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Low-Complexity Near-ML Decoding of Large Non-Orthogonal STBCs using Reactive Tabu Search

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***Abstract*—Non-orthogonal space-time block codes (STBC) with *large dimensions* are attractive because they can simultaneously achieve both high spectral efficiencies (same spectral efficiency as in V-BLAST for a given number of transmit antennas) *as well as* full transmit diversity. Decoding of non-orthogonal STBCs with large dimensions has been a challenge. In this paper, we present a reactive tabu search (RTS) based algorithm for decoding non-orthogonal STBCs from cyclic division algebras (CDA) having large dimensions. Under i.i.d fading and perfect channel state information at the receiver (CSIR), our simulation results show that RTS based decoding of 12×12 STBC from CDA and 4-QAM with 288 real dimensions achieves $i)$ 10^{-3} uncoded BER at an SNR of just 0.5 dB away from SISO AWGN performance, and $ii)$ a coded BER performance close to within about 5 dB of the theoretical MIMO capacity, using rate-3/4 turbo code at a spectral efficiency of 18 bps/Hz. RTS is shown to achieve near SISO AWGN performance with less number of dimensions than with LAS algorithm (which we reported recently) at some extra complexity than LAS. We also report good BER performance of RTS when i.i.d fading and perfect CSIR assumptions are relaxed by considering a spatially correlated MIMO channel model, and by using a training based iterative RTS decoding/channel estimation scheme.**

I. INTRODUCTION

MIMO systems that employ non-orthogonal space-time block codes (STBC) from cyclic division algebras (CDA) for arbitrary number of transmit antennas, N_t, are attractive because they can simultaneously provide both *full-rate* (i.e., N_t complex symbols per channel use, which is same as in V-BLAST) as well as *full transmit diversity* [1],[2]. The 2×2 Golden code is a well known non-orthogonal STBC from CDA for 2 transmit antennas [3]. High spectral efficiencies of the order of tens of bps/Hz can be achieved using large non-orthogonal STBCs. For e.g., a 16×16 STBC from CDA has 256 complex symbols in it with 512 real dimensions; with 16-QAM and rate-3/4 turbo code, this system offers a high spectral efficiency of 48 bps/Hz. Decoding of non-orthogonal STBCs with such large dimensions, however, has been a challenge. Sphere decoder and its low-complexity variants are prohibitively complex for decoding such STBCs with hundreds of dimensions. Recently, we proposed a low-complexity near-ML achieving algorithm to decode large non-orthogonal STBCs from CDA; this algorithm, which is based on bit-flipping approach, is termed as likelihood ascent search (LAS) algorithm [4]-[6]. In this paper, we present a *reactive tabu search (RTS) based approach* to near-ML decoding of non-orthogonal STBCs with large dimensions.

Key attractive features of the proposed RTS based decoding are its low-complexity and near-ML performance in systems with large dimensions (e.g., hundreds of dimensions). While creating hundreds of dimensions in space alone (e.g., V-BLAST) requires hundreds of antennas, use of non-orthogonal STBCs from CDA can create hundreds of dimensions with just tens of antennas (space) and tens of channel uses (time). Given that 802.11 smart WiFi products with 12 transmit antennas[1] at 2.5 GHz are now commercially available [7] (which establishes that issues related to placement of many antennas and RF/IF chains can be solved in large aperture communication terminals like set-top boxes/laptops), large non-orthogonal STBCs (e.g., 16×16 STBC from CDA) in combination with large dimension near-ML decoding using RTS can enable communications at increased spectral efficiencies of the order of tens of bps/Hz (note that current standards achieve only < 10 bps/Hz using only up to 4 tx antennas).

Tabu search (TS), a heuristic originally designed to obtain approximate solutions to combinatorial optimization problems [8]-[10], is increasingly applied in communication problems [11]-[13]. For e.g., in [11], design of constellation label maps to maximize asymptotic coding gain is formulated as a quadratic assignment problem (QAP), which is solved using RTS [10]. RTS approach is shown to be effective in terms of BER performance and efficient in terms of computational complexity in CDMA multiuser detection [12]. In [13], a fixed TS based detection in V-BLAST is presented. In this paper, we establish that RTS based decoding of non-orthogonal STBCs can achieve excellent BER performance (near-ML and near-capacity performance) in large dimensions at practically affordable low-complexities. We also present a stopping-criterion for the RTS algorithm. RTS for large dimension non-orthogonal STBC decoding has not been reported so far. Our results in this paper can be summarized as follows:

- Under i.i.d fading and perfect channel state information at the receiver (CSIR), our simulation results show that RTS based decoding of 12×12 STBC from CDA and 4-QAM (288 real dimensions) achieves $i)$ 10^{-3} uncoded BER at an SNR of just 0.5 dB away from SISO AWGN performance, and $ii)$ a coded BER performance close to within about 5 dB of the theoretical capacity using rate-3/4 turbo code at a spectral efficiency of 18 bps/Hz.
- Compared to the LAS algorithm we reported recently in [4]-[6], RTS achieves near-SISO AWGN performance with less number of dimensions than with LAS; this is achieved at some extra complexity compared to LAS.
- We report good BER performance when i.i.d fading and perfect CSIR assumptions are relaxed by adopting a spatially correlated MIMO channel model, and a training based iterative RTS decoding/channel estimation scheme.

II. NON-ORTHOGONAL STBC MIMO SYSTEM MODEL

Consider a STBC MIMO system with multiple transmit and receive antennas. An (n, p, k) STBC is represented by a ma-

[1] 12 antennas in these products are now used only for beamforming. Single-beam multi-antenna approaches can offer range increase and interference avoidance, but not spectral efficiency increase.

trix $\mathbf{X}_c \in \mathbb{C}^{n\times p}$, where n and p denote the number of transmit antennas and number of time slots, respectively, and k denotes the number of complex data symbols sent in one STBC matrix. The (i,j)th entry in $\mathbf{X}_c$ represents the complex number transmitted from the ith transmit antenna in the jth time slot. The rate of an STBC is $\frac{k}{p}$. Let N_r and $N_t = n$ denote the number of receive and transmit antennas, respectively. Let $\mathbf{H}_c \in \mathbb{C}^{N_r\times N_t}$ denote the channel gain matrix, where the (i,j)th entry in $\mathbf{H}_c$ is the complex channel gain from the jth transmit antenna to the ith receive antenna. We assume that the channel gains remain constant over one STBC matrix duration. Assuming rich scattering, we model the entries of $\mathbf{H}_c$ as $\mathcal{CN}(0,1)$. The received space-time signal matrix, $\mathbf{Y}_c \in \mathbb{C}^{N_r\times p}$, can be written as

$$\mathbf{Y}_c = \mathbf{H}_c\mathbf{X}_c + \mathbf{N}_c, \tag{1}$$

where $\mathbf{N}_c \in \mathbb{C}^{N_r\times p}$ is the noise matrix at the receiver and its entries are modeled as i.i.d $\mathcal{CN}\big(0,\sigma^2 = \frac{N_tE_s}{\gamma}\big)$, where E_s is the average energy of the transmitted symbols, and γ is the average received SNR per receive antenna [14], and the (i,j)th entry in $\mathbf{Y}_c$ is the received signal at the ith receive antenna in the jth time-slot. Consider linear dispersion STBCs, where $\mathbf{X}_c$ can be written in the form [14]

$$\mathbf{X}_c \;=\; \sum_{i=1}^{k} x_c^{(i)}\mathbf{A}_c^{(i)}, \tag{2}$$

where $x_c^{(i)}$ is the ith complex data symbol, and $\mathbf{A}_c^{(i)} \in \mathbb{C}^{N_t\times p}$ is its corresponding weight matrix. The received signal model in (1) can be written in an equivalent V-BLAST form as

$$\mathbf{y}_c \;=\; \sum_{i=1}^{k} x_c^{(i)}\,(\widehat{\mathbf{H}}_c\,\mathbf{a}_c^{(i)}) + \mathbf{n}_c \;=\; \widetilde{\mathbf{H}}_c\mathbf{x}_c + \mathbf{n}_c, \tag{3}$$

where $\mathbf{y}_c \in \mathbb{C}^{N_rp\times 1} = vec\,(\mathbf{Y}_c)$, $\widehat{\mathbf{H}}_c \in \mathbb{C}^{N_rp\times N_tp} = (\mathbf{I}_p \otimes \mathbf{H}_c)$, $\mathbf{I}_p$ is $p\times p$ identity matrix, $\mathbf{a}_c^{(i)} \in \mathbb{C}^{N_tp\times 1} = vec\,(\mathbf{A}_c^{(i)})$, $\mathbf{n}_c \in \mathbb{C}^{N_rp\times 1} = vec\,(\mathbf{N}_c)$, $\mathbf{x}_c \in \mathbb{C}^{k\times 1}$ whose ith entry is the data symbol $x_c^{(i)}$, and $\widetilde{\mathbf{H}}_c \in \mathbb{C}^{N_rp\times k}$ whose ith column is $\widehat{\mathbf{H}}_c\,\mathbf{a}_c^{(i)}$, $i = 1,2,\cdots,k$. Each element of $\mathbf{x}_c$ is an M-PAM/M-QAM symbol. M-PAM symbols take discrete values from $\mathbb{A} \triangleq \{a_q, q = 1,\cdots,M\}$, where $a_q = (2q-1-M)$, and M-QAM is nothing but two PAMs in quadrature. Let $\mathbf{y}_c$, $\widetilde{\mathbf{H}}_c$, $\mathbf{x}_c$, $\mathbf{n}_c$ be decomposed into real and imaginary parts as:

$$\mathbf{y}_c = \mathbf{y}_I + j\mathbf{y}_Q, \quad \mathbf{x}_c = \mathbf{x}_I + j\mathbf{x}_Q,$$
$$\mathbf{n}_c = \mathbf{n}_I + j\mathbf{n}_Q, \quad \widetilde{\mathbf{H}}_c = \mathbf{H}_I + j\mathbf{H}_Q. \tag{4}$$

Further, we define $\mathbf{H}_r \in \mathbb{R}^{2N_rp\times 2k}$, $\mathbf{y}_r \in \mathbb{R}^{2N_rp\times 1}$, $\mathbf{x}_r \in \mathbb{A}^{2k\times 1}$, and $\mathbf{n}_r \in \mathbb{R}^{2N_rp\times 1}$ as

$$\mathbf{H}_r = \begin{pmatrix} \mathbf{H}_I & -\mathbf{H}_Q \\ \mathbf{H}_Q & \mathbf{H}_I \end{pmatrix}, \quad \mathbf{y}_r = [\mathbf{y}_I^T \;\; \mathbf{y}_Q^T]^T, \tag{5}$$

$$\mathbf{x}_r = [\mathbf{x}_I^T \;\; \mathbf{x}_Q^T]^T, \quad \mathbf{n}_r = [\mathbf{n}_I^T \;\; \mathbf{n}_Q^T]^T. \tag{6}$$

Now, (3) can be written as

$$\mathbf{y}_r \;=\; \mathbf{H}_r\mathbf{x}_r + \mathbf{n}_r. \tag{7}$$

Henceforth, we work with the real-valued system in (7). For notational simplicity, we drop subscripts r in (7) and write

$$\mathbf{y} \;=\; \mathbf{H}\mathbf{x} + \mathbf{n}, \tag{8}$$

where $\mathbf{H} = \mathbf{H}_r \in \mathbb{R}^{2N_rp\times 2k}$, $\mathbf{y} = \mathbf{y}_r \in \mathbb{R}^{2N_rp\times 1}$, $\mathbf{x} = \mathbf{x}_r \in \mathbb{A}^{2k\times 1}$, and $\mathbf{n} = \mathbf{n}_r \in \mathbb{R}^{2N_rp\times 1}$. We assume that the channel coefficients are known at the receiver but not at the transmitter. The ML solution is given by

$$\mathbf{x}_{ML} \;=\; \mathop{\arg\min}_{\mathbf{x}\,\in\,\mathbb{A}^{2k}} \; \mathbf{x}^T\mathbf{H}^T\mathbf{H}\mathbf{x} - 2\mathbf{y}^T\mathbf{H}\mathbf{x}, \tag{9}$$

whose complexity is exponential in k.

A. High-rate Non-orthogonal STBCs from CDA

We focus on the decoding of square (i.e., $n = p = N_t$), full-rate (i.e., $k = pn = N_t^2$), circulant (where the weight matrices $\mathbf{A}_c^{(i)}$'s are permutation type), non-orthogonal STBCs from CDA [1], whose construction for arbitrary number of transmit antennas n is given by the matrix in Eqn.(9.a) given at the bottom of this column. In (9.a), $\omega_n = e^{\frac{\mathbf{j}2\pi}{n}}$, $\mathbf{j} = \sqrt{-1}$, and $d_{u,v}$, $0 \le u,v \le n-1$ are the n^2 data symbols from a QAM alphabet. When $\delta = e^{\sqrt{5}\mathbf{j}}$ and $t = e^{\mathbf{j}}$, the STBC in (9.a) achieves full transmit diversity (under ML decoding) as well as information-losslessness [1]. When $\delta = t = 1$, the code ceases to be of full-diversity (FD), but continues to be information-lossless (ILL). High spectral efficiencies with large n can be achieved using this code construction. However, since these STBCs are non-orthogonal, ML detection gets increasingly impractical for large n. Consequently, a key challenge in realizing the benefits of these large STBCs in practice is that of achieving near-ML performance for large n at low decoding complexities. The RTS based decoding algorithm we present in the following section essentially addresses this challenge.

III. RTS Algorithm for Large Non-Orthogonal STBC Decoding

In this section, we present the RTS algorithm, which is an iterative local search algorithm, for decoding non-orthogonal STBCs. The goal is to get $\widehat{\mathbf{x}}$, an estimate of $\mathbf{x}$, given $\mathbf{y}$ and $\mathbf{H}$.

Neighborhood Definition: Let $a_q \in \mathbb{A}$, $q = 1,\cdots,M$. Define a set $\mathcal{N}(a_q)$ as a fixed subset of $\mathbb{A}\backslash a_q$, which we refer to as the *symbol neighborhood* of a_q. We choose the cardinality of this set to be the same for all a_q, $q = 1,\cdots,M$; i.e., we take $|\mathcal{N}(a_q)| = N$, $\forall q$. Note that the maximum and minimum values of N are $M-1$ and 1, respectively. For e.g., $\mathbb{A} = \{-3,-1,1,3\}$ for 4-PAM, and choosing N to be 2, $\mathcal{N}(-3) = \{-1,1\}$, $\mathcal{N}(-1) = \{-3,1\}$, $\mathcal{N}(1) = \{-1,3\}$, $\mathcal{N}(3) = \{1,-1\}$ are possible symbol neighborhoods. Let $w_v(a_q)$, $v = 1,\cdots,N$ denote the vth element in $\mathcal{N}(a_q)$; i.e., we say $w_v(a_q)$ is the vth symbol neighbor of a_q.

Let $\mathbf{x}^{(m)} = [x_1^{(m)}\, x_2^{(m)} \cdots x_{2k}^{(m)}]$ denote the data vector belonging to the solution space, in the mth iteration, where $x_i^{(m)} = a_q$, $q \in \{1,\cdots,M\}$. We refer to the vector

$$\mathbf{z}^{(m)}(u,v) \;=\; \big[z_1^{(m)}(u,v) \;\; z_2^{(m)}(u,v) \;\cdots\; z_{2k}^{(m)}(u,v)\big], \tag{10}$$

as the (u,v)th *vector neighbor* $\big($or simply the (u,v)th neighbor$\big)$ of $\mathbf{x}^{(m)}$, $u = 1,\cdots,2k$, $v = 1,\cdots,N$, if $i)$ $\mathbf{x}^{(m)}$ differs from $\mathbf{z}^{(m)}(u,v)$ in the uth coordinate, and $ii)$ the uth element of $\mathbf{z}^{(m)}(u,v)$ is the vth symbol neighbor of $x_u^{(m)}$. That is,

$$z_i^{(m)}(u,v) \;=\; \begin{cases} x_i^{(m)} & \text{for } \; i \neq u \\ w_v(x_u^{(m)}) & \text{for } \; i = u. \end{cases} \tag{11}$$

So we will have $2kN$ vectors which differ from a given vector

$$\begin{bmatrix}
\sum_{i=0}^{n-1} d_{0,i}\, t^i & \delta\sum_{i=0}^{n-1} d_{n-1,i}\,\omega_n^i\, t^i & \cdots & \delta\sum_{i=0}^{n-1} d_{1,i}\,\omega_n^{(n-1)i}\, t^i \\
\sum_{i=0}^{n-1} d_{1,i}\, t^i & \sum_{i=0}^{n-1} d_{0,i}\,\omega_n^i\, t^i & \cdots & \delta\sum_{i=0}^{n-1} d_{2,i}\,\omega_n^{(n-1)i}\, t^i \\
\sum_{i=0}^{n-1} d_{2,i}\, t^i & \sum_{i=0}^{n-1} d_{1,i}\,\omega_n^i\, t^i & \cdots & \delta\sum_{i=0}^{n-1} d_{3,i}\,\omega_n^{(n-1)i}\, t^i \\
\vdots & \vdots & \vdots & \vdots \\
\sum_{i=0}^{n-1} d_{n-2,i}\, t^i & \sum_{i=0}^{n-1} d_{n-3,i}\,\omega_n^i\, t^i & \cdots & \delta\sum_{i=0}^{n-1} d_{n-1,i}\,\omega_n^{(n-1)i}\, t^i \\
\sum_{i=0}^{n-1} d_{n-1,i}\, t^i & \sum_{i=0}^{n-1} d_{n-2,i}\,\omega_n^i\, t^i & \cdots & \sum_{i=0}^{n-1} d_{0,i}\,\omega_n^{(n-1)i}\, t^i
\end{bmatrix} \tag{9.a}$$

in the solution space in only one coordinate. These $2kN$ vectors form the neighborhood of the given vector. We note that neighborhood definition based on bit-flipping [4] is a special case of the above neighborhood definition for $N = 1, M = 2$.

The algorithm is said to execute a move (u, v) if $\mathbf{x}^{(m+1)} = \mathbf{z}^{(m)}(u, v)$. The number of candidates to be considered for a move in the mth iteration is $2kN$. Since the coordinate that changes in a move can take M possible values for M-PAM, the total number of possible moves is $2kMN$. The tabu value of a move, which is a non-negative integer, means that the move cannot be considered for that many number of subsequent iterations, unless certain conditions are satisfied.

Tabu Matrix: A *tabu_matrix* of size $2kM \times N$ is the matrix whose entries denote the tabu values of moves. The (r, s)th entry of the *tabu_matrix* corresponds to the move (u, v) from $\mathbf{x}^{(m)}$ when $u = \lfloor \frac{r-1}{M} \rfloor + 1$, $v = s$ and $x_u^{(m)} = a_q$, where $q = mod(r-1, M) + 1$.

RTS Algorithm: Let $\mathbf{g}^{(m)}$ be the vector which has the least ML cost found till the mth iteration of the algorithm. Let l_{rep} be the average length (in number of iterations) between two successive occurrences of the same solution vector (repetitions), at the end of an iteration. Tabu period, P, a dynamic non-negative integer parameter, is defined. If a move is marked as tabu in an iteration, it will remain as tabu for P subsequent iterations. The algorithm starts with an initial solution vector $\mathbf{x}^{(0)}$, which, for e.g., could be the MMSE or MF output vector. Set $\mathbf{g}^{(0)} = \mathbf{x}^{(0)}$, $l_{rep} = 0$, and $P = P_0$. All the entries of the *tabu_matrix* are set to zero. The following steps 1) to 3) are performed in each iteration. Consider mth iteration in the algorithm, $m \geq 0$.

Step 1): Define $\mathbf{y}_{mf} \stackrel{\triangle}{=} \mathbf{H}^T\mathbf{y}$, $\mathbf{R} \stackrel{\triangle}{=} \mathbf{H}^T\mathbf{H}$, and $\mathbf{f}^{(m)} \stackrel{\triangle}{=} \mathbf{R}\mathbf{x}^{(m)} - \mathbf{y}_{mf}$. Let $\mathbf{e}^{(m)}(u, v) = \mathbf{z}^{(m)}(u, v) - \mathbf{x}^{(m)}$. The ML costs of the $2kN$ neighbors of $\mathbf{x}^{(m)}$, namely, $\mathbf{z}^{(m)}(u, v)$, $u = 1, \cdots, 2k$, $v = 1, \cdots, N$, are computed as

$$\begin{aligned}
\phi(\mathbf{z}^{(m)}(u, v)) &= \big(\mathbf{x}^{(m)} + \mathbf{e}^{(m)}(u, v)\big)^T \mathbf{R} \big(\mathbf{x}^{(m)} + \mathbf{e}^{(m)}(u, v)\big) \\
&\quad -2\big(\mathbf{x}^{(m)} + \mathbf{e}^{(m)}(u, v)\big)^T \mathbf{y}_{mf} \\
&= \phi(\mathbf{x}^{(m)}) + 2\big(\mathbf{e}^{(m)}(u, v)\big)^T \mathbf{R}\,\mathbf{x}^{(m)} \\
&\quad + \big(\mathbf{e}^{(m)}(u, v)\big)^T \mathbf{R}\,\mathbf{e}^{(m)}(u, v) - 2\big(\mathbf{e}^{(m)}(u, v)\big)^T \mathbf{y}_{mf} \\
&= \phi(\mathbf{x}^{(m)}) + \underbrace{2\, e_u^{(m)}(u, v)\, f_u^{(m)} + \big(e_u^{(m)}(u, v)\big)^2 \mathbf{R}_{u,u}}_{\stackrel{\triangle}{=} C\left(e_u^{(m)}(u,v)\right)}, \quad (12)
\end{aligned}$$

where $e_u^{(m)}(u, v)$ is the uth element of $\mathbf{e}^{(m)}(u, v)$, $f_u^{(m)}$ is uth element of $\mathbf{f}^{(m)}$, and $\mathbf{R}_{u,u}$ is the (u, u)th element of $\mathbf{R}$. $\phi(\mathbf{x}^{(m)})$ on the RHS in (12) can be dropped since it will not affect the cost minimization. Let

$$(u_1, v_1) = \underset{u,\, v}{\arg\min}\;\; C\big(e_u^{(m)}(u, v)\big). \quad (13)$$

The move (u_1, v_1) is accepted if any one of the following two conditions is satisfied:

$i)$ $\phi(\mathbf{z}^{(m)}(u_1, v_1)) < \phi(\mathbf{g}^{(m)})$

$ii)$ *tabu_matrix*$((u_1-1)M+q, v_1) = 0$ where $q : x_{u_1}^{(m)} = a_q \in \mathbb{A}$.

If move (u_1, v_1) is accepted, then make

$$\mathbf{x}^{(m+1)} = \mathbf{x}^{(m)} + \mathbf{e}^{(m)}(u_1, v_1). \quad (14)$$

If move (u_1, v_1) is not accepted (i.e., neither of conditions $i)$ and $ii)$ is satisfied), find (u_2, v_2) such that

$$(u_2, v_2) = \underset{u,\, v\,:\, u \neq u_1,\, v \neq v_1}{\arg\min}\;\; C\big(e_u^{(m)}(u, v)\big), \quad (15)$$

and check for acceptance of the (u_2, v_2) move. If this also cannot be accepted, repeat the procedure for (u_3, v_3), and so on. If all the $2kN$ moves are tabu, then all the *tabu_matrix* entries are decremented by the minimum value in the *tabu_matrix*; this goes on till one of the moves becomes permissible. Let (u', v') be the index of the neighbor with the minimum cost for which the move is permitted. The variables q', q'', v'' are implicitly defined by $x_{u'}^{(m)} = a_{q'} = w_{v''}(x_{u'}^{(m+1)})$, and $x_{u'}^{(m+1)} = a_{q''}$, where $a_{q'}, a_{q''} \in \mathbb{A}$.

Step 2: After a move is done, the new solution vector is checked for repetition. For the channel model in (8), repetition can be checked by comparing the ML costs of the solutions in the previous iterations. If there is a repetition, the length of the repetition from the previous occurrence is found, the average length, l_{rep}, is updated, and the tabu period P is modified as $P = P + 1$. If the number of iterations elapsed since the last change of the value of P exceeds βl_{rep}, for a fixed $\beta > 0$, make $P = P - 1$. The minimum value of P, however, will be 1. Note that this step, if executed, also qualifies as the one which changed P. After a move (u', v') is accepted, if $\phi(\mathbf{x}^{(m+1)}) < \phi(\mathbf{g}^{(m)})$, make

$$\begin{aligned}
\textit{tabu_matrix}\,((u'-1)M + q', v') &= 0, \\
\textit{tabu_matrix}\,((u'-1)M + q'', v'') &= 0, \quad (16)
\end{aligned}$$

and $\mathbf{g}^{(m+1)} = \mathbf{x}^{(m+1)}$; else,

$$\begin{aligned}
\textit{tabu_matrix}\,((u'-1)M + q', v') &= P + 1, \\
\textit{tabu_matrix}\,((u'-1)M + q'', v'') &= P + 1, \quad (17)
\end{aligned}$$

and $\mathbf{g}^{(m+1)} = \mathbf{g}^{(m)}$.

Step 3): Update the entries of the *tabu_matrix* as

$$\textit{tabu\_matrix}\,(r, s) = \max\{\textit{tabu\_matrix}\,(r, s) - 1, 0\}, \quad (18)$$

for $r = 1, \cdots, 2kM$, $s = 1, \cdots, N$. $\mathbf{f}^{(m)}$ is updated as

$$\mathbf{f}^{(m+1)} = \mathbf{f}^{(m)} + e_{u'}^{(m)}(u', v')\mathbf{R}_{u'}, \quad (19)$$

where $\mathbf{R}_{u'}$ is the u'th column of $\mathbf{R}$.

Stopping criterion: The algorithm can be stopped based on a fixed number of iterations. Though convergence can be slow at low SNRs (typ. hundreds of iterations), it can be fast (typ. tens of iterations) at moderate to high SNRs. So rather than fixing a large number of iterations to stop the algorithm irrespective of the SNR, we use an efficient stopping criterion which makes use of the knowledge of the best ML cost in a given iteration, as follows.

Since the ML criterion is to minimize $\|\mathbf{H}\mathbf{x} - \mathbf{y}\|^2$, the minimum value of the objective function $\mathbf{x}^T\mathbf{H}^T\mathbf{H}\mathbf{x} - 2\mathbf{x}^T\mathbf{H}^T\mathbf{y}$, is always greater than $-\mathbf{y}^T\mathbf{y}$. We stop the algorithm when the least ML cost achieved in an iteration is within certain range of the global minimum, which is $-\mathbf{y}^T\mathbf{y}$. We stop the algorithm in the mth iteration, if the condition

$$\frac{|\phi(\mathbf{g}^{(m)}) - (-\mathbf{y}^T\mathbf{y})|}{|-\mathbf{y}^T\mathbf{y}|} < \alpha_1 \quad (20)$$

is met with at least *min_iter* iterations being completed to make sure the search algorithm has 'settled.' The bound is gradually relaxed as the number of iterations increase and the algorithm is terminated when

$$\frac{|\phi(\mathbf{g}^{(m)}) - (-\mathbf{y}^T\mathbf{y})|}{|-\mathbf{y}^T\mathbf{y}|} < m\alpha_2. \tag{21}$$

In (20) and (21), α_1 and α_2 are positive constants. In addition, we terminate the algorithm whenever the number of repetitions of solutions exceeds *max_rep*. Also, the maximum number of iterations is set to *max_iter*. We have found that use of the following stopping criterion parameters results in low complexity without compromising much on the performance (compared to a fixed number of iterations of 300) for 4-QAM: *min_iter* $= 20$, *max_iter* $= 300$, *max_rep* $= 75$, $\alpha_1 = 0.05$, and $\alpha_2 = 0.0005$.

IV. Simulation Results

We evaluated the uncoded/coded BER performance of the RTS algorithm in decoding non-orthogonal STBCs with $\delta = t = 1$ (i.e., ILL) and $\delta = e^{\sqrt{5}\mathrm{j}}$, $t = e^{\mathrm{j}}$ (i.e., FD-ILL[2]) through simulations. The following RTS parameters are used in all the simulations: MMSE initial vector, $P_0 = 2, \beta = 1, 0.1, \alpha_1 = 5\%, \alpha_2 = 0.05\%$, *max_rep*=75, *max_iter* $= 300$, *min_iter* $= 20$.

A. Uncoded BER performance of RTS:

RTS versus LAS Performance: In Fig. 1, we plot the uncoded BER of the RTS algorithm as a function of average received SNR per receive antenna, γ, in decoding 4×4 (32 dimensions), 8×8 (128 dimensions) and 12×12 (288 dimensions) non-orthogonal ILL STBCs for 4-QAM and $N_t = N_r$. Perfect CSIR and i.i.d fading are assumed. For the same settings, performance of the LAS algorithm in [4]-[6] are also plotted for comparison. MMSE initial vector is used in both RTS and LAS. As a reference, we have plotted the BER performance on a SISO AWGN channel as well. From Fig. 1, the following interesting observations can be made:

- the BER of RTS algorithm improves and approaches SISO AWGN performance as $N_t = N_r$ (i.e., STBC size) is increased; e.g., performance close to within 0.5 dB from SISO AWGN performance is achieved at 10^{-3} uncoded BER in decoding 12×12 STBC with 288 real dimensions.
- *RTS algorithm performs better than LAS algorithm* (see RTS and LAS BER plots for 4×4 and 8×8 STBCs). Further, while both RTS and LAS algorithms exhibit large system behavior (i.e., BER improves as $N_t = N_r$ is increased), RTS is able to achieve nearness to SISO AWGN performance at 10^{-3} BER with less number of dimensions than with LAS. This is evident by observing that, while LAS requires 512 dimensions (16×16 STBC) to achieve 1 dB closeness to SISO AWGN performance at 10^{-3} BER, RTS is able to achieve even 0.5 dB closeness with just 288 dimensions (12×12 STBC). RTS is able to achieve this better performance because, while the bit/symbol-flipping strategies are similar in both RTS and LAS, the inherent escape strategy in RTS allows it to move out of local minimas and move towards better solutions. Consequently, RTS incurs some extra complexity compared to LAS, without increase in the order of complexity.

[2]Our simulation results show that the BER performance of FD-ILL and ILL STBCs with RTS decoding are almost the same.

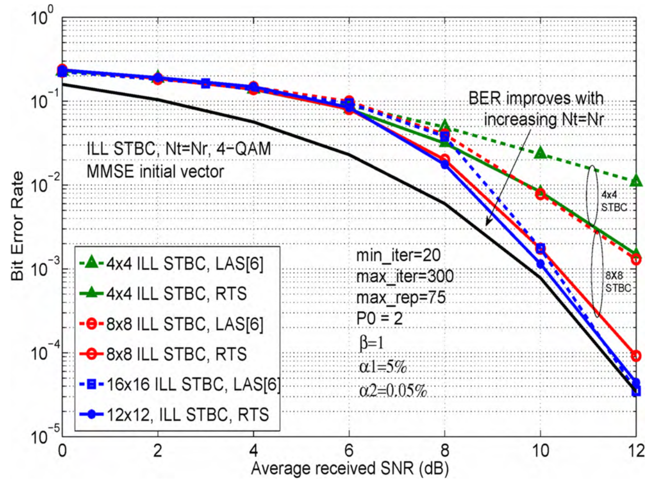

Fig. 1. Uncoded BER of RTS decoding of 4×4, 8×8 and 12×12 non-orthogonal STBCs from CDA. $N_t = N_r$, ILL STBCs ($\delta = t = 1$), 4-QAM. *RTS achieves near SISO AWGN performance for increasing $N_t = N_r$ (i.e., STBC size). RTS performs better than LAS.*

RTS performance in V-BLAST: A similar observation can be made with uncoded BER of RTS detection in V-BLAST in Fig. 2 for $N_t = N_r$ and 4-QAM. From Fig. 2, it is seen that LAS requires 128 dimensions (64×64 V-BLAST) to achieve performance within 1 dB of SISO AWGN performance at 10^{-3} BER, whereas RTS is able to achieve even better closeness with just 64 dimensions (32×32 V-BLAST). In summary, the ability to achieve near SISO AWGN performance at less dimensions than LAS is an attractive feature of RTS.

B. Turbo coded BER performance of RTS

Figure 3 shows the rate-3/4 turbo coded BER of RTS decoding of 12×12 non-orthogonal ILL STBC with $N_t = N_r$ and 4-QAM (corresponding to a spectral efficiency of 18 bps/Hz), under perfect CSIR and i.i.d fading. The theoretical minimum SNR required to achieve 18 bps/Hz spectral efficiency on a $N_t = N_r = 12$ MIMO channel with perfect CSIR and i.i.d fading is 4.27 dB (obtained through simulation of the ergodic capacity formula [14]). From Fig. 3, it is seen that RTS decoding is able to achieve vertical fall in coded BER close to within about 5 dB from the theoretical minimum SNR, which is good nearness to capacity performance. This nearness to capacity can be further improved by 1 to 1.5 dB if soft decision values, proposed in [5], are fed to the turbo decoder.

C. Iterative RTS Decoding/Channel Estimation

Next, we relax the perfect CSIR assumption by considering a training based iterative RTS decoding/channel estimation scheme. Transmission is carried out in frames, where one $N_t \times N_t$ pilot matrix (for training purposes) followed by N_d data STBC matrices are sent in each frame. One frame length, T, (taken to be the channel coherence time) is $T = (N_d + 1)N_t$ channel uses. The proposed scheme works as follows: i) obtain an MMSE estimate of the channel matrix during the pilot phase, ii) use the estimated channel matrix to decode the data STBC matrices using RTS algorithm, and iii) iterate between channel estimation and RTS decoding for a certain

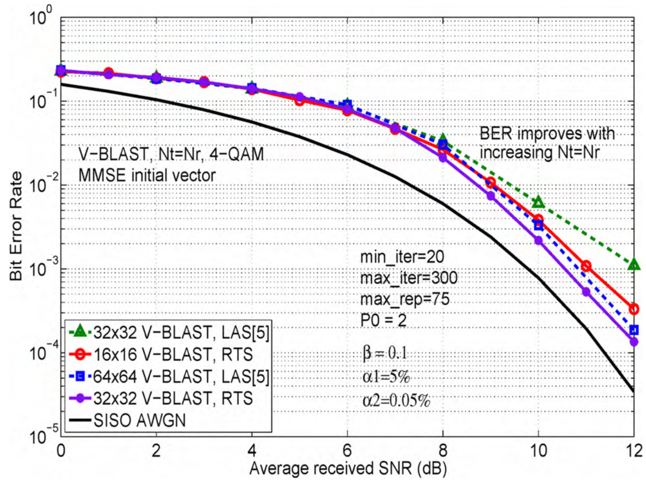

Fig. 2. Uncoded BER of RTS detection of V-BLAST with $N_t = N_r$ and 4-QAM. *RTS achieves near SISO AWGN performance for increasing $N_t = N_r$. RTS performs better than LAS.*

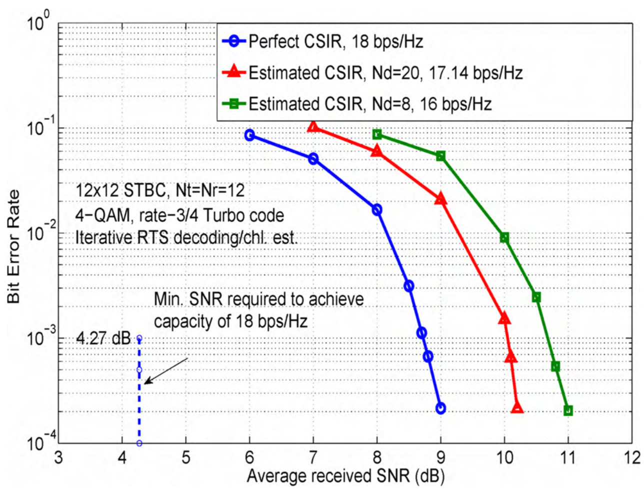

Fig. 3. Turbo coded BER of RTS decoding of 12×12 non-orthogonal ILL STBC with $N_t = N_r$, 4-QAM, rate-3/4 turbo code, and 18 bps/Hz. *BER of RTS with estimated CSIR approaches close to that with perfect CSIR for increasing N_d (i.e., slow fading).*

number of times. For 12×12 ILL STBC, in addition to perfect CSIR performance, Fig. 3 also shows the performance with CSIR estimated using the above iterative RTS decoding/channel estimation scheme for $N_d = 8$ and $N_d = 20$. 2 iterations between RTS decoding and channel estimation are used. With $N_d = 20$ (which corresponds to large coherence times, i.e., slow fading) the BER and bps/Hz with estimated CSIR get closer to those with perfect CSIR.

D. Effect of MIMO Spatial Correlation

In Figs. 1 to 3, we assumed i.i.d fading. But spatial correlation at transmit/receive antennas and the structure of scattering and propagation environment can affect the rank structure of the MIMO channel resulting in degraded performance [15],[16]. We relaxed the i.i.d. fading assumption by considering the correlated MIMO channel model proposed by Gesbert et al in [16], which takes into account carrier frequency (f_c), spacing between antenna elements (d_t, d_r), distance between tx and rx antennas (R), and scattering environment. In Fig. 4, we plot the uncoded BER of RTS decoding of 12×12 FD-ILL STBC with perfect CSIR in i) i.i.d. fading, and ii) correlated MIMO fading model in [16]. It is seen that, compared to i.i.d fading, there is a loss in diversity order in spatial correlation for $N_t = N_r = 12$; further, use of more receive antennas ($N_r = 14, N_t = 12$) alleviates this loss in performance. Finally, we note that have carried out simulations of RTS decoding for 16-QAM as well, where similar results reported here for 4-QAM are observed. The RTS decoding can be used to decode *perfect codes* of large dimensions as well.

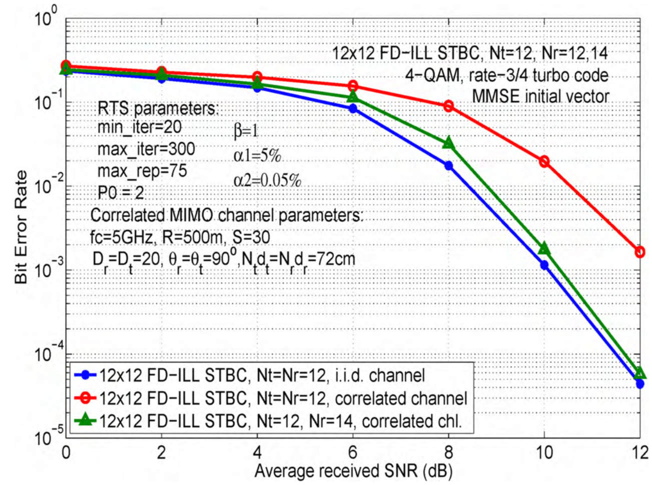

Fig. 4. Effect of spatial correlation on the performance of RTS decoding of 12×12 FD-ILL STBC with $N_t = 12$, $N_r = 12, 14$, 4-QAM, rate-3/4 turbo code, 18 bps/Hz. $f_c = 5$ GHz, $R = 500$ m, $S = 30$, $D_t = D_r = 20$ m, $\theta_t = \theta_r = 90°$, $N_r d_r = N_t d_t = 72$ cm. *Spatial correlation degrades achieved diversity order compared to i.i.d. Increasing N_r alleviates this performance loss.*

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