SIR Analysis and Interference Cancellation in Uplink OFDMA with Large Carrier Frequency/Timing Offsets

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***Abstract*—In uplink orthogonal frequency division multiple access (OFDMA), large timing offsets (TO) and/or carrier frequency offsets (CFO) of other users with respect to a desired user can cause significant multiuser interference (MUI). In this letter, we analytically characterize the degradation in the average output signal-to-interference ratio (SIR) due to the combined effect of both TOs as well as CFOs in uplink OFDMA. Specifically, we derive closed-form expressions for the average SIR at the DFT output in the presence of large CFOs and TOs. The analytical expressions derived for the signal and various interference terms at the DFT output are used to devise an interference cancelling receiver to mitigate the effect of CFO/TO-induced interferences.**

***Index Terms*—Carrier frequency offset, multiuser interference, timing offset, uplink OFDMA.**

I. INTRODUCTION

THE performance of uplink OFDMA depends to a large extent on how well the orthogonality among different subcarriers is maintained at the receiver. Factors including $i)$ timing offsets (TO) of different users caused due to path delay differences between different users and imperfect timing synchronization, and $ii)$ carrier frequency offsets (CFO) of different users induced by Doppler effects and/or poor oscillator alignments, can destroy the orthogonality among subcarriers at the receiver and cause multiuser interference (MUI) [1]. The detrimental effect of TO-induced orthogonality loss can be alleviated by $i)$ providing adequate guard interval (expensive in terms of throughput for large TOs), or $ii)$ use of GPS timing (expensive in terms of hardware at the mobiles), or $iii)$ closed-loop timing correction between mobile transmitters and the base station (BS) receiver (expensive in terms of feedback bandwidth, pilot power, and oscillator cost). For example, in IEEE 802.16e standard, the detrimental effects of CFOs and TOs are reduced through tight closed-loop frequency/timing correction between the mobile transmitters and the BS receiver [2]. An alternate open-loop approach to handle the effects of large TOs and CFOs in uplink OFDMA is to employ interference cancellation (IC) techniques at the BS receiver. Receivers employing the IC approach to handle the effects of CFOs alone, assuming ideal time synchronization and sampling (in other words, assuming zero TOs) have been proposed in [1],[3],[4]. Effect of TOs alone, assuming zero CFOs, on the performance of OFDM/OFDMA has been reported in [5],[6],[7]. Estimation of CFOs and TOs in uplink OFDMA has been investigated in [8],[9],[10]. To our knowledge, an analytical characterization of the effect of both large CFOs as well as large TOs on the performance in uplink OFDMA has not been reported so far. In this letter, we analytically characterize the degradation in the average output signal-to-interference ratio (SIR) due to the combined effect of both CFOs as well as TOs in uplink OFDMA. We derive closed-form expressions for the average SIR at the DFT output in the presence of large CFOs and TOs. Numerical results show that the SIR degradation is severe for large CFOs and TOs. We show that this degradation in performance can be alleviated through interference cancelling receivers devised using the expressions derived for the signal and various interference terms at the DFT output.

II. SYSTEM MODEL

We consider an uplink OFDMA system with K users, where each user communicates with a BS through an independent multipath channel. We assume that there are N subcarriers in each OFDM symbol and one subcarrier can be allocated to only one user. The information symbol for the uth user on the kth subcarrier is denoted by $X_k^{(u)}$, $k \in S_u$, where S_u is the set of subcarriers assigned to the uth user and $E\left[\left|X_k^{(u)}\right|^2\right] = 1$. Then, $\bigcup_{u=1}^{K} S_u = \{0, 1, \ldots, N-1\}$ and $S_u \bigcap S_v = \phi$ for $u \neq v$. The length of the cyclic prefix added is N_g sampling periods[1], and is assumed to be longer than the maximum channel delay spread, $L-1$, normalized by the sampling period (i.e., $N_g \geq L-1$). After IDFT processing and cyclic prefix insertion at the transmitter, the time-domain sequence of the uth user, $x_n^{(u)}$, is given by

$$x_n^{(u)} = \frac{1}{N} \sum_{k \in S_u} X_k^{(u)} e^{\frac{j2\pi nk}{N}}, \quad -N_g \leq n \leq N-1. \tag{1}$$

If cyclic prefix is not added, then $N_g = 0$. The uth user's signal at the receiver input, after passing through the channel, is given by

$$s_n^{(u)} = x_n^{(u)} \star h_n^{(u)}, \tag{2}$$

where $\star$ denotes linear convolution and $h_n^{(u)}$ is the uth user's channel impulse response. It is assumed that $h_n^{(u)}$ is non-zero

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[1]Let T denote one OFDM symbol period including the cyclic prefix duration. Then $T_s = \frac{T}{N+N_g}$ denotes one sampling period.

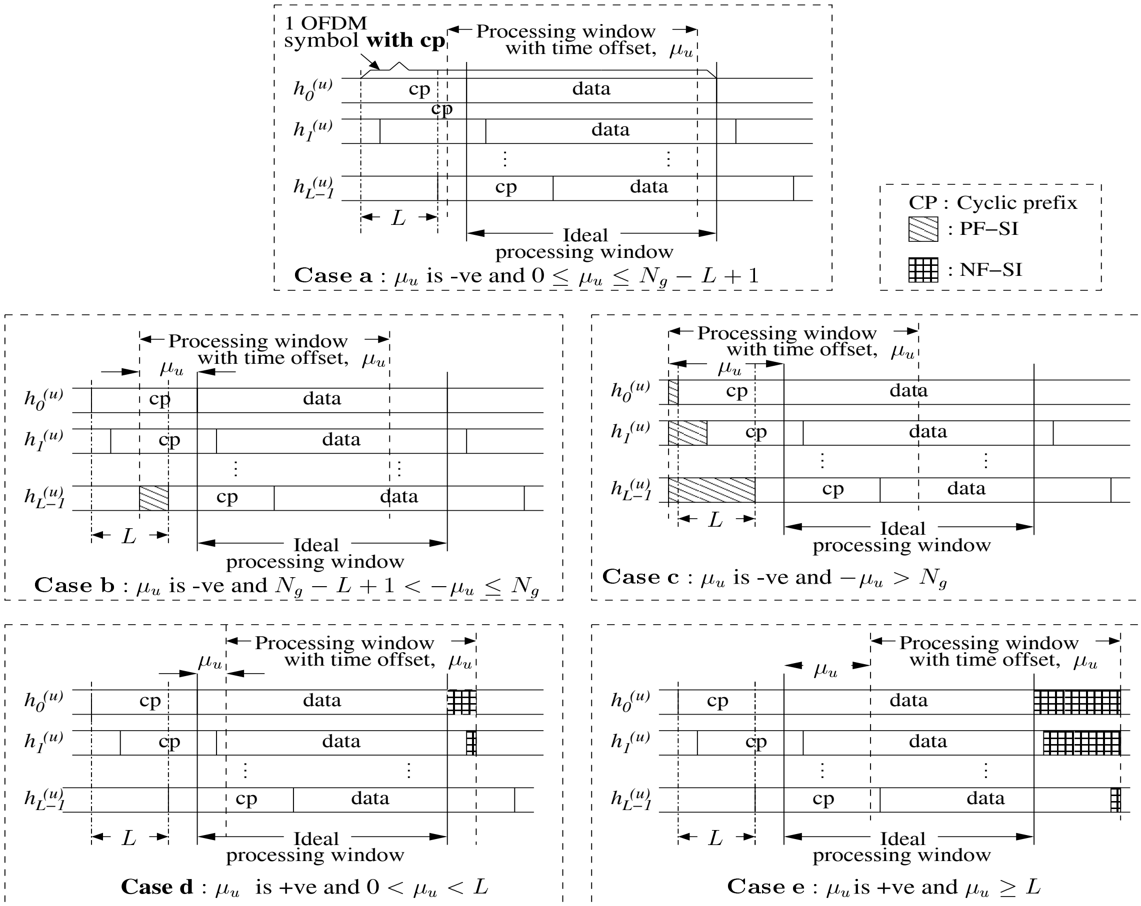

Fig. 1. Different timing misalignment cases *a)* to *e)* for uplink OFDMA with a single user u. Only self interferences.

only for $n = 0, \ldots, L-1$, and that all users' channels are statistically independent. We assume that $h_n^{(u)}$'s are i.i.d. complex Gaussian with zero mean and $E\Big[\Big(h_{n,I}^{(u)}\Big)^2\Big] = E\Big[\Big(h_{n,Q}^{(u)}\Big)^2\Big] = 1/2L$, where $h_{n,I}^{(u)}$ and $h_{n,Q}^{(u)}$ are the real and imaginary parts of $h_n^{(u)}$. The channel coefficient in frequency-domain, $H_k^{(u)}$, is given by

$$H_k^{(u)} = \sum_{n=0}^{L-1} h_n^{(u)} e^{\frac{-j2\pi nk}{N}}, \quad \text{and} \quad E\Big[\Big|H_k^{(u)}\Big|^2\Big] = 1. \qquad (3)$$

Let ϵ_u, $u = 1, 2, \cdots, K$ denote uth user's residual CFO normalized by the subcarrier spacing, $|\epsilon_u| \le 0.5, \forall u$, and let μ_u, $u = 1, 2, \cdots, K$ denote uth user's residual TO in number of sampling periods at the receiver. As in [5], these TOs are considered to be introduced by an erroneous detection of the start of an OFDM symbol of each user, which can occur, for example, due to an erroneous initial frame synchronization. Consequently, the processing window of each user at the receiver gets shifted by the corresponding user's TO with respect to the ideal processing window. These TOs can be positive ($\mu_u > 0$) or negative ($\mu_u < 0$) as illustrated in Figs. 1 and 2. TOs and CFOs cause loss of orthogonality among the subcarriers at the receiver, thus degrading the performance. In order to fully characterize the effect of TOs analytically, different cases of timing misalignment as itemized in the following subsection need to be considered.

A. Different Cases of Timing Misalignment

Let us first consider the different cases of timing misalignment in a system with a single user u. In order to highlight the effect of non-zero TOs alone (i.e., $|\mu_u| > 0$), consider zero CFO (i.e., $\epsilon_u = 0$). For $\mu_u < 0$, depending on the magnitude of μ_u compared to delay spread $L-1$ and cyclic prefix length N_g, interference from previous frame data (which we refer to as *Previous Frame Self Interference* (PF-SI)) and inter-carrier interference due to loss of some samples of the current frame in the processing window (which we refer to as *Current Frame Self Interference* (CF-SI)) may or may not occur. We need to consider the following three cases for $\mu_u \le 0$.

- **Case a):** $0 \le -\mu_u \le N_g - L + 1$, where there will be no loss of orthogonality, and hence there is no TO-induced interference.
- **Case b):** $N_g - L + 1 < -\mu_u \le N_g$, where PF-SI is caused by some paths. The number of such paths causing PF-SI in this case will be $L - [(N_g + \mu_u) + 1]$. In addition, since up to $L - [(N_g + \mu_u) + 1]$ samples of the current frame are lost in the processing window, it results in loss of orthogonality due to TO, and hence in CF-SI.
- **Case c):** $-\mu_u > N_g$, where PF-SI is caused by all the L paths. In addition, CF-SI also occurs due to loss of up to L samples of the current frame.

For $\mu_u > 0$, any value of $\mu_u > 0$ will cause both interference from next frame data, which we refer to as *Next Frame Self*

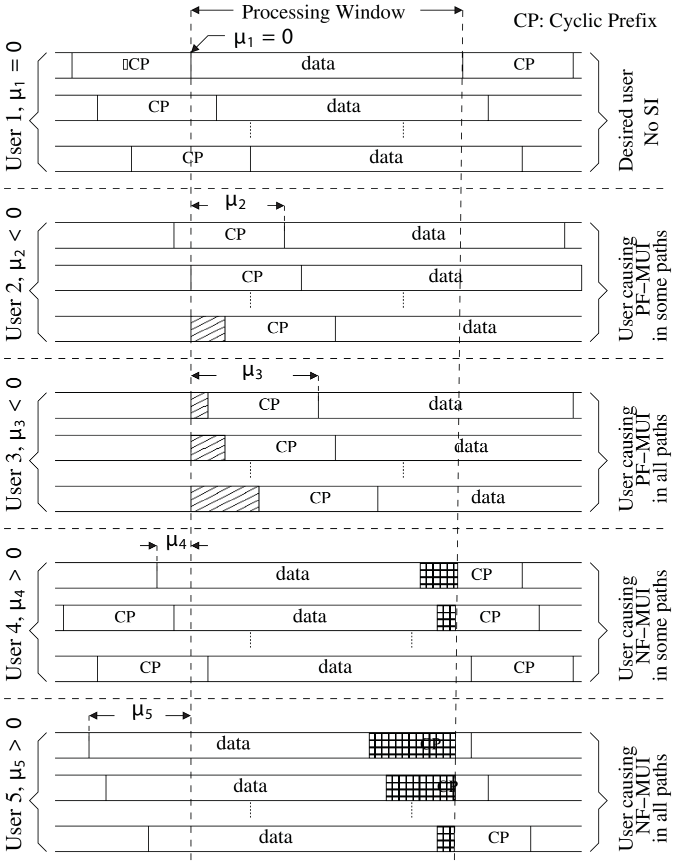

Fig. 2. Different timing misalignment scenarios in uplink OFDMA with multiple users.

Interference (NF-SI), as well as CF-SI due to loss of some samples in the current frame. The following two cases need to be considered for $\mu_u > 0$.

- **Case d):** $0 < \mu_u < L$, where NF-SI is caused by some paths. The number of such paths causing NF-SI in this case is μ_u. In addition, CF-SI also will occur due to loss of up to μ_u samples in the current frame.
- **Case e):** $\mu_u \geq L$, where NF-SI is caused by all L paths. CF-SI will occur in this case also.

In Fig. 1, we illustrate timing misalignment scenarios for the above five cases a) to e) for a given user u in the absence of other users, in which case the interferences are essentially self interferences. In addition to the above self interferences, other user interferences will occur in the multiuser case. In the multiuser case also, the same five cases apply, and the corresponding multiuser interference (MUI) terms caused by previous, current and next frame data symbols of other users are denoted by *Previous Frame MUI* (PF-MUI), *Current Frame MUI* (CF-MUI), and *Next Frame MUI* (NF-MUI), respectively. In Fig. 2, we illustrate a possible timing misalignment scenario for the multiuser case where the desired user is perfectly aligned (i.e., no self interferences) and the other users are misaligned (causing PF-MUI, CF-MUI, NF-MUI). In the presence of both TOs as well as CFOs, additional CFO-induced interference will be generated. When there is no TO-induced interference on a given path, we refer to the interference generated by the desired user CFO as *CFO-induced Self Interference* (CFO-SI) on that path, and those generated by the other user CFOs as *CFO-induced MUI* (CFO-MUI) on that path. On all the paths that experience TO-induced interferences, non-zero CFOs will affect PF-SI/MUI, CF-SI/MUI and NF-SI/MUI.

III. SIR Analysis

In this section, we derive analytical expressions for the average SIR at the DFT output of the receiver in the presence of both TOs (μ_u's), and CFOs (ϵ_u's). We first obtain the analytical expressions for signal and interference terms at the DFT output for different time offset cases, and use these expressions to obtain the expressions for the average output SIR. We note that the DFT output on a given subcarrier consists of three components; a desired signal component, self interference components, and multiuser interference components.

A. DFT Output Expressions

We consider that among the K users in the system, K_α users, $0 \leq K_\alpha \leq K$, belong to timing offset case $\alpha \in \mathcal{T}$, where $\mathcal{T} = \{a, b, c, d, e\}$ denotes the set of time offset cases a) to e) described in Sec. II-A, such that

$$\sum_{\alpha \in \mathcal{T}} K_\alpha \quad = \quad K. \tag{4}$$

Let the desired user u belong to the time offset case $\lambda \in \mathcal{T}$, and each of the other users belong to any of the time offset cases in $\mathcal{T}$. Since DFT is a linear operation, the desired signal, self interference, and multiuser interference terms at the DFT output can be written individually for different cases of time offsets, as follows. Notation-wise, we use (DS), (SI), and (MI) in the superscript to denote the desired signal, self interference and multiuser interference, respectively.

1) Expressions for DS, SI, and MUI: Desired Signal: Let $Y_{k,\alpha}^{(u)(\mathrm{DS})}$ denote the desired signal component at the DFT output on the kth subcarrier of the desired user u, belonging to time offset case $\alpha \in \mathcal{T}$. Define

$$\Gamma_{qk}^{(u)(l)}(n_1, n_2) \quad \triangleq \quad \frac{1}{N} \sum_{n=n_1}^{n_2} e^{\frac{j2\pi n(q+\epsilon_u-k)}{N}}. \tag{5}$$

The desired user u, belonging to case α), will have the following desired signal output:

$$Y_{k,\alpha}^{(u),(\mathrm{DS})} = X_k^{(u)} \underbrace{\left(e^{\frac{j2\pi\mu_u k}{N}} \sum_{l=0}^{L-1} h_l^{(u)} e^{\frac{-j2\pi lk}{N}} \Gamma_{kk}^{(u)(l)}(n_{\alpha_1}, n_{\alpha_2}) \right)}_{\triangleq \mathcal{H}_{k,\alpha}^{(u)}}, \tag{6}$$

where $(n_{\alpha_1}, n_{\alpha_2})$ corresponding to different cases are given by

$$(n_{a_1}, n_{a_2}) \quad = \quad (0, N-1), \tag{7}$$

$$(n_{b_1}, n_{b_2}) = \begin{cases} (0, N-1), & \text{for } 0 \leq l \leq N_g + \mu_u \\ (-\mu_u - N_g + l, N-1), & \text{for } l > N_g + \mu_u, \end{cases} \tag{8}$$

$$(n_{c_1}, n_{c_2}) \quad = \quad (-\mu_u - N_g + l, N-1), \tag{9}$$

$$(n_{d_1}, n_{d_2}) = \begin{cases} (0, N-1-\mu_u+l), & \text{for } 0 \le l \le \mu_u - 1 \\ (0, N-1), & \text{for } l \ge \mu_u, \end{cases} \quad (10)$$

and

$$(n_{e_1}, n_{e_2}) \;=\; (0, N-1-\mu_u+l). \quad (11)$$

Note that $\mathcal{H}_{k,\alpha}^{(u)}$ in (6) can be viewed as an *effective channel coefficient* in frequency-domain in the case of non-zero time and carrier frequency offsets. When time/carrier synchronization is perfect (i.e., $\mu_u = \epsilon_u = 0$), $\mathcal{H}_{k,\alpha}^{(u)}$ simply becomes the fade coefficient $H_k^{(u)}$ given in (3).

Self Interference: Let $Y_{k,\alpha}^{(u)(\mathrm{SI})}$ denote the self interference component at the DFT output of the desired user u, belonging to time offset case $\alpha \in \mathcal{T}$, on the kth subcarrier. Self interference can be due to data in previous frame (PF-SI) or next frame (NF-SI). In addition, SI can be even due to current frame data because of non-zero CFO (CFO-SI) or because of the number of samples of the current frame in the processing window being less than N (CF-SI). Denoting the previous frame data symbol and the next frame data symbol of the uth user on the qth subcarrier by $X_q^{(u)(p)}$ and $X_q^{(u)(n)}$, respectively, we obtain the expressions for the self interference components at the DFT output of the desired user u on the kth subcarrier for the different cases of time offsets $a)$ to $e)$; these expressions are given in Table I.

Multiuser Interference: Let $Y_{k,\lambda,\alpha}^{(u,v)(\mathrm{MI})}$ denote the multiuser interference component at the DFT output of the desired user u belonging to any time offset $\lambda \in \mathcal{T}$, due to any other user v belonging to any time offset case $\alpha \in \mathcal{T}$. The expressions for the MUI components due to other user v belonging to different cases of time offsets $a)$ to $e)$ are also given in Table I.

The overall DFT output on the kth subcarrier of the desired user u belonging to time offset case $\lambda \in \mathcal{T}$, denoted by $Y_{k,\lambda}^{(u)}$, is given by

$$Y_{k,\lambda}^{(u)} \;=\; Y_{k,\lambda}^{(u),(\mathrm{DS})} + Y_{k,\lambda}^{(u),(\mathrm{SI})} + \sum_{\alpha \in \mathcal{T}} \sum_{\substack{v=1 \\ v \ne u}}^{K_\alpha} Y_{k,\lambda,\alpha}^{(u,v),(\mathrm{MI})} + Z_k^{(u)}, \quad (12)$$

where $Y_{k,\lambda}^{(u),(\mathrm{DS})}$ for different λ are given by (6), the expressions for $Y_{k,\lambda}^{(u),(\mathrm{SI})}$ and $Y_{k,\lambda,\alpha}^{(u,v),(\mathrm{MI})}$ for different λ and α are given in Table I, and $Z_k^{(u)}$ is the noise term which is complex Gaussian with zero mean and variance σ_n^2.

2) Average SINR Expressions: From the DFT output expressions presented in the above, the expressions for the average SINR at the output of the DFT can be obtained as follows. We denote the average output SINR on the kth subcarrier of the desired user u belonging to the time offset case $\lambda \in \mathcal{T}$ as $\overline{SINR}_{k,\lambda}^{(u)}$, which can be written in the following form

$$\overline{SINR}_{k,\lambda}^{(u)} \;=\; \frac{P_\lambda^{(u),(\mathrm{DS})}}{\left(\sigma_{k,\lambda}^{(u),(\mathrm{SI})}\right)^2 + \sum_{\lambda \in \mathcal{T}} \left(\sigma_{k,\lambda}^{(u),(\mathrm{MI})}\right)^2 + \sigma_n^2}, \quad (13)$$

where $P_\lambda^{(u),(\mathrm{DS})}$ is the desired signal power, and $\left(\sigma_{k,\lambda}^{(u),(\mathrm{SI})}\right)^2$, $\left(\sigma_{k,\lambda}^{(u),(\mathrm{MI})}\right)^2$, and σ_n^2 denote the variances of the self interference, multiuser interference, and noise components, respectively, at the DFT output on the kth subcarrier of the desired user u belong to time offset case λ. The expressions for

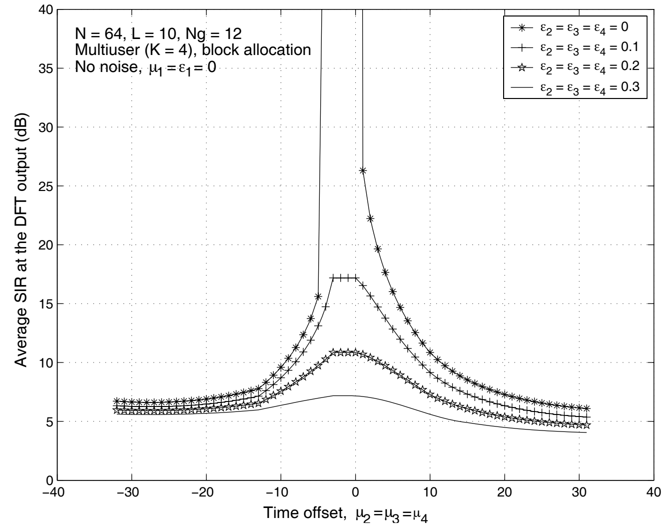

Fig. 3. Average SIR at the output of the DFT in multiuser uplink OFDMA with CFOs and TOs. $K = 4$, $N = 64$, $L = 10$, $N_g = 12$, no noise, block allocation of subcarriers, $\epsilon_1 = \mu_1 = 0$, $\epsilon_2 = \epsilon_3 = \epsilon_4$, $\mu_2 = \mu_3 = \mu_4$.

$P_\lambda^{(u),(\mathrm{DS})}$, $\left(\sigma_{k,\lambda}^{(u),(\mathrm{SI})}\right)^2$, and $\left(\sigma_{k,\lambda}^{(u),(\mathrm{MI})}\right)^2$ are summarized in Table II.

IV. Results and Discussions

We computed the average SIR at the DFT output using the expressions derived in the previous subsection. In Fig. 3, we present numerical results that illustrate the SIR degradation due to SI and MUI components for $K = 4$, $N = 64$, $L = 10$, $N_g = 12$, no noise, and block allocation of subcarriers to users, where subcarriers 1 to 16 are allotted to user 1, subcarriers 17 to 32 are allotted to user 2, and so on. We take the desired user 1 to be perfectly aligned in time and frequency (i.e., $\mu_1 = \epsilon_1 = 0$), and the remaining three users to have the same offsets (i.e., $\epsilon_2 = \epsilon_3 = \epsilon_4$ and $\mu_2 = \mu_3 = \mu_4$). The other user CFO values used are $0, 0.1, 0.2, 0.3$. In the ideal case of μ_i's $= \epsilon_i$'s $= 0$, the SIR is infinite (since no noise), and for non-zero μ_i's and ϵ_i's, the SIR degrades due to the various interference terms caused by CFOs and TOs. Note that even in the case of perfect timing alignment (i.e., μ_i's $= 0$), SIR degrades due to non-zero CFOs (i.e., ϵ_i's $\neq 0$). For example, with μ_i's $= 0$, the output SIR degrades to about 7.5 dB for ϵ_i's $= 0.3$. Non-zero μ_i's degrade the SIR even further such that for μ_i's > 15, the SIR falls below 5 dB. Also, in the case of perfect frequency alignment (ϵ_i's $= 0$), SIR degrades due to imperfect timing alignment (μ_i's $\neq 0$). While the SIR starts degrading for μ_i's $= 1$ itself due to NF-SI/MUI caused in case of μ_i's > 0, there is no SIR degradation due to negative time offsets up to $-(N_g - L + 1)$ (i.e., up to μ_i's $= -3$ in this example) since PF-SI/MUI gets introduced only for μ_i's $< -(N_g - L + 1)$.

Interference Cancellation: We note that the performance degradation due to large CFOs/TOs can be alleviated through the use of interference cancellation techniques at the receiver; e.g., a parallel interference canceller (PIC) can be devised using the estimates of the present and previous data symbols of the desired as well as other users, and the estimates of CFO and TO values. Assuming BPSK modulation, the bit decision

TABLE I
SELF INTERFERENCE AND MULTIUSER INTERFERENCE EXPRESSIONS FOR DIFFERENT TIME OFFSET CASES a) TO e).

Case $\lambda \in \mathcal{T}$	***Self Interference Expressions***	***Interference Type***
$\lambda = a$	$Y_{k,a}^{(u),(\mathrm{SI})} = \sum_{\substack{q\in S_u\\ q\neq k}} X_q^{(u)} e^{\frac{j2\pi\mu_u q}{N}} \sum_{l=0}^{L-1} h_l^{(u)} e^{\frac{-j2\pi lq}{N}} \Gamma_{qk}^{(u)(l)}(0, N-1)$	CFO-SI
$\lambda = b$	$Y_{k,b}^{(u),(\mathrm{SI})} = \sum_{\substack{q\in S_u\\ q\neq k}} X_q^{(u)} e^{\frac{j2\pi\mu_u q}{N}} \sum_{l=0}^{N_g+\mu_u} h_l^{(u)} e^{\frac{-j2\pi lq}{N}} \Gamma_{qk}^{(u)(l)}(0, N-1)$	1st term: CFO-SI
	$+ \sum_{\substack{q\in S_u\\ q\neq k}} X_q^{(u)} e^{\frac{j2\pi\mu_u q}{N}} \sum_{l=N_g+\mu_u+1}^{L-1} h_l^{(u)} e^{\frac{-j2\pi lq}{N}} \Gamma_{qk}^{(u)(l)}(n_{b_1}, n_{b_2})$	2nd term: CF-SI
	$+ \sum_{\substack{q\in S_u\\ q\neq k}} X_q^{(u)(p)} e^{\frac{j2\pi(\mu_u+N_g)q}{N}} \sum_{l=N_g+\mu_u+1}^{L-1} h_l^{(u)} e^{\frac{-j2\pi lq}{N}} \Gamma_{qk}^{(u)(l)}(0, n_{b_1}-1)$	3rd term: PF-SI
$\lambda = c$	$Y_{k,c}^{(u),(\mathrm{SI})} = \sum_{\substack{q\in S_u\\ q\neq k}} X_q^{(u)} e^{\frac{j2\pi\mu_u q}{N}} \sum_{l=0}^{L-1} h_l^{(u)} e^{\frac{-j2\pi lq}{N}} \Gamma_{qk}^{(u)(l)}(n_{c_1}, n_{c_2})$	1st term: CF-SI
	$+ \sum_{q\in S_u} X_q^{(u)(p)} e^{\frac{j2\pi(\mu_u+N_g)q}{N}} \sum_{l=0}^{L-1} h_l^{(u)} e^{\frac{-j2\pi lq}{N}} \Gamma_{qk}^{(u)(l)}(0, n_{c_1}-1)$	2nd term: PF-SI
$\lambda = d$	$Y_{k,d}^{(u),(\mathrm{SI})} = \sum_{\substack{q\in S_u\\ q\neq k}} X_q^{(u)} e^{\frac{j2\pi\mu_u q}{N}} \sum_{l=0}^{\mu_u-1} h_l^{(u)} e^{\frac{-j2\pi lq}{N}} \Gamma_{qk}^{(u)(l)}(n_{d_1}, n_{d_2})$	1st term: CF-SI
	$+ \sum_{\substack{q\in S_u\\ q\neq k}} X_q^{(u)} e^{\frac{j2\pi\mu_u q}{N}} \sum_{l=\mu_u}^{L-1} h_l^{(u)} e^{\frac{-j2\pi lq}{N}} \Gamma_{qk}^{(u)(l)}(n_{d_1}, n_{d_2})$	2nd term: CFO-SI
	$+ \sum_{q\in S_u} X_q^{(u)(n)} e^{\frac{-j2\pi(N_g-\mu_u)q}{N}} \sum_{l=0}^{\mu_u-1} h_l^{(u)} e^{\frac{-j2\pi lq}{N}} \Gamma_{qk}^{(u)(l)}(n_{d_2}+1, N-1)$	3rd term: NF-SI
$\lambda = e$	$Y_{k,e}^{(u),(\mathrm{SI})} = \sum_{\substack{q\in S_u\\ q\neq k}} X_q^{(u)} e^{\frac{j2\pi\mu_u q}{N}} \sum_{l=0}^{L-1} h_l^{(u)} e^{\frac{-j2\pi lq}{N}} \Gamma_{qk}^{(u)(l)}(n_{e_1}, n_{e_2})$	1st term: CF-SI
	$+ \sum_{q\in S_u} X_q^{(u)(n)} e^{\frac{-j2\pi(N_g-\mu_u)q}{N}} \sum_{l=0}^{L-1} h_l^{(u)} e^{\frac{-j2\pi lq}{N}} \Gamma_{qk}^{(u)(l)}(n_{e_2}+1, N-1)$	2nd term: NF-SI

Case $\lambda \in \mathcal{T}$	***Multiuser Interference Expressions***	***Interference Type***
$\lambda = a$	$Y_{k,\lambda,a}^{(u,v),(\mathrm{MI})} = \sum_{\substack{q\in S_v\\ v\neq u}} X_q^{(v)} e^{\frac{j2\pi\mu_v q}{N}} \sum_{l=0}^{L-1} h_l^{(v)} e^{\frac{-j2\pi lq}{N}} \Gamma_{qk}^{(v)(l)}(0, N-1)$	CFO-MUI
$\lambda = b$	$Y_{k,\lambda,b}^{(u,v),(\mathrm{MI})} = \sum_{\substack{q\in S_v\\ v\neq u}} X_q^{(v)} e^{\frac{j2\pi\mu_v q}{N}} \sum_{l=0}^{N_g+\mu_v} h_l^{(v)} e^{\frac{-j2\pi lq}{N}} \Gamma_{qk}^{(v)(l)}(0, N-1)$	1st term: CFO-MUI
	$+ \sum_{\substack{q\in S_v\\ v\neq u}} X_q^{(v)} e^{\frac{j2\pi\mu_v q}{N}} \sum_{l=N_g+\mu_v+1}^{L-1} h_l^{(v)} e^{\frac{-j2\pi lq}{N}} \Gamma_{qk}^{(v)(l)}(n_{b_1}, n_{b_2})$	2nd term: CF-MUI
	$+ \sum_{\substack{q\in S_v\\ v\neq u}} X_q^{(v)(p)} e^{\frac{j2\pi(\mu_v+N_g)q}{N}} \sum_{l=N_g+\mu_v+1}^{L-1} h_l^{(v)} e^{\frac{-j2\pi lq}{N}} \Gamma_{qk}^{(v)(l)}(0, n_{b_1}-1)$	3rd term: PF-MUI
$\lambda = c$	$Y_{k,\lambda,c}^{(u,v),(\mathrm{MI})} = \sum_{\substack{q\in S_v\\ v\neq u}} X_q^{(v)} e^{\frac{j2\pi\mu_v q}{N}} \sum_{l=0}^{L-1} h_l^{(v)} e^{\frac{-j2\pi lq}{N}} \Gamma_{qk}^{(v)(l)}(n_{c_1}, n_{c_2})$	1st term: CF-MUI
	$+ \sum_{\substack{q\in S_v\\ v\neq u}} X_q^{(v)(p)} e^{\frac{j2\pi(\mu_v+N_g)q}{N}} \sum_{l=0}^{L-1} h_l^{(v)} e^{\frac{-j2\pi lq}{N}} \Gamma_{qk}^{(v)(l)}(0, n_{c_1}-1)$	2nd term: PF-MUI
$\lambda = d$	$Y_{k,\lambda,d}^{(u,v),(\mathrm{MI})} = \sum_{\substack{q\in S_v\\ v\neq u}} X_q^{(v)} e^{\frac{j2\pi\mu_v q}{N}} \sum_{l=0}^{\mu_v-1} h_l^{(v)} e^{\frac{-j2\pi lq}{N}} \Gamma_{qk}^{(v)(l)}(n_{d_1}, n_{d_2})$	1st term: CF-MUI
	$+ \sum_{\substack{q\in S_v\\ v\neq u}} X_q^{(v)} e^{\frac{j2\pi\mu_v q}{N}} \sum_{l=\mu_v}^{L-1} h_l^{(v)} e^{\frac{-j2\pi lq}{N}} \Gamma_{qk}^{(v)(l)}(n_{d_1}, n_{d_2})$	2nd term: CFO-MUI
	$+ \sum_{\substack{q\in S_v\\ v\neq u}} X_q^{(v)(n)} e^{\frac{-j2\pi(N_g-\mu_v)q}{N}} \sum_{l=0}^{\mu_v-1} h_l^{(u)} e^{\frac{-j2\pi lq}{N}} \Gamma_{qk}^{(v)(l)}(n_{d_2}+1, N-1)$	3rd term: NF-MUI
$\lambda = e$	$Y_{k,\lambda,e}^{(u,v),(\mathrm{MI})} = \sum_{\substack{q\in S_v\\ v\neq u}} X_q^{(v)} e^{\frac{j2\pi\mu_v q}{N}} \sum_{l=0}^{L-1} h_l^{(v)} e^{\frac{-j2\pi lq}{N}} \Gamma_{qk}^{(v)(l)}(n_{e_1}, n_{e_2})$	1st term: CF-MUI
	$+ \sum_{\substack{q\in S_v\\ v\neq u}} X_q^{(v)(n)} e^{\frac{-j2\pi(N_g-\mu_v)q}{N}} \sum_{l=0}^{L-1} h_l^{(v)} e^{\frac{-j2\pi lq}{N}} \Gamma_{qk}^{(v)(l)}(n_{e_2}+1, N-1)$	2nd term: NF-MUI

TABLE II
DESIRED SIGNAL POWER AND VARIANCES OF SI AND MUI COMPONENTS AT THE DFT OUTPUT IN UPLINK OFDMA FOR DIFFERENT TIME OFFSET CASES a) TO e).

Case λ $\lambda \in \mathcal{T}$	$P_{k\lambda}^{(u),(\mathrm{DS})}$	$(\sigma_{k,\lambda}^{(u),(\mathrm{SI})})^2$	$(\sigma_{k,\lambda}^{(u),(\mathrm{MI})})^2$
$\lambda = a$	$\sum_{l=0}^{L-1} \left\lvert \Gamma_{kk}^{(u)(l)}(n_{a_1}, n_{a_2}) \right\rvert^2$	$\sum_{\substack{q \in S_u \\ q \neq k}} \sum_{l=0}^{L-1} \left\lvert \Gamma_{qk}^{(u)(l)}(n_{a_1}, n_{a_2}) \right\rvert^2$	$\sum_{\substack{v=1 \\ v \neq u}}^{K_a} \sum_{\substack{q \in S_v \\ v \neq u}} \sum_{l=0}^{L-1} \left\lvert \Gamma_{qk}^{(v)(l)}(n_{a_1}, n_{a_2}) \right\rvert^2$
$\lambda = b$	$\sum_{l=0}^{L-1} \left\lvert \Gamma_{kk}^{(u)(l)}(n_{b_1}, n_{b_2}) \right\rvert^2$	$\sum_{\substack{q \in S_u \\ q \neq k}} \sum_{l=0}^{L-1} \left\lvert \Gamma_{qk}^{(u)(l)}(n_{b_1}, n_{b_2}) \right\rvert^2 + \sum_{q \in S_u} \sum_{l=N_g+\mu_u+1}^{L-1} \left\lvert \Gamma_{qk}^{(u)(l)}(0, n_{b_1}-1) \right\rvert^2$	$\sum_{\substack{v=1 \\ v \neq u}}^{K_b} \Big[\sum_{\substack{q \in S_v \\ v \neq u}} \sum_{l=0}^{L-1} \left\lvert \Gamma_{qk}^{(v)(l)}(n_{b_1}, n_{b_2}) \right\rvert^2 + \sum_{\substack{q \in S_v \\ v \neq u}} \sum_{l=N_g+\mu_v+1}^{L-1} \left\lvert \Gamma_{qk}^{(v)(l)}(0, n_{b_1}-1) \right\rvert^2 \Big]$
$\lambda = c$	$\sum_{l=0}^{L-1} \left\lvert \Gamma_{kk}^{(u)(l)}(n_{c_1}, n_{c_2}) \right\rvert^2$	$\sum_{\substack{q \in S_u \\ q \neq k}} \sum_{l=0}^{L-1} \left\lvert \Gamma_{qk}^{(u)(l)}(n_{c_1}, n_{c_2}) \right\rvert^2 + \sum_{q \in S_u} \sum_{l=0}^{L-1} \left\lvert \Gamma_{qk}^{(u)(l)}(0, n_{c_1}-1) \right\rvert^2$	$\sum_{\substack{v=1 \\ v \neq u}}^{K_c} \Big[\sum_{\substack{q \in S_v \\ v \neq u}} \sum_{l=0}^{L-1} \left\lvert \Gamma_{qk}^{(v)(l)}(n_{c_1}, n_{c_2}) \right\rvert^2 + \sum_{\substack{q \in S_v \\ v \neq u}} \sum_{l=0}^{L-1} \left\lvert \Gamma_{qk}^{(v)(l)}(0, n_{c_1}-1) \right\rvert^2 \Big]$
$\lambda = d$	$\sum_{l=0}^{L-1} \left\lvert \Gamma_{kk}^{(u)(l)}(n_{d_1}, n_{d_2}) \right\rvert^2$	$\sum_{\substack{q \in S_u \\ q \neq k}} \sum_{l=0}^{L-1} \left\lvert \Gamma_{qk}^{(u)(l)}(n_{d_1}, n_{d_2}) \right\rvert^2 + \sum_{q \in S_u} \sum_{l=0}^{\mu_u-1} \left\lvert \Gamma_{qk}^{(u)(l)}(n_{d_2}+1, N-1) \right\rvert^2$	$\sum_{\substack{v=1 \\ v \neq u}}^{K_d} \Big[\sum_{\substack{q \in S_v \\ v \neq u}} \sum_{l=0}^{L-1} \left\lvert \Gamma_{qk}^{(v)(l)}(n_{d_1}, n_{d_2}) \right\rvert^2 + \sum_{\substack{q \in S_v \\ v \neq u}} \sum_{l=0}^{\mu_v-1} \left\lvert \Gamma_{qk}^{(v)(l)}(n_{d_2}+1, N-1) \right\rvert^2 \Big]$
$\lambda = e$	$\sum_{l=0}^{L-1} \left\lvert \Gamma_{kk}^{(u)(l)}(n_{e_1}, n_{e_2}) \right\rvert^2$	$\sum_{\substack{q \in S_u \\ q \neq k}} \sum_{l=0}^{L-1} \left\lvert \Gamma_{qk}^{(u)(l)}(n_{e_1}, n_{e_2}) \right\rvert^2 + \sum_{q \in S_u} \sum_{l=0}^{L-1} \left\lvert \Gamma_{qk}^{(u)(l)}(n_{e_2}+1, N-1) \right\rvert^2$	$\sum_{\substack{v=1 \\ v \neq u}}^{K_e} \Big[\sum_{\substack{q \in S_v \\ v \neq u}} \sum_{l=0}^{L-1} \left\lvert \Gamma_{qk}^{(v)(l)}(n_{e_1}, n_{e_2}) \right\rvert^2 + \sum_{\substack{q \in S_v \\ v \neq u}} \sum_{l=0}^{L-1} \left\lvert \Gamma_{qk}^{(v)(l)}(n_{e_2}+1, N-1) \right\rvert^2 \Big]$

at the first stage output of the PIC can be obtained using a matched filter detector, as

$$\widehat{X}_{k,(1)}^{(u)} = \operatorname{sgn}\left\{ \Re\left[\left(\widehat{\mathcal{H}}_{k,\lambda}^{(u)} \right)^* \widetilde{Y}_{k,\lambda}^{(u)} \right] \right\}, \tag{14}$$

where the (1) in $\widehat{X}_{k,(1)}^{(u)}$ denotes the stage index, $\widetilde{Y}_{k,\lambda}^{(u)}$ is the DFT output corresponding to the input signal compensated for the CFO of the desired user in time domain; in other words, $\widetilde{Y}_{k,\lambda}^{(u)}$ is nothing but $Y_{k,\lambda}^{(u)}$ with ϵ_i replaced with $\epsilon_i - \widehat{\epsilon}_u$, and $\widehat{\mathcal{H}}_{k,\lambda}^{(u)}$ is an estimate of $\mathcal{H}_{k,\lambda}^{(u)}$ with μ_u and ϵ_u replaced with $\widehat{\mu}_u$ and 0, respectively. This bit decision operation is carried out on subcarriers of all users. In general, for any stage-m, $m \geq 2$, the bit decision at the mth stage output is given by

$$\begin{aligned} \widehat{X}_{k,(m)}^{(u)} = \operatorname{sgn}\Bigg\{ \Re\Bigg[\left(\widehat{\mathcal{H}}_{k,\lambda}^{(u)} \right)^* \Bigg(\widetilde{Y}_{k,\lambda}^{(u)} - \widehat{Y}_{k,\lambda}^{(u),(\mathrm{SI})} \\ - \sum_{v=1}^{K_a} \widehat{Y}_{k,\lambda,a}^{(u,v),(\mathrm{MI})} - \sum_{v=1}^{K_b} \widehat{Y}_{k,\lambda,b}^{(u,v),(\mathrm{MI})} - \sum_{v=1}^{K_c} \widehat{Y}_{k,\lambda,c}^{(u,v),(\mathrm{MI})} \\ - \sum_{v=1}^{K_d} \widehat{Y}_{k,\lambda,d}^{(u,v),(\mathrm{MI})} - \sum_{v=1}^{K_e} \widehat{Y}_{k,\lambda,e}^{(u,v),(\mathrm{MI})} \Bigg) \Bigg] \Bigg\}, \end{aligned} \tag{15}$$

where $\lambda \in \mathcal{T}$, $\widehat{Y}_{k,\lambda}^{(u),(\mathrm{SI})}$ is given by $Y_{k,\lambda}^{(u),(\mathrm{SI})}$ in Table I with $X_q^{(u)}$ replaced with the bit estimate in the $(m-1)$th stage, $\widehat{X}_{q,(m-1)}^{(u)}$, and μ_u and ϵ_u replaced with $\widehat{\mu}_u$ and 0, respectively. Similarly, $\widehat{Y}_{k,\lambda,\alpha}^{(u,v),(\mathrm{MI})}$ is obtained by replacing μ_v and ϵ_v in $Y_{k,\lambda,\alpha}^{(u,v),(\mathrm{MI})}$ with $\widehat{\mu}_v$ and $\widehat{\epsilon}_v - \widehat{\epsilon}_u$, respectively. It is noted that to detect a bit in stage m, i.e., to get $\widehat{X}_{k,(m)}^{(u)}$ from (15), estimates of the previous, current, and next bits, i.e., $\widehat{X}_{q,(m-1)}^{(u),(p)}$, $\widehat{X}_{q,(m-1)}^{(u)}$, and $\widehat{X}_{q,(m-1)}^{(u),(n)}$, are needed. The requirement of the next bit estimate would introduce one extra bit delay in the overall detection, since cancellation in a stage is done after the next bit is detected in the previous stage. In Fig. 4, we present the simulated BER performance at the 2nd and 3rd stage outputs of the PIC receiver in an uplink OFDMA system with $K = 4$ users, $N = 64$ subcarriers, interleaved allocation of subcarriers (where user 1 is allotted subcarriers $1, 5, 9, \cdots$, and user 2 is allotted subcarriers $2, 6, 10, \cdots$, and so on), BPSK, $L = 2$, $N_g = 4$, $[\mu_1, \mu_2, \mu_3, \mu_4] = [-2, -5, 1, 5]$ and $[\epsilon_1, \epsilon_2, \epsilon_3, \epsilon_4] = [0.1, -0.3, -0.25, 0.15]$, assuming perfect estimates of μ_i's and ϵ_i's. It can be seen that, even at these large values of TOs/CFOs, the PIC with 3 stages is able to improve the BER performance close to that without interference, highlighting the effectiveness of the IC approach to handle the effect of large CFOs/TOs.

Uplink OFDMA without Cyclic Prefix (CP): Since the IC approach illustrated in the above can handle different types of interference including MUI and SI caused due to TOs in excess of the CP length (N_g), the CP can be dispensed with in uplink OFDMA if an IC approach is adopted at the receiver. The expressions for signal and interference terms at the DFT output of uplink OFDMA 'with CP' derived in the previous section can be specialized for the case 'without CP' as follows.

- For $\mu = 0$ without CP, the DS, SI, MUI expressions are obtained by substituting $N_g = 0$ in the corresponding expressions 'with CP' for $\mu = N_g$ in Case b).
- For $-\mu > 0$ without CP, the DS, SI, MUI expressions are obtained by substituting $N_g = 0$ in the corresponding expressions 'with CP' in Case c).

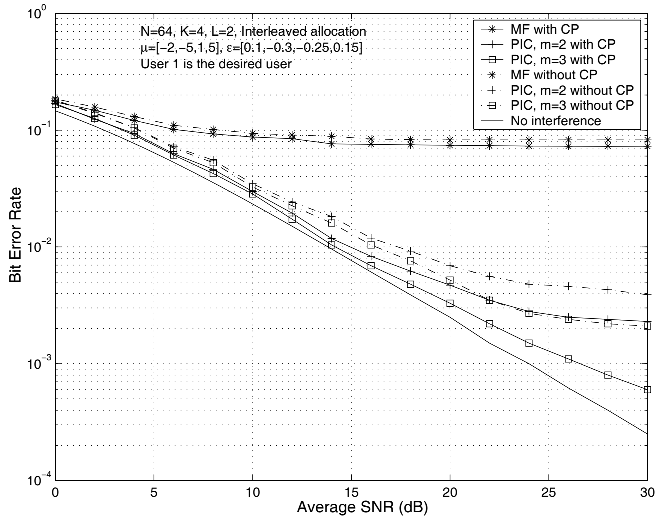

Fig. 4. BER performance of the proposed PIC receiver with and without cyclic prefix. $K = 4$, $N = 64$, interleaved allocation, BPSK, $L = 2$, $[\mu_1, \mu_2, \mu_3, \mu_4] = [-2, -5, 1, 5]$, $[\epsilon_1, \epsilon_2, \epsilon_3, \epsilon_4] = [0.1, -0.3, -0.25, 0.15]$, $N_g = 4$ for system with cyclic prefix.

- For $\mu \geq L$ without CP, the DS, SI, MUI expressions are obtained by substituting $N_g = 0$ in the corresponding expressions 'with CP' in Case e).
- For $0 < \mu < L$ without CP, $i)$ for $0 \leq l \leq \mu - 1$, only CF-SI/MUI and NF-SI/MUI will occur and the expressions for them and the DS are obtained by substituting $N_g = 0$ in the corresponding expressions 'with CP' in Case d), $ii)$ for $l = \mu$, only CFO-SI/MUI will occur and the expressions for them and the DS are obtained by substituting $N_g = 0$ in the corresponding expressions 'with CP' in Case d), and $iii)$ for $l > \mu$, instead of only CFO-SI/MUI that occur in the case of 'with CP', we get CF-SI/MUI and PF-SI/MUI in the case 'without CP' (since there is no CP to avoid PF and CF interferences); the expressions for the resulting CF-SI and CF-MUI without CP are given by $\sum_{\substack{q \in S_u \\ q \neq k}} X_q^{(u)} e^{\frac{j2\pi\mu_u q}{N}} \sum_{l=\mu_u+1}^{L-1} h_l^{(u)} e^{\frac{-j2\pi l q}{N}} \Gamma_{qk}^{(u)(l)}(n_{d_1}, n_{d_2})$, $\sum_{\substack{q \in S_v \\ v \neq u}} X_q^{(v)} e^{\frac{j2\pi\mu_v q}{N}} \sum_{l=\mu_v+1}^{L-1} h_l^{(v)} e^{\frac{-j2\pi l q}{N}} \Gamma_{qk}^{(v)(l)}(n_{d_1}, n_{d_2})$, respectively, where $(n_{d_1}, n_{d_2}) = (l - \mu, N - 1)$; PF-SI/PF-MUI expressions are given by $\sum_{q \in S_u} X_q^{(u)(p)} e^{\frac{j2\pi\mu_u q}{N}} \sum_{l=\mu_u+1}^{L-1} h_l^{(u)} e^{\frac{-j2\pi l q}{N}} \Gamma_{qk}^{(u)(l)}(0, n_{d_1} - 1)$, $\sum_{\substack{q \in S_v \\ v \neq u}} X_q^{(v)(p)} e^{\frac{j2\pi\mu_v q}{N}} \sum_{l=\mu_v+1}^{L-1} h_l^{(v)} e^{\frac{-j2\pi l q}{N}} \Gamma_{qk}^{(v)(l)}(0, n_{d_1} - 1)$, respectively; and DS expression is obtained by using $(n_{d_1}, n_{d_2}) = (l - \mu, N - 1)$ in (6).

The above expressions specialized for 'without CP' can be used in the computation of the average output SIR for the system without CP, and in the implementation of the PIC receiver when CP is not employed. In Fig. 4, in addition to the BER performance of the PIC 'with CP,' we plot the corresponding BER performance 'without CP' as well. From Fig. 4, it can be seen that, for a given PIC stage, because there can be more interference when CP is not employed, the system without CP performs poorer compared to the system with CP. This performance degradation due to lack of CP can be alleviated by using additional PIC stages; e.g., in Fig. 4, the performance achieved with $m = 2$ with CP can be achieved without CP by increasing the number of cancellation stages to $m = 3$.

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