This full text paper was peer reviewed at the direction of IEEE Communications Society subject matter experts for publication in the WCNC 2010 proceedings.

Robust Relay Precoder Design for MIMO-Relay Networks

P. Ubaidulla and A. Chockalingam
Department of ECE, Indian Institute of Science, Bangalore 560012, INDIA

***Abstract*—In this paper, we consider a *robust* design of MIMO-relay precoder and receive filter for the destination nodes in a non-regenerative multiple-input multiple-output (MIMO) relay network. The network consists of multiple source-destination node pairs assisted by a single MIMO-relay node. The source and destination nodes are single antenna nodes, whereas the MIMO-relay node has multiple transmit and multiple receive antennas. The channel state information (CSI) available at the MIMO-relay node for precoding purpose is assumed to be *imperfect*. We assume that the norms of errors in CSI are upper-bounded, and the MIMO-relay node knows these bounds. We consider the robust design of the MIMO-relay precoder and receive filter based on the minimization of the total MIMO-relay transmit power with constraints on the mean square error (MSE) at the destination nodes. We show that this design problem can be solved by solving an alternating sequence of minimization and worst-case analysis problems. The minimization problem is formulated as a convex optimization problem that can be solved efficiently using interior-point methods. The worst-case analysis problem can be solved analytically using an approximation for the MSEs at the destination nodes. We demonstrate the robust performance of the proposed design through simulations.**

I. Introduction

Relay-assisted wireless communication systems have been studied widely [1]–[3]. Improvement in link quality and reliability, and increase in coverage area are some of the benefits resulting from the use of relaying in wireless systems. Various relaying schemes have been proposed in the literature. Among them, regenerative and non-regenerative schemes have been studied widely [1], [2]. In regenerative relaying, the relay nodes decode the received signal, re-encode and then transmit it to the destination nodes. Whereas, in non-regenerative relaying, the relay nodes scale the received signal and transmit it to the destination nodes. The non-regenerative relaying is of practical interest as the signal processing involved is less complex and is easier to implement. A widely studied wireless relay-assisted system consists of a single source-destination pair and multiple relay nodes. Relay precoder designs for such a system have been reported in [4]–[6]. All the studies in [1]–[6] consider relay nodes having a single antenna. Use of relay nodes with multiple-input multiple-output (MIMO) capability has the potential for further enhancing the spectral efficiency and the link reliability. Recently, studies on the application of MIMO techniques in relay networks have been reported in [7]–[9]. In [7], a relay precoder design that maximizes the capacity between the source and destination nodes in a non-regenerative relay system with single MIMO source-destination pair, and a MIMO-relay is considered. In [8], a MIMO point-to-multi-point system with a MIMO-relay is studied. In [9], a source and relay precoder design based on the minimization of mean-square error (MSE) for a three-node multicarrier MIMO-relay network is reported.

This work in part was supported by Indo-French Centre for the Promotion of Advanced Research (IFCPAR) Project No. 4000-IT-1.

Most of the works mentioned above assume the availability of perfect channel state information (CSI) at the relay node. In practice, the CSI available at the relay node is usually imperfect due to different factors such as estimation error, quantization, feedback delay, etc. Moreover, the performance of the precoders designed based on the assumption of perfect CSI degrades in the presence of errors in the CSI . Hence, it is of interest to develop relay precoder designs that are robust to errors in CSI. Relay precoder designs for single-antenna nodes with partial/imperfect CSI have been studied in [10]–[12]. The robust precoder designs proposed in [10] are based on the second-order statistics of the CSI. These are stochastically robust designs, and do not guarantee quality-of-service (QoS) for all realizations of the CSI. Whereas, in [11], the robust designs consider only large-scale fading. A robust relay precoder for minimizing total relay transmit power under an SINR constraint at the destination node is considered in [12]. Very recently, a study on robust MIMO-relay precoder design with SINR constraints for a multipoint-to-multipoint relay network has been reported in [13].

In this paper, we propose a robust precoder/receive filter design for a *MIMO-relay* system using non-regenerative relaying with *imperfect* CSI at the MIMO-relay. More specifically, we consider a system with multiple source-destination pairs assisted by a single MIMO-relay. The source and destination nodes are single antenna nodes, whereas the relay node has multiple receive and transmit antennas. The relay precoder design is based on the minimization of the total relay transmit power under MSE constraints at the destination nodes. Our robust design takes into account the imperfections in the instantaneous CSI, and ensures that the QoS constraints are satisfied for all realizations of CSI errors belonging to a given uncertainty set. This is ensured by adopting a worst-case (minimax) design. Although the system model considered here and in the robust design in [13] are the same, we consider an MSE-constrained design whereas [13] considers an SINR-constrained design. Our formulation leads to a optimization with second order cone (SOC) constraints which are computationally less complex compared to the semi-definite program (SDP) formulations in [13]. We show that the proposed robust

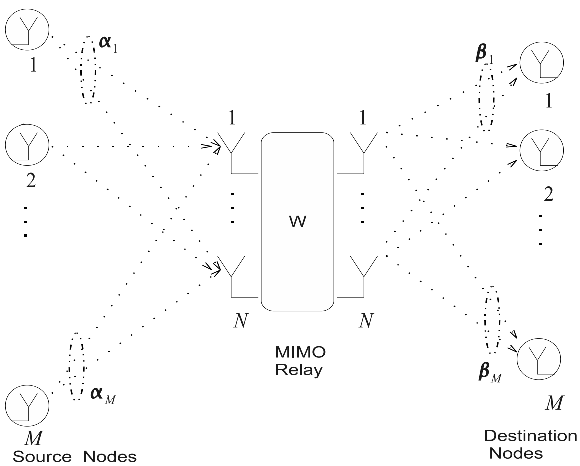

Fig. 1. MIMO-relay system model

precoder/receive filter design problem can be solved by solving a sequence of minimization and worst-case analysis problems. The minimization problem is formulated as a convex optimization problem that can be solved efficiently using interior-point methods [14]. The worst-case analysis problem can be solved analytically. Further, for the special case of relay precoder and receive filter design with perfectly known CSI, our formulation leads to a convex optimization problem with SOC constraints.

The rest of the paper is organized as follows. The system model is presented in Section II. The proposed robust precoder/receive filter design is presented in Section III. Section IV presents simulation results and comparisons. Conclusions are presented in Section V.

II. System Model

We consider a wireless relay system with M source-destination pairs, and a MIMO-relay node having N transmit and receive antennas, as shown in Fig. 1. The source and destination nodes are equipped with single antenna. We assume that there is no direct link between the transmit and the destination nodes. We consider a non-regenerative relaying scheme with half-duplex relay mode. In this mode, during the first time slot, the ith source node transmits the symbol[1] $x_i \in \mathbb{C}$ with $\mathbb{E}\{|x_i|^2\} = 1$, where $\mathbb{E}\{\cdot\}$ denotes the expectation operator. Let $\alpha_{i,j} \in \mathbb{C}$ represent the channel gain from the ith source node to the jth receive antenna of the relay node. Define $\boldsymbol{\alpha}_i = [\alpha_{i,1}\ \alpha_{i,2} \cdots \alpha_{i,N}]^T$, $1 \le i \le M$, and $\mathbf{A} = [\boldsymbol{\alpha}_1\ \boldsymbol{\alpha}_2 \cdots \boldsymbol{\alpha}_M]$. The signal vector received at the relay node is given by

$$\mathbf{y} \;=\; \mathbf{A}\mathbf{x} + \boldsymbol{\mu}, \tag{1}$$

[1] Vectors are denoted by boldface lowercase letters, and matrices are denoted by boldface uppercase letters. $[\cdot]^T$ denotes transpose operation, $[\cdot]^H$ denotes Hermitian operation, and $[\cdot]^\star$ denotes complex conjugation. $\otimes$ denotes the Kronecker product. $\Re(\cdot)$ and $\Im(\cdot)$ denote real part and imaginary part of the argument respectively. vec$(\cdot)$ stacks all the columns of the argument into a single column vector. diag$(\cdot)$ generates a diagonal matrix with the argument on the diagonal. $\mathbf{I}_N$ denotes a $N \times N$ identity matrix.

where $\mathbf{y} = [y_1\ y_2 \cdots y_N]^T$, $\mathbf{x} = [x_1\ x_2 \cdots x_M]^T$, $\boldsymbol{\mu} = \mu_1\ \mu_2 \cdots \mu_N]^T$, $\mu_k \in \mathbb{C}$ is the noise at the kth receive antenna of the relay node. The elements of $\boldsymbol{\mu}$ are independent and complex Gaussian random variables with zero mean and $\mathbb{E}\{|\mu_k|^2\} = \sigma^2_{\mu_k}$, $1 \le k \le M$. During the second time slot, the relay node transmits the received signal vector after multiplying it by a precoding matrix $\mathbf{W} \in \mathbb{C}^{N\times N}$. Let $\beta_{i,j} \in \mathbb{C}$ represent the channel gain from the the jth transmit antenna of the relay node to ith destination node. Define $\boldsymbol{\beta}_i = [\beta_{i,1}\ \beta_{i,2} \cdots \beta_{i,N}]$, $1 \le i \le M$, and $\mathbf{B} = \left[\boldsymbol{\beta}_1^T\ \boldsymbol{\beta}_2^T \cdots \boldsymbol{\beta}_M^T\right]^T$. The signals received by the destination nodes, $z_i,\ 1 \le i \le M$, can be represented vectorially as

$$\mathbf{z} \;=\; \mathbf{B}\mathbf{W}\mathbf{y} + \boldsymbol{\nu}, \tag{2}$$

where $\mathbf{z} = [z_1\ z_2 \cdots z_M]^T$, $\boldsymbol{\nu} = [\nu_1\ \nu_2 \cdots \nu_M]^T$, and $\nu_i \in \mathbb{C}$ is the noise at the ith destination node. The elements of $\boldsymbol{\nu}$ are independent and complex Gaussian with zero mean and $\mathbb{E}\{|\nu_k|^2\} = \sigma^2_{\nu_k}$, $1 \le k \le M$. Let $\boldsymbol{\Gamma} = \text{diag}(\gamma_1, \gamma_2, \cdots, \gamma_M)$, where γ_i is the receive scaling factor at the ith destination node. The estimate of the transmitted signal vector can be expressed as

$$\begin{aligned}\widehat{\mathbf{x}} &= \boldsymbol{\Gamma}\mathbf{z} \\ &= \boldsymbol{\Gamma}\mathbf{B}\mathbf{W}\mathbf{y} + \boldsymbol{\nu} \\ &= \boldsymbol{\Gamma}\mathbf{B}\mathbf{W}\mathbf{A}\mathbf{x} + \boldsymbol{\Gamma}\mathbf{B}\mathbf{W}\boldsymbol{\mu} + \boldsymbol{\Gamma}\boldsymbol{\nu},\end{aligned} \tag{3}$$

where $\widehat{\mathbf{x}} = [\widehat{x}_1,\ \widehat{x}_2 \cdots \widehat{x}_M]^T$, and $\widehat{x}_i$ is the signal estimate at the ith destination node. We consider CSI uncertainties that can be modeled as

$$\boldsymbol{\alpha}_i \;=\; \widehat{\boldsymbol{\alpha}}_i + \boldsymbol{\phi}_i, \quad 1 \le i \le M, \tag{4}$$

$$\boldsymbol{\beta}_i \;=\; \widehat{\boldsymbol{\beta}}_i + \boldsymbol{\pi}_i, \quad 1 \le i \le M, \tag{5}$$

where $\boldsymbol{\alpha}_i,\ \boldsymbol{\beta}_i,\ 1 \le i \le M$ are the true CSI, $\widehat{\boldsymbol{\alpha}}_i,\ \widehat{\boldsymbol{\beta}}_i,\ 1 \le i \le M$, are the imperfect CSI available at the relay node, and $\boldsymbol{\phi}_i,\ \boldsymbol{\pi}_i,\ 1 \le i \le M$ represent the additive errors in the CSI. Further, we assume that $\|\boldsymbol{\phi}_i\| \le \delta_{\alpha_i}$, and $\|\boldsymbol{\pi}_i\| \le \delta_{\beta_i}$, $1 \le i \le M$. Equivalently, $\boldsymbol{\alpha}_i$ belongs to the uncertainty set $\mathcal{R}_{\alpha_i}$, and $\boldsymbol{\beta}_i$ belongs to the uncertainty set $\mathcal{R}_{\beta_i}$, where

$$\mathcal{R}_{\alpha_i} = \{\zeta \big| \zeta = \widehat{\boldsymbol{\alpha}}_i + \boldsymbol{\phi}_i, \|\boldsymbol{\phi}_i\| \le \delta_{\alpha_i}\}, \tag{6}$$

and

$$\mathcal{R}_{\beta_i} = \{\zeta \big| \zeta = \widehat{\boldsymbol{\beta}}_i + \boldsymbol{\pi}_i, \|\boldsymbol{\pi}_i\| \le \delta_{\beta_i}\}. \tag{7}$$

III. Proposed Robust Relay Precoder and Receive Filter Design

In this section, we describe the proposed robust design of the relay precoding matrix $\mathbf{W}$ and the receive filter $\boldsymbol{\Gamma}$, that minimize the total relay transmit power while meeting MSE constraints at all the destination nodes in the presence of imperfect CSI at the relay node. The CSI required by the relay node for the purpose of precoder and receive filter design consists of $\boldsymbol{\alpha}_i,\ \boldsymbol{\beta}_i,\ 1 \le i \le M$. When the CSI available at the relay is imperfect with errors of bounded norm, we can make the relay precoder and the receive filter robust by ensuring that the precoder and the receive filter satisfy the

MSE constraints for all possible CSI errors satisfying the norm bound. Mathematically, this problem can be represented as

$$\begin{aligned} \min_{\mathbf{W},\mathbf{\Gamma}} \quad & P \\ \text{subject to} \quad & \epsilon_i \leq \eta_i, \\ & \boldsymbol{\alpha}_i \in \mathcal{R}_{\alpha_i}, \boldsymbol{\beta}_i \in \mathcal{R}_{\beta_i} \quad 1 \leq i \leq M, \end{aligned} \tag{8}$$

where P is the total relay transmit power, ϵ_i is the actual MSE at the ith destination node, and η_i is the maximum allowed MSE (MSE target) at the ith destination node. The problem above is equivalent to

$$\begin{aligned} \min_{\mathbf{W},\mathbf{\Gamma}} \quad & \max_{\boldsymbol{\alpha}_i \in \mathcal{R}_{\alpha_i}, \forall i} P \\ \text{subject to} \quad & \max_{\boldsymbol{\alpha}_i \in \mathcal{R}_{\alpha_i}, \boldsymbol{\beta}_i \in \mathcal{R}_{\beta_i}, \forall i} \epsilon_i \leq \eta_i. \end{aligned} \tag{9}$$

Though an exact solution to this problem is difficult, we propose a tractable solution based on the cutting-set method [15]. The proposed solution involves solving an alternating sequence of two sub-problems, viz., i) precoder/receive filter design with fixed channel vectors, and ii) computation of worst-case channel vectors for a fixed precoder and receive filter. We note that the precoder/receive filter design with perfect CSI is a special case of the problem described above, which involves only solving once the first sub-problem using the perfectly known channel vectors. The following subsections describe the solutions of the first and second sub-problems and the iterative algorithm to solve the overall robust design problem.

A. Precoder/Receive Filter Design for Given Channel Vectors

The first sub-problem in the proposed robust design is the computation of the relay precoder $\mathbf{W}$ and the receive filter $\mathbf{\Gamma}$ for a given set of channel vectors $\boldsymbol{\alpha}_i, \boldsymbol{\beta}_i,\ 1 \leq i \leq M$. This computation involves the minimization of the total relay transmit power under MSE constraints at the destination nodes. Mathematically, this problem can be written as

$$\begin{aligned} \min_{\mathbf{W},\mathbf{\Gamma}} \quad & P \\ \text{subject to} \quad & \epsilon_i \leq \eta_i, \quad 1 \leq i \leq M. \end{aligned} \tag{10}$$

The total relay transmit power can be expressed as

$$\begin{aligned} P &= \mathbb{E}\{\|\mathbf{W}\mathbf{y}\|^2\} \\ &= \mathbb{E}\{\widetilde{\mathbf{w}}^H(\mathbf{I}_{N^2} \otimes \mathbf{y}^T)^H(\mathbf{I}_{N^2} \otimes \mathbf{y}^T)\widetilde{\mathbf{w}}\} \\ &= \widetilde{\mathbf{w}}^H(\mathbf{I}_{N^2} \otimes (\mathbb{E}\{(\mathbf{y}^T)^H\mathbf{y}^T\})\widetilde{\mathbf{w}} \\ &= \widetilde{\mathbf{w}}^H\left(\mathbf{I}_{N^2} \otimes (\mathbf{A}\mathbf{A}^H + \mathbf{R}_\mu)^T\right)\widetilde{\mathbf{w}}, \end{aligned} \tag{11}$$

where $\widetilde{\mathbf{w}} = \text{vec}(\mathbf{W}^T)$, and $\mathbf{R}_\mu = \mathbb{E}\{\boldsymbol{\mu}\boldsymbol{\mu}^H\}$. The estimate of the transmitted signal x_i at the ith destination node can be written as

$$\begin{aligned} \widehat{x}_i &= \gamma_i\boldsymbol{\beta}_i\mathbf{W}\boldsymbol{\alpha}_i x_i + \gamma_i\sum_{k=1,k\neq i}^{M}\boldsymbol{\beta}_i\mathbf{W}\boldsymbol{\alpha}_k x_k \\ &\quad + \gamma_i\boldsymbol{\beta}_i\mathbf{W}\boldsymbol{\mu} + \gamma_i\nu_i \end{aligned} \tag{12}$$

$$\begin{aligned} &= \gamma_i\boldsymbol{\beta}_i(\mathbf{I}_N \otimes \boldsymbol{\alpha}_i^T)\widetilde{\mathbf{w}}x_i + \sum_{k=1,k\neq i}^{M}\gamma_i\boldsymbol{\beta}_i(\mathbf{I}_N \otimes \boldsymbol{\alpha}_k^T)\widetilde{\mathbf{w}}x_k \\ &\quad + \sum_{k=1}^{N}\gamma_i\boldsymbol{\beta}_i(\mathbf{I}_N \otimes \mathbf{e}_k^T)\widetilde{\mathbf{w}}\mu_k + \gamma_i\nu_i, \end{aligned} \tag{13}$$

where $\mathbf{e}_k$ is the kth column of $\mathbf{I}_N$. From the expression given above, the MSE at the ith destination node can be written as

$$\begin{aligned} \epsilon_i &= \mathbb{E}\{|\widehat{x}_i - x|^2\} \\ &= |\gamma_i\boldsymbol{\beta}_i(\mathbf{I}_N \otimes \boldsymbol{\alpha}_i^T)\widetilde{\mathbf{w}} - 1|^2 + |\gamma_i|^2\sum_{k=1,k\neq i}^{M}|\boldsymbol{\beta}_i(\mathbf{I}_N \otimes \boldsymbol{\alpha}_k^T)\widetilde{\mathbf{w}}|^2 \\ &\quad + |\gamma_i|^2\sum_{k=1}^{N}|\boldsymbol{\beta}_i(\mathbf{I}_N \otimes \mathbf{e}_k^T)\widetilde{\mathbf{w}}|^2\sigma_{\mu_k}^2 + |\gamma_i|^2\sigma_{\nu_i}^2. \end{aligned} \tag{14}$$

The MSE constraints at the destination nodes can be written as

$$\begin{aligned} &|\gamma_i\boldsymbol{\beta}_i(\mathbf{I}_N \otimes \boldsymbol{\alpha}_i^T)\widetilde{\mathbf{w}} - 1|^2 + |\gamma_i|^2\sum_{k=1,k\neq i}^{M}|\boldsymbol{\beta}_i(\mathbf{I}_N \otimes \boldsymbol{\alpha}_k^T)\widetilde{\mathbf{w}}|^2 \\ &\quad + |\gamma_i|^2\sum_{k=1}^{N}|\boldsymbol{\beta}_i(\mathbf{I}_N \otimes \mathbf{e}_k^T)\widetilde{\mathbf{w}}|^2\sigma_{\mu_k}^2 + |\gamma_i|^2\sigma_{\nu_i}^2 \leq \eta_i, \quad \forall i \\ &\Rightarrow |\boldsymbol{\beta}_i(\mathbf{I}_N \otimes \boldsymbol{\alpha}_i^T)\widetilde{\mathbf{w}} - \psi_i|^2 + \sum_{k=1,k\neq i}^{M}|\boldsymbol{\beta}_i(\mathbf{I}_N \otimes \boldsymbol{\alpha}_k^T)\widetilde{\mathbf{w}}|^2 \\ &\quad + \sum_{k=1}^{N}|\boldsymbol{\beta}_i(\mathbf{I}_N \otimes \mathbf{e}_k^T)\widetilde{\mathbf{w}}|^2\sigma_{\mu_k}^2 + \sigma_{\nu_i}^2 \leq \eta_i|\psi_i|^2, \quad \forall i, \end{aligned} \tag{15}$$

where $\psi_i = \frac{1}{\gamma_i}$. The precoder/receive filter design problem as obtained by substituting (15) in (10) is not a convex optimization problem. But we can transform this problem into a convex optimization problem as follows. Let

$$\mathbf{p}^i = \left[p_1^i \cdots p_{i-1}^i \ \ (p_i^i - \psi_i) \ \ p_{i+1}^i \cdots p_M^i\right], \tag{16}$$

where $p_k^i = \boldsymbol{\beta}_i(\mathbf{I}_N \otimes \boldsymbol{\alpha}_k^T)\widetilde{\mathbf{w}}$. Let $\mathbf{q}^i = \left[q_1^i \ \ q_2^i \cdots q_M^i\right]$, where $q_k^i = \boldsymbol{\beta}_i(\mathbf{I}_N \otimes \mathbf{e}_k^T)\widetilde{\mathbf{w}}\sigma_{\mu_i}$. The constraints in (15) can be reformulated as

$$\left\|\left[\mathbf{p}^i \quad \mathbf{q}^i \quad \sigma_{\nu_i}\right]\right\|^2 \leq \eta_i|\psi_i|^2, \quad \forall i. \tag{17}$$

The precoder design that minimizes the total relay transmit power under MSE constraints at the destination nodes can be expressed as

$$\begin{aligned} \min_{\widetilde{\mathbf{w}},\{\psi_i\}} \quad & \widetilde{\mathbf{w}}^H\left(\mathbf{I}_{N^2} \otimes (\mathbf{A}\mathbf{A}^H + \mathbf{R}_\mu)^T\right)\widetilde{\mathbf{w}} \\ & \left\|\left[\mathbf{p}^i \quad \mathbf{q}^i \quad \sigma_{\nu_i}\right]\right\| \leq \sqrt{\eta_i}\psi_i, \quad 1 \leq i \leq M, \end{aligned} \tag{18}$$

where we have assumed $\psi_i,\ 1 \leq i \leq M$, are non-negative real numbers. This assumption is valid, as any phase factor of ψ_i can be absorbed into $\mathbf{W}$ without affecting the MSE. The objective function in the problem in (18) is a convex quadratic function, and the constraints are SOC constraints. Hence, the precoder/receive filter design has been formulated as a convex optimization problem that can be solved efficiently using interior-point methods [14], [16].

Further, as we described earlier, if the CSI is known perfectly at the relay node, then the relay precoder and the receive filter can be designed by solving (18).

B. Computation of Worst-Case Channels

The second sub-problem in the proposed robust design involves the computation of the worst-case channels, i.e., those channel vectors which belong to the uncertainty region and maximize the total relay transmit power and the MSEs at the destination nodes.

First, we consider the computation of the worst case channels that maximize the MSEs for a given precoder and receive filter. If the worst-case analysis problem can be solved exactly, then the exact robust optimal solution to the problem above is possible. But, in the present problem, it turns out that an exact solution to the worst-case analysis, i.e., the computation of $\boldsymbol{\alpha}_i,\ \boldsymbol{\beta}_i,\ 1 \le i \le M$ that maximize P and $\epsilon_i,\ 1 \le i \le M$, is not possible due to the non-convexity of this problem. Hence, we propose an approximate solution to the worst-case analysis problem. We can express the MSEs, $\epsilon_i,\ 1 \le i \le M$, as

$$\begin{aligned}\epsilon_i &= |\gamma_i(\widehat{\boldsymbol{\beta}}_i + \boldsymbol{\pi}_i)\mathbf{W}(\widehat{\boldsymbol{\alpha}}_i + \boldsymbol{\phi}_i) - 1|^2 \\ &+ |\gamma_i|^2 \sum_{k=1,k\neq i}^{M} |(\widehat{\boldsymbol{\beta}}_i + \boldsymbol{\pi}_i)\mathbf{W}(\widehat{\boldsymbol{\alpha}}_k + \boldsymbol{\phi}_k)|^2 \\ &+ |\gamma_i|^2(\widehat{\boldsymbol{\beta}}_i + \boldsymbol{\pi}_i)\mathbf{W}\mathbf{R}_\mu\mathbf{W}^H(\widehat{\boldsymbol{\beta}}_i + \boldsymbol{\pi}_i)^H + |\gamma_i|^2\sigma_\nu^2. \end{aligned} \tag{19}$$

Let $\overline{\boldsymbol{\phi}}_k^i,\ 1 \le k \le M$, and $\overline{\boldsymbol{\pi}}^i$ be the optimal solution of the following problem:

$$\begin{aligned} \max_{\{\boldsymbol{\phi}_k\}_{k=1}^M, \boldsymbol{\pi}_i} \quad & \epsilon_i \\ \text{subject to} \quad & \|\boldsymbol{\phi}_k\|^2 \le \delta_{\alpha_k}, \quad 1 \le k \le M, \\ & \|\boldsymbol{\pi}_i\|^2 \le \delta_{\beta_i}. \end{aligned} \tag{20}$$

Then $\overline{\boldsymbol{\alpha}}_k^i = \widehat{\boldsymbol{\alpha}}_i + \overline{\boldsymbol{\phi}}_k^i,\ 1 \le k \le M$, and $\overline{\boldsymbol{\beta}}^i = \widehat{\boldsymbol{\beta}}_i + \overline{\boldsymbol{\pi}}^i$ correspond to the worst-case channels, which give rise to the worst-case MSE at the ith destination node, given the imperfect CSI $(\{\widehat{\boldsymbol{\alpha}}_k\}_{k=1}^M, \widehat{\boldsymbol{\beta}}_i)$ at the relay, and the CSI error norm bounds $\{\delta_{\alpha_k}\}_{k=1}^M$ and δ_{β_i}. Note that the MSE at the ith destination node is a function of the source-to-relay channel vectors of all the source nodes, i.e., $\boldsymbol{\alpha}_k,\ 1 \le k \le M$. From (19), we can see that solving (20) exactly is quite difficult. To significantly simply the problem, we approximate ϵ_i in (20) by neglecting those terms in (19) which involve second and higher orders of $\{\boldsymbol{\phi}_k\}_{k=1}^M$ and $\boldsymbol{\pi}_i$. We can approximate the MSE at the ith source node as

$$\begin{aligned}\epsilon_i &\approx 2\Re\{(\theta_{ii}^\star - 1)\boldsymbol{\beta}_i\mathbf{W}\boldsymbol{\phi}_i\} + 2\Re\{(\theta_{ii}^\star - 1)\boldsymbol{\pi}_i\mathbf{W}\boldsymbol{\alpha}_i\} \\ &+ 2\sum_{k=1,k\neq i}^{M} \Re\{\theta_{ik}^\star\boldsymbol{\beta}_i\mathbf{W}\boldsymbol{\phi}_k\} \\ &+ 2\Re\Big\{\boldsymbol{\pi}_i\Big(\sum_{k=1,k\neq i}^{M}\theta_{ik}^\star\mathbf{W}\boldsymbol{\alpha}_k + \mathbf{W}\mathbf{R}_\mu\mathbf{W}\boldsymbol{\beta}^H\Big)\Big\}, \end{aligned} \tag{21}$$

where $\theta_{ij} = \boldsymbol{\beta}_i\mathbf{W}\boldsymbol{\alpha}_j$. Considering the terms involving $\boldsymbol{\phi}_k$ and $\boldsymbol{\pi}_k$ in (21), and applying Cauchy-Schwarz inequality, we can determine the worst-case CSI error vectors as follows:

$$\overline{\boldsymbol{\phi}}_i^i = \frac{\delta_{\alpha_i}}{\|(\theta_{ii}-1)\mathbf{W}^H\boldsymbol{\beta}_i^H\|}(\theta_{ii}-1)\mathbf{W}^H\boldsymbol{\beta}_i^H, \quad \forall i \tag{22}$$

$$\overline{\boldsymbol{\phi}}_k^i = \frac{\delta_{\alpha_k}}{\|\theta_{ik}\mathbf{W}^H\boldsymbol{\beta}_i^H\|}\theta_{ik}\mathbf{W}^H\boldsymbol{\beta}_i^H, \quad \forall i, \forall k \neq i \tag{23}$$

$$\overline{\boldsymbol{\pi}}^i = \frac{\delta_{\beta_i}}{\|\mathbf{f}_i\|}\mathbf{f}_i, \quad \forall i, \tag{24}$$

where

$$\mathbf{f}_i = (\theta_{ii}^\star - 1)\mathbf{W}\boldsymbol{\alpha}_i + \Big(\sum_{k=1,k\neq i}^{M}\theta_{ik}^\star\mathbf{W}\boldsymbol{\alpha}_k + \mathbf{W}\mathbf{R}_\mu\mathbf{W}\boldsymbol{\beta}^H\Big).$$

Next, we consider the computation of the worst case channels that maximize the total relay transmit power. The total relay transmit power can be expressed as

$$P = \sum_{k=1}^{M}(\widehat{\boldsymbol{\alpha}}_k + \boldsymbol{\phi}_k)^H\mathbf{W}^H\mathbf{W}(\widehat{\boldsymbol{\alpha}} + \boldsymbol{\phi}_k)_k + \text{trace}(\mathbf{W}\mathbf{R}_\mu\mathbf{W}^H). \tag{25}$$

As the last term in (25) does not depend on the CSI error, the worst-case channel vectors maximizing the total relay transmit power is obtained by solving

$$\max_{\{\boldsymbol{\phi}_i : \|\boldsymbol{\phi}_i\| \le \delta_{\alpha_i}\}_{i=1}^M} \sum_{k=1}^{M}(\widehat{\boldsymbol{\alpha}}_k + \boldsymbol{\phi}_k)^H\mathbf{W}^H\mathbf{W}(\widehat{\boldsymbol{\alpha}}_k + \boldsymbol{\phi}_k). \tag{26}$$

The problem in (27) is equivalent to the following individual problems for $1 \le k \le M$:

$$\max_{\boldsymbol{\phi}_k : \|\boldsymbol{\phi}_k\| \le \delta_{\alpha_k}} (\widehat{\boldsymbol{\alpha}}_k + \boldsymbol{\phi}_k)^H\mathbf{W}^H\mathbf{W}(\widehat{\boldsymbol{\alpha}}_k + \boldsymbol{\phi}_k). \tag{27}$$

The constraint in (27) is always active. So, the optimality conditions [14] can be written as

$$\nabla\mathcal{L}_k = 0 \tag{28a}$$

$$\boldsymbol{\phi}_k^H\boldsymbol{\phi}_k - \delta_{\alpha_k}^2 = 0, \tag{28b}$$

$$\rho > 0, \tag{28c}$$

where ∇ is the gradient operator, ρ is the Lagrange multiplier, and $\mathcal{L}_k$ is the Lagrangian associated with (27)

$$\mathcal{L}_k = (\widehat{\boldsymbol{\alpha}}_k + \mathbf{a}_k)^H\mathbf{W}^H\mathbf{W}(\widehat{\boldsymbol{\alpha}}_k + \boldsymbol{\phi}_k) + \rho(\boldsymbol{\phi}_k^H\boldsymbol{\phi}_k - \delta_{\alpha_k}^2). \tag{29}$$

From (28a) and (29), we get

$$\boldsymbol{\phi}_k = -(\mathbf{W}^H\mathbf{W} + \rho\mathbf{I}_N)^{-1}\mathbf{W}^H\mathbf{W}\boldsymbol{\alpha}_k. \tag{30}$$

Let $\mathbf{W}^H\mathbf{W} = \mathbf{U}\boldsymbol{\Lambda}\mathbf{V}^H$ be the singular value decomposition of $\mathbf{W}\mathbf{W}^H$, where $\mathbf{U}$ and $\mathbf{V}$ are unitary matrices, and $\boldsymbol{\Lambda}$ is a diagonal matrix containing the singular values $\lambda_i\ 1 \le i \le N$. Then, based on (30) and (28b), we can show that

$$\begin{aligned}\boldsymbol{\phi}_k^H\boldsymbol{\phi}_k &= \mathbf{g}_k^H\boldsymbol{\Lambda}(\boldsymbol{\Lambda} + \rho\mathbf{I}_N)^{-2}\boldsymbol{\Lambda}\mathbf{g}_k = \delta_{\alpha_k}^2 \\ \Rightarrow & \sum_{i=1}^{N}\frac{|g_{ki}|^2\lambda_i^2}{(\lambda_i + \rho)^2} - \delta_{\alpha_k}^2 = 0, \end{aligned} \tag{31}$$

where $\mathbf{g}_k = \mathbf{V}^H\boldsymbol{\alpha}_k$, and g_{ki} is the ith element of $\mathbf{g}_k$. The Lagrange multiplier ρ can be determined by solving (31).

This full text paper was peer reviewed at the direction of IEEE Communications Society subject matter experts for publication in the WCNC 2010 proceedings.

C. Iterative Algorithm for the Robust Design

The proposed robust precoder design involves iterating over a sequence of minimization and worst-case analysis steps described in the previous two subsections till a stopping criterion is met. We start with the set $\mathcal{S}$ of channel vectors, which initially contains only the imperfect CSI $\widehat{\boldsymbol{\alpha}}_i, \widehat{\boldsymbol{\beta}}_i$, $1 \leq i \leq M$ available at the relay node. The first step involves the solution of the optimization problem in (10) for all elements of the set $\mathcal{S}$. This step computes $\mathbf{W}$ and $\mathbf{\Gamma}$ for a given $\mathcal{S}$. The second step is the worst-case analysis as described in the previous subsection. If the resulting worst-case channels violate the MSE constraints at the destination nodes, these channel vectors are added to $\mathcal{S}$. So, during the worst-case analysis step in each iteration, the set $\mathcal{S}$ of the worst-case channels may be expanded depending on the MSE constraint violations. During the minimization step in each iteration, the precoder and the receive filter are optimized to meet MSE constraints for increasing number of worst-case channels resulting in increased robustness. These two steps are iterated till maximum constraint violation $\max_i(\bar{\epsilon}_i - \eta_i)$, where $\bar{\epsilon}_i$ is the worst-case MSE at the ith destination node, becomes less than a certain threshold. When the worst-case analysis problem has an exact solution, these iterations lead to the robust optimal solution [15]. For the problem considered here, the worst-case analysis is approximate, and the iteration is not guaranteed to lead to the robust optimal solution. However, our simulations show that the proposed design is robust to the CSI errors.

IV. Simulation Results

In this section, we illustrate the performance of the proposed robust design of the MIMO-relay precoder and receive filter, evaluated through simulations. We compare the performance of the proposed robust design with that of the non-robust design. The non-robust design of the precoder and receive filter are obtained by solving (18) using $\widehat{\boldsymbol{\alpha}}_k$, $\widehat{\boldsymbol{\beta}}_k$, $1 \leq k \leq M$. The channel fading is modeled as Rayleigh, with the channel vectors $\boldsymbol{\alpha}_k$, $\boldsymbol{\beta}_k$, $1 \leq k \leq M$, comprising of independent and identically distributed (i.i.d) samples of a complex Gaussian process with zero mean and unit variance. The noise at each node is assumed to be zero-mean complex Gaussian random variable. In all the simulations, we have assumed $\sigma^2_{\mu_i} = \sigma^2_\mu$, $\sigma^2_{\nu_i} = \sigma^2_\nu$, $\eta_i = \eta$, $\delta_{\alpha_i} = \delta_{\beta_i} = \delta$, $1 \leq i \leq M$.

For a comparison of the performance of the proposed robust design with that of the non-robust design, first we consider the cumulative distribution of achieved MSEs in the presence of CSI errors. For this purpose, we consider a system with $M = 4$ transmit nodes communicating with $M = 4$ destination nodes, and a MIMO-relay with $N = 4$ Tx/Rx antennas. The target MSE is set as 0.2 for all destination nodes. To estimate the cumulative distribution, we use $\boldsymbol{\phi}_k$, $\boldsymbol{\pi}_k$, $1 \leq k \leq M$, satisfying the norm constraints. The results are shown in Fig. 2. The results show that the non-robust design fails to meet the MSE target with higher probabilities for larger values of the CSI error bounds. The robust design results in MSE less than the target MSE even in the presence of CSI errors. We also evaluate the performance of the proposed design in the presence of CSI errors that are Gaussian distributed. For this study, we consider a system with $M = N = 4$, and the target

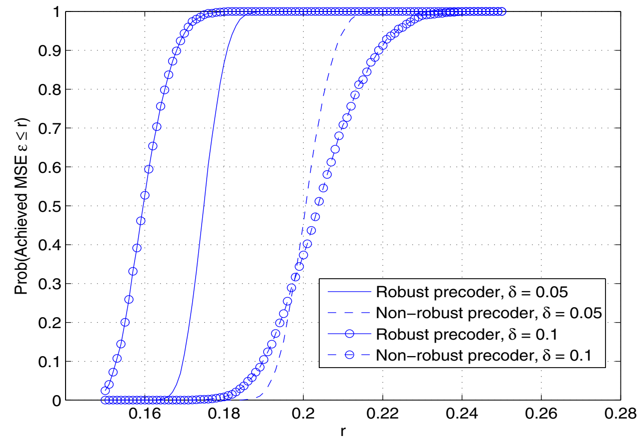

Fig. 2. Cumulative distribution of achieved MSE $\epsilon_i = \epsilon$, $1 \leq i \leq M$. Target MSE $\eta = 0.2$, $\delta = 0.05, 0.1$, $M = N = 4$, $\sigma^2_\mu = \sigma^2_\nu = 0.1$.

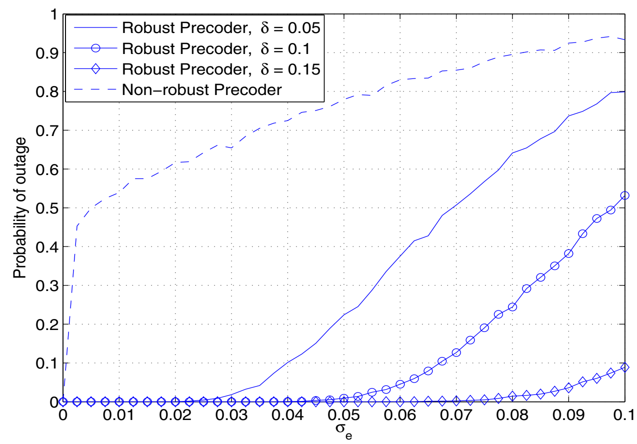

Fig. 3. Probability of outage $Pr\{\epsilon_i > \eta_i\}$ versus channel estimation error variance. Target MSE $\eta = 0.1$, $\delta = 0.05, 0.1, 0.15$, $M = N = 4$, $\sigma^2_\mu = \sigma^2_\nu = 0.1$.

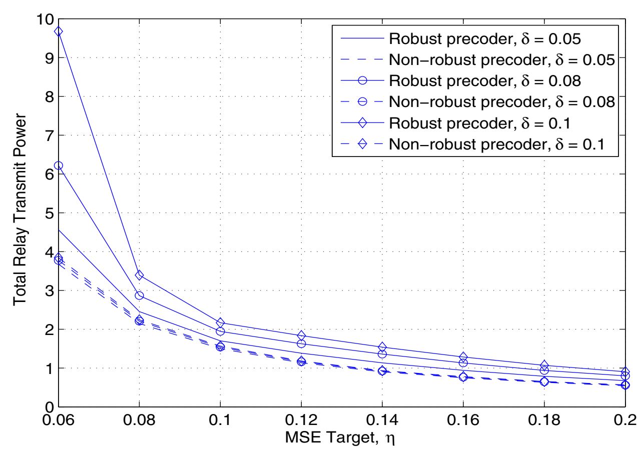

Fig. 4. Total relay transmit power P versus maximum allowed MSEs at the destination relays. $\delta = 0.05, 0.1$, $M = N = 4$, $\sigma^2_\mu = \sigma^2_\nu = 0.1$.

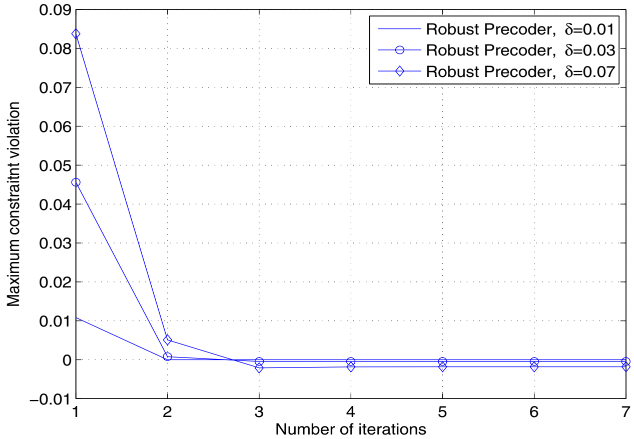

Fig. 5. Convergence of the proposed robust design. $M = N = 6$, $\sigma_\mu^2 = \sigma_\nu^2 = 0.1$, $\eta = 0.3$, $1 \leq i \leq M$, $\delta = 0.01, 0.03, 0.07$.

MSE $\eta = 0.1$ for all destination nodes. The components of the CSI error vectors $\boldsymbol{\phi}_k$, and $\boldsymbol{\pi}_k$ are generated as independent and identically distributed complex Gaussian random variables with zero mean and variance σ_e^2. We compare the performance of the non-robust design and robust design in terms of the probability of outage defined as $Pr\{\epsilon_i > \eta_i\}$ versus CSI estimation error variance. Probability of outage for the non-robust design and the robust design with $\delta = 0.05,\ 0.1,\ 0.15$ are shown in Fig. 3. The probability of outage of the non-robust design significantly increases with increase in the CSI error variance, whereas the robust design result in zero or very low outage depending on design value of δ. For example, when $\sigma_e = 0.06$, the probability of outage is negligibly small for the robust precoder designed with $\delta = 0.15$, and it is 0.045 for $\delta = 0.1$, whereas it is 0.83 for the non-robust design.

Next, we study the performance of the proposed design in terms of total relay transmit power versus MSE target for different values of CSI error bounds. For this purpose, we consider a set-up with system parameters set as $M = N = 4$, $\sigma_\mu^2 = \sigma_\nu^2 = 0.1$. The total relay transmit power resulting from the robust and the non-robust designs in the presence of CSI errors is estimated through simulations. The results are shown in Fig. 4. The results show that the total relay transmit power required to achieve a given MSE target increases with increase in the CSI error norm bound. Comparing with the results in Fig. 2, we observe that this increase in transmit power is the price to pay for ensuring that the MSE constraints are satisfied in the presence of CSI errors. Finally, the convergence behaviour of the proposed design is shown in Fig. 5. We consider a set-up with $M = 6$ source-destination pairs and a MIMO-relay with $N = 6$ Tx/Rx antennas. The target MSE is 0.3 at all the destination nodes. We have shown the convergence results for CSI error norms $\delta = 0.01,\ 0.03,$ and 0.07. From the results, we can observe that the algorithm converges in less than 4 iterations of the minimization and worst case analysis steps.

V. CONCLUSIONS

We presented a robust MIMO-relay precoder/receive filter design with MSE constraints for wireless relay networks in the presence of imperfect CSI at the relay. The proposed robust design was based on total relay transmit power minimization under MSE constraints at the destination nodes. We showed that this design problem can be solved by solving a sequence of minimization and worst-case analysis problems. The minimization problem is formulated as a convex optimization problem that can be solved efficiently. The worst-case analysis problem is solved analytically using an approximation for the MSEs. We presented simulation results which illustrate the improved performance of the proposed robust design compared to the non-robust design in the presence of CSI imperfections at the MIMO-relay.

REFERENCES

[1] A. Sendonaris, E. Erkip, and B. Aazhang, “User cooperation diversity. Part I. system description,” *IEEE Trans. Commun.*, vol. 51, pp. 1927–1938, Nov. 2003.

[2] J. N. Laneman, D. N. C. Tse, and G. W. Wornell, “Cooperative diversity in wireless networks: Efficient protocols and outage behavior,” *IEEE Trans. Inf. Theory*, vol. 50, pp. 3062–3080, Dec. 2004.

[3] Y. W. Hong, W. J. Huang, F. H. Chiu, and C. C. J. Kuo, “Cooperative communications in resource-constrained wireless networks,” *IEEE Signal Process. Mag.*, vol. 24, pp. 47–57, May 2007.

[4] Y. Jing and H. Jafarkhani, “Network beamforming using relays with perfect channel information,” in *Proc. ICASSP’2007*, Apr. 2007, pp. III–473–III–476.

[5] G. Zheng, K. Wong, A. Paulraj, and B. Ottersten, “Collaborative-relay beamforming with perfect CSI: Optimum and distributed implementation,” *IEEE Signal Process. Letters*, vol. 16, pp. 257–260, April 2009.

[6] Z. Yi and I. Kim, “Joint optimization of relay-precoders and decoders with partial channel side information in cooperative networks,” *IEEE J. Sel. Areas Commun.*, vol. 25, pp. 447–458, Feb. 2007.

[7] X. Tang and Y. Hua, “Optimal design of non-regenerative MIMO wireless relays,” *IEEE Trans. Wireless Commun.*, vol. 6, pp. 1398–1407, Apr. 2007.

[8] C. B. Chae, T. Tang, R. W. Heath, and S. Cho, “MIMO relaying with linear processing for multiuser transmission in fixed relay networks,” *IEEE Trans. Signal Process.*, vol. 56, pp. 727–738, Feb. 2008.

[9] Y. Rong, “Non-regenerative multicarrier MIMO relay communications based on minimization of mean-squared error,” in *Proc. ICC’2009*, Jun. 2009.

[10] V. H. Nassab, S. Shahbazpanahi, A. Grami, and Z. Q. Luo, “Distributed beamforming for relay networks based on second-order statistics of the channel state information,” *IEEE Trans. Signal Process.*, vol. 56, pp. 4306–4306, Sep. 2008.

[11] T. Q. S. Quek and H. S. M. Z. Win, “Robust wireless relay networks: Slow power allocation with guaranteed QoS,” *IEEE J. Sel. Topics in Signal Processing*, vol. 1, pp. 700–713, Dec. 2007.

[12] P. Ubaidulla and A. Chockalingam, “Robust distributed beamforming for wireless relay networks,” in *Proc. IEEE PIMRC’2009*, Sep. 2009.

[13] B. K. Chalise and L. Vandendorpe, “MIMO relay design for multipoint-to- multipoint communications with imperfect channel state information,” *IEEE Trans. Signal Process.*, vol. 57, pp. 2785–2796, Jul. 2009.

[14] S. Boyd and L. Vandenberghe, *Convex Optimization*. Cambridge University Press, 2004.

[15] A. Mutapcic and S. Boyd, “Cutting-set methods for robust convex optimization with pessimizing oracles,” *Optimization methods and software*, vol. 24, pp. 381–406, Jun. 2009.

[16] J. Sturm, “Using SeDuMi 1.02, a MATLAB toolbox or optimization over symmetric cones,” *Optimization Methods and Software*, vol. 11, pp. 625–653, 1999.