Joint Channel Estimation/Detection in MIMO Systems Using Belief Propagation

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***Abstract*—We study the problem of joint channel estimation and detection in MIMO systems using belief propagation (BP). We propose a BP based algorithm which jointly estimates the MIMO channel matrix and detects the data symbols in a frame (consisting of pilot and data symbols) through message passing in two passes; one pass from the start to the end of a frame and another from the end to the start. A message comprises a small number of parameters that characterize the computed densities of the marginal probabilities. We study the performance of the proposed algorithm in V-BLAST and STBC MIMO systems for varying pilot density and pilot/data power allocation in a frame. Our simulation results show that a close to perfect CSIR performance is achieved by the proposed algorithm using a non-uniform pilot/data power allocation scheme based on the knowledge of the average channel SNR. We show that the proposed BP based joint channel estimation/detection scheme performs better than MMSE channel estimation followed by maximum-likelihood (ML) detection in slow fading.**

Keywords – *Joint channel estimation/detection, MIMO systems, belief propagation, message passing, V-BLAST, Alamouti code, Golden code.*

I. INTRODUCTION

Use of multiple antennas at the transmitter can offer the benefits of transmit diversity (e.g., using space-time coding) and high data rates (e.g., using spatial multiplexing - V-BLAST) [1],[2],[3]. Crucial receiver functions in such multiple-input multiple-output (MIMO) systems include channel estimation and MIMO detection. Works that view channel estimation and detection in MIMO systems as separate problems have been widely reported. Techniques that employ iterations between channel estimation and detection can offer improved performance. Iterative receiver algorithms are attractive to achieve a good trade off between performance and complexity [4]. Receivers that iterate between channel estimation, multiuser detection and channel decoding in CDMA systems are presented in [5],[6]. Similar iterative techniques in the context of MIMO and MIMO-OFDM systems are presented in [7]-[10]. Our focus in this paper is on the application of *belief propagation* (BP) of *joint* channel estimation and detection in MIMO systems, which, to our knowledge, has not been reported in the literature so far.

BP is a technique that solves inference problems using graphical models [11]-[13]. More precisely, BP is an algorithm used to compute the marginalization of functions by passing messages on a graphical model. The algorithm was initially formalized for trees, and, in the case of trees, is known to solve the inference problem exactly [11]. It is also empirically found to be working on many loopy graphs [14]. BP is known to be well suited in several communication problems [13]; e.g., decoding of turbo codes and LDPC codes [15],[16], multiuser detection [17], signal detection in ISI channels [18], and MIMO detection [19],[20].

Recently, BP based approaches to carry out combined channel estimation and detection in communication systems are gaining importance [4],[21]-[23]. In [21],[22], BP based joint channel estimation and detection has been proposed for single-user fading channels. In [23], a BP based joint channel estimation and detection scheme is proposed for a two-user system with a desired user and a co-channel interferer, where pilot symbols are assumed to be available for the desired user, whereas no pilot is available from the interferer.

In this paper, we present BP based joint channel estimation and detection in the context of MIMO systems, which has not been reported so far. Specifically, we propose a BP based algorithm which jointly estimates the MIMO channel matrix and detects the data symbols in a frame (each frame consists of pilot and data symbols) through message passing. The message passing is performed in two passes, one pass from the start to the end of a frame and another from the end to the start. Each message is made up of a small number of parameters that characterize the computed densities of the marginal probabilities. We study the bit error performance of the proposed algorithm in V-BLAST and STBC (Alamouti code [2] and Golden code [3]) MIMO systems through simulations. Our simulation results show that a close to perfect channel state information at the receiver (CSIR) performance is achieved by the proposed algorithm using a non-uniform pilot/data power allocation scheme based on the knowledge of the average channel SNR. Our results further show that the proposed BP based joint channel estimation/detection scheme performs better than MMSE channel estimation followed by maximum-likelihood (ML) detection in slow fading.

The rest of the paper is organized as follows. The MIMO system model considered is presented in Section II. The proposed BP based joint channel estimation/detection algorithm is presented in Section III. Simulation results and discussions are presented in Section IV. Conclusions are presented in Section V.

II. SYSTEM MODEL

Consider a point-to-point MIMO link with N_t antennas at the transmitter and N_r antennas at the receiver. Transmission is carried out in frames, where N_p pilot matrices (for training purposes) are interleaved by N_d data STBC matrices in each frame as shown in Fig. 1. The ith transmitted space-time matrix, $\mathbf{X}_i$, can be either a $N_t \times N_t$ pilot matrix, or a space-time code matrix $\big($in which case, $\mathbf{X}_i \in \mathbb{C}^{N_t \times T}$, where T is the number of channel uses in the STBC$\big)$, or a V-BLAST

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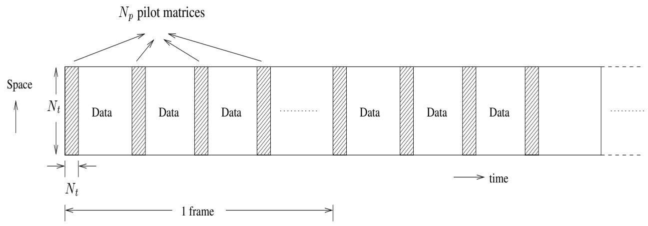

Fig. 1. Transmission scheme with N_p pilot matrices and N_d data STBC matrices in each frame.

vector $\big($in which case, $\mathbf{X}_i \in \mathbb{C}^{N_t}\big)$. Let the channel gain matrix corresponding to the ith STBC matrix be denoted by $\mathbf{H}_i$, where the (j,k)th entry in $\mathbf{H}_i$ is the complex channel gain from the kth transmit to the jth receive antenna. The received signal matrix $\mathbf{Y}_i \in \mathbb{C}^{N_r \times T}$ ($T = 1$ in the case of V-BLAST) can be written as

$$\mathbf{Y}_i \;=\; \mathbf{H}_i\mathbf{X}_i + \mathbf{N}_i, \quad i = 1, 2, \cdots, L, \tag{1}$$

where $L = N_p + N_d$ is the number of STBC matrices in one frame, and $\mathbf{N}_i \in \mathbb{C}^{N_r \times T}$ is the noise matrix at the receiver. The entries of $\mathbf{N}_i$ are modeled as i.i.d. $\mathcal{CN}\left(0, \sigma^2 = \frac{N_t E_s}{\gamma}\right)$, where E_s is the average energy of the transmitted symbols, and γ is the average received SNR per receive antenna. Assuming that the entries of $\mathbf{H}_i$ are generated by a Gauss Markov process (e.g., as in slow fading), $\mathbf{H}_i$ can be written as

$$\mathbf{H}_i \;=\; \alpha\mathbf{H}_{i-1} + \sqrt{1-\alpha^2}\,\mathbf{W}_i, \tag{2}$$

where α denotes the correlation between channel gains in consecutive space-time matrices. The entries of $\mathbf{W}_i$ are modeled as i.i.d. $\mathcal{CN}(0,1)$. $\alpha = 1$ denotes a block fading model. Applying $vec(.)$ operation on transpose of (1), we can write

$$vec\left(\mathbf{Y}_i^t\right) \;=\; \left(\mathbf{I}_{N_r} \otimes \mathbf{X}_i^t\right) vec\left(\mathbf{H}_i^t\right) + vec\left(\mathbf{N}_i^t\right), \tag{3}$$

where $(.)^t$ denotes the transpose operation, and $\mathbf{I}_n$ denotes the $n \times n$ identity matrix. Further, define $\mathbf{y}_i \stackrel{\triangle}{=} vec\left(\mathbf{Y}_i^t\right)$, $\mathbf{C}_i \stackrel{\triangle}{=} (\mathbf{I}_{N_r} \otimes \mathbf{X}_i^t)$, $\mathbf{h}_i \stackrel{\triangle}{=} vec\left(\mathbf{H}_i^t\right)$ and $\mathbf{n}_i \stackrel{\triangle}{=} vec\left(\mathbf{N}_i^t\right)$. With these definitions, (3) can be written as

$$\mathbf{y}_i \;=\; \mathbf{C}_i\mathbf{h}_i + \mathbf{n}_i, \quad i = 1, 2, \cdots, L, \tag{4}$$

which is the system model we will use henceforth.

III. Proposed BP based Joint Channel Estimation/Detection

In this section, we present the proposed BP based algorithm for joint channel estimation/detection for the MIMO system model described in the previous section. The goal is to determine $\mathbf{X}_i$'s, given $\mathbf{y}_i$'s and the knowledge of σ^2. The optimum solution to this problem is given by the MAP detector, which maximizes $p(\mathbf{X}_i|\mathbf{y}_1^L)$ where $\mathbf{y}_1^L \stackrel{\triangle}{=} [\mathbf{y}_1, \mathbf{y}_2, \cdots, \mathbf{y}_L]$, and $p(\mathbf{X}_i|\mathbf{y}_1^L)$ can be written as

$$\begin{aligned} p\left(\mathbf{X}_i|\mathbf{y}_1^L\right) \;&=\; \int p\left(\mathbf{C}_i, \mathbf{h}_i|\mathbf{y}_1^L\right) d\mathbf{h}_i \\ &\propto\; \int p\left(\mathbf{C}_i, \mathbf{y}_1^{i-1}, \mathbf{y}_i, \mathbf{y}_{i+1}^L, \mathbf{h}_i\right) d\mathbf{h}_i. \end{aligned} \tag{5}$$

Conditioned on $\mathbf{h}_i$, the above marginal probability can be written as

$$\begin{aligned} p\left(\mathbf{X}_i|\mathbf{y}_1^L\right) &\propto \int p(\mathbf{y}_i, \mathbf{C}_i, \mathbf{y}_1^{i-1}, \mathbf{y}_{i+1}^L|\mathbf{h}_i)\,p\left(\mathbf{h}_i\right) d\mathbf{h}_i \\ &\propto \int p(\mathbf{y}_i, \mathbf{C}_i|\mathbf{h}_i)p(\mathbf{y}_1^{i-1}|\mathbf{h}_i)p(\mathbf{y}_{i+1}^L|\mathbf{h}_i)\,p\left(\mathbf{h}_i\right) d\mathbf{h}_i \\ &\propto \int p(\mathbf{y}_i, \mathbf{C}_i|\mathbf{h}_i)p\big(\mathbf{h}_i|\mathbf{y}_1^{i-1}\big)p\big(\mathbf{h}_i|\mathbf{y}_{i+1}^L\big)/p\big(\mathbf{h}_i\big) d\mathbf{h}_i \\ \propto p(\mathbf{C}_i) &\int p\big(\mathbf{y}_i|\mathbf{h}_i, \mathbf{C}_i\big)p\big(\mathbf{h}_i|\mathbf{y}_1^{i-1}\big)p\big(\mathbf{h}_i|\mathbf{y}_{i+1}^L\big)/p\big(\mathbf{h}_i\big) d\mathbf{h}_i, \end{aligned} \tag{6}$$

where line 2 in (6) is obtained from line 1 due to conditional independence of $\mathbf{y}_i$'s given $\mathbf{h}_i$. The probabilities $p\big(\mathbf{h}_i|\mathbf{y}_1^{i-1}\big)$ and $p\big(\mathbf{h}_i|\mathbf{y}_{i+1}^L\big)$ in (6), referred to as forward and backward probabilities, respectively, can be computed as follows.

A. Computation of forward probabilities, $p\big(\mathbf{h}_i|\mathbf{y}_1^{i-1}\big)$

The forward probability, $p\big(\mathbf{h}_i|\mathbf{y}_1^{i-1}\big)$, can be written as

$$\begin{aligned} p\big(\mathbf{h}_i|\mathbf{y}_1^{i-1}\big) \;&=\; \int p\big(\mathbf{h}_i|\mathbf{h}_{i-1}\big)p\big(\mathbf{h}_{i-1}|\mathbf{y}_{i-1}, \mathbf{y}_1^{i-2}\big) d\mathbf{h}_i \\ &\propto \int p\big(\mathbf{h}_i|\mathbf{h}_{i-1}\big)p\big(\mathbf{h}_{i-1}, \mathbf{y}_{i-1}, \mathbf{y}_1^{i-2}\big) d\mathbf{h}_i \\ &= \int p\big(\mathbf{h}_i|\mathbf{h}_{i-1}\big)p\big(\mathbf{h}_{i-1}\big)p\big(\mathbf{y}_{i-1}, \mathbf{y}_1^{i-2}|\mathbf{h}_{i-1}\big) d\mathbf{h}_i \\ &= \int p\big(\mathbf{h}_i|\mathbf{h}_{i-1}\big)p\big(\mathbf{h}_{i-1}\big)p\big(\mathbf{y}_{i-1}|\mathbf{h}_{i-1}\big)p\big(\mathbf{y}_1^{i-2}|\mathbf{h}_{i-1}\big) d\mathbf{h}_i \\ &= \int p\big(\mathbf{h}_i|\mathbf{h}_{i-1}\big)p\big(\mathbf{y}_{i-1}|\mathbf{h}_{i-1}\big)p\big(\mathbf{y}_1^{i-2}, \mathbf{h}_{i-1}\big) d\mathbf{h}_i \\ &\propto \int p\big(\mathbf{h}_i|\mathbf{h}_{i-1}\big)p\big(\mathbf{h}_{i-1}|\mathbf{y}_1^{i-2}\big)p\big(\mathbf{y}_{i-1}|\mathbf{h}_{i-1}\big) d\mathbf{h}_i, \end{aligned} \tag{7}$$

where line 3 in (7) is obtained from line 2 due to conditional independence of $\mathbf{y}_i$'s given $\mathbf{h}_{i-1}$. Now, $p\big(\mathbf{y}_{i-1}|\mathbf{h}_{i-1}\big)$ in (7) can be written as

$$\begin{aligned} p\big(\mathbf{y}_{i-1}|\mathbf{h}_{i-1}\big) \;&=\; \sum_{\mathbf{C}_{i-1}} p\big(\mathbf{y}_{i-1}|\mathbf{C}_{i-1}, \mathbf{h}_{i-1}\big)p\big(\mathbf{C}_{i-1}\big) \\ &\propto \sum_{\mathbf{C}_{i-1}} p(\mathbf{C}_{i-1})\,\mathcal{F}_{\mathcal{N}}\big(\mathbf{y}_{i-1}, \mathbf{C}_{i-1}\mathbf{h}_{i-1}, \sigma^2\mathbf{I}_{TN_r}\big), \end{aligned} \tag{8}$$

where the function $\mathcal{F}_{\mathcal{N}}\big(\mathbf{a}, \mathbf{m}, \mathbf{K}\big)$ is defined as

$$\mathcal{F}_{\mathcal{N}}\big(\mathbf{a}, \mathbf{m}, \mathbf{K}\big) \stackrel{\triangle}{=} exp\Big\{-\left(\mathbf{a}-\mathbf{m}\right)^H \mathbf{K}^{-1}\big(\mathbf{a}-\mathbf{m}\big)\Big\}, \tag{9}$$

and $(.)^H$ denotes the conjugate transpose operation. The summation in (8) is over all possible code words. Combining (7) and (8), $p\big(\mathbf{h}_i|\mathbf{y}_1^{i-1}\big)$ can be recursively obtained as

$$\begin{aligned} p\big(\mathbf{h}_i|\mathbf{y}_1^{i-1}\big) \;&\propto\; \int p\big(\mathbf{h}_{i-1}|\mathbf{y}_1^{i-2}\big)\mathcal{F}_{\mathcal{N}}\big(\mathbf{h_i}, \alpha\mathbf{h}_{i-1}, \left(1-\alpha^2\right)\mathbf{I}\big) \\ &\cdot \sum_{\mathbf{C}_{i-1}} p(\mathbf{C}_{i-1})\,\mathcal{F}_{\mathcal{N}}\big(\mathbf{y}_{i-1}, \mathbf{C}_{i-1}\mathbf{h}_{i-1}, \sigma^2\mathbf{I}_{TN_r}\big) d\mathbf{h}_i. \end{aligned} \tag{10}$$

B. Computation of backward probabilities, $p\big(\mathbf{h}_i|\mathbf{y}_{i+1}^L\big)$

Similar to the forward probabilities, the backward probability, $p\big(\mathbf{h}_i|\mathbf{y}_{i+1}^L\big)$, can be written as

$$\begin{aligned} p\big(\mathbf{h}_i|\mathbf{y}_{i+1}^L\big) \;&=\; \int p\big(\mathbf{h}_i|\mathbf{h}_{i+1}\big)p\big(\mathbf{h}_{i+1}|\mathbf{y}_{i+1}, \mathbf{y}_{i+2}^L\big) d\mathbf{h}_i \\ &\propto \int p\big(\mathbf{h}_i|\mathbf{h}_{i+1}\big)p\big(\mathbf{h}_{i+1}|\mathbf{y}_{i+2}^L\big)p\big(\mathbf{y}_{i+1}|\mathbf{h}_{i+1}\big) d\mathbf{h}_i, \end{aligned} \tag{11}$$

where

$$p(\mathbf{y}_{i+1}|\mathbf{h}_{i+1}) = \sum_{\mathbf{C}_{i+1}} p(\mathbf{y}_{i+1}|\mathbf{C}_{i+1},\mathbf{h}_{i+1})p(\mathbf{C}_{i+1})$$
$$\propto \sum_{\mathbf{C}_{i+1}} p(\mathbf{C}_{i+1})\mathcal{F}_{\mathcal{N}}(\mathbf{y}_{i+1},\mathbf{C}_{i+1}\mathbf{h}_{i+1},\sigma^2\mathbf{I}_{TN_r}). \quad (12)$$

Combining (11) and (12), the backward probability can be recursively computed as

$$p(\mathbf{h}_i|\mathbf{y}_{i+1}^L) \propto \int p(\mathbf{h}_{i+1}|\mathbf{y}_{i+2}^L)\mathcal{F}_{\mathcal{N}}(\mathbf{h_i},\alpha\mathbf{h}_{i+1},(1-\alpha^2)\,\mathbf{I})$$
$$\cdot \sum_{\mathbf{C}_{i+1}} p(\mathbf{C}_{i+1})\,\mathcal{F}_{\mathcal{N}}(\mathbf{y}_{i+1},\mathbf{C}_{i+1}\mathbf{h}_{i+1},\sigma^2\mathbf{I}_{TN_r})d\mathbf{h}_i. \quad (13)$$

C. Implementation of the Proposed BP Algorithm

Computation of the APP in (6) involves the following steps: $i)$ computation of the forward probabilities given by (10), $ii)$ computation of the backward probabilities given by (13), $iii)$ computation of the integrand in (6), and $iv)$ evaluation of the integral in (6). For a given i, $i = 1, 2, \cdots, L$, the APP expression in (6) is computed for all possible $\mathbf{X}_i$'s, and the $\mathbf{X}_i$ with the maximum APP is declared as the detected data matrix.

The computations in steps $i)$ and $ii)$ involve the densities of the continuous random vectors $\mathbf{h}_i$, $i = 1, 2, \cdots, L$. Since $\mathbf{h}_i$'s are Gaussian random vectors, the forward or backward probabilities can be fully characterized by the parameters $\mathbf{m}$, $\mathbf{K}$, and a constant amplitude which multiplies the function $\mathcal{F}_{\mathcal{N}}(\mathbf{a},\mathbf{m},\mathbf{K})$. These three parameters comprise the message which is passed to the next step in the recursion involved in the computation of the forward and backward probabilities. The computation of these three parameters require the use of the property in Lemma 1 (Lemmas 1,2,3 are given in Appendix). The number of these 3-tuple parameters passed is limited to a small fixed k, in order to limit the computational complexity of the algorithm, which otherwise would require exponential complexity. The k best 3-tuple parameters with the highest amplitude constants are chosen to construct the message to be passed. The computation of the integrand in step $iii)$ involves the summation of terms of the form $\mathcal{F}_{\mathcal{N}}(\mathbf{a},\mathbf{m}_1,\mathbf{K}_1)\mathcal{F}_{\mathcal{N}}(\mathbf{a},\mathbf{m}_2,\mathbf{K}_2)\mathcal{F}_{\mathcal{N}}(\mathbf{b},\mathbf{Ga},\sigma^2\mathbf{I})$, which, using the properties in Lemma 1 and 2, can be simplified to a summation of terms of the form $\mathcal{F}_{\mathcal{N}}(\mathbf{a},\mathbf{m},\mathbf{K})$. Finally, the resulting integral in step $iv)$ is computed using the property in Lemma 3.

IV. Simulation Results and Discussions

We evaluated the BER performance of the proposed BP-based algorithm for the following three MIMO architectures: $i)$ 2×2 V-BLAST, $ii)$ 2×2 orthogonal STBC (Alamouti Code), and $iii)$ 2×2 non-orthogonal STBC (Golden Code). Each frame consists of K channel uses, out of which K_p channel uses are for pilot and $K_d = K - K_p$ channel uses are for data. The ratio K_p/K refers to the pilot density in a frame. The total power per frame is P, out of which P_p is allotted for the pilot part and $P_d = P - P_p$ is allotted for the data part. We compare the performance of the proposed scheme with the performance under the assumption of perfect CSIR and ML detection. We use BPSK modulation in all our simulations (although the proposed algorithm works for QAM as well).

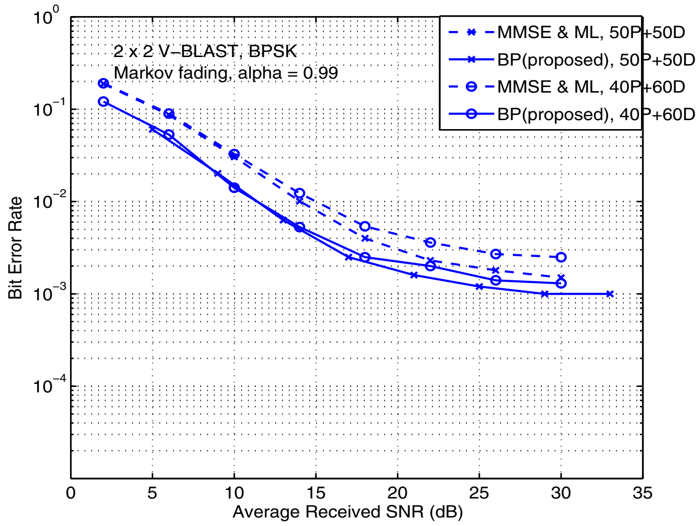

Fig. 2. Comparison of the BER performance of the proposed BP based joint channel estimation/detection scheme with that of a scheme that employs MMSE channel estimation and ML detection in a 2×2 V-BLAST system.

A. 2×2 V-BLAST System

First, in Fig. 2, we compare the uncoded BER performance of the proposed joint BP approach with that of a scheme that separately uses MMSE for channel estimation and ML for detection in a 2×2 V-BLAST system under slow fading. The value of the Markov fading parameter α is 0.99. We consider different pilot densities in a frame, keeping the frame length fixed at $K = 100$. For a given K, we denote the pilot density in a frame using the notation K_pP+K_dD, which indicates K_p pilot channel uses and K_d data channel uses per frame. We use uniform power allocation, where the power per channel use is kept constant throughout the frame; i.e., $P_p/K_p = P_d/K_d = P/K = c$, where c is a function of the average SNR. From Fig. 2, we can see the the proposed BP approach outperforms the MMSE channel estimation/ML detection scheme by about 3 to 4 dB at an uncoded BER of 5×10^{-2}, which illustrates the effectiveness of the proposed BP based scheme.

Next, we note that increased pilot channel uses per frame (K_p) causes increased loss in spectral efficiency (e.g., with 50P+50D, the spectral efficiency gets halved). In Fig. 3, we illustrate that such loss in spectral efficiency can be reduced (while achieving good BER performance) using non-uniform power allocation between pilot and data, based on the knowledge of the average channel SNR at the transmitter.

Figure 3 shows the BER performance of the 2×2 V-BLAST system with 30P+70D (i.e.,$K_p = 30, K_d = 70, K = 100$, and a spectral efficiency of 1.4 bps/Hz) for non-uniform power allocation between pilot and data under block fading (i.e., $\alpha = 1$). We denote the pilot/data power allocation profile in a frame using the notation $m\%$P+$n\%$D, where $m = \frac{P_p}{P} \times 100$

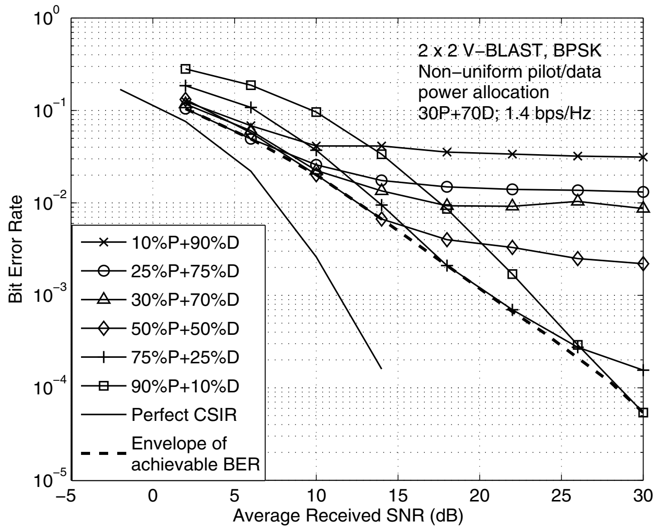

Fig. 3. BER performance of the proposed BP based joint channel estimation/detection algorithm in a 2×2 V-BLAST system for varying pilot/data power allocation. Pilot density: 30P+70D.

% of the total power is allotted equally among the K_p pilot channel uses, and $n = \frac{P_d}{P} \times 100$ % of the total power is allotted equally among the K_d data channel uses. The uniform power allocation employed in Fig. 2 becomes a special case of this non-uniform allocation when $m = \frac{P_p}{P} = \frac{K_p}{K}$ and $n = \frac{P_d}{P} = \frac{K_d}{K}$. For the considered 30P+70D pilot density in Fig. 3, the 30%P+70%D power allocation profile corresponds to the uniform power allocation. All other allocation profiles shown in Fig. 3 are non-uniform profiles. From Fig. 3, it can be observed that a higher pilot power helps at higher SNRs while a higher data power is advantageous at lower SNRs (in fact, there is a crossover between the BER curves corresponding to different power allocation profiles). For a given non-uniform power allocation profile, the BER performance exhibits flooring. It is noted, however, that this flooring happens at different SNR ranges for different profiles. This observation could be exploited by way of choosing the power allocation profile depending on the operating average channel SNR. If such an average SNR based selection of power allocation profile is done, then a performance which is shown as the 'envelope of the achievable SNR' (which denotes the envelope of the best possible BER performances) can be achieved.

B. 2×2 *Orthogonal STBC System (Alamouti Code)*

In Fig. 4, we present the uncoded BER performance of the proposed BP algorithm for the Alamouti 2×2 MIMO system whose spectral efficiency is 1 bps/Hz (in the case of no pilots and no channel coding) for block fading ($\alpha = 1$) and uniform power allocation. We consider the case of 10P+90D and 4P+96D, where the spectral efficiencies are 0.9 bps/Hz and 0.96 bps/Hz, respectively. Compared to the 1 bps/Hz in the ideal case, this spectral efficiency loss due to pilots is very small. From Fig. 4, it can be seen that, in the case of Alamouti code, the proposed scheme is very effective in achieving close to perfect CSIR performance (to within about 1 dB) incurring only a small loss in spectral efficiency due to pilots.

C. 2×2 *Non-orthogonal STBC System (Golden Code)*

In Fig. 5, we present the uncoded BER performance of the proposed BP algorithm for the 2×2 Golden Code with BPSK and block fading at 2 bps/Hz spectral efficiency in the ideal case. We consider 20+80D training, which corresponds to a spectral efficiency of 1.8 bps/Hz. We plot the BERs of the proposed scheme for varying pilot/data power allocation. ML performance of the Golden code with perfect CSIR is also plotted. As in the case of 2×2 V-BLAST in Fig. 3, it is observed here also that performance can be significantly improved with non-uniform power allocation. By choosing the power allocation profile based on the knowledge of average receive SNR at the transmitter, the proposed scheme can achieve performance quite close to that with perfect CSIR (see the envelope of the achievable BER).

D. *Computational Complexity*

The proposed algorithm consists of three steps, namely, $i)$ computation of forward probabilities, $ii)$ computation of backward probabilities, and $iii)$ computation of the APPs. For an $N_t \times N_r$ MIMO system, the length of $\mathbf{h}_i$ is $N_t N_r$. Assuming that the number of messages is limited to k (defined in Sec. III-C), and denoting the cardinality of the set containing all possible codewords of $\mathbf{C}_i$ by $|\mathcal{C}|$, the complexity of computing $p(\mathbf{h}_i|\mathbf{y}_1^{i-1})$ is $\mathcal{O}(N_r^2 T^2 k |\mathcal{C}| L)$, where L is the number of STBC matrices in one frame. The $N_r^2 T^2$ term is due to the matrix inversion that is required when using the property in Lemma 1. We point out that the above complexity expression is for the case when the number of pilot STBCs is zero in a frame. For the case of N_p pilot STBCs and N_d data STBCs in a frame, the complexity order is $\mathcal{O}(N_r^2 T^2 k |\mathcal{C}| N_d)$. The computation of $p(\mathbf{h}_i|\mathbf{y}_{i+1}^L)$ requires a similar complexity of $\mathcal{O}(N_r^2 T^2 k |\mathcal{C}| N_d)$. The computation of the APPs requires a complexity of $\mathcal{O}(N_r^2 N_t^2 k^2 |\mathcal{C}| N_d)$, where $N_r^2 N_t^2$ is due to matrix inversion. Therefore, the total complexity is $\mathcal{O}(N_r^2 (T^2 k + N_t^2 k^2) |\mathcal{C}| N_d)$. As the number of bits in a frame is $N_d \log_2 |\mathcal{C}|$, the total complexity of the algorithm per bit is given by $\mathcal{O}(N_r^2 (T^2 k + N_t^2 k^2) |\mathcal{C}| / \log_2 |\mathcal{C}|)$.

For the special case of V-BLAST and orthogonal STBCs, the matrix inversion reduces to an inversion of a scaled identity matrix. The total complexity of the algorithm per bit then drastically reduces to $\mathcal{O}((k + k^2) |\mathcal{C}| / \log_2 |\mathcal{C}|)$.

V. Conclusions

We proposed a belief propagation based approach to joint channel estimation and detection in the context of MIMO systems, including V-BLAST and orthogonal/non-orthogonal STBCs. The performance of the proposed algorithm was shown to be close to that of perfect CSIR, particularly with non-uniform power allocation among pilot and data symbols in frame. Also, the proposed BP approach of joint channel estimation/detection was shown to outperform (by about 3 to 4 dB) individual MMSE based channel estimation followed by ML detection. The proposed approach can be extended

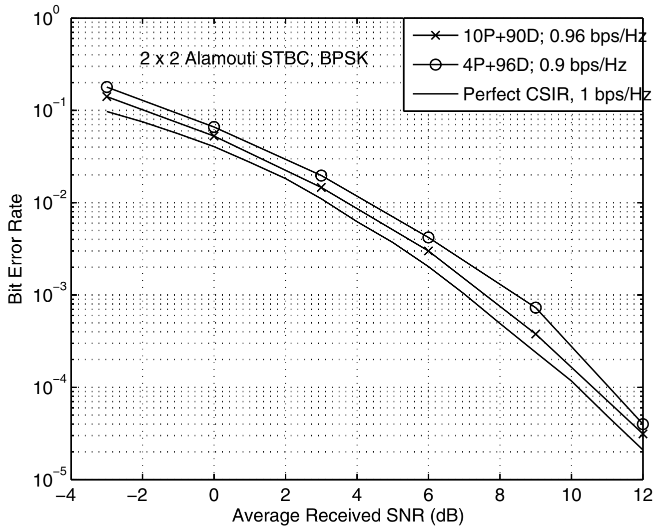

Fig. 4. BER performance of the proposed BP based joint channel estimation/detection algorithm for 2×2 Alamouti code.

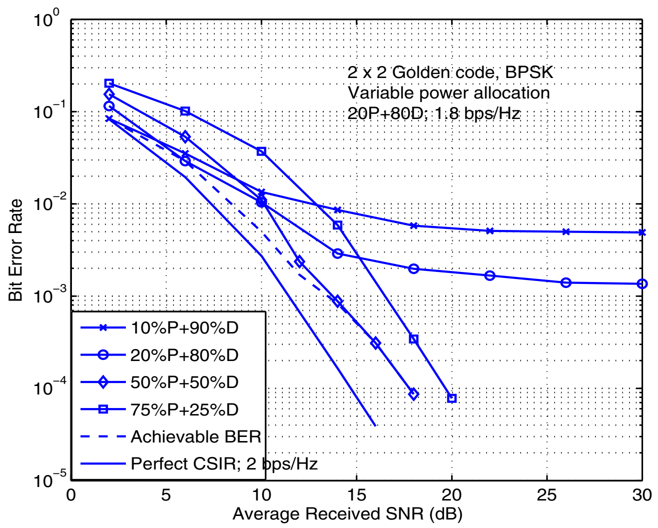

Fig. 5. BER performance of the proposed BP based joint channel estimation/detection algorithm for 2×2 Golden code.

to coded systems where channel estimation, MIMO detection and channel decoding can be carried out jointly.

Appendix

The following three properties of $\mathcal{F}_{\mathcal{N}}(\cdot,\cdot,\cdot)$ in Lemmas 1 to 3 are used in the implementation of the proposed algorithm. Due to page limit, we state these lemmas skipping the proof.

Lemma 1:

$$\mathcal{F}_{\mathcal{N}}(\mathbf{a},\mathbf{m},\mathbf{K})\mathcal{F}_{\mathcal{N}}(\mathbf{b},\mathbf{Ga},\sigma^2\mathbf{I}) = \mathcal{F}_{\mathcal{N}}(\mathbf{a},\mathbf{m}',\mathbf{K}')\mathcal{F}_{\mathcal{N}}(\mathbf{b},\mathbf{Gm},\sigma^2\mathbf{I}+\mathbf{GKG}^H), \quad (14)$$

where $\mathbf{K}' = \mathbf{K} - \mathbf{KG}^H(\sigma^2\mathbf{I}+\mathbf{GKG}^H)^{-1}\mathbf{GK}$ and $\mathbf{m}' = \mathbf{m} - \mathbf{KG}^H(\sigma^2\mathbf{I}+\mathbf{GKG}^H)^{-1}\mathbf{Gm} + \mathbf{K}'\frac{\mathbf{G}^H\mathbf{b}}{\sigma^2}$. □

Lemma 2:

$$\mathcal{F}_{\mathcal{N}}(\mathbf{a},\mathbf{m}_1,\mathbf{K}_1)\mathcal{F}_{\mathcal{N}}(\mathbf{a},\mathbf{m}_2,\mathbf{K}_2) = \mathcal{F}_{\mathcal{N}}(\mathbf{m}_1,\mathbf{m}_2,\mathbf{K}_1+\mathbf{K}_2)\mathcal{F}_{\mathcal{N}}(\mathbf{a},\mathbf{m}_3,\mathbf{K}_3), \quad (15)$$

where $\mathbf{K}_3 = \mathbf{K}_1(\mathbf{K}_1+\mathbf{K}_2)^{-1}\mathbf{K}_2$ and $\mathbf{m}_3 = \mathbf{K}_3(\mathbf{K}_1^{-1}\mathbf{m}_1 + \mathbf{K}_2^{-1}\mathbf{m}_2)$. □

Lemma 3:

$$\int \frac{\mathcal{F}_{\mathcal{N}}(\mathbf{a},\mathbf{m},\mathbf{K})}{\mathcal{F}_{\mathcal{N}}(\mathbf{a},\mathbf{0},\sigma^2\mathbf{I})}d\mathbf{a} \propto \frac{1}{\det\left(\mathbf{K}^{-1} - \frac{\mathbf{I}}{\sigma^2}\right)} exp\left\{\mathbf{m}^H\left(\sigma^2\mathbf{I} - \mathbf{K}\right)^{-1}\mathbf{m}\right\}. \quad (16)$$

□

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