDMA/CDMA system having L subspectra ($L = 1$ being the "wide-band" CDMA case) be denoted with subscripts "L;" then it is postulated that

$$\begin{aligned}\left(\frac{E_b}{\eta_0}\right)_L &= \left(\frac{E_b}{\eta_0}\right)_1 \sum_{i=0}^{L-1} e^{-\delta i}\\ M_L &= M_1/L\\ \delta_L &= L\delta_1\\ N_L &= N_1/L.\end{aligned}$$

These relationships are based on some reasonable assumptions concerning the number of paths that a channel of a given bandwidth is able to resolve and the conservation of path powers when unresolvable paths merge.

The total capacity for a FDMA/CDMA system with L subspectra, given the parameters associated with wide-band CDMA, may then be calculated as $L \times C((E_b/\eta_0)_L, M_L, \delta_L, N_L)$.

IV. NUMERICAL RESULTS

Table I lists the estimated capacities of FDMA/CDMA for $L = 1$, 2, 3, 4, 6, and 12, $N_1 = 1023$, $M_1 = 12$, $(E_b/\eta_0)_1 =$ 10 dB, and $\delta_1 = 0.0$ and 0.1, respectively. The amount of phase noise present is characterized by the loop SNR gain factor, which is defined as the ratio of the average SNR within the loop bandwidth of the PLL to the effective SNR of the FDMA/CDMA system. The system corresponding to Table I has a loop gain factor of 20 dB. For comparison purposes, analogous capacity calculations for the perfect reference cases taken from [1] are listed in Table II. The capacity estimates in Table I (and Table III) are obtained using equations in [3, Sec. VII], and the capacities in Table II are obtained using [1, eqs. [4(a)–(c)].

TABLE I

ESTIMATED CAPACITANCES OF FDMA/CDMA WITH PHASE NOISE. $N = 1023, M = 12$, LOOP GAIN FACTOR = 20 dB

L	System Capacity							
	$\delta_1 = 0.0$				$\delta_1 = 0.1$			
	10^{-1}	10^{-2}	10^{-3}	10^{-4}	10^{-1}	10^{-2}	10^{-3}	10^{-4}
1	929	319	171	106	919	305	158	93
2	1012	298	138	72	990	278	120	56
3	1017	261	102	42	987	237	84	27
4	992	220	72	20	956	196	52	8
6	906	150	30	0	870	132	12	0
12	612	24	0	0	600	24	0	0

TABLE II

ESTIMATED CAPACITIES OF FDMA/CDMA WITH PERFECT PHASE REFERENCE. $N = 1023, M = 12$

L	System Capacity							
	$\delta_1 = 0.0$				$\delta_1 = 0.1$			
	10^{-1}	10^{-2}	10^{-3}	10^{-4}	10^{-1}	10^{-2}	10^{-3}	10^{-4}
1	1758	480	241	145	1738	464	226	131
2	1656	412	182	94	1628	390	164	78
3	1560	348	135	57	1521	321	114	39
4	1464	292	96	28	1420	264	72	12
6	1290	198	42	0	1236	174	24	0
12	840	48	0	0	840	36	0	0

TABLE III

ESTIMATED CAPACITIES OF FDMA/CDMA WITH PHASE NOISE. $N_1 = 1023, M_1 = 12, \delta = 0.1$

L	System Capacity							
	Loop gain factor = 16 dB				Loop gain factor = 10 dB			
	10^{-1}	10^{-2}	10^{-3}	10^{-4}	10^{-1}	10^{-2}	10^{-3}	10^{-4}
1	691	247	131	77	385	153	82	48
2	782	234	102	46	468	154	66	28
3	798	204	72	21	495	135	45	9
4	784	168	44	4	496	112	28	0
6	726	108	12	0	468	72	0	0
12	504	12	0	0	336	0	0	0

From Table I, it is seen that, with the exception of the case where the required BER is 10^{-1}, the results indicate that the total capacity decreases as the available spectrum is more finely divided. Thus, wide-band CDMA is seen to provide the most efficient usage of available spectrum for the majority of the cases. For the 10^{-1} BER case, the total capacity first increases as L is increased, then eventually decreases as the spectrum is further divided. The greatest increase in capacity is seen when L is changed from $L = 1$ to $L = 2$ in both the $\delta_1 = 0.0$ and $\delta_1 = 0.1$ cases; this increase, however, is less than 10%. Also, it is seen that this apparent advantage of two- and three-segment FDMA/CDMA systems becomes less pronounced when δ_1 is increased from 0.0 to 0.1.

In general, the behavior of the total capacity for the different combinations of BER and L is similar to that observed for noncoherent demodulation [1], which is not surprising since the most notable effect of phase noise on receiver performance is the loss of coherence. To further illustrate this point, the estimated capacities for the systems with the same parameters as those used in Table I, but with greater amounts of phase noise, are listed in Table III. The amount of phase noise present is characterized by the loop SNR gain factor, which is defined as the ratio of the PLL SNR to the system SNR. The system corresponding to Table I has a gain factor of 20 dB, whereas

TABLE IV
SIMULATED CAPACITIES WITH PHASE NOISE. $N = 127, M = 12, \delta = 0.1$

L	System Capacity			
	Loop gain factor = 10 dB		Loop gain factor = 0 dB	
	10^{-1}	10^{-2}	10^{-1}	10^{-2}
1	188	52	135	47
2	164	44	122	40
4	152	36	116	32

the systems corresponding to Table III have loop gain factors of 10 and 16 dB.

It is seen that as the loop gain factor decreases, the system behaves increasingly like a noncoherent system, where wide-band CDMA does not have a clear advantage over the hybrid schemes for all BER's. Specifically, when the loop gain factor is 10 dB, wide-band CDMA no longer provides the greatest capacity for 10^{-2} BER, and is far from optimal for 10^{-1} BER.

V. COMPUTER SIMULATION

To verify the above observations, and to gauge the accuracy of the bound-based capacity estimates, a Monte Carlo computer simulation has been set up according to the model described above. Due to limitations on the computing resources, the simulation assumes no AWGN, and the processing gains used for the simulation are smaller than the ones used in the analytical estimates; specifically, the wide-band processing gain N_1, previously assumed to be 1023, is set equal to 127 for the simulation. Also, the Tikhonov density is approximated by a Gaussian density. The validity of this approximation for moderate to high PLL SNR's is well established [6].

Table IV shows the simulation results for $\delta_1 = 0.1, M_1 = 12$, BER values of 10^{-1} and 10^{-2}, loop gain factors of 0 and 10 dB, and $L = 1, 2$, and 4. The simulation results indicate that the wide-band system provides greater capacity than any of the hybrid FDMA/CDMA systems, even when the loop gain factor is as low as 0 dB. The percentage of the difference in capacity, however, does decrease as more phase noise is added, which is consistent with the previous observation (from the analytical results) that hybrid systems compare more favorably when there is more phase noise.

VI. CONCLUSIONS

Capacity estimates for hybrid FDMA/CDMA systems operating over a multipath Rayleigh channel in the presence of AWGN and phase noise have been presented. Numerical results indicate that the manner in which the capacity is decreased (relative to that of coherent demodulation with perfect phase reference) is similar to that observed for noncoherent demodulation and that the degree of similarity (to a noncoherent system) depends on the amount of phase noise present, as indicated by the PLL SNR gain factor.

Using the analytical estimates based on an approximate BER upper bound, and for the particular set of parameters shown, when the loop gain factor is 20 dB, wide-band CDMA yields greater capacity than FDMA/CDMA for all but the highest BER case (10^{-1}), and even then the wide-band system yields over 90% of the maximum achievable capacity. On the other hand, when the loop gain factor is 10 dB, wide-band CDMA has greatest capacity only for 10^{-3} and 10^{-4} BER's, and compares less favorably for the higher BER's.

Simulation results also indicate that the advantage of the wide-band system is lessened as phase noise is increased, but the capacity of the wide-band system remains greater than those of the hybrids for the range of loop gain factors considered. Thus, the effect of the bounding error seems to be an overestimation of the amount of phase noise required for the hybrid systems to surpass the wide-band system in capacity.

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