LLR based BER Analysis of Orthogonal STBCs using QAM on Rayleigh Fading Channels

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***Abstract*—In this paper, we derive analytical expressions for the bit error rate (BER) of space-time block codes (STBC) from complex orthogonal designs (COD) with quadrature amplitude modulation (QAM) on Rayleigh fading channels. We take a bit log-likelihood ratio (LLR) based approach to derive the BER expressions. We first derive the LLRs for the various bits forming the QAM symbol, and use these LLRs to derive analytical expressions for the error rate of the individual bits forming the QAM symbol, and hence the average BER of the system. The approach presented in this paper can be used in the BER analysis of any STBC from COD with linear processing, for any value of M in a M-QAM system. Here, we present the BER analysis and results for a 16-QAM system with i) (2-Tx, L-Rx) antennas using Alamouti code (rate-1 STBC), ii) (3-Tx, L-Rx) antennas using a rate-1/2 STBC, and iii) (5-Tx, L-Rx) antennas using a rate-7/11 STBC. The LLRs derived can also be used as soft inputs to decoders for various coded QAM schemes, including turbo coded QAM with space-time coding.**

Keywords – *STBC, complex orthogonal design, QAM, BER analysis, log-likelihood ratio.*

I. Introduction

The potential capacity gains achieved by using multiple antenna systems has led to considerable attention in the area of space-time coding [1]. Space-time block codes (STBC) from complex orthogonal designs (COD) are of interest as they can be used for complex constellations such as QAM to achieve higher data rates in wireless communication systems [2],[3]. Recent works have reported analytical expressions for the symbol error rate (SER) and the bit error rate (BER) of orthogonal STBCs. In [4], Shin and Lee derived expressions for the SER of orthogonal STBCs on Rayleigh fading channels. They derived the SER by converting the multiple input multiple output (MIMO) system model to an equivalent single input single output (SISO) model. Recently, Simon in [5], and Taricco and Biglieri in [6], have reported exact expressions for the pairwise error probability (PEP) as well as approximate expressions for the BER for space-time codes.

Our key contribution in this paper is the derivation of analytical expressions for the BER for linear STBCs from COD with *QAM* modulation on Rayleigh fading channels. We adopt a *bit log-likelihood ratio (LLR)* based approach, where we first derive expressions for the LLRs of the individual bits forming the M-QAM symbol, and then use these LLRs to obtain the BER expressions. We present the BER analysis for 16-QAM systems with i) (2-Tx, L-Rx) antennas using the rate-1 Alamouti code, ii) (3-Tx, L-Rx) antennas using a rate-1/2 code, and iii) (5-Tx, L-Rx) antennas using a rate-7/11 code. Although we present the analysis and results only for 16-QAM in this paper, the approach applies for any value of M and for any arbitrary mapping of bits to the M-QAM symbol. In addition, the LLRs derived can also be used as soft inputs to decoders for various coded QAM schemes, including turbo coded QAM with space-time coding.

The rest of the paper is organized as follows. We present the MIMO system model in Section II. In Section III, we derive the LLRs for the various bits forming a 16-QAM symbol. In Section IV, we derive the analytical expressions for the BER. Numerical results and discussions are presented in Section V. Section VI presents the conclusions.

II. System Model

We consider a wireless communication system with L_t transmit and L_r receive antennas. The channel is assumed to be a flat, slowly varying (quasi-static), Rayleigh fading channel. We consider space-time block codes, where each codeword is a matrix with P rows and L_t columns, with complex valued symbols as its entries. Here, P is the number of time slots required to transmit one codeword. For some K information symbols, $s_1, s_2, \cdots, s_K$, which are selected from the 16-QAM constellation (see Fig. 1)[1], the entries of the codeword $\mathbf{X} = \{x_t^i,\ t = 1, 2, \cdots, P;\ i = 1, 2, \cdots, L_t\}$ are a linear combination of the information symbols $s_k,\ k = 1, 2, \cdots, K$ and their complex conjugates. At time slot t, $t = 1, 2, \cdots, P$, the t^{th} row of the codeword $\mathbf{X}$ (i.e., $x_t^1, x_t^2, \cdots, x_t^{L_t}$) is transmitted simultaneously from L_t antennas. The symbol transmission rate, R, is defined as the number of information symbols transmitted per time slot, i.e., $R = K/P$. The received codeword, $\mathbf{Y}$, can be written as

$$\mathbf{Y} \;=\; \mathbf{X}\,\mathbf{H} + \mathbf{N}, \tag{1}$$

where $\mathbf{Y} = \{y_t^j\ :\ t = 1, 2, \cdots, P;\ j = 1, 2, \cdots, L_r\}$ is a matrix of size $P \times L_r$, whose entry y_t^j is the signal received at antenna j at time slot t; $\mathbf{H} = \{h_{i,j}\}$ is the channel matrix of size $L_t \times L_r$, whose entry $h_{i,j}$ is the complex channel gain from the transmit antenna i to the receive antenna j. The

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[1] 4 bits, (r_1, r_2, r_3, r_4) are mapped on to a complex symbol $s_k = s_{kI} + j s_{kQ}$. The horizontal/vertical line pieces in Fig. 1 denote that all bits under these lines take the value 1, and the rest take the value 0.

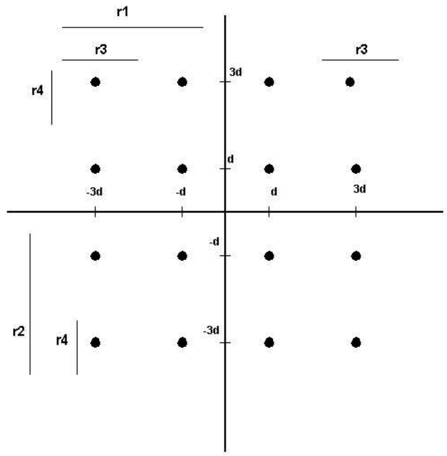

Fig. 1. 16-QAM Constellation

random variables $|h_{i,j}|$'s are assumed to be i.i.d Rayleigh distributed with $E(|h_{i,j}|^2) = \Omega$; $\mathbf{N} = \{n_t^j\}$ is the noise matrix of size $P \times L_r$, whose entries are i.i.d complex Gaussian noise with zero mean and variance σ^2.

Let $\mathcal{C}(.)$ be a mapping from a K-tuple complex message vector $\mathbf{s} = (s_1, s_2, \cdots, s_K)$ to the columnwise orthogonal $P \times L_t$ codeword $\mathbf{X} = \mathcal{C}(\mathbf{s})$. Due to the columnwise orthogonality of the linear orthogonal space-time block codes considered, the $L_t \times L_t$ matrix $\mathcal{C}(s)^H \, \mathcal{C}(s)$ is given by

$$\mathcal{C}(s)^H \, \mathcal{C}(s) = diag\left\{ \sum_{k=1}^{K} (g_{k,1} \cdot |s_k|^2), \cdots, \sum_{k=1}^{K} (g_{k,L_t} \cdot |s_k|^2) \right\}, \quad (2)$$

where $(.)^H$ denotes the Hermitian operator, and $\mathbf{G} = \{g_{m,n}\}$ is a matrix of size $K \times L_t$ whose entries can take non-negative integer values (for example, for the Alamouti code $g_{m,n} = 1, \forall m, n$). Assuming perfect knowledge of the channel coefficients at the receiver, the combined signal output for the symbol s_k is given by

$$\hat{s_k} \;=\; \Delta_k \, s_k + \zeta_k, \quad (3)$$

where

$$\Delta_k = \sum_{j=1}^{L_r} \left[g_{k,1} |h_{1,j}|^2 + g_{k,2} |h_{2,j}|^2 + \cdots + g_{k,L_t} |h_{L_t,j}|^2 \right], \quad (4)$$

and ζ_k is a complex Gaussian random variable with zero mean and variance $\Delta_k \, \sigma^2$.

III. Bit Log-Likelihood Ratios

We define the LLR for the bit $r_i,\ i = 1, 2, 3, 4$ of symbol s_k, $k = 1, 2, \cdots, K$, as

$$\begin{aligned} LLR_{s_k}(r_i) &= \log \left\{ \frac{Pr\left(r_i = 1 | \mathbf{Y}, \mathbf{H}\right)}{Pr\left(r_i = 0 | \mathbf{Y}, \mathbf{H}\right)} \right\} \\ &= \log \left\{ \frac{Pr\left(r_i = 1 | \hat{s_k}, \mathbf{H}\right)}{Pr\left(r_i = 0 | \hat{s_k}, \mathbf{H}\right)} \right\}. \end{aligned} \quad (5)$$

Assuming that all the symbols are equally likely and that the fading is independent of the transmitted symbols, using Bayes' rule, we have

$$LLR_{s_k}(r_i) = \log \left\{ \frac{\sum_{\alpha \in S_i^{(1)}} f_{\hat{s}_k | \mathbf{H}, s_k} \left(\hat{s}_k | \mathbf{H}, s_k = \alpha \right)}{\sum_{\beta \in S_i^{(0)}} f_{\hat{s}_k | \mathbf{H}, s_k} \left(\hat{s}_k | \mathbf{H}, s_k = \beta \right)} \right\}. \quad (6)$$

Since $f_{\hat{s}_k | \mathbf{H}, s_k} \left(\hat{s}_k | \mathbf{H}, s_k = \alpha \right) = \frac{1}{\pi \hat{\sigma_k}^2} \exp\left(\frac{-1}{\hat{\sigma_k}^2} \left\| \hat{s_k} - \Delta_k \, \alpha \right\|^2 \right)$, where $\hat{\sigma_k}^2 = \Delta_k \, \sigma^2$, (6) can be written as

$$LLR_{s_k}(r_i) = \log \left(\frac{\sum_{\alpha \in S_i^{(1)}} \exp\left(\frac{-1}{\hat{\sigma_k}^2} \|\hat{s}_k - \Delta_k \alpha\|^2 \right)}{\sum_{\beta \in S_i^{(0)}} \exp\left(\frac{-1}{\hat{\sigma_k}^2} \|\hat{s}_k - \Delta_k \beta\|^2 \right)} \right). \quad (7)$$

Using the approximation, $\log(\sum_j \exp(-X_j)) \approx -\min_j(X_j)$, we can approximate $LLR_{s_k}(r_i)$ as[2]

$$LLR_{s_k}(r_i) = \frac{1}{\hat{\sigma_k}^2} \left\{ \min_{\beta \in S_i^{(0)}} \left\| \hat{s}_k - \Delta_k \, \beta \right\|^2 - \min_{\alpha \in S_i^{(1)}} \left\| \hat{s}_k - \Delta_k \, \alpha \right\|^2 \right\}. \quad (8)$$

Define k complex variables, $\hat{z}_k$, $k = 1, 2, \cdots, K$, as

$$\hat{z}_k \triangleq \frac{\hat{s}_k}{\Delta_k}. \quad (9)$$

Using (9) in (8) and normalizing by $4/\hat{\sigma_k}^2$, $LLR_{s_k}(r_i)$ is written as

$$LLR_{s_k}(r_i) = \frac{\Delta_k}{4} \left\{ \min_{\beta \in S_i^{(0)}} \|\hat{z}_k - \beta\|^2 - \min_{\alpha \in S_i^{(1)}} \|\hat{z}_k - \alpha\|^2 \right\}. \quad (10)$$

Note that the set partitions $S_i^{(1)}$ and $S_i^{(0)}$ are delimited by horizontal or vertical boundaries. As a consequence, two symbols in different sets closest to the received symbol always lie either on the same row (if the delimiting boundaries are vertical) or on the same column (if the delimiting boundaries are horizontal). Using the above fact, the log-likelihood ratios for each of the bits forming the symbol, s_k, are given as

$$LLR_{s_k}(r_1) = \begin{cases} -\Delta_k \, \hat{z}_{kI} d & |\hat{z}_{kI}| \leq 2d \\ 2\Delta_k \, d(d - \hat{z}_{kI}) & \hat{z}_{kI} > 2d \\ -2\Delta_k \, d(d + \hat{z}_{kI}) & \hat{z}_{kI} < -2d, \end{cases} \quad (11)$$

$$LLR_{s_k}(r_2) = \begin{cases} -\Delta_k \, \hat{z}_{kQ} d & |\hat{z}_{kQ}| \leq 2d \\ 2\Delta_k \, d(d - \hat{z}_{kQ}) & \hat{z}_{kQ} > 2d \\ -2\Delta_k \, d(d + \hat{z}_{kQ}) & \hat{z}_{kQ} < -2d, \end{cases} \quad (12)$$

$$LLR_{s_k}(r_3) = \Delta_k \, d\{|\hat{z}_{kI}| - 2d\}, \quad (13)$$

$$LLR_{s_k}(r_4) = \Delta_k \, d\{|\hat{z}_{kQ}| - 2d\}. \quad (14)$$

In the above equations, $\hat{z}_{kI}$ and $\hat{z}_{kQ}$ are the real and imaginary parts of $\hat{z}_k$, respectively, and $2d$ is the minimum distance between pairs of signal points.

[2] This is quite a standard approximation [9], and, as we will see in Sec. V, the analytical BER evaluated using this approximate LLR is almost the same as the BER evaluated through simulations without this approximation.

IV. Derivation of BER

In this section, we derive the probability of error for the bit $r_i, i = 1, 2, 3, 4$, forming a 16-QAM symbol. The probability of error for bit r_1 in symbol s_k, P^k_{b1}, can be written as

$$P^k_{b1} = P^k_{b1|s_{kI}=-d}.\Pr\{s_{kI} = -d\} + P^k_{b1|s_{kI}=-3d}.\Pr\{s_{kI} = -3d\} + P^k_{b1|s_{kI}=d}.\Pr\{s_{kI} = d\} + P^k_{b1|s_{kI}=3d}.\Pr\{s_{kI} = 3d\}, \quad (15)$$

where s_{kI} represents the real part of s_k. Let us first consider $P^k_{b1|s_{kI}=-d}$, which is given by

$$P^k_{b1|s_{kI}=-d} = \overline{P^k_{b1|s_{kI}=-d,\mathbf{H}}}, \quad (16)$$

where the overline indicates averaging over the complex random variables $\{h_{i,j}\}$. $P^k_{b1|s_{kI}=-d,\mathbf{H}}$ can be written as

$$\begin{aligned} P^k_{b1|s_{kI}=-d,\mathbf{H}} &= \Pr\Big(LLR_{s_k}(r_1) < 0 \,|\, s_{kI} = -d, \mathbf{H}\Big) \\ &= \Pr\left(\frac{\zeta_{kI}}{\Delta_k} \geq d\right) \\ &= Q\left(\frac{d(\sqrt{\Delta_k})}{\sigma_I}\right), \end{aligned} \quad (17)$$

where $\sigma_I^2 = \sigma^2/2$. Let us define

$$\xi = \frac{1}{P}\sum_{i=1}^{L_t}\sum_{k=1}^{K} g_{k,i}. \quad (18)$$

We then have $\frac{d}{\sigma_I} = \sqrt{\frac{4E_b\,R}{5N_oL_r\,\xi}}$, where E_b is the energy per bit per transmit antenna and R is the rate of the STBC used. From the above, we can write

$$P^k_{b1|s_{kI}=-d,\mathbf{H}} = Q\left(\sqrt{\frac{4E_b\,R\,\Delta_k}{5N_oL_r\,\xi}}\right). \quad (19)$$

To obtain $P^k_{b1|s_{kI}=-d}$, we need to uncondition $P^k_{b1|s_{kI}=-d,\mathbf{H}}$ w.r.t Δ_k, which is given by

$$\begin{aligned} \Delta_k &= \sum_{j=1}^{L_r}\Big(g_{k,1}|h_{1,j}|^2 + g_{k,2}|h_{2,j}|^2 + \cdots + g_{k,L_t}|h_{L_t,j}|^2\Big) \\ &= g_{k,1}\left(\sum_{j=1}^{L_r}|h_{1,j}|^2\right) + \cdots + g_{k,L_t}\left(\sum_{j=1}^{L_r}|h_{L_t,j}|^2\right). \end{aligned} \quad (20)$$

Let us define $\theta_n = \sum_{j=1}^{L_r}|h_{n,j}|^2, n = 1, 2, \cdots, L_t$. Since $|h_{i,j}|^2$ are i.i.d exponential with mean Ω, the random variables θ_n are i.i.d Gamma random variables with density function

$$f_{\theta_n}(x) = \frac{1}{\Gamma(L_r)\Omega^{L_r}}\exp\Big(-\frac{x}{\Omega}\Big)\,x^{L_r-1}, \quad (21)$$

and the moment generating function[3] (MGF) is given by

$$\mathcal{M}_{\theta_n}(s) = \left(\frac{1}{1+s\Omega}\right)^{L_r}. \quad (22)$$

Since $\Delta_k = \sum_{n=1}^{L_t} g_{k,n}\theta_n$, its MGF, $\mathcal{M}_{\Delta_k}$, is given by

$$\mathcal{M}_{\Delta_k} = \prod_{n=1}^{L_t}\left(\frac{1}{1+s\Omega g_{k,n}}\right)^{L_r}. \quad (23)$$

Using the above and Craig's formula [10], we can show that

$$\begin{aligned} P^k_{b1|s_{kI}=-d} &= \overline{Q\left(\sqrt{\frac{4E_b\,R\,\Delta_k}{5N_oL_r\,\xi}}\right)} \\ &= \frac{1}{\pi}\int_{\phi=0}^{\frac{\pi}{2}}\prod_{n=1}^{L_t}\left(\frac{\sin^2\phi}{\sin^2\phi + \mu_1 g_{k,n}}\right)^{L_r} d\phi, \end{aligned} \quad (24)$$

where $\mu_1 = \frac{2\,\gamma_b\,R}{5L_r\xi}$ and $\gamma_b = \frac{\Omega E_b}{N_o}$. Note that the expression $\frac{1}{\pi}\int_{\phi=0}^{\frac{\pi}{2}}\prod_{n=1}^{L_t}\left(\frac{\sin^2\phi}{\sin^2\phi+\mu_1 g_{k,n}}\right)^{L_r} d\phi$ in the above can be evaluated numerically and accurately using Gauss-Chebyshev Quadrature rule. Similarly, the conditional error probability $P^k_{b1|s_{kI}=-3d,\mathbf{H}}$ is given by

$$\begin{aligned} P^k_{b1|s_{kI}=-3d,\mathbf{H}} &= \Pr\Big(LLR_{s_k}(r_1) < 0 \,|\, s_{kI} = -3d, \mathbf{H}\Big) \\ &= \Pr\left(\frac{\zeta_{kI}}{\Delta_k} \geq 3d\right) \\ &= Q\left(\sqrt{\frac{36E_b\,R\,\Delta_k}{5N_oL_r\,\xi}}\right). \end{aligned} \quad (25)$$

Unconditioning $P^k_{b1|s_{kI}=-3d,\mathbf{H}}$ w.r.t Δ_k, it can be shown that

$$\begin{aligned} P^k_{b1|s_{kI}=-d} &= \overline{Q\left(\sqrt{\frac{36E_b\,R\,\Delta_k}{5N_oL_r\,\xi}}\right)} \\ &= \frac{1}{\pi}\int_{\phi=0}^{\frac{\pi}{2}}\prod_{n=1}^{L_t}\left(\frac{\sin^2\phi}{\sin^2\phi + \mu_2 g_{k,n}}\right)^{L_r} d\phi, \end{aligned} \quad (26)$$

where $\mu_2 = \frac{18\,\gamma_b\,R}{5L_r\xi}$. It can further be shown that $P^k_{b1|a_I=-d} = P^k_{b1|a_I=d}$ and $P^k_{b1|a_I=-3d} = P^k_{b1|a_I=3d}$. Moreover, for the 16-QAM constellation considered, it can be shown that $P^k_{b1} = P^k_{b2}$ and $P^k_{b3} = P^k_{b4}$. With the above, the BER expressions for the bits r_1, r_2, r_3, r_4 of the symbol s_k can be written as

$$\begin{aligned} P^k_{b1} = P^k_{b2} &= \frac{1}{2}\Big(P^k_1 + P^k_2\Big) \\ P^k_{b3} = P^k_{b4} &= \frac{1}{2}\Big(2P^k_1 + P^k_2 - P^k_3\Big), \end{aligned} \quad (27)$$

where P^k_j, $j = 1, 2, 3$, are given by

[3]The moment generating function, $\mathcal{M}_{\theta_n}(s)$ is defined as $\mathcal{M}_{\theta_n}(s) = E[\exp(-s\theta_n)]$

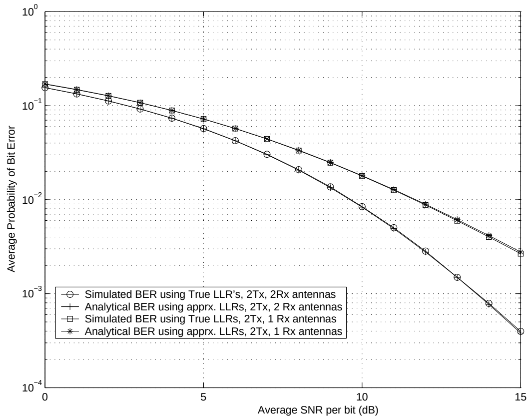

Fig. 2. Comparison of the analytical BER evaluated using approximate LLRs vs the simulated BER using the LLRs without approximation. 16-QAM with rate-1 STBC (Alamouti code). 2Tx/2Rx and 2Tx/1Rx antennas.

$$P_j^k = \frac{1}{\pi}\int_{\phi=0}^{\frac{\pi}{2}} \prod_{n=1}^{L_t} \left(\frac{\sin^2\phi}{\sin^2\phi + \mu_j g_{k,n}}\right)^{L_r} d\phi, \quad (28)$$

and

$$\mu_1 = \frac{2\,\gamma_b\,R}{5L_r\xi}, \quad \mu_2 = \frac{18\,\gamma_b\,R}{5L_r\xi}, \quad \mu_3 = \frac{10\,\gamma_b\,R}{L_r\xi}. \quad (29)$$

Again, note that (28) can be evaluated numerically and accurately using the Gauss-Chebyshev Quadrature rule. The average BER for symbol s_k, $k = 1, 2, \cdots, K$, P_b^k, is given by

$$P_b^k = \frac{1}{4}\left(P_{b1}^k + P_{b2}^k + P_{b3}^k + P_{b4}^k\right). \quad (30)$$

Finally, the average BER of the system, P_b, is given by

$$P_b = \frac{1}{K}\sum_{k=1}^{K} P_b^k. \quad (31)$$

V. Results and Discussion

We computed the BER performance of 16-QAM as a function of average SNR for the following space time block codes:

$$\mathcal{C}_1 = \begin{pmatrix} s_1 & s_2 \\ -s_2^* & s_1^* \end{pmatrix}, \qquad \mathcal{C}_2 = \begin{pmatrix} s_1 & s_2 & s_3 \\ -s_2 & s_1 & -s_4 \\ -s_3 & s_4 & s_1 \\ -s_4 & -s_3 & s_2 \\ s_1^* & s_2^* & s_3^* \\ -s_2^* & s_1^* & -s_4^* \\ -s_3^* & s_4^* & s_1^* \\ -s_4^* & -s_3^* & s_2^* \end{pmatrix},$$

and

$$\mathcal{C}_3 = \begin{pmatrix} s_1 & s_2 & s_3 & 0 & s_4 \\ -s_2^* & s_1^* & 0 & s_3 & s_5 \\ s_3^* & 0 & -s_1^* & s_2 & s_6 \\ 0 & s_3^* & -s_2^* & -s_1 & s_7 \\ s_4^* & 0 & 0 & -s_7^* & -s_1^* \\ 0 & s_4^* & 0 & s_6^* & -s_2^* \\ 0 & 0 & s_4^* & s_5^* & -s_3^* \\ 0 & -s_5^* & -s_6^* & 0 & s_1 \\ s_5^* & 0 & s_7^* & 0 & s_2 \\ -s_6^* & -s_7^* & 0 & 0 & s_3 \\ s_7 & -s_6 & -s_5 & s_4 & 0 \end{pmatrix}.$$

$\mathcal{C}_1$ is the well known Alamouti code with parameters $P = K = L_t = 2$, $R = 1$, and $\mathcal{C}_1^H\mathcal{C}_1$ is a 2×2 diagonal matrix with the $(i,i)^{th}$ diagonal element, $D(i,i)$, of the form $\sum_{k=1}^{2} \|s_k\|^2$.

$\mathcal{C}_2$ is a rate-1/2 STBC with parameters $P = 8, K = 4, L_t = 3$, $R = 1/2$, and $\mathcal{C}_2^H\mathcal{C}_2$ is a 3×3 diagonal matrix with the $(i,i)^{th}$ diagonal element, $D(i,i)$, of the form $\sum_{k=1}^{4}(2 \cdot \|s_k\|^2)$.

$\mathcal{C}_3$ is a rate-7/11 STBC with parameters $P = 11, K = 7, L_t = 5$, $R = 7/11$, and $\mathcal{C}_3^H\mathcal{C}_3$ is a 5×5 diagonal matrix with the $(i,i)^{th}$ diagonal element, $D(i,i)$, of the form

$$D(1,1) = D(2,2) = D(3,3) = D(4,4) = \sum_{k=1}^{7} \|s_k\|^2, \quad (32)$$

$$D(5,5) = \sum_{k=1}^{3}(2.\|s_k\|^2) + \sum_{k=3}^{7} \|s_k\|^2. \quad (33)$$

In Fig. 2, we compare the analytical BER evaluated using the approximate LLRs derived versus the simulated BER using the LLRs without approximation, for 16-QAM rate-1 STBC (Alamouti code) for 2Tx/2Rx and 2Tx/1Rx antennas. It is observed that the analytically computed BER is almost the same as the simulated BER, indicating that the approximation to the LLRs results in insignificant difference between the analytically computed BER and the true BER.

Figures 3, 4, 5 provide the analytical results of the average BER performance as a function of the average SNR, γ_b, for different STBCs $\mathcal{C}_1$, $\mathcal{C}_2$ and $\mathcal{C}_3$, respectively. The number of receive antennas considered include $L = 1, 2, 4, 10$. Figure 6 presents the comparative BER performance of the different STBCs $\mathcal{C}_1$, $\mathcal{C}_2$ and $\mathcal{C}_3$ when the number of receive antennas $L = 2$. The performance in AWGN is also shown for comparison. We also point out that the LLRs derived can also be used as soft inputs to decoders for various coded QAM schemes, including turbo coded QAM with space-time coding.

VI. Conclusions

Using a bit LLR based approach, we derived analytical expressions for the BER of STBCs from complex orthogonal designs with QAM on Rayleigh fading. We first derived the LLRs for the various bits forming the QAM symbol, and used these LLRs to derive analytical expressions for the error rate

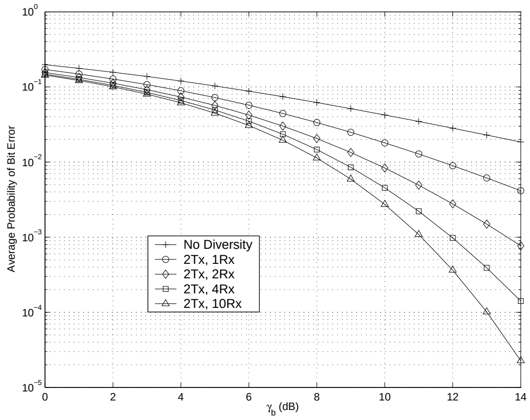

Fig. 3. BER performance of 16-QAM with 2 transmit antennas and $L_r = 1, 2, 4, 10$ receive antennas using rate-1 STBC (Alamouti code).

of the individual bits forming the QAM symbol, and hence the average BER of the system. Although the analysis was given only for 16-QAM in this paper, the approach applies to the BER analysis of M-QAM systems for any value of M (any arbitrary mapping of bits to QAM symbols) for any STBC from COD with linear processing. We presented the analytical BER results for 16-QAM with i) (2-Tx, L-Rx) antennas using Alamouti code (rate-1 STBC), ii) (3-Tx, L-Rx) antennas using a rate-1/2 STBC and iii) (5-Tx, L-Rx) antennas using a rate-7/11 STBC. The LLRs derived can also be used as soft inputs to decoders for various coded QAM schemes, including turbo coded QAM with space-time coding.

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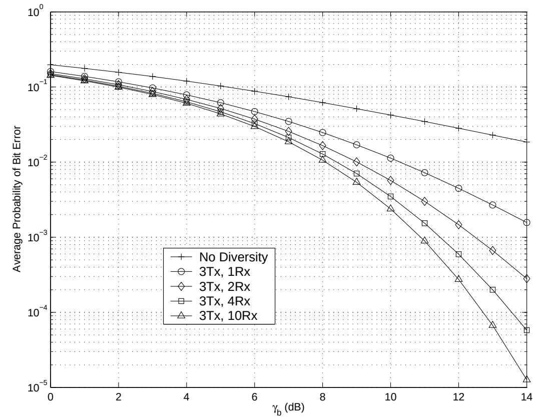

Fig. 4. BER performance of 16-QAM with 3 transmit antennas and $L_r = 1, 2, 4, 10$ receive antennas using rate-1/2 STBC.

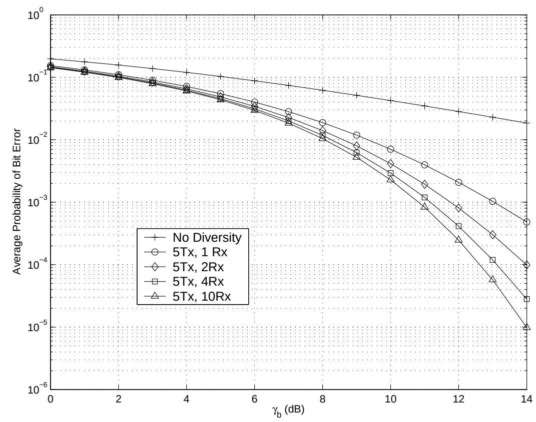

Fig. 5. BER performance of 16-QAM with 5 transmit antennas and $L_r = 1, 2, 4, 10$ receive antennas using rate-7/11 STBC.

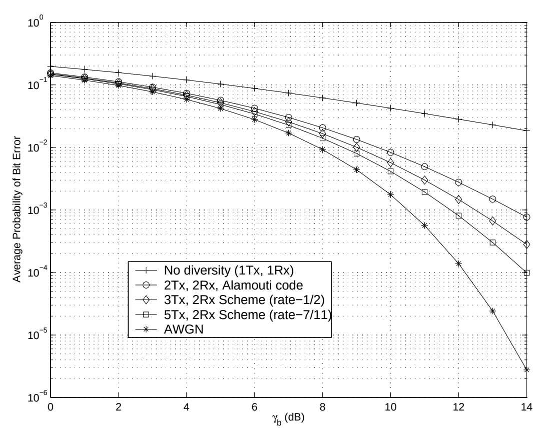

Fig. 6. BER performance of 16-QAM with different STBCs – i) 2 Tx antennas using rate-1 STBC (Alamouti code), ii) 3 Tx antennas using rate-1/2 STBC, iii) 5 Tx antennas using rate-7/11 STBC. Number of receive antennas, $L_2 = 2$.