High-rate, Single-Symbol Decodable Distributed STBCs for Partially-Coherent Cooperative Networks

D. Sreedhar, A. Chockalingam, and B. Sundar Rajan
Department of ECE, Indian Institute of Science, Bangalore 560012, INDIA

***Abstract*—Space-time block codes (STBCs) that are single-symbol decodable (SSD) in a co-located multiple antenna setting need not be SSD in a distributed cooperative communication setting. A relay network with N relays and a single source-destination pair is called a partially-coherent relay channel (PCRC) if the destination has perfect channel state information (CSI) of all the channels and the relays have only the phase information of the source-to-relay channels. In our earlier work, we had derived a set of necessary and sufficient conditions for a distributed STBC (DSTBC) to be SSD for a PCRC. Using these conditions, in this paper we show that the possibility of *channel phase compensation* operation at the relay nodes using partial CSI at the relays increases the possible rate of SSD DSTBCs from $\frac{2}{N}$ when the relays do not have CSI to $\frac{1}{2}$, which is independent of N. We also show that when a DSTBC is SSD for a PCRC, then arbitrary coordinate interleaving of the in-phase and quadrature-phase components of the variables does not disturb its SSD property. Using this property we are able to construct codes that are SSD and have higher rate than $\frac{2}{N}$ but giving full diversity only for signal constellations satisfying certain conditions.**

Keywords – *Cooperative communications, amplify-and-forward protocol, distributed STBC, single-symbol decoding.*

I. Introduction

The problem of fading and the ways to combat it through spatial diversity techniques have been an active area of research. Multiple-input multiple-output (MIMO) techniques have become popular in realizing spatial diversity and high data rates through the use of multiple transmit antennas. For such co-located multiple transmit antenna systems low maximum-likelihood (ML) decoding complexity space-time block codes (STBCs) have been studied by several researchers [1]-[10] which include the well known complex orthogonal designs (CODs) and their generalizations. Recent research has shown that the advantages of spatial diversity could be realized in single-antenna user nodes through user cooperation [11],[12] via relaying. A simple wireless relay network of $N + 2$ nodes consists of a single source-destination pair with N relays. For such relay channels, use of CODs has been studied in [13]. CODs are attractive for cooperative communications for the following reasons: i) they offer full diversity gain and coding gain, ii) they are 'scale free' in the sense that deleting some rows does not affect the orthogonality, iii) entries are linear combination of the information symbols and their conjugates which means only linear processing is required at the relays, and iv) they admit very fast ML decoding (single-symbol decoding (SSD)). However, it should be noted that the last property applies only to the decode-and-forward (DF) policy at the relay node.

In a scenario where the relays amplify and forward (AF) the signal, it is known that the orthogonality is lost, and hence the destination has to use a complex multi-symbol ML decoding or sphere decoding [13],[14]. It should be noted that the AF policy is attractive for two reasons: i) the complexity at the relay is greatly reduced, and ii) the restrictions on the rate because the relay has to decode is avoided [15]. In order to avoid the complex ML decoding at the destination, in [16], the authors propose an alternative code design strategy and propose a SSD code for 2 and 4 relays. For arbitrary number of relays, recently in [17], distributed orthogonal STBCs (DOSTBCs) have been studied and it is shown that if the destination has the complete channel state information (CSI) of all the source-to-relay channels and the relay-to-destination channels, then the maximum possible rate is upper bounded by $\frac{2}{N}$ complex symbols per channel use for N relays. Towards improving the rate of transmission and achieving simultaneously both full-diversity as well as SSD at the destination, in our earlier work [18], we study relay channels with the assumption that the relays have the phase information of the source-to-relay channels and the destination has the CSI of all the channels. We derived a set of necessary and sufficient conditions for a DSTBC to be SSD in PCRC.

In this paper, we use the conditions developed in [18] and show the following results regarding the high rate codes.

- It is shown that the possibility of *channel phase compensation* operation at the relay nodes using partial CSI at the relays increases the possible rate of SSD DSTBCs from $\frac{2}{N}$ when the relays do not have CSI to $\frac{1}{2}$, which is independent of N.
- It is shown that when a DSTBC is SSD for a PCRC, then arbitrary coordinate interleaving of the in-phase and quadrature-phase components of the variables does not disturb its SSD property for PCRC. This property enables construction of codes that are SSD and have higher rate than $\frac{2}{N}$ but offer full diversity only for signal constellations satisfying certain conditions.

A unified treatment of the results of [18] and of this paper is available as [20].

The remaining part of the paper is organized as follows: In

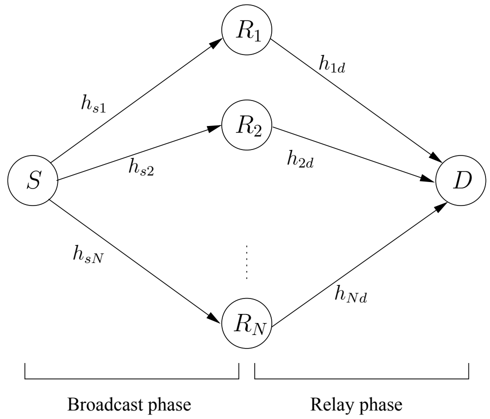

Fig. 1. A cooperative relay network.

Section II, the signal model for a PCRC is developed. In Section III, we recapitulate the necessary and sufficient conditions derived in [18] for a DSTBC to be SSD in a PCRC is obtained. In Section IV, we prove that the rate-halving codes from real orthogonal designs are SSD for PCRC. Then, in Section V, we prove that arbitrary coordinate interleaving of the in-phase and quadrature-phase components of the variables does not disturb its SSD property for PCRC. Concluding remarks are given in Section VI.

II. System Model

Consider a wireless network with $N+2$ nodes consisting of a source, a destination, and N relays, as shown in Fig. 1. All nodes are half-duplex nodes, i.e., a node can either transmit or receive at a time on a specific frequency. We consider amplify-and-forward (AF) transmission at the relays. Transmission from the source to the destination is carried out in two phases. In the first phase, the source transmits information symbols $x^{(i)}, 1 \leq i \leq T_1$ in T_1 time slots. All the N relays receive these T_1 symbols. This phase is called the *broadcast phase*. In the second phase, all the N relays[1] perform distributed space-time block encoding on their received vectors and transmit the resulting encoded vectors in T_2 time slots. That is, each relay will transmit a column (with T_2 entries) of a distributed STBC matrix of size $T_2 \times N$. The destination receives a faded and noise added version of this matrix. This phase is called the *relay phase*. We assume that the source-to-relay channels remain static over T_1 time slots, and the relay-to-destination channels remain static over T_2 time slots.

The received signal at the jth relay, $j = 1, \cdots, N$, in the ith time slot, $i = 1, \cdots, T_1$, denoted by $v_j^{(i)}$, can be written as[2]

$$v_j^{(i)} = \sqrt{E_1} h_{sj} x^{(i)} + z_j^{(i)}, \qquad (1)$$

where h_{sj} is the complex channel gain from the source s to the jth relay, $z_j^{(i)}$ is additive white Gaussian noise at relay j with zero mean and unit variance, E_1 is the transmit energy per symbol in the broadcast phase, and $\mathbb{E}\left[\left(x^{(i)}\right)^* x^{(i)}\right] = 1$. We assume that each relay has the knowledge of the channel phase on the link between the source and itself in the broadcast phase. That is, defining the channel gain from source to relay j as $h_{sj} = \alpha_{sj} e^{\mathbf{j}\theta_{sj}}$, we assume that relay j has perfect knowledge of only θ_{sj} and does not have the knowledge of α_{sj}. We perform a channel phase compensation operation and amplification of the received signals at the relays before performing space-time encoding. That is, we multiply $v_j^{(i)}$ in (1) by $e^{-\mathbf{j}\theta_{sj}}$ as well as by an amplification factor G . The amplified and phase compensated ith received signal at the jth relay can be written as

$$\widehat{v}_j^{(i)} = \underbrace{\sqrt{\frac{E_2}{E_1+1}}}_{\triangleq G} e^{-\mathbf{j}\theta_{sj}} v_j^{(i)}, \qquad (2)$$

where E_2 is the transmit energy per transmission of a symbol in the relay phase, and the amplification factor G at the relay, makes the total transmission energy per symbol in the relay phase to be equal to E_2. Let E_t denote the total energy per symbol in both the phases put together. Then, it is shown in [15] that the optimum energy allocation that maximizes the receive SNR at the destination is when half the energy is spent in the broadcast phase and the remaining half in the relay phase when the time allocations for the relay and broadcast phase are same i.e., $T_1 = T_2$. We also assume that the energy is distributed equally i.e., $E_1 = \frac{E_t}{2}$ and $E_2 = \frac{E_t}{2M}$, where M is the number of transmissions per symbol in the STBC. For the unequal-time allocation ($T_1 \neq T_2$) this distribution might not be optimal.

At relay j, a $2T_1 \times 1$ real vector $\widehat{\mathbf{v}}_j$ given by

$$\widehat{\mathbf{v}}_j = \left[\widehat{v}_{jI}^{(1)}, \widehat{v}_{jQ}^{(1)}, \widehat{v}_{jI}^{(2)}, \widehat{v}_{jQ}^{(2)}, \cdots, \widehat{v}_{jI}^{(T_1)}, \widehat{v}_{jQ}^{(T_1)}\right]^T, \qquad (3)$$

is formed, where $\widehat{v}_{jI}^{(i)}$ and $\widehat{v}_{jQ}^{(i)}$, respectively, are the in-phase (real part) and quadrature (imaginary part) components of $\widehat{v}_j^{(i)}$. Now, (3) can be written in the form

$$\widehat{\mathbf{v}}_j = G\sqrt{E_1}\,|h_{sj}|\,\mathbf{x} + \widehat{\mathbf{z}}_j. \qquad (4)$$

where $\mathbf{x}$ is the $2T_1 \times 1$ data symbol real vector, given by

$$\mathbf{x} = \left[x_I^{(1)}, x_Q^{(1)}, x_I^{(2)}, x_Q^{(2)}, \cdots, x_I^{(T_1)}, x_Q^{(T_1)}\right]^T, \qquad (5)$$

$\widehat{\mathbf{z}}_j$ is the $2T_1 \times 1$ noise vector, given by

$$\widehat{\mathbf{z}}_j = \left[\widehat{z}_{jI}^{(1)}, \widehat{z}_{jQ}^{(1)}, \widehat{z}_{jI}^{(2)}, \widehat{z}_{jQ}^{(2)}, \cdots, \widehat{z}_{jI}^{(T_1)}, \widehat{z}_{jQ}^{(T_1)}\right]^T,$$

where $\widehat{z}_j^{(i)} = G\, e^{-\mathbf{j}\theta_{sj}} z_j^{(i)}$. Let

$$\mathbf{C} = \left[\mathbf{c}_1, \mathbf{c}_2, \cdots, \mathbf{c}_N\right] \qquad (6)$$

[1] Here, we assume that all the N relays participate in the cooperative transmission. It is also possible that some relays do not participate in the transmission based on whether the channel is in outage or not. We do not consider such a partial participation scenario here.

[2] We use the following notation: Vectors are denoted by boldface lowercase letters, and matrices are denoted by boldface uppercase letters. Superscripts T and $\mathcal{H}$ denote transpose and conjugate transpose operations, respectively and $*$ denotes matrix conjugation operation.

denote the $T_2 \times N$ distributed STBC matrix to be sent in the relay phase jointly by all N relays, where $\mathbf{c}_j$ denotes the jth column of $\mathbf{C}$. The jth column $\mathbf{c}_j$ is manufactured by the jth relay as

$$\begin{aligned} \mathbf{c}_j &= \mathbf{A}_j \widehat{\mathbf{v}}_j \\ &= G\sqrt{E_1}\mathbf{A}_j\,|h_{sj}|\,\mathbf{x} + \mathbf{A}_j\,\widehat{\mathbf{z}}_j, \end{aligned} \quad (7)$$

where $\mathbf{A}_j$ is the complex processing matrix of size $T_2 \times 2T_1$ for the jth relay, called the *relay matrix*. For example, for the 2-relay case (i.e., $N = 2$), with $T_1 = T_2 = 2$, using Alamouti code, the relay matrices are given by

$$\mathbf{A}_1 = \begin{bmatrix} 1 & \mathbf{j} & 0 & 0 \\ 0 & 0 & -1 & \mathbf{j} \end{bmatrix} \text{ and } \mathbf{A}_2 = \begin{bmatrix} 0 & 0 & 1 & \mathbf{j} \\ 1 & -\mathbf{j} & 0 & 0 \end{bmatrix}. \quad (8)$$

Let $\mathbf{y}$ denote the $T_2 \times 1$ received signal vector at the destination in T_2 time slots. Then, $\mathbf{y}$ can be written as

$$\mathbf{y} = \sum_{j=1}^{N} h_{jd}\mathbf{c}_j + \mathbf{z}_d, \quad (9)$$

where h_{jd} is the complex channel gain from the jth relay to the destination, and $\mathbf{z}_d$ is the AWGN noise vector at the destination with zero mean and $E[\mathbf{z}_d\,\mathbf{z}_d^{\mathcal{H}}] = \mathbf{I}$. Substituting (7) in (9), we can write

$$\mathbf{y} = G\sqrt{E_1}\left(\sum_{j=1}^{N} h_{jd}|h_{sj}|\mathbf{A}_j\right)\mathbf{x} + \underbrace{\sum_{j=1}^{N} h_{jd}\mathbf{A}_j\widehat{\mathbf{z}}_j + \mathbf{z}_d}_{\tilde{\mathbf{z}}_d\,:\,\text{total noise}}. \quad (10)$$

Such systems will be referred as *partially-coherent relay channels* (PCRC). A distributed STBC which is SSD for a PCRC will be referred as SSD-DSTBC-PCRC.

III. Conditions for SSD in PCRC

In [18], we have proved the following two theorems.

Theorem 1: For co-located MIMO with N transmit antennas, the linear STBC as given in (6) is SSD *iff*

$$\begin{aligned} &\mathbf{A}_{jI}^T\mathbf{A}_{jI} + \mathbf{A}_{jQ}^T\mathbf{A}_{jQ} = \mathbf{D}_{jj}^{(1)}; \forall j, \\ &\mathbf{A}_{jI}^T\mathbf{A}_{iI} + \mathbf{A}_{jQ}^T\mathbf{A}_{iQ} + \mathbf{A}_{iI}^T\mathbf{A}_{jI} + \mathbf{A}_{iQ}^T\mathbf{A}_{jQ} = \mathbf{D}_{ij}^{(2)}; \forall i,j, i \neq j, \\ &\mathbf{A}_{jI}^T\mathbf{A}_{iQ} + \mathbf{A}_{jQ}^T\mathbf{A}_{iI} - \mathbf{A}_{iI}^T\mathbf{A}_{jQ} - \mathbf{A}_{iQ}^T\mathbf{A}_{jI} = \mathbf{D}_{ij}^{(3)}; \forall i,j, i \neq j, \end{aligned} \quad (11)$$

where $\mathbf{A}_j = \mathbf{A}_{jI} + \mathbf{j}\mathbf{A}_{jQ},\ j = 1,2,\cdots,N$, where $\mathbf{A}_{jI}$ and $\mathbf{A}_{jQ}$ are real matrices, and $\mathbf{D}_{jj}^{(1)}$, $\mathbf{D}_{ij}^{(2)}$ and $\mathbf{D}_{ij}^{(3)}$ are block diagonal matrices of the form

$$\mathbf{D}_{ij}^{(k)} = \begin{bmatrix} \mathbf{D}_{ij,1}^{(k)} & \mathbf{0} & \cdots & \mathbf{0} \\ \mathbf{0} & \mathbf{D}_{ij,2}^{(k)} & \cdots & \mathbf{0} \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{0} & \cdots & \cdots & \mathbf{D}_{ij,T_1}^{(k)} \end{bmatrix}, \quad (12)$$

where $\mathbf{D}_{ij,l}^{(k)} = \begin{bmatrix} a_{ij,l}^{(k)} & b_{ij,l}^{(k)} \\ b_{ij,l}^{(k)} & c_{ij,l}^{(k)} \end{bmatrix}$ and it is understood that whenever the superscript is (1) as in $\mathbf{D}_{ij}^{(1)}$, then $i = j$.

Theorem 2: A code as given by (7) is SSD-DSTBC-PCRC *iff* the relay matrices $\mathbf{A}_j,\ j = 1,2,\cdots,N$, satisfy (11) (i.e., the code is SSD for a co-located MIMO set up), and, in addition,

$$\begin{aligned} &{\mathbf{A}_{j_1I}}^T\left(\mathbf{A}_{j_2I}{\mathbf{A}_{j_2I}}^T + \mathbf{A}_{j_2Q}{\mathbf{A}_{j_2Q}}^T\right)\mathbf{A}_{j_3I} + \\ &{\mathbf{A}_{j_3I}}^T\left(\mathbf{A}_{j_2I}{\mathbf{A}_{j_2I}}^T + \mathbf{A}_{j_2Q}{\mathbf{A}_{j_2Q}}^T\right)\mathbf{A}_{j_1I} + \\ &{\mathbf{A}_{j_1Q}}^T\left(\mathbf{A}_{j_2I}{\mathbf{A}_{j_2I}}^T + \mathbf{A}_{j_2Q}{\mathbf{A}_{j_2Q}}^T\right)\mathbf{A}_{j_3Q} + \\ &{\mathbf{A}_{j_3Q}}^T\left(\mathbf{A}_{j_2I}{\mathbf{A}_{j_2I}}^T + \mathbf{A}_{j_2Q}{\mathbf{A}_{j_2Q}}^T\right)\mathbf{A}_{j_1Q} = \mathbf{D}'_{j_1,j_2,j_3}, \\ &\qquad \forall j_1, j_2, j_3, \end{aligned} \quad (13)$$

$$\begin{aligned} &{\mathbf{A}_{j_1I}}^T\left(\mathbf{A}_{j_2I}{\mathbf{A}_{j_2Q}}^T + \mathbf{A}_{j_2Q}{\mathbf{A}_{j_2I}}^T\right)\mathbf{A}_{j_3Q} + \\ &{\mathbf{A}_{j_3Q}}^T\left(\mathbf{A}_{j_2I}{\mathbf{A}_{j_2Q}}^T + \mathbf{A}_{j_2Q}{\mathbf{A}_{j_2I}}^T\right)\mathbf{A}_{j_1I} + \\ &{\mathbf{A}_{j_1Q}}^T\left(\mathbf{A}_{j_2I}{\mathbf{A}_{j_2Q}}^T + \mathbf{A}_{j_2Q}{\mathbf{A}_{j_2I}}^T\right)\mathbf{A}_{j_3I} + \\ &{\mathbf{A}_{j_3I}}^T\left(\mathbf{A}_{j_2I}{\mathbf{A}_{j_2Q}}^T + \mathbf{A}_{j_2Q}{\mathbf{A}_{j_2I}}^T\right)\mathbf{A}_{j_1Q} = \mathbf{D}''_{j_1,j_2,j_3}, \\ &\qquad \forall j_1, j_2, j_3, \end{aligned} \quad (14)$$

where $\mathbf{D}'_{j_1,j_2,j_3}$ and $\mathbf{D}''_{j_1,j_2,j_3}$, $1 \leq j_1, j_2, j_3 \leq N$, are block diagonal matrices of the form in (12).

We will use these results to derive our new results in the following two sections.

IV. A class of rate-$\frac{1}{2}$ SSD DSTBCs

It is well known that the rate of square SSD codes for co-located MIMO systems falls exponentially as the number of antennas increases. In this section, it is shown that if non-square designs are used then SSD codes for PCRCs can be achieved with rate $\frac{1}{2}$ for any number of antennas.

It is well known [1] that real orthogonal designs (RODs) with rate one exist for any number of antennas and these are non-square designs for more than 2 antennas and the delay increases exponentially with the number of antennas. Using these RODs, in [1], a class of rate $\frac{1}{2}$ complex orthogonal designs for any number of antennas is obtained as follows: If $\mathbf{G}$ is a $p \times N$ rate one ROD, where p denotes the delay and N denotes the number of antennas with variables $x_1, x_2, \cdots, x_p$, then, denoting by $\mathbf{G}^*$ the complex design obtained by replacing x_i with x_i^*, $i = 1,2,\cdots,p$, the design $\begin{bmatrix} \mathbf{G} \\ \mathbf{G}^* \end{bmatrix}$ is a $2p \times N$ rate-$\frac{1}{2}$ COD. We refer to this construction as stacking construction. The following theorem asserts that the rate $\frac{1}{2}$ CODs by stacking construction are SSD for PCRC.

Theorem 3: The rate-1/2 CODs, constructed from rate one RODs by stacking construction [1] are SSD-DSTBC-PCRC.

Proof: Let $\mathbf{G}_c$ be the rate-1/2 COD obtained from a $p \times N$ ROD $\mathbf{G}$ by stacking construction, i.e.,

$$\mathbf{G}_c = \begin{bmatrix} \mathbf{G} \\ \mathbf{G}^* \end{bmatrix}. \quad (15)$$

Let the $p \times p$ real matrices $\hat{\mathbf{A}}_j\ j = 1,\cdots,N$ generate the columns of $\mathbf{G}$, i.e.,

$$\mathbf{G} = \left[\hat{\mathbf{A}}_1\mathbf{x}, \hat{\mathbf{A}}_2\mathbf{x}, \cdots, \hat{\mathbf{A}}_N\mathbf{x}\right], \quad (16)$$

where $\mathbf{x}$ is the $p \times 1$ real data vector and the matrices $\hat{\mathbf{A}}_j$ denote the column vector representation matrices used in [5]. By the definition of RODs, $\mathbf{G}^T\mathbf{G} = \left(\mathbf{x}^T\mathbf{x}\right)\mathbf{I}$. This implies that

$$\begin{aligned} \hat{\mathbf{A}}_j^T\hat{\mathbf{A}}_j &= \mathbf{I},\ j = 1, \cdots, N \\ \hat{\mathbf{A}}_j^T\hat{\mathbf{A}}_i &= -\hat{\mathbf{A}}_i^T\hat{\mathbf{A}}_j,\ i, j = 1, \cdots, N, i \neq j. \end{aligned} \quad (17)$$

It is noted that the Hurwitz-Radon family of matrices satisfy (17) and explicit construction for any N is given in [1]. It is noted that the representation in [1] is different from the column vector representation used in this paper. An important consequence is that the Hurwitz-Radon family of matrices satisfy the conditions

$$\begin{aligned} \hat{\mathbf{A}}_j^T\hat{\mathbf{A}}_j &= \mathbf{I},\ j = 1, \cdots, N \\ \hat{\mathbf{A}}_j^T &= -\hat{\mathbf{A}}_j,\ j = 1, \cdots, N \\ \hat{\mathbf{A}}_j\hat{\mathbf{A}}_i &= -\hat{\mathbf{A}}_i\hat{\mathbf{A}}_j,\ i, j = 1, \cdots, N, i \neq j, \end{aligned} \quad (18)$$

and hence $\hat{\mathbf{A}}_j\hat{\mathbf{A}}_j^T = \mathbf{I}\ \forall j$, which we will use in our proof. Viewing $\mathbf{G}_c$ as a $T_2 \times N$ distributed STBC with $T_1 = p$ and $T_2 = 2p$, the $T_2 \times 2T_1$ relay matrices $\mathbf{A}_j$ of $\mathbf{G}_c$ have the structure

$$\mathbf{A}_{jI} = \begin{pmatrix} \mathbf{U}_j \\ \mathbf{U}_j \end{pmatrix} \text{ and } \mathbf{A}_{jQ} = \begin{pmatrix} \mathbf{V}_j \\ -\mathbf{V}_j \end{pmatrix}. \quad (19)$$

Since $\mathbf{G}_c$ is constructed from a ROD, the coefficients of real and imaginary components are same, i.e., the matrices $\mathbf{U}_j$ and $\mathbf{V}_j$ have the form

$$\begin{aligned} \mathbf{U}_j &= \left[\gamma_{1,j}, \mathbf{0}, \gamma_{2,j}, \mathbf{0}, \cdots, \gamma_{T_1,j}, \mathbf{0}\right] \\ \mathbf{V}_j &= \left[\mathbf{0}, \gamma_{1,j}, \mathbf{0}, \gamma_{2,j}, \cdots, \mathbf{0}, \gamma_{T_1,j}\right], \end{aligned} \quad (20)$$

with $\gamma_{i,j}$ are column vectors of $\hat{\mathbf{A}}_j$. Since $\hat{\mathbf{A}}_j\hat{\mathbf{A}}_j^T = \mathbf{I}\ \forall j$, it is easily verified that $\mathbf{U}_j\mathbf{U}_j^T = \mathbf{I}$ and $\mathbf{V}_j\mathbf{V}_j^T = \mathbf{I}\ \forall j$. It is also easily seen that $\mathbf{U}_j\mathbf{V}_j^T = \mathbf{0}$ and $\mathbf{V}_j\mathbf{U}_j^T = \mathbf{0}$. Hence, we have

$$\begin{aligned} \mathbf{A}_{jI}{\mathbf{A}_{jI}}^T + \mathbf{A}_{jQ}{\mathbf{A}_{jQ}}^T &= 2\mathbf{I} \\ \mathbf{A}_{jI}{\mathbf{A}_{jQ}}^T + \mathbf{A}_{jQ}{\mathbf{A}_{jI}}^T &= \mathbf{0} \end{aligned} \quad (21)$$

Substituting this in (13), we get the left hand side of (13) to be

$$2\left(\mathbf{A}_{j_1I}{}^T\mathbf{A}_{j_3I} + \mathbf{A}_{j_3}{}^T\mathbf{A}_{j_1I} + \mathbf{A}_{j_1Q}{}^T\mathbf{A}_{j_3Q} + \mathbf{A}_{j_3Q}{}^T\mathbf{A}_{j_3Q}\right), \quad (22)$$

which, by (11), is always a block diagonal matrix of the form (12). Also the left hand side of (14) is $\mathbf{0}$. Hence, $\mathbf{G}_c$ is SSD for PCRC. □

In [17], it is shown that if the N relays do not have any CSI and the destination has all the CSI, then an upper bound on the rate of distributed SSD codes is $\frac{2}{N}$, which decreases rapidly as the number of relays increases. However, Theorem 3 shows that, if the relay knows only the phase information of the source-relay channels then the lower bound on the rate of the distributed SSD codes is $\frac{1}{2}$ which is independent of the number of relays. For example, the ROD part of such rate-1/2 SSD DSTBCs for PCRC for 10 relays using Hurwitz-Radon construction yields a 32×10 matrix and for 12 relays it yields a 64×12 matrix.

V. Invariance of SSD under coordinate interleaving

In this section, we show that the property of SSD of a DSTBC for PCRC is invariant under coordinate interleaving of the data symbols. To illustrate the usefulness of this result we first show the following lemma.

Lemma 1: If $\mathbf{G}(x_1, \cdots, x_{T_1})$ is a SSD design in T_1 variables and N transmit nodes that satisfies (11), (13) and (14), then the design in $2T_1$ variables and $2N$ transmit nodes given by

$$\bar{\mathbf{G}}(x_1, \cdots, x_{2T_1}) = \begin{bmatrix} \mathbf{G}(x_1, \cdots, x_{T_1}) & \mathbf{0} \\ \mathbf{0} & \mathbf{G}(x_{T_1+1}, \cdots, x_{2T_1}) \end{bmatrix} \quad (23)$$

also satisfies (11), (13) and (14).

Proof: If $\mathbf{A}_j,\ 1 \leq j \leq N$ are the relay matrices of $\mathbf{G}$, then the corresponding $\bar{\mathbf{A}}_j$ matrices for $\bar{\mathbf{G}}$ are $\bar{\mathbf{A}}_j = \begin{bmatrix} \mathbf{A}_j & \mathbf{0} \\ \mathbf{0} & \mathbf{0} \end{bmatrix}$, $1 \leq j \leq N$ and $\bar{\mathbf{A}}_j = \begin{bmatrix} \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{A}_j \end{bmatrix}$, $N+1 \leq j \leq 2N$. It is easily verified that if $\mathbf{A}_j$ satisfies (11), (13) and (14), then so do the matrices $\bar{\mathbf{A}}_j$. □

As an example, if we choose $\mathbf{G}(x_1, x_2)$ to be the Alamouti code in the lemma above then we get the code

$$\begin{bmatrix} x_1 & x_2 & 0 & 0 \\ -x_2^* & x_1^* & 0 & 0 \\ 0 & 0 & x_3 & x_4 \\ 0 & 0 & -x_4^* & x_3^* \end{bmatrix}. \quad (24)$$

This code is SSD for PCRC. Note that a 4-antenna COD has only rate only $\frac{3}{4}$ whereas this code has rate 1. However, it is easily shown that this code does not give full-diversity. But, coordinate interleaving for this example results in $CIOD_4$ which gives full-diversity for any signal set with coordinate product distance zero, and we have already seen that $CIOD_4$ has the SSD property for PCRC.

The following theorem shows that it is the property of coordinate interleaving to leave the SSD property of any arbitrary STBC for PCRC intact.

Theorem 4: If an STBC with K variables $x_1, x_2, \cdots, x_K$, satisfy (11), (13) and (14), the SSD property is unaffected by doing arbitrary coordinate interleaving among all real and imaginary components of x_i.[3]

Proof: The data-symbol vector in (5) after interleaving can be written as

$$\tilde{\mathbf{x}} = \tilde{\mathbf{I}}\mathbf{x}$$

where $\tilde{\mathbf{I}}$ is the interleaving matrix which is a permutation matrix obtained by permuting the rows (/columns) of the identity matrix $\mathbf{I}$ to reflect the coordinate interleaving operation. It can be easily checked that $\tilde{\mathbf{I}}^2 = \mathbf{I}$. Also, if $\mathbf{D}$ is a block diagonal matrix of the form (12), then so is the matrix $\tilde{\mathbf{I}}\mathbf{D}\tilde{\mathbf{I}}$. Hence, for PCRC with co-ordinate interleaving (7) can be written as

$$\begin{aligned} \mathbf{c}_j &= \mathbf{A}_j\widehat{\mathbf{v}}_j \\ &= G\sqrt{E_1}\mathbf{A}_j|h_{sj}|\tilde{\mathbf{I}}\mathbf{x} + \mathbf{A}_j\widehat{\mathbf{z}}_j, \end{aligned} \quad (25)$$

[3] It should be noted that neither the source nor the relay does an explicit interleaving, but the net effect of the relay matrices is such that the output of relays is an interleaved version of the information symbols.

which means that after interleaving, the equivalent linear processing matrix is $\mathbf{A}_j\tilde{\mathbf{I}}$. It is easily verified that if $\mathbf{A}_j$ satisfies (11), (13) and (14), then so does $\mathbf{A}_j\tilde{\mathbf{I}}$ also. □

As an example, consider the Alamouti code $\begin{bmatrix} x_1 & x_2 \\ -x_2^* & x_1^* \end{bmatrix}$, whose relay matrices are given by (8). For this case, $N = T_1 = T_2 = 2$. The permutation matrix $\tilde{\mathbf{I}}$ for the coordinate interleaving operation is $\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}$. The relay matrices for the coordinate interleaved code are

$$\mathbf{A}_1\tilde{\mathbf{I}} = \begin{bmatrix} 1 & 0 & 0 & \mathbf{j} \\ 0 & \mathbf{j} & -1 & 0 \end{bmatrix} \text{ and } \mathbf{A}_2\tilde{\mathbf{I}} = \begin{bmatrix} 0 & \mathbf{j} & 1 & 0 \\ 1 & 0 & 0 & -\mathbf{j} \end{bmatrix}, \quad (26)$$

and the resulting code is $\begin{bmatrix} x_{1I} + jx_{2Q} & x_{2I} + jx_{1Q} \\ -x_{2I} + jx_{1Q} & x_{1I} - jx_{2Q} \end{bmatrix} = \begin{bmatrix} \tilde{x}_1 & \tilde{x}_2 \\ -\tilde{x}_2^* & \tilde{x}_1^* \end{bmatrix}$. Also, for the code in (24) which is SSD for PCRC, if we choose the permutation matrix $\tilde{\mathbf{I}}$ as

$$\tilde{\mathbf{I}} = \begin{bmatrix} 1&0&0&0&0&0&0&0 \\ 0&0&0&0&0&1&0&0 \\ 0&0&1&0&0&0&0&0 \\ 0&0&0&0&0&0&0&1 \\ 0&0&0&0&1&0&0&0 \\ 0&1&0&0&0&0&0&0 \\ 0&0&0&0&0&0&1&0 \\ 0&0&0&1&0&0&0&0 \end{bmatrix}, \quad (27)$$

the resulting code is given by

$$\begin{bmatrix} x_{1I} + jx_{3Q} & x_{2I} + jx_{4Q} & 0 & 0 \\ -x_{2I} + jx_{4Q} & x_{1I} - jx_{3Q} & 0 & 0 \\ 0 & 0 & x_{3I} + jx_{1Q} & x_{4I} + jx_{2Q} \\ 0 & 0 & -x_{4I} + jx_{2Q} & x_{3I} - jx_{1Q} \end{bmatrix} = \begin{bmatrix} \tilde{x}_1 & \tilde{x}_2 & 0 & 0 \\ -\tilde{x}_2^* & \tilde{x}_1^* & 0 & 0 \\ 0 & 0 & \tilde{x}_3^* & \tilde{x}_4^* \\ 0 & 0 & -\tilde{x}_4^* & \tilde{x}_3^* \end{bmatrix}, \quad (28)$$

which is the complex interleaved orthogonal design with 4 antennas ($CIOD_4$) in [8]. Hence, $CIOD_4$ is also SSD for PCRC. In general, if we have a code with K complex information symbols which is SSD for PCRC, then we can generate $(2K)!$ codes which are SSD for PCRC by coordinate interleaving. Also, by using the construction in (23) and coordinate interleaving we can construct SSD codes for PCRC that have higher rates. But it is noted that, complex interleaved orthogonal designs give full diversity only under restricted constellations [8].

VI. Conclusions

In this paper, we have proved that rate-halving codes from real orthogonal designs are single-symbol decodable in a partially coherent relay channel. We have also proved that arbitrary coordinate interleaving of the in-phase and quadrature-phase components of the variables does not disturb its single-symbol decodability property in a partially coherent relay channel.

Acknowledgment

This work was partly supported by the Swarnajayanti Fellowship to A. Chockalingam from the Department of Science and Technology, Government of India (Project Ref. No: 6/3/2002-S.F.), and by the Council of Scientific & Industrial Research (CSIR), India, through Research Grant (22(0365)/04/EMR-II) to B. S. Rajan. This work was also partly supported by the DRDO-IISc Program on Advanced Research in Mathematical Engineering through research grants to and A. Chockalingam and B. S. Rajan.

References

[1] V. Tarokh, H. Jafarkhani, and A. R. Calderbank, "Space-time block codes from orthogonal designs," *IEEE Trans. Inform. Theory,* vol. 45, pp. 1456-1467, July 1999.

[2] O. Tirkkonen and A. Hottinen, "Square matrix embeddable space-time block codes for complex signal constellations," *IEEE Trans. Inform. Theory,* vol. 48, no. 2, pp. 384-395, February 2002.

[3] W. Su and X.-G. Xia, "Signal constellations for quasi-orthogonal space-time block codes with full diversity," *IEEE Trans. Inform. Theory,* vol. 50, no. 10, pp. 2331-2347, October 2004.

[4] W. Su, X.-G. Xia, and K. J. R. Liu, "Systematic design of complex orthogonal space-time block codes with high rates," *Proc. IEEE WCNC'2004,* pp. 1442-1445, March 2004.

[5] X. B. Liang, "Orthogonal designs with maximal rates," *IEEE Trans. Inform. Theory,* vol.49, no. 10, pp. 2468-2503, October 2003.

[6] C. Yuen, Y. L. Guan, and T. T. Tjhung, "A class of four-group quasi-orthogonal space-time block code achieving full rate and full diversity for any number of antennas," *Proc. IEEE PIMRC2005,* vol. 1, pp. 92-96, September 2005.

[7] C. Yuen, Y. L. Guan, and T. T. Tjhung, "Quasi-orthogonal STBC with minimum decoding complexity," *IEEE Trans. Wireless Commun.,* vol. 4, no. 5, pp. 2089-2094, September 2005.

[8] Md. Z. A. Khan and B. S. Rajan,"Single-symbol maximum-likelihood decodable linear STBCs," *IEEE Trans. Inform. Theory,* vol. 52, pp. 2062-2091, May 2006.

[9] D. N. Dao, C. Yuen, C. Tellambura, Y. L. Guan, and T. T. Tjhung, "Four-group decodable space-time block codes," available on line in arXiv:0707.3959v1 [cs.IT], 26 July 2007. Also to appear in *IEEE Trans. on Signal Processing.*

[10] S. Karamakar and B. S. Rajan, "Minimum-decoding complexity maximum-rate space-time block codes from Clifford algebras" *Proc. IEEE ISIT'2006,* July 2006.

[11] A. Sendonaris, E. Erkip, and B. Aazhang, "User cooperative diversity - Part I: System description," *IEEE Trans. Commun.,* vol. 51, no. 11, pp. 1927-1948, November 2003.

[12] J. N. Laneman and G. W. Wornell, "Distributed space-time-coded protocols for exploiting cooperative diversity," *IEEE Trans. Inform. Theory.,* vol. 49, no. 10, pp. 2415-2425, October 2003.

[13] Y. Jing and H. Jafarkhani, "Using orthogonal and quasi-orthogonal designs in wireless relay networks," *Proc. IEEE GLOBECOM'2006,* December 2006. Also to appear in *IEEE Trans. Inform. Theory.*

[14] G. S. Rajan and B. S. Rajan. "A non-orthogonal distributed space-time coded protocol - Part 1: Signal model and design criteria" *Proc. IEEE ITW'06,* 2006.

[15] Y. Jing and B. Hassibi, "Distributed space-time coding in wireless relay networks," *IEEE Trans. Wireless Commun.,* July 2004.

[16] G. S. Rajan and B. S. Rajan, "A non-orthogonal distributed space-time coded protocol - Part 2: Code construction and DM-G tradeoff," *Proc. IEEE ITW'06,* 2006.

[17] Z. Yi and I.-M. Kim, "Single-symbol ML decodable distributed STBCs for cooperative networks," *IEEE Trans. Inform. Theory,* vol. 53, no. 8, pp. 2977-2985, August 2007.

[18] D. Sreedhar, A. Chockalingam and B. S. Rajan, "Single-Symbol ML Decodable Distributed STBCs for Partially-Coherent Cooperative Networks," submitted to *IEEE ICC 2008.*

[19] B. Hassibi and B. Hochwald, "High rate codes that are linear in space and time," *IEEE Trans. Inform. Theory*, vol. 48, pp. 1804-1824, July 2002.

[20] D. Sreedhar, A. Chockalingam, and B. S. Rajan, "Single-symbol ML decodable distributed STBCs for partially-coherent cooperative networks," *submitted to IEEE Trans. Inform. Theory,* August 2007. Also available online at arXiv:0708.3019v1 [cs.IT] 22 Aug 2007.