Uplink OFDMA without Cyclic Prefix: An Interference Cancellation Approach

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***Abstract*—In uplink OFDMA, timing offsets (TO) and/or carrier frequency offsets (CFO) of other users with respect to a desired user can cause multiuser interference (MUI). In practical uplink OFDMA systems, effect of this MUI is made acceptably small by requiring that timing/frequency alignment be achieved at the base station (BS) receiver with high precision. Alternatively, an ability to handle large TOs and CFOs using interference cancellation techniques at the BS receiver can offer the benefits of $i)$ allowing low-cost transmit carrier frequency oscillators to be used at all the mobiles in an open-loop mode which can reduce the mobile cost and complexity, and $ii)$ dropping the cyclic prefix (CP), which increases the throughput. In this paper, we first analytically characterize the degradation in the average output signal-to-interference ratio (SIR) due to the combined effect of both TOs as well as CFOs in uplink OFDMA without CP, and subsequently propose a parallel interference canceller (PIC) to mitigate the effect of interferences due to no CP and large TO/CFO. We illustrate the bit error performance of the proposed PIC for the cases of with and without CP. Simulation results show that performance close to that of 'with CP' can be achieved 'without CP' using increased number of PIC stages.**

I. Introduction

In uplink OFDMA [1], factors including $i)$ timing offsets (TO) of different users caused due to path delay differences between different users and imperfect timing synchronization, and $ii)$ carrier frequency offsets (CFO) of different users induced by Doppler effects and/or poor oscillator alignments, can destroy the orthogonality among subcarriers and cause multiuser interference (MUI). The detrimental effect of TO-induced orthogonality loss can be alleviated by $i)$ providing adequate cyclic prefix (expensive in terms of throughput for large TOs), or $ii)$ use of GPS timing (expensive in terms of hardware at the mobiles), or $iii)$ closed-loop timing correction between mobile transmitters and the base station (BS) receiver (expensive in terms of feedback bandwidth, pilot power, and oscillator cost). IEEE 802.16e standard [1], for example, mandates that the TO be brought within 1/8th of the cyclic prefix (CP) duration, and this necessitates the use of a tight closed-loop timing correction approach. Likewise, it is required that the CFOs be brought within 1% of the subcarrier spacing through closed-loop frequency correction [1].

An alternate open-loop approach to handle the effects of large TOs and CFOs in uplink OFDMA is to employ interference cancelling (IC) receivers at the BS. An ability to handle large TOs and CFOs using IC techniques can offer the benefits of $i)$ allowing low-cost transmit carrier frequency oscillators to be used at all the mobiles in an open-loop mode which can reduce the mobile cost and complexity, and $ii)$ dropping the CP, which increases the throughput. It is noted that if the CP is not added, the subcarrier orthogonality gets lost even when the timing is perfect (i.e., zero TO), and the resulting interference increases with increasing TO. However, an IC receiver can handle this interference as well.

Receivers employing the IC approach to handle the effects of CFOs alone, assuming ideal time synchronization and sampling (in other words, assuming no orthogonality loss due to timing offsets) have been proposed in [2],[3]. Effect of TOs on OFDM/OFDMA performance have been reported in several papers [4],[5]. Estimation CFOs and TOs in OFDM has been investigated in [6],[7]. These papers, however, are not concerned with cancellation of interference caused by both TOs as well as CFOs in uplink OFDMA. In this paper, we are concerned with the analytical characterization the interferences caused by both TOs as well as CFOs in uplink OFDMA, and the cancellation of these interferences *when CP is not added*. We point out that getting rid of CP becomes a possibility in uplink OFDMA if an open-loop cancellation approach can handle all sources of interference (CFO-/TO-induced multiuser interference, and self interferences), and provision of CP – which is primarily meant for handling self interference (i.e., ISI) – in that case can become redundant. So the open-loop cancellation approach allows the removal of the CP, in addition to allowing the use of low-cost oscillators at the mobile transmitters. We first analytically characterize the degradation in the average output signal-to-interference ratio (SIR) due to the combined effect of both TOs as well as CFOs in uplink OFDMA without CP. Using the knowledge of CFOs and TOs at the receiver, we then propose a parallel interference canceller (PIC) to mitigate the TO-/CFO-induced interference effects. Simulation results show that performance close to the case of 'with CP' can be achieved in the case of 'without CP' using increased number of PIC stages.

II. System Model

We consider an uplink OFDMA system with K users, where each user communicates with a BS through an independent multipath channel. We assume that there are N subcarriers in each OFDM symbol and one subcarrier can be allocated to only one user. The information symbol for the uth user on the kth subcarrier is denoted by $X_k^{(u)}$, $k \in S_u$, where

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S_u is the set of subcarriers assigned to the uth user and $E\left[\left|X_k^{(u)}\right|^2\right] = 1$. Then, $\bigcup_{u=1}^{K} S_u = \{0, 1, \ldots, N-1\}$ and $S_u \bigcap S_v = \phi$ for $u \neq v$. The length of the cyclic prefix added is N_g sampling periods[1], and is assumed to be longer than the maximum channel delay spread, L, normalized by the sampling period (i.e., $N_g > L$). After IDFT processing and cyclic prefix insertion at the transmitter, the time-domain sequence of the uth user, $x_n^{(u)}$, is given by

$$x_n^{(u)} = \frac{1}{N} \sum_{k \in S_u} X_k^{(u)} e^{\frac{j2\pi nk}{N}}, \quad -N_g \leq n \leq N-1. \tag{1}$$

When cyclic prefix is not added, then $N_g = 0$. The uth user's signal at the receiver input, after passing through the channel, is given by

$$s_n^{(u)} = x_n^{(u)} \star h_n^{(u)}, \tag{2}$$

where $\star$ denotes linear convolution and $h_n^{(u)}$ is the uth user's channel impulse response. It is assumed that $h_n^{(u)}$ is non-zero only for $n = 0, \ldots, L-1$, and that all users' channels are statistically independent. We assume that $h_n^{(u)}$'s are i.i.d. complex Gaussian with zero mean and $E\left[\left(h_{n,I}^{(u)}\right)^2\right] = E\left[\left(h_{n,Q}^{(u)}\right)^2\right] = 1/2L$, where $h_{n,I}^{(u)}$ and $h_{n,Q}^{(u)}$ are the real and imaginary parts of $h_n^{(u)}$. The channel coefficient in frequency-domain, $H_k^{(u)}$, is given by

$$H_k^{(u)} = \sum_{n=0}^{L-1} h_n^{(u)} e^{\frac{-j2\pi nk}{N}}, \quad \text{and} \quad E\left[\left|H_k^{(u)}\right|^2\right] = 1. \tag{3}$$

Let ϵ_u, $u = 1, 2, \cdots, K$ denote uth user's residual CFO normalized by the subcarrier spacing, $|\epsilon_u| \leq 0.5, \forall\, u$, and let μ_u, $u = 1, 2, \cdots, K$ denote uth user's residual TO in number of sampling periods at the receiver. The timing offsets μ_u's can be positive ($\mu_u > 0$) or negative ($\mu_u < 0$).

A. Cases of Timing Misalignment Without Cyclic Prefix

Let us consider the different cases of timing misalignment with no CP (i.e., $N_g = 0$) for OFDMA with a single user u. In order to highlight the effect of non-zero TOs (i.e., $|\mu_u| > 0$), consider $\epsilon_u = 0$ (i.e., zero CFO). For $\mu_u < 0$, depending on the magnitude of μ_u compared to delay spread L and cyclic prefix duration N_g, interference from previous frame data (which we refer to as *Previous Frame Self Interference* (PF-SI)) and inter-carrier interference due to loss of some samples of the current frame in the processing window (which we refer to as *Current Frame Self Interference* (CF-SI)) may or may not occur. We have to consider different cases of timing misalignment when there is no CP.

For $\mu_u \leq 0$, the following two cases of timing misalignment need to be considered.

- **Case a):** $\mu_u = 0$, where there is no interference from the first path, and there is PF-SI and CF-SI from the remaining $L-1$ paths (i.e., because there is no CP, there will be interference even when timing synchronization is perfect). This is in contrast with OFDMA with CP, where there will be no interference when $\mu_u = 0$.
- **Case b):** $-\mu_u > 0$, where there is PF-SI and CF-SI from all the L paths. This is in contrast with OFDMA with CP, where PF-SI/CF-SI does not occur for $-\mu_u \leq N_g - L + 1$.

For $\mu_u > 0$, the following two cases need to be considered.

- **Case c):** $0 < \mu_u < L$, where PF-SI can occur in addition to NF-SI and CF-SI. This is in contrast with the case of $N_g \neq 0$, where the non-zero CP will disallow PF-SI. For $N_g = 0$, however, NF-SI is caused by the first μ_u paths, PF-SI is caused by the last $L - \mu_u - 1$ paths, and the $(\mu_u + 1)$th path causes no interference. In addition, CF-SI also will occur due to loss of up to μ_u samples in the current frame.
- **Case d):** $\mu_u \geq L$, where NF-SI is caused by all paths. CF-SI will occur in this case as well.

Figure 1 illustrates the timing misalignment scenarios without CP, for the above four cases a) to d) for a given user u in the absence of other users. In addition to the above self interferences, other user interference will occur in the multiuser case. In the multiuser case also, the same four cases apply, and the corresponding MUI terms caused by previous, current and next frame data symbols of other users are denoted by *Previous Frame MUI* (PF-MUI), *Current Frame MUI* (CF-MUI), and *Next Frame MUI* (NF-MUI), respectively. Figure 2 illustrates a possible time misalignment scenario without CP for the multiuser case where the desired user is perfectly aligned (i.e., no self interferences) and the other users are misaligned (causing PF-MUI, CF-MUI, NF-MUI).

In the presence of both TOs as well as CFOs, additional CFO-induced interference will be generated. When there is no TO-induced interference on a given path, we refer to the interference generated by the desired user CFO as *CFO-induced Self Interference* (CFO-SI) on that path, and those generated by the other user CFOs as *CFO-induced MUI* (CFO-MUI) on that path. On all the paths that experience TO-induced interferences, non-zero CFOs (i.e., ϵ_u's) will affect PF-SI/MUI, CF-SI/MUI and NF-SI/MUI.

III. SIR ANALYSIS

In this section, we derive analytical expressions for the average SIR at the DFT output of the receiver in the presence of both TOs, μ_u's, and CFOs, ϵ_u's. We first obtain the analytical expressions for the output of the DFT for different time offset cases, and use these expressions to obtain the expressions for the average output SIR. We note that the DFT output corresponding to a given subcarrier consists of three components; a desired signal component, self interference components, and multiuser interference components.

A. DFT Output Expressions Without Cyclic Prefix

We consider that among the K users in the system, K_α users, $0 \leq K_\alpha \leq K$, belong to timing offset case $\alpha \in \mathcal{T}_{ncp}$,

[1]Let T denote one OFDM symbol period including the cyclic prefix duration. Then $T_s = \frac{T}{N+N_g}$ denotes one sampling period.

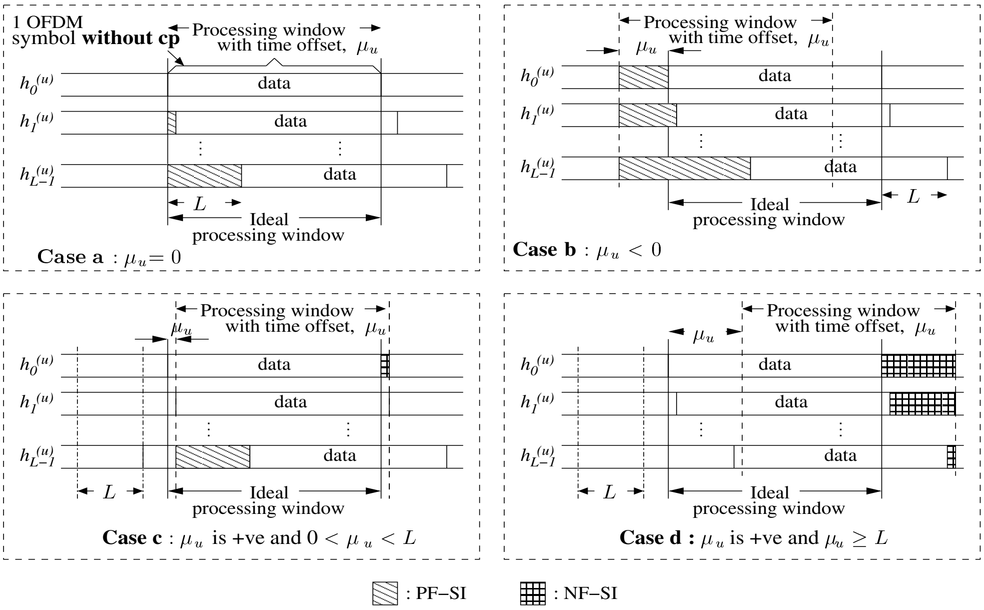

Fig. 1. Different timing misalignment scenarios for uplink OFDMA with a single user u **without cyclic prefix**. Only self interferences.

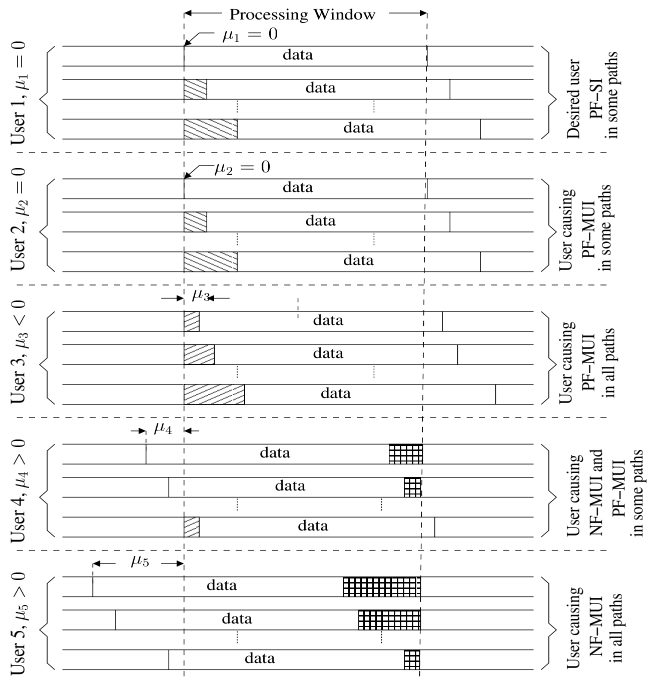

Fig. 2. Different timing misalignment scenarios in uplink OFDMA with multiple users **without cyclic prefix**.

where $\mathcal{T}_{ncp} = \{a, b, c, d\}$ denotes the set of time offset cases a) to d) without CP described in Sec. II-A, such that

$$\sum_{\alpha = a,b,c,d} K_\alpha = K. \quad (4)$$

Let the desired user u belong to the time offset case $\lambda \in \mathcal{T}_{ncp}$, and each of the other users belong to any of the time offset cases in $\mathcal{T}_{ncp}$. Since DFT is a linear operation, the desired signal, SI and MUI terms at the DFT output can be written individually for different cases of time offsets, as follows. Notation-wise, we use (DS), (SI), and (MI) in the superscript to denote the desired signal, SI and MUI, respectively.

Expressions for DS, SI, and MUI

Desired Signal: Let $Y_{k,\alpha}^{(u)(\mathrm{DS})}$ denote the desired signal component at the DFT output on the kth subcarrier of the desired user u, belonging to time offset case $\alpha \in \mathcal{T}_{ncp}$. Define

$$\Gamma_{qk}^{(u)(l)}(n_1, n_2) \;\triangleq\; \frac{1}{N}\sum_{n=n_1}^{n_2} e^{\frac{j2\pi n(q+\epsilon_u-k)}{N}}. \quad (5)$$

The desired user u, if belongs to case α), will have the following desired signal output:

$$Y_{k,\alpha}^{(u),(\mathrm{DS})} = X_k^{(u)} \underbrace{\left(e^{\frac{j2\pi\mu_u k}{N}} \sum_{l=0}^{L-1} h_l^{(u)} e^{\frac{-j2\pi lk}{N}} \, \Gamma_{kk}^{(u)(l)}(n_{\alpha_1}, n_{\alpha_2}) \right)}_{\triangleq\, \mathcal{H}_{k,\alpha}^{(u)}} \quad (6)$$

where $(n_{\alpha_1}, n_{\alpha_2})$ corresponding to different cases are given by

$$(n_{a_1}, n_{a_2}) = (l, N-1), \quad (7)$$

$$(n_{b_1}, n_{b_2}) = (l-\mu_u, N-1), \quad (8)$$

$$(n_{c_1}, n_{c_2}) = \begin{cases} (0, N-1-\mu_u+l), & \text{for } 0 \leq l \leq \mu_u \\ (l-\mu_u, N-1), & \text{for } l \geq \mu_u+1, \end{cases} \quad (9)$$

$$(n_{d_1}, n_{d_2}) = (0, N-1-\mu_u+l). \quad (10)$$

SI and MUI: Let $Y_{k,\alpha}^{(u)(\mathrm{SI})}$ denote the SI component at the DFT output of the desired user u, belonging to time offset case $\alpha \in \mathcal{T}_{ncp}$, on the kth subcarrier. Likewise, let $Y_{k,\lambda,\alpha}^{(u,v)(\mathrm{MI})}$ denote

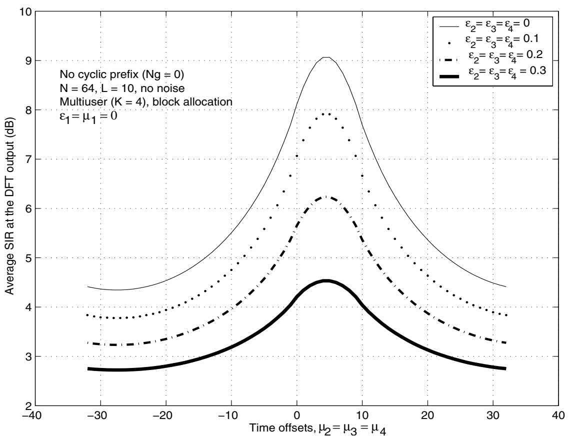

Fig. 3. Average SIR at the DFT output in multiuser uplink OFDMA with CFOs and TOs, and **without cyclic prefix** ($N_g = 0$). $K = 4$, $N = 64$, block allocation of subcarriers, $L = 10$, $\epsilon_1 = \mu_1 = 0$, $\epsilon_2 = \epsilon_3 = \epsilon_4$, $\mu_2 = \mu_3 = \mu_4$, no noise.

the MUI component at the DFT output of the desired user u belonging to any time offset $\lambda \in \mathcal{T}_{ncp}$, due to any other user v belonging to any time offset case $\alpha \in \mathcal{T}_{ncp}$. The expressions for $Y_{k,\alpha}^{(u)(\mathrm{SI})}$ and $Y_{k,\lambda,\alpha}^{(u,v)(\mathrm{MI})}$ for all $\lambda, \alpha \in \{a, b, c, d\}$ are given in Table-II. The overall DFT output on the kth subcarrier of the desired user u belonging to time offset case $\lambda \in \mathcal{T}_{ncp}$, denoted by $Y_{k,\lambda}^{(u)}$, is given by

$$Y_{k,\lambda}^{(u)} = Y_{k,\lambda}^{(u),(\mathrm{DS})} + Y_{k,\lambda}^{(u),(\mathrm{SI})} + \sum_{\alpha=a,b,c,d} \sum_{\substack{v=1 \\ v\neq u}}^{K_\alpha} Y_{k,\lambda,\alpha}^{(u,v),(\mathrm{MI})} + Z_k^{(u)}, \quad (11)$$

where $Z_k^{(u)}$ is the noise term which is complex Gaussian with zero mean and variance σ_n^2. The average output SINR on the kth subcarrier of the desired user u belonging to the time offset case $\lambda \in \mathcal{T}_{ncp}$ is then given by

$$\overline{SINR}_{k,\lambda}^{(u)} = \frac{P_\lambda^{(u),(\mathrm{DS})}}{\left(\sigma_{k,\lambda}^{(u),(\mathrm{SI})}\right)^2 + \sum_{\lambda=a,b,c,d}\left(\sigma_{k,\lambda}^{(u),(\mathrm{MI})}\right)^2 + \sigma_n^2}, \quad (12)$$

where the expressions for desired signal power $P_\lambda^{(u),(\mathrm{DS})}$, and the variances of the SI and MUI terms $\left(\sigma_{k,\lambda}^{(u),(\mathrm{SI})}\right)^2$, and $\left(\sigma_{k,\lambda}^{(u),(\mathrm{MI})}\right)^2$ are given in Table I. Figure 3 shows the SIR performance without cyclic prefix (i.e., $N_g = 0$) for $K = 4$ users, $N = 64$ subcarriers, and $L = 10$, as a function of TOs for various values of CFOs. User 1 is taken to be the desired user with $\mu_1 = \epsilon_1 = 0$. It is observed that the SIR without CP degrades even when ϵ's and μ's are zero, and the degradation gets increasingly severe for large CFOs and TOs. This loss can be alleviated through the use of IC techniques at the receiver.

IV. PIC RECEIVER

We note that IC receivers can be devised for the considered uplink OFDMA system using estimates of the current, previous and next data symbols of the desired as well as the other users. An example of such a receiver using parallel interference cancellation is illustrated in this section. Figure 4 shows the proposed PIC receiver for cancelling the CFO- and TO- induced interferences. The receiver first performs CFO compensation (by multiplying the received signal with $\exp(-j2\pi\hat{\epsilon}_u n/N)$, where $\hat{\epsilon}_u$ is an estimate of ϵ_u, followed by DFT operation and multistage interference cancellation. Following the CFO compensation, estimates of the current, previous and next data symbols on all subcarriers at the DFT output are made, which, along with the estimates of the channel coefficients, are then used to reconstruct the interference terms. The reconstructed interference terms are then subtracted from the received signal. These steps are repeated in multiple stages of cancellation. The cancellation operation in the proposed receiver is explained as follows.

We use the matched filter (i.e., single user detector) as the first stage of the PIC. For example, for BPSK modulation, the bit estimate at the output of the matched filter (MF) detector of user u belonging to time offset case λ on the kth subcarrier, denoted by $\widehat{X}_{k,(1)}^{(u)}$, is given by

$$\widehat{X}_{k,(1)}^{(u)} = \mathrm{sgn}\left\{\Re\left[\left(\widehat{\mathcal{H}}_{k,\lambda}^{(u)}\right)^* Y_{k,\lambda}^{(u)}\right]\right\}, \quad (13)$$

where the (1) in $\widehat{X}_{k,(1)}^{(u)}$ denotes the stage index, and $\widehat{\mathcal{H}}_{k,\lambda}^{(u)}$ is an estimate of $\mathcal{H}_{k,\lambda}^{(u)}$ with μ_u and ϵ_u replaced with their respective estimates $\hat{\mu}_u$ and $\hat{\epsilon}_u$. This bit decision operation is carried out on subcarriers of all users. In the second stage of the PIC, on each subcarrier of each user, the bit decisions from the first stage output using (13) are used to reconstruct the various interference components, cancel them from the MF output, and make bit decisions on the cancelled outputs. In general, for any stage-m, $m \geq 2$, the bit decision at the mth stage output is given by

$$\widehat{X}_{k,(m)}^{(u)} = \mathrm{sgn}\Bigg\{\Re\Bigg[\left(\widehat{\mathcal{H}}_{k,\lambda}^{(u)}\right)^*\Bigg(Y_{k,\lambda}^{(u)} - \widehat{Y}_{k,\lambda}^{(u),(\mathrm{SI})} - \sum_{v=1}^{K_a} \widehat{Y}_{k,\lambda,a}^{(u,v),(\mathrm{MI})}$$
$$- \sum_{v=1}^{K_b} \widehat{Y}_{k,\lambda,b}^{(u,v),(\mathrm{MI})} - \sum_{v=1}^{K_c} \widehat{Y}_{k,\lambda,c}^{(u,v),(\mathrm{MI})} - \sum_{v=1}^{K_d} \widehat{Y}_{k,\lambda,d}^{(u,v),(\mathrm{MI})}\Bigg)\Bigg]\Bigg\}, \quad (14)$$

where $\widehat{Y}_{k,\lambda}^{(u),(\mathrm{SI})}$ is given by $Y_{k,\lambda}^{(u),(\mathrm{SI})}$ in Table II with $X_q^{(u)}$ replaced with the bit estimate in the $(m-1)$th stage, $\widehat{X}_{q,(m-1)}^{(u)}$, and μ_u and ϵ_u replaced with $\hat{\mu}_u$ and $\hat{\epsilon}_u$, respectively. Similar replacements with bit, CFO, and TO estimates are done in $Y_{k,\lambda,\alpha}^{(u,v),(\mathrm{MI})}$ given in Table II to get the corresponding $\widehat{Y}_{k,\lambda,\alpha}^{(u,v),(\mathrm{MI})}$, $\lambda, \alpha \in \{a, b, c, d\}$. It is noted that to detect a bit in stage m, i.e., to get $\widehat{X}_{k,(m)}^{(u)}$ from (14), estimates of the previous, current, and next bits, i.e., $\widehat{X}_{q,(m-1)}^{(u),(p)}$, $\widehat{X}_{q,(m-1)}^{(u)}$, and $\widehat{X}_{q,(m-1)}^{(u),(n)}$, are needed. The requirement of the next bit estimate would introduce one extra bit delay in the overall detection, since cancellation in a stage is done after the next bit is detected in the previous stage. PIC receivers for M-ary modulation can be obtained likewise.

BER Performance of the Proposed PIC: In Fig.5, we present the BER performance of the 2nd and 3rd stages of the PIC receiver in uplink OFDMA with $K = 4$ users, $N = 64$ subcarriers, interleaved allocation of equal-power subcarriers, BPSK, and $L = 2$. For the system with cyclic prefix N_g is taken to be 4. The TOs and CFOs considered are $[\mu_1, \mu_2, \mu_3, \mu_4] = [-2, -5, 1, 5]$ and $[\epsilon_1, \epsilon_2, \epsilon_3, \epsilon_4] = [0.1, -0.3, -0.25, 0.15]$.

Case λ	$P_{k\lambda}^{(u),(\text{DS})}$	$\left(\sigma_{k,\lambda}^{(u),(\text{SI})}\right)^2$	$\left(\sigma_{k,\lambda}^{(u),(\text{MI})}\right)^2$
$\lambda \in \mathcal{T}_{ncp}$			
$\lambda = a$	$\sum_{l=0}^{L-1} \left\vert\Gamma_{kk}^{(u)(l)}(n_{a_1}, n_{a_2})\right\vert^2$	$\sum_{q\in S_u, q\neq k} \sum_{l=1}^{L-1} \left\vert\Gamma_{qk}^{(u)(l)}(n_{a_1}, n_{a_2})\right\vert^2 + \sum_{q\in S_u, q\neq k} \sum_{l=1}^{L-1} \left\vert\Gamma_{qk}^{(u)(l)}(0, n_{a_1}-1)\right\vert^2$	$\sum_{v=1, v\neq u}^{K_a} \Big[\sum_{q\in S_v, v\neq u} \sum_{l=1}^{L-1} \left\vert\Gamma_{qk}^{(v)(l)}(n_{a_1}, n_{a_2})\right\vert^2 + \sum_{q\in S_v, v\neq u} \sum_{l=1}^{L-1} \left\vert\Gamma_{qk}^{(v)(l)}(0, n_{a_1}-1)\right\vert^2\Big]$
$\lambda = b$	$\sum_{l=0}^{L-1} \left\vert\Gamma_{kk}^{(u)(l)}(n_{b_1}, n_{b_2})\right\vert^2$	$\sum_{q\in S_u, q\neq k} \sum_{l=0}^{L-1} \left\vert\Gamma_{qk}^{(u)(l)}(n_{b_1}, n_{b_2})\right\vert^2 + \sum_{q\in S_u} \sum_{l=0}^{L-1} \left\vert\Gamma_{qk}^{(u)(l)}(0, n_{b_1}-1)\right\vert^2$	$\sum_{v=1, v\neq u}^{K_b} \Big[\sum_{q\in S_v, v\neq u} \sum_{l=0}^{L-1} \left\vert\Gamma_{qk}^{(v)(l)}(n_{b_1}, n_{b_2})\right\vert^2 + \sum_{q\in S_v, v\neq u} \sum_{l=0}^{L-1} \left\vert\Gamma_{qk}^{(v)(l)}(0, n_{b_1}-1)\right\vert^2\Big]$
$\lambda = c$	$\sum_{l=0}^{L-1} \left\vert\Gamma_{kk}^{(u)(l)}(n_{c_1}, n_{c_2})\right\vert^2$	$\sum_{q\in S_u, q\neq k} \sum_{l=0}^{L-1} \left\vert\Gamma_{qk}^{(u)(l)}(n_{c_1}, n_{c_2})\right\vert^2 + \sum_{q\in S_u} \sum_{l=\mu_u+1}^{L-1} \left\vert\Gamma_{qk}^{(u)(l)}(0, n_{c_1}-1)\right\vert^2 + \sum_{q\in S_u} \sum_{l=0}^{\mu_u-1} \left\vert\Gamma_{qk}^{(u)(l)}(n_{c_2}+1, N-1)\right\vert^2$	$\sum_{v=1, v\neq u}^{K_c} \Big[\sum_{q\in S_v, v\neq u} \sum_{l=0}^{L-1} \left\vert\Gamma_{qk}^{(v)(l)}(n_{c_1}, n_{c_2})\right\vert^2 + \sum_{q\in S_v, v\neq u} \sum_{l=\mu_u+1}^{L-1} \left\vert\Gamma_{qk}^{(v)(l)}(0, n_{c_1}-1)\right\vert^2 + \sum_{q\in S_v, v\neq u} \sum_{l=0}^{\mu_u-1} \left\vert\Gamma_{qk}^{(v)(l)}(n_{c_2}+1, N-1)\right\vert^2\Big]$
$\lambda = d$	$\sum_{l=0}^{L-1} \left\vert\Gamma_{kk}^{(u)(l)}(n_{d_1}, n_{d_2})\right\vert^2$	$\sum_{q\in S_u, q\neq k} \sum_{l=0}^{L-1} \left\vert\Gamma_{qk}^{(u)(l)}(n_{d_1}, n_{d_2})\right\vert^2 + \sum_{q\in S_u} \sum_{l=0}^{L-1} \left\vert\Gamma_{qk}^{(u)(l)}(n_{d_2}+1, N-1)\right\vert^2$	$\sum_{v=1, v\neq u}^{K_d} \Big[\sum_{q\in S_v, v\neq u} \sum_{l=0}^{L-1} \left\vert\Gamma_{qk}^{(v)(l)}(n_{d_1}, n_{d_2})\right\vert^2 + \sum_{q\in S_v, v\neq u} \sum_{l=0}^{L-1} \left\vert\Gamma_{qk}^{(v)(l)}(n_{d_2}+1, N-1)\right\vert^2\Big]$

TABLE I

DESIRED SIGNAL POWER AND VARIANCES OF SI AND MUI COMPONENTS AT THE DFT OUTPUT IN UPLINK OFDMA **without cyclic prefix**.

We assume perfect estimates of all ϵ_i's, μ_i's and channel coefficients in the simulations. These channel coefficients are assumed to be constant in the processing window. User 1 is taken to be the desired user. The performance of the MF detector (i.e., no cancellation) as well as single user (i.e., no interference) performance are also plotted for comparison. From Fig. 5, we can make the following observations. The BER achieved with a conventional MF detector is quite high (high error floors close to 0.1 BER); this is because the values of TOs and CFOs considered are high which result in high interference. With MF detector, the BER with and without CP are about the same because other CFO- and TO-induced interference dominate compared to ISI induced by the lack of CP. With the proposed PIC, the BER improves significantly. With CP, the PIC with three stages (i.e., $m = 3$) is able to achieve a performance close to the single user performance. At a given cancellation stage, the performance without CP is worse compared to that with CP. For example, for $m = 2$, the error floor occurs at 2×10^{-3} BER with CP, whereas the flooring occurs at 4×10^{-3}. However, this loss in performance due to lack of CP can be recovered by using additional PIC stages. For example, the performance achieved for $m = 2$ with CP can be achieved without CP by increasing the number of cancellation stages to $m = 3$. Thus, because of its ability to handle high CFOs and TOs even without CP, the proposed PIC approach can potentially reduce the mobile transmitter cost and complexity and avoid the overhead due to CP.

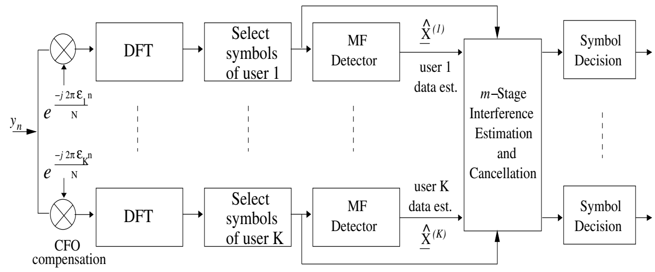

Fig. 4. PIC receiver for cancellation of interferences due to CFOs and TOs in uplink OFDMA without cyclic prefix.

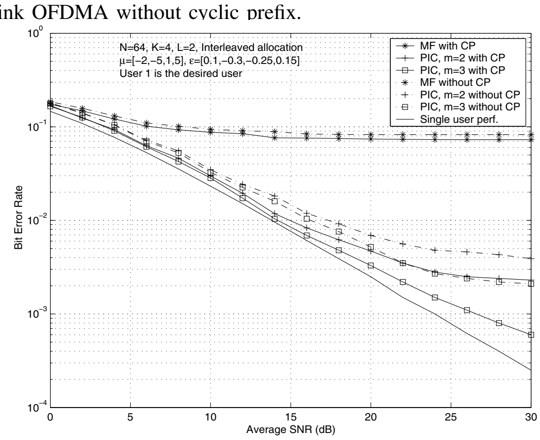

Fig. 5. BER performance of the PIC with and without CP. $K = 4$, $N = 64$, interleaved allocation, BPSK, $L = 2$, $[\mu_1, \mu_2, \mu_3, \mu_4] = [-2, -5, 1, 5]$, $[\epsilon_1, \epsilon_2, \epsilon_3, \epsilon_4] = [0.1, -0.3, -0.25, 0.15]$, $N_g = 4$ for system with CP.

REFERENCES

[1] IEEE 802.16e-2005: IEEE standard for Local and Metropolitan Area Networks. Part 16: Air Interface for Fixed and Mobile Broadband Wireless Access Systems. December 2005.

[2] D. Huang and K. B. Letaief, "An interference cancellation scheme for carrier frequency offsets correction in OFDMA systems," *IEEE Trans. on Commun.,* vol. 53, no. 7, pp. 1155-1165, July 2005.

[3] S. Manohar, V. Tikiya, D. Sreedhar, and A. Chockalingam, "Cancellation of multiuser interference due to carrier frequency offsets in uplink OFDMA," *IEEE Trans. Wireless Commun.,* vol. 6, no. 7, June 2007.

[4] Y. Mostofi and D. C. Cox, "Mathematical analysis of the impact of timing synchronization errors on the performance of an OFDM system," *IEEE Trans. on Commun.,* vol. 54, no. 2, pp. 226-230, February 2006.

[5] M. Park, K. Ko, H. Yoo, and D. Kong, "Performance analysis of OFDMA uplink systems with symbol timing misalignment," *IEEE Commun. Letters*, vol. 7, no. 8, pp. 376-378, August 2003.

[6] P. H. Moose, "A technique for orthogonal frequency division multiplexing frequency offset correction," *IEEE Trans. Commun.*, vol. 42, pp. 2908-2914, October 1994.

[7] J. J. van de Beek, M. Sandell, and P. O. Borjesson, "ML estimation of time and frequency offset in OFDM systems," *IEEE Trans. Sig. Proc.*, vol. 45, no. 7, pp. 18001805, 1997.

Case λ $\lambda \in \mathcal{T}_{ncp}$	***Self Interference Expressions***	***Interference Type***
$\lambda = a$	$Y^{(u),(\mathrm{SI})}_{k,a} = \sum_{\substack{q\in S_u \\ q\neq k}} X^{(u)}_q h^{(u)}_0 \, \Gamma^{(u)(l)}_{qk}(0, N-1)$ $+ \sum_{\substack{q\in S_u \\ q\neq k}} X^{(u)}_q \sum_{l=1}^{L-1} h^{(u)}_l e^{\frac{-j2\pi lq}{N}} \, \Gamma^{(u)(l)}_{qk}(n_{a_1}, n_{a_2})$ $+ \sum_{\substack{q\in S_u \\ q\neq k}} X^{(u)(p)}_q \sum_{l=1}^{L-1} h^{(u)}_l e^{\frac{-j2\pi lq}{N}} \, \Gamma^{(u)(l)}_{qk}(0, n_{a_1}-1)$	1st term: CFO-SI 2nd term: CF-SI 3rd term: PF-SI
$\lambda = b$	$Y^{(u),(\mathrm{SI})}_{k,b} = \sum_{\substack{q\in S_u \\ q\neq k}} X^{(u)}_q e^{\frac{j2\pi\mu_u q}{N}} \sum_{l=0}^{L-1} h^{(u)}_l e^{\frac{-j2\pi lk}{N}} \, \Gamma^{(u)(l)}_{qk}(n_{b_1}, n_{b_2})$ $+ \sum_{q\in S_u} X^{(u)(p)}_q e^{\frac{j2\pi\mu_u q}{N}} \sum_{l=0}^{L-1} h^{(u)}_l e^{\frac{-j2\pi lq}{N}} \, \Gamma^{(u)(l)}_{qk}(0, n_{b_1}-1)$	1st term: CF-SI 2nd term: PF-SI
$\lambda = c$	$Y^{(u),(\mathrm{SI})}_{k,c} = \sum_{q\in S_u} X^{(u)(n)}_q e^{\frac{j2\pi\mu_u q}{N}} \sum_{l=0}^{\mu_u-1} h^{(u)}_l e^{\frac{-j2\pi lq}{N}} \, \Gamma^{(u)(l)}_{qk}(n_{c_2}+1, N-1)$ $+ \sum_{\substack{q\in S_u \\ q\neq k}} X^{(u)}_q e^{\frac{j2\pi\mu_u q}{N}} \sum_{l=0}^{L-1} h^{(u)}_l e^{\frac{-j2\pi lq}{N}} \, \Gamma^{(u)(l)}_{qk}(n_{c_1}, n_{c_2})$ $+ \sum_{q\in S_u} X^{(u)(p)}_q e^{\frac{j2\pi\mu_u q}{N}} \sum_{l=\mu_u+1}^{L-1} h^{(u)}_l e^{\frac{-j2\pi lq}{N}} \, \Gamma^{(u)(l)}_{qk}(0, n_{c_1}-1)$	1st term: NF-SI 2nd term: CF-SI,CFO-SI 3rd term: PF-SI
$\lambda = d$	$Y^{(u),(\mathrm{SI})}_{k,d} = \sum_{\substack{q\in S_u \\ q\neq k}} X^{(u)}_q e^{\frac{j2\pi\mu_u q}{N}} \sum_{l=0}^{L-1} h^{(u)}_l e^{\frac{-j2\pi lq}{N}} \, \Gamma^{(u)(l)}_{qk}(n_{d_1}, n_{d_2})$ $+ \sum_{q\in S_u} X^{(u)(n)}_q e^{\frac{j2\pi\mu_u q}{N}} \sum_{l=0}^{L-1} h^{(u)}_l e^{\frac{-j2\pi lq}{N}} \, \Gamma^{(u)(l)}_{qk}(n_{d_2}+1, N-1)$	1st term: CF-SI 2nd term: NF-SI

Case λ $\lambda \in \mathcal{T}_{ncp}$	***Multiuser Interference Expressions***	***Interference Type***
$\lambda = a$	$Y^{(u,v),(\mathrm{MI})}_{k,\lambda,a} = \sum_{\substack{q\in S_v \\ v\neq u}} X^{(v)}_q h^{(v)}_0 \, \Gamma^{(v)(l)}_{qk}(0, N-1)$ $+ \sum_{\substack{q\in S_v \\ v\neq u}} X^{(v)}_q \sum_{l=1}^{L-1} h^{(v)}_l e^{\frac{-j2\pi lq}{N}} \, \Gamma^{(v)(l)}_{qk}(n_{a_1}, n_{a_2})$ $+ \sum_{\substack{q\in S_v \\ v\neq u}} X^{(v)(p)}_q \sum_{l=1}^{L-1} h^{(v)}_l e^{\frac{-j2\pi lq}{N}} \, \Gamma^{(v)(l)}_{qk}(0, n_{a_1}-1)$	1st term: CFO-MUI 2nd term: CF-MUI 3rd term: PF-MUI
$\lambda = b$	$Y^{(u,v),(\mathrm{MI})}_{k,\lambda,b} = \sum_{\substack{q\in S_v \\ v\neq u}} X^{(v)}_q e^{\frac{j2\pi\mu_v q}{N}} \sum_{l=0}^{L-1} h^{(v)}_l e^{\frac{-j2\pi lk}{N}} \, \Gamma^{(v)(l)}_{qk}(n_{b_1}, n_{b_2})$ $+ \sum_{\substack{q\in S_v \\ v\neq u}} X^{(v)(p)}_q e^{\frac{j2\pi\mu_v q}{N}} \sum_{l=0}^{L-1} h^{(v)}_l e^{\frac{-j2\pi lq}{N}} \, \Gamma^{(v)(l)}_{qk}(0, n_{b_1}-1)$	1st term: CF-MUI 2nd term: PF-MUI
$\lambda = c$	$Y^{(u,v),(\mathrm{MI})}_{k,\lambda,c} = \sum_{\substack{q\in S_v \\ v\neq u}} X^{(v)(n)}_q e^{\frac{j2\pi\mu_v q}{N}} \sum_{l=0}^{\mu_v-1} h^{(v)}_l e^{\frac{-j2\pi lq}{N}} \, \Gamma^{(v)(l)}_{qk}(n_{c_2}+1, N-1)$ $+ \sum_{\substack{q\in S_v \\ v\neq u}} X^{(v)}_q e^{\frac{j2\pi\mu_v q}{N}} \sum_{l=0}^{L-1} h^{(v)}_l e^{\frac{-j2\pi lq}{N}} \, \Gamma^{(v)(l)}_{qk}(n_{c_1}, n_{c_2})$ $+ \sum_{\substack{q\in S_v \\ v\neq u}} X^{(v)(p)}_q e^{\frac{j2\pi\mu_v q}{N}} \sum_{l=\mu_v+1}^{L-1} h^{(v)}_l e^{\frac{-j2\pi lq}{N}} \, \Gamma^{(v)(l)}_{qk}(0, n_{c_1}-1)$	1st term: NF-MUI 2nd term: CF-MUI,CFO-MUI 3rd term: PF-MUI
$\lambda = d$	$Y^{(u,v),(\mathrm{MI})}_{k,\lambda,d} = \sum_{\substack{q\in S_v \\ v\neq u}} X^{(v)}_q e^{\frac{j2\pi\mu_v q}{N}} \sum_{l=0}^{L-1} h^{(v)}_l e^{\frac{-j2\pi lq}{N}} \, \Gamma^{(v)(l)}_{qk}(n_{d_1}, n_{d_2})$ $+ \sum_{\substack{q\in S_v \\ v\neq u}} X^{(v)(n)}_q e^{\frac{j2\pi\mu_v q}{N}} \sum_{l=0}^{L-1} h^{(v)}_l e^{\frac{-j2\pi lq}{N}} \, \Gamma^{(v)(l)}_{qk}(n_{d_2}+1, N-1)$	1st term: CF-SI 2nd term: NF-SI

TABLE II

SELF INTERFERENCE AND MULTIUSER INTERFERENCE EXPRESSIONS FOR DIFFERENT CASES OF TIME OFFSETS **without cyclic prefix**.