ROBUST JOINT PRECODER/RECEIVE FILTER DESIGNS FOR MULTIUSER MIMO DOWNLINK

P. Ubaidulla and A. Chockalingam

Department of ECE, Indian Institute of Science, Bangalore 560012, INDIA

ABSTRACT

In this paper, we consider robust joint linear precoder/receive filter designs for multiuser multi-input multi-output (MIMO) downlink that minimize the sum mean square error (SMSE) *in the presence of imperfect channel state information at the transmitter (CSIT)*. The base station (BS) is equipped with multiple transmit antennas, and each user terminal is equipped with *one or more receive antennas*. We consider a stochastic error (SE) model and a norm-bounded error (NBE) model for the CSIT error. In the case of CSIT error following SE model, we compute the desired downlink precoder/receive filter matrices by solving the simpler uplink problem by exploiting the uplink-downlink duality for the MSE region. In the case of the CSIT error following the NBE model, we consider the worst-case SMSE as the objective function, and propose an iterative algorithm for the robust transceiver design. The robustness of the proposed algorithms to imperfections in CSIT is illustrated through simulations.

1. INTRODUCTION

There has been considerable interest in multiuser multi-input multi-output (MIMO) wireless communication systems in view of their potential to offer the benefits of transmit diversity and increased channel capacity [1, 2]. In multiuser MIMO systems, multiuser interference at the receiver is a crucial issue. As a means to mitigate multiuser interference, transmit-side processing in the form of precoding has been studied widely [2]. An important criterion that has been frequently used in precoder designs for multiuser MIMO downlink is sum mean square error (SMSE) [3, 4]. However, most studies in multiuser MIMO, like in [3, 4], assume availability of perfect CSIT. But, in practice, CSIT suffers from inaccuracies caused by errors in channel estimation and/or limited, delayed or erroneous feedback. The performance of precoding schemes is sensitive to such inaccuracies [5]. Hence, it is of interest to develop *transceiver designs that are robust to errors in CSIT*. Two approaches to robust designs are generally adopted. One approach is based on minimax or worst case performance [6], applicable when the parameter uncertainties belong to a predefined uncertainty set. The other approach is based on a stochastic measure of the performance, applicable when the distribution of the parameter uncertainty is available. A few studies on robust precoding for multiuser multiple-input single-output MISO downlink with imperfect CSIT have been reported in the literature [7, 8]. The studies on robust precoder design in [7, 8], however, are only for user terminals with single receive antenna.

Recently, a robust precoder design for multiuser MIMO downlink based on total BS transmit power minimization under individual user mean square error (MSE) constraints has been reported in [9]. We, in this paper, propose robust joint designs of the precoder and receive filter for multiuser MIMO downlink with imperfect CSIT. We consider two models for the CSIT error, *viz.*, stochastic error (SE) model and norm-bounded error (NBE) model, and propose robust transceiver designs valid under each model. The proposed transceiver design under the SE model is based on minimizing a modified function of SMSE under a total transmit power constraint. We show that the uplink-downlink duality for the MSE region (which holds when CSIT is perfect [4]) is also valid when the CSIT error follows the SE model. We exploit this duality and propose a robust transceiver design algorithm.The proposed transceiver design under the NBE model is based on minimizing the worst-case SMSE under a total transmit power constraint. For this model, we propose an iterative optimization algorithm, wherein each iteration involves the solution of a pair of convex optimization problems that can be solved efficiently. The robustness of the proposed algorithm to imperfections in CSIT is illustrated through simulations.

The rest of the paper is organized as follows. The system model and the CSIT error models are given in Section 2. The proposed robust transceiver designs for SE and NBE models of CSIT error are given in Section 3 and Section 4 respectively. Simulation results and comparisons are presented in Section 5. Conclusions are presented in Section 6.

2. SYSTEM MODEL

We consider a multiuser MIMO downlink, where a base station (BS) communicates with M users on the downlink. The BS employs N_t transmit antennas and the kth user is equipped with N_{r_k} receive antennas, $1 \leq k \leq M$. Let $\mathbf{u}_k$ denote[1] the $L_k \times 1$ data symbol vector for the kth user, where L_k, $k = 1, 2, \cdots, M$, is the number of data streams for the kth

[1] Vectors are denoted by boldface lowercase letters, and matrices are denoted by boldface uppercase letters. $[.]^T$ denotes transpose, and $[.]^H$ denotes Hermitian. vec(.) operator stacks the columns of the input matrix into one column-vector. $\mathbf{A} \succeq \mathbf{B}$ denotes $\mathbf{A} - \mathbf{B}$ is positive semi-definite.

user. Stacking the data vectors for all the users, we get the global data vector $\mathbf{u} = [\mathbf{u}_1^T, \cdots, \mathbf{u}_M^T]^T$. Let $\mathbf{B}_k \in \mathbb{C}^{N_t \times L_k}$ represent the precoding matrix for the kth user. The global precoding matrix $\mathbf{B} = [\mathbf{B}_1, \mathbf{B}_2, \cdots, \mathbf{B}_M]$. The transmit vector is given by

$$\mathbf{x} = \mathbf{B}\mathbf{u}. \quad (1)$$

The kth component of the transmit vector $\mathbf{x}$ is transmitted from the kth transmit antenna. The overall channel matrix is

$$\mathbf{H} = [\mathbf{H}_1^T \quad \mathbf{H}_2^T \cdots \mathbf{H}_M^T]^T, \quad (2)$$

where $\mathbf{H}_k$ is the $N_{r_k} \times N_t$ channel matrix of the kth user. The entries of the channel matrices are assumed to be zero-mean, unit-variance complex Gaussian random variables. The received signal vectors are given by

$$\mathbf{y}_k = \mathbf{H}_k \mathbf{B}\mathbf{u} + \mathbf{n}_k, \quad 1 \leq k \leq M. \quad (3)$$

The kth user estimates its data vector as

$$\widehat{\mathbf{u}}_k = \mathbf{C}_k \mathbf{H}_k \Big(\sum_{j=1}^{M} \mathbf{B}_j \mathbf{u}_j \Big) + \mathbf{C}_k \mathbf{n}_k, \ 1 \leq k \leq M, \quad (4)$$

where $\mathbf{C}_k \in \mathbb{C}^{L_k \times N_{r_k}}$ is the receive filter of the kth user, and $\mathbf{n}_k$ is the zero-mean noise vector with $\mathbb{E}\{\mathbf{n}_k \mathbf{n}_k^H\} = \sigma_n^2 \mathbf{I}$. Stacking the estimated vectors of all users, the global estimate can be written as

$$\widehat{\mathbf{u}} = \mathbf{C}\mathbf{H}\mathbf{B}\mathbf{u} + \mathbf{C}\mathbf{n}, \quad (5)$$

where $\mathbf{C}$ is a block diagonal matrix with $\mathbf{C}_k, 1 \leq k \leq M$ on the diagonal, and $\mathbf{n} = [\mathbf{n}_1^T, \cdots, \mathbf{n}_M^T]^T$. The global receive matrix $\mathbf{C}$ has block diagonal structure as the receivers are non-cooperative. The MSE between the symbol vector $\mathbf{u}_k$ and the estimate $\widehat{\mathbf{u}}_k$ at the kth user is given by

$$\epsilon_k = \mathbb{E}\{\|\widehat{\mathbf{u}}_k - \mathbf{u}_k\|^2\}, \ 1 \leq k \leq M, \quad (6)$$

and the sum-MSE (SMSE) is given by

$$\texttt{smse} = \mathbb{E}\{\|\widehat{\mathbf{u}} - \mathbf{u}\|^2\} = \sum_{k=1}^{M} \epsilon_k, \quad (7)$$

where $\mathbb{E}\{\cdot\}$ denotes expectation operator.

2.1. CSIT Error Models

In this paper, we consider two models for the CSIT error. In both the models, the true channel matrix of the kth user, $\mathbf{H}_k$, is represented as

$$\mathbf{H}_k = \widehat{\mathbf{H}}_k + \mathbf{E}_k, \quad 1 \leq k \leq M, \quad (8)$$

where $\widehat{\mathbf{H}}_k$ is the transmit CSIT of the kth user, and $\mathbf{E}_k$ is the CSIT error matrix. The overall channel matrix can be written as

$$\mathbf{H} = \widehat{\mathbf{H}} + \mathbf{E}, \quad (9)$$

where $\widehat{\mathbf{H}} = [\widehat{\mathbf{H}}_1^T \quad \widehat{\mathbf{H}}_2^T \cdots \widehat{\mathbf{H}}_M^T]^T$, and $\mathbf{E} = [\mathbf{E}_1^T \quad \mathbf{E}_2^T \cdots \mathbf{E}_M^T]^T$. In a stochastic error (SE) model, $\mathbf{E}_k$ is the estimation error matrix. The error matrix $\mathbf{E}_k$ is assumed to be Gaussian distributed with zero mean and $\mathbb{E}\{\mathbf{E}_k \mathbf{E}_k^H\} = \sigma_{\mathbf{E}}^2 \mathbf{I}_{N_{r_k} N_{r_k}}$. An alternate error model is a norm-bounded error (NBE) model, where

$$\|\mathbf{E}_k\|_F \leq \delta_k, \quad 1 \leq k \leq M, \quad (10)$$

or, equivalently, the true channel $\mathbf{H}_k$ belongs to the uncertainty set $\mathcal{R}_k$ given by

$$\mathcal{R}_k = \{\zeta | \zeta = \widehat{\mathbf{H}}_k + \mathbf{E}_k, \|\mathbf{E}_k\|_F \leq \delta_k\}, \ 1 \leq k \leq M, \quad (11)$$

where δ_k is the CSIT *uncertainty size*, and $\|\cdot\|_F$ denotes the Frobenius norm. When the transmitter performs the channel estimation in systems where channel reciprocity holds (e.g., as in TDD systems), it is suitable to adopt the SE model for CSIT error. But, when the transmitter obtains the CSIT through a feedback channel from the receiver (e.g., as in FDD systems), the CSIT error is mainly due to quantization. In this case, the NBE model is suitable. Both models have been employed in robust precoder designs reported in the literature [3,4,8]. In this paper, we use the SE model in Section 3, and the NBE model in Section 4.

3. PROPOSED ROBUST TRANSCEIVER DESIGN WITH SE MODEL

In this section, we consider the robust transceiver design based on minimizing the SMSE under a total BS transmit power constraint. The CSIT error is assumed to follow the SE model. First we show that the MSE duality between the downlink and the equivalent uplink, which holds when the CSIT is perfect [3,4], is also valid when the CSIT follows the SE model. We use this MSE duality to transform the downlink robust transceiver design problem to a simpler problem in the equivalent uplink. We obtain the desired solution for the downlink by an appropriate transformation of the solution for the uplink.

3.1. Uplink-Downlink Duality Under SE Model

In this subsection, we show that the downlink and the equivalent uplink achieve the same MSE averaged over the CSIT error under a total transmit power constraint. In the multiuser MIMO downlink, when the CSIT error follow the SE model (8), the downlink MSE at the kth user terminal averaged over $\mathbf{E}_k$ can be written as

$$\mu_k = \mathbb{E}_{\mathbf{E_k}}\{\epsilon_k\} = \mathrm{Tr}\Big(\mathbf{I} + \mathbf{C}_k \widehat{\mathbf{H}}_k \Big(\sum_{j=1}^{M} \mathbf{B}_j \mathbf{B}_j^H \Big) \widehat{\mathbf{H}}_k^H \mathbf{C}_k - 2\Re\big(\mathbf{C}_k \widehat{\mathbf{H}}_k \mathbf{B}_k\big)\Big) + \|\mathbf{C}_k\|_F^2 \left(\sigma_E^2 \|\mathbf{B}\|_F^2 + \sigma_n^2\right). \quad (12)$$

For the corresponding dual uplink, we have

$$\widehat{\mathbf{u}}_k = \mathbf{C}_k^U \mathbf{H}_k^H \Big(\sum_{j=1}^{M} \mathbf{B}_j^U \mathbf{u}_j \Big) + \mathbf{C}_k^U \mathbf{n}_k, \ \forall k, \quad (13)$$

where $\mathbf{C}_k^U$ denotes the receive filter, $\mathbf{B}_k^U$ denotes the precoder, and $\mathbf{H}_k^U$ denotes the dual uplink channel matrix of the

kth user. The dual uplink MSE at the kth user terminal averaged over $\mathbf{E}_k$ can be written as

$$\mu_k^U = \mathbb{E}_{\mathbf{E_k}}\{\epsilon_k^U\} = \text{Tr}\Big(\mathbf{I} + \mathbf{C}_k^U \Big(\sum_{j=1}^{M} \widehat{\mathbf{H}}_j^H \mathbf{B}_j^U (\mathbf{B}_j^U)^H \widehat{\mathbf{H}}_j\Big) \mathbf{C}_k^U - 2\Re\Big(\mathbf{C}_k^U \widehat{\mathbf{H}}_k^H \mathbf{B}_k^U\Big)\Big) + \|\mathbf{C}_k^U\|_F^2 \left(\sigma_E^2 \|\mathbf{B}^U\|_F^2 + \sigma_n^2\right). \quad (14)$$

We claim that, for a given uplink system with precoder matrix $\mathbf{B}^U$ and receive filter $\mathbf{C}^U$, we can obtain a precoder matrix $\mathbf{B}$ and a receive filter $\mathbf{C}$ for the equivalent downlink such that $\mu_k^U = \mu_k$, $1 \le k \le M$, and $\|\mathbf{B}\|_F^2 = \|\mathbf{B}^U\|_F^2$. Let us consider the following transformations from the uplink to the dual downlink [4]:

$$\mathbf{B}_k = \sqrt{a_k}(\mathbf{C}_k^U)^H, \quad \mathbf{C}_k = \sqrt{(1/a_k)}(\mathbf{B}_k^U)^H. \quad (15)$$

Substituting the above in (12) and setting $\mu_k^U = \mu_k$, $1 \le k \le M$, we have

$$\mathbf{\Gamma a} = \sigma_n^2 \mathbf{P}, \quad (16)$$

where

$$\mathbf{a} = \begin{bmatrix} a_1 & a_2 & \dots & a_M \end{bmatrix}^T, \quad (17)$$

$$\mathbf{P} = \begin{bmatrix} \|\mathbf{B}_1^U\|_F^2 & \dots & \|\mathbf{B}_M^U\|_F^2 \end{bmatrix}^T, \quad (18)$$

and

$$\mathbf{\Gamma}_{k,j} = \begin{cases} \sum_{i \ne k} \|\mathbf{C}_k^U \mathbf{H}_i^H \mathbf{B}_i^U\|_F^2 + \widetilde{\sigma}^2 \|\mathbf{C}_k^U\|_F^2 & \text{if } k = j, \\ -\|\mathbf{C}_j^U \mathbf{H}_k \mathbf{B}_k^U\|_F^2 - \sigma_E^2 \|\mathbf{B}\|_F^2 \|\mathbf{C}_k^U\|_F^2 & \text{if } k \ne j, \end{cases} \quad (19)$$

where $\widetilde{\sigma}^2 = \sigma_n^2 + \sigma_E^2\|\mathbf{B}\|_F^2$. As $\mathbf{\Gamma}$ is real-valued and has strictly dominant positive diagonal elements and negative off-diagonal elements, $\mathbf{\Gamma}^{-1}$ exists and it has non-negative elements. Hence, there exists a solution for (16) such that $a_k > 0$, $1 \le k \le M$. Further, adding all the individual equations in (16), we have

$$\sum_{k=1}^{M} \|\mathbf{B}_k\|_F^2 = \sum_{k=1}^{M} a_k \|\mathbf{C}_k^U\|_F^2 = \sum_{k=1}^{M} \|\mathbf{B}_k^U\|^2, \quad (20)$$

which implies the equality of total transmit power in the downlink and the equivalent uplink. We observe that, the arguments here essentially extends the result in [4] to the case with imperfect CSIT.

3.2. Robust Precoder/Receive Filter Design

The uplink-downlink duality enables us to compute robust downlink precoder and receive filter by appropriate transformations of the precoder and receive filter for the virtual uplink, the computation of which is often simpler. In this subsection, we describe the design of the robust precoder and receive filter for the uplink. For a fixed precoder matrix $\mathbf{B}$, the optimum receive filter can be obtained by setting the derivative of the SMSE averaged over the CSIT error, $\mu^U = \sum_{k=1}^M \mu_k^U$, with respect to the $\mathbf{C}_k^U$, $1 \le k \le M$ equal to zero. Performing this operation, we obtain

$$\mathbf{C}_k^U = (\mathbf{B}_k^U)^H \mathbf{H}_k^U \Big(\sum_{k=1}^{M} (\mathbf{H}_k^U)^H \mathbf{B}_k^U (\mathbf{B}_k^U)^H \mathbf{H}_k + \widetilde{\sigma}^2 \mathbf{I}\Big)^{-1}, \forall k. \quad (21)$$

Substituting the expression for $\mathbf{C}_k^U$ in (14), and after a few algebraic manipulations, the uplink SMSE, averaged over the CSIT error, can be represented as

$$\mu^U = \sum_{k=1}^{M} \mu_k^U = \sum_{k=1}^{M} L_k - N_T + \sigma_n^2 \text{tr}(\mathbf{Z}^{-1}), \quad (22)$$

where $\mathbf{Z} = \Big(\sum_{k=1}^M \mathbf{H}_k^H \mathbf{B}_k^U (\mathbf{B}_k^U)^H \mathbf{H}_k + \widetilde{\sigma}^2 \mathbf{I}\Big)$. Now, we can reformulate the robust uplink precoder design problem as

$$\min_{\{\mathbf{B}_k^U\}_{k=1}^M} \quad f \stackrel{\triangle}{=} \text{tr}(\mathbf{Z}^{-1})$$
$$\text{subject to} \quad \sum_{k=1}^{M} \|\mathbf{B}_k^U\|^2 \le P_T, \quad (23)$$

where P_T is the upper limit on the BS transmit power. We solve this constrained optimization using the projected version of conjugate gradient method [10]. The proposed algorithm is shown in Table I. The optimum value of the coefficient α in the table I can be obtained by a line search [10]. $f_k' = \mathbf{H}_k \mathbf{Z}^{-2} \mathbf{H}_k^H \mathbf{B}_k^U$ is the derivative of f with respect to $\left(\mathbf{B}_k^U\right)^H$.

TABLE I
1) $\mathrm{j} = 0$, $\mathbf{B}_k^{U^{\mathrm{j}}} = \sqrt{\frac{P_T}{\sum_{i=1}^M L_k}}\mathbf{I}$, $\mathbf{D}_k^{\mathrm{j}} = -f_k'$, $1 \le k \le M$.
2) $\mathrm{j} = 2$, $\mathbf{B}_k^{U^{\mathrm{j}+1}} = \mathbf{B}_k^{U^{\mathrm{j}}} + \alpha \mathbf{D}_k^{\mathrm{j}}$, $\forall k$
3) $\mathbf{D}_k^{\mathrm{j}} = -f_k' + \frac{\|\mathbf{D}_k^{\mathrm{j}}\|^2}{\|\mathbf{D}_k^{\mathrm{j}-1}\|^2} \mathbf{D}_k^{\mathrm{j}-1}$, $\forall k$
4) $\mathbf{B}_k^{U^{\mathrm{j}+1}} = \mathbf{B}_k^{U^{\mathrm{j}}} + \alpha \mathbf{D}_k^{\mathrm{j}}$, $\forall k$
5) $\mathbf{B}_k^{U^{\mathrm{j}+1}} = \sqrt{\frac{P_T}{\sum_{i=1}^M \|\mathbf{B}_k^{U^{\mathrm{j}+1}}\|^2}} \mathbf{B}_k^{U^{\mathrm{j}+1}}$, $\forall k$
6) Stop if sufficient accuracy is reached; else $\mathrm{j} = \mathrm{j} + 1$, go to step 3.

We can obtain the desired robust precoder and receive filter matrices for the downlink by applying the transformation in (15).

4. PROPOSED ROBUST TRANSCEIVER DESIGN WITH NBE MODEL

In this section we consider the robust transceiver design when the CSIT follows the NBE model. In this case, we consider a minimax design, wherein the robust transceiver design seeks to minimize the worst case SMSE under a total BS transmit power constraint. This problem can be written as

$$\min_{\mathbf{B},\mathbf{C}} \quad \max_{\mathbf{E}_k : \|\mathbf{E}_k\| \le \delta_k, \forall k} \quad \texttt{smse}(\mathbf{B}, \mathbf{C}, \mathbf{E}) \quad (24)$$
$$\text{subject to} \quad \text{Tr}(\mathbf{B}\mathbf{B}^H) \le P_T.$$

The optimization problem given above, is a semi-infinite optimization problem, which in general is intractable [6]. We show, in the following, that an appropriate transformation makes the problem in (24) tractable.

We note that the problem in (24) can be written as

$$\begin{aligned} \min_{\mathbf{B},\mathbf{C},t} \quad & \sum_{k=1}^{M} t_k \\ \text{subject to} \quad & \|\mathbf{D}_k(\widehat{\mathbf{h}}_k + \mathbf{e}_k) - \mathbf{f}_k\|^2 + \sigma_n^2\|\mathbf{c}_k\|^2 \le t_k, \\ & \forall \|\mathbf{e}_k\| \le \delta_k, \quad \forall k \\ & \|\mathbf{b}\|^2 \le P_T, \end{aligned} \tag{25}$$

where t_k, $1 \le k \le M$ are the dummy variables, $\mathbf{D}_k = \mathbf{B}^T \otimes \mathbf{C}_k$, $\widehat{\mathbf{h}}_k = \text{vec}(\widehat{\mathbf{H}}_k)$, $\mathbf{e}_k = \text{vec}(\mathbf{E}_k)$,$\mathbf{c}_k = \text{vec}(\mathbf{C}_k)$, $\mathbf{f}_k = \text{vec}(\mathbf{I}_{L_k \times L_k})$, and $\mathbf{b} = \text{vec}(\mathbf{B})$. The first constraint in (25) is convex in $\mathbf{B}$ for a fixed value of $\mathbf{C}_k$ and vice versa, but not jointly convex in $\mathbf{B}$ and $\mathbf{C}_k$. Hence, to design the transceiver, we propose an iterative algorithm, each iteration of which involves only the computation of $\mathbf{B}$ or $\mathbf{C}_k$.

4.1. Robust Precoder Design

For the design of the precoder matrix $\mathbf{B}$, the second term in the left hand side of the first constraint in (25) is not relevant, hence we drop this term. The robust precoder design problem for a fixed value of $\mathbf{C}$ can be represented as

$$\begin{aligned} \min_{\mathbf{B},t} \quad & \sum_{k=1}^{M} t_k \\ \text{subject to} \quad & \begin{bmatrix} t_k & [\mathbf{D}_k\mathbf{h}_k - \mathbf{f}_k]^H \\ [\mathbf{D}_k\mathbf{h}_k - \mathbf{f}_k] & \mathbf{I} \end{bmatrix} \succeq \mathbf{0}, \\ & \forall \|\mathbf{e}_k\| \le \delta_k, \quad 1 \le k \le M, \\ & \|\mathbf{b}\| \le \sqrt{P_T}, \end{aligned} \tag{26}$$

where $\mathbf{h}_k = \widehat{\mathbf{h}}_k + \mathbf{e}_k$. In the above formulation, we have invoked the Schur Complement Lemma [11] to reformulate the first constraint in (26) as a linear matrix inequality (LMI). The first constraint in (26) can be written as

$$\mathbf{A} \succeq \mathbf{P}^H\mathbf{X}\mathbf{Q} + \mathbf{Q}^H\mathbf{X}^H\mathbf{P}, \tag{27}$$

where

$$\mathbf{A} = \begin{bmatrix} t_k & \left[\mathbf{D}_k\widehat{\mathbf{h}}_k - \mathbf{f}_k\right]^H \\ \left[\mathbf{D}_k\widehat{\mathbf{h}}_k - \mathbf{f}_k\right] & \mathbf{I} \end{bmatrix}, \tag{28}$$

$\mathbf{P} = \begin{bmatrix}\mathbf{0} & \mathbf{D}_k^H\end{bmatrix}$, $\mathbf{X} = \mathbf{e}_k$, and $\mathbf{Q} = -\begin{bmatrix}1 & \mathbf{0}\end{bmatrix}$. Having reformulated the constraint as in (27), we can invoke the following Lemma [12] to solve the problem in (26):

Lemma 1: Given matrices $\mathbf{P}$, $\mathbf{Q}$, $\mathbf{A}$ with $\mathbf{A} = \mathbf{A}^H$,

$$\mathbf{A} \succeq \mathbf{P}^H\mathbf{X}\mathbf{Q} + \mathbf{Q}^H\mathbf{X}^H\mathbf{P}, \quad \forall \mathbf{X} : \|\mathbf{X}\| \le \rho \tag{29}$$

if and only if $\exists \lambda \ge 0$ such that

$$\begin{bmatrix} \mathbf{A} - \lambda\mathbf{Q}^H\mathbf{Q} & -\rho\mathbf{P}^H \\ -\rho\mathbf{P} & \lambda\mathbf{I} \end{bmatrix} \succeq \mathbf{0}. \tag{30}$$

Applying Lemma 1, we can formulate the robust precoder design problem in (26) as the following convex optimization problem:

$$\begin{aligned} \min_{\mathbf{B},t,\lambda} \quad & \sum_{k=1}^{M} t_k \\ \text{subject to} \quad & \mathbf{M}_k \succeq \mathbf{0}, \ \lambda_k > 0, \quad \forall k, \\ & \|\mathbf{b}\| \le \sqrt{P_T}, \end{aligned} \tag{31}$$

where

$$\mathbf{M}_k = \begin{bmatrix} t_k - \lambda_k & (\mathbf{D}_k\widehat{\mathbf{h}}_k - \mathbf{f}_k)^H & \mathbf{0} \\ (\mathbf{D}_k\widehat{\mathbf{h}}_k - \mathbf{f}_k) & \mathbf{I} & -\rho\mathbf{D}_k \\ \mathbf{0} & -\rho\mathbf{D}_k^H & \lambda_k\mathbf{I} \end{bmatrix}. \tag{32}$$

4.2. Robust Receive filter Design

Here, we consider the design of robust receive filter for a given precoder filter $\mathbf{B}$. This problem can be written as

$$\begin{aligned} \min_{\mathbf{C},t} \quad & \sum_{k=1}^{M} t_k \\ \text{subject to} \quad & \|\mathbf{D}_k(\widehat{\mathbf{h}}_k + \mathbf{e}_k) - \mathbf{f}_k\|^2 + \sigma_n^2\|\mathbf{c}_k\|^2 \le t_k, \\ & \forall \|\mathbf{e}_k\| \le \delta_k, \quad 1 \le k \le M. \end{aligned} \tag{33}$$

Applying the Schur Complement Lemma to the first constraint in (33), we can formulate the robust receive filter design as the following convex optimization problem:

$$\min_{\mathbf{C},t,\lambda} \quad \sum_{k=1}^{M} t_k \tag{34}$$

$$\text{subject to} \quad \mathbf{N}_k \succeq \mathbf{0}, \ \lambda_k > 0, \quad \forall k, \tag{35}$$

where

$$\mathbf{N}_k = \begin{bmatrix} t_k - \lambda_k & \begin{bmatrix}(\mathbf{D}_k\widehat{\mathbf{h}}_k - \mathbf{f}_k \\ \sigma_n\mathbf{c}_k\end{bmatrix}^H & \mathbf{0} \\ \begin{bmatrix}\mathbf{D}_k\widehat{\mathbf{h}}_k - \mathbf{f}_k \\ \sigma_n\mathbf{c}_k\end{bmatrix} & \mathbf{I} & -\rho\mathbf{D}_k \\ \mathbf{0} & -\rho\mathbf{D}_k^H & \lambda_k\mathbf{I} \end{bmatrix}. \tag{36}$$

4.3. Iterative Algorithm for Solving (24)

Here we present the proposed iterative algorithm for the minimization of the SMSE under a constraint on the total BS transmit power, when the CSIT error follows NBE model. At the $(n+1)$th iteration, the value of $\mathbf{B}$, denoted by $\mathbf{B}^{n+1}$, is the solution to problem (31) and hence satisfies the BS transmit power constraint. Having computed $\mathbf{B}^{n+1}$, $\mathbf{C}^{n+1}$ is the solution to the problem in (34). So $J(\mathbf{B}^{n+1}, \mathbf{C}^{n+1}) \le J(\mathbf{B}^{n+1}, \mathbf{C}^{n}) \le J(\mathbf{B}^{n}, \mathbf{C}^{n})$, where

$$J(\mathbf{B}, \mathbf{C}) = \max_{\|\mathbf{E}_k\| < \delta_k, \forall k} \texttt{smse}(\mathbf{B}, \mathbf{C}, \mathbf{E}). \tag{37}$$

The monotonically decreasing nature of $J(\mathbf{B}^n, \mathbf{C}^n)$, together with the fact that $J(\mathbf{B}^n, \mathbf{C}^n)$ is lower-bounded, implies that the proposed algorithm converges to a limit as $n \to \infty$. The iteration is terminated when the norm of the difference in the results of consecutive iterations are below a threshold or when the maximum number of iterations is reached. We note that proposed algorithm is not guaranteed to converge to the global minimum.

5. SIMULATION RESULTS

In this section, we illustrate the performance of the proposed robust transceiver algorithms, evaluated through simulations. We compare the performance of the proposed robust designs with the nonrobust transceiver designs reported in [4]. First,

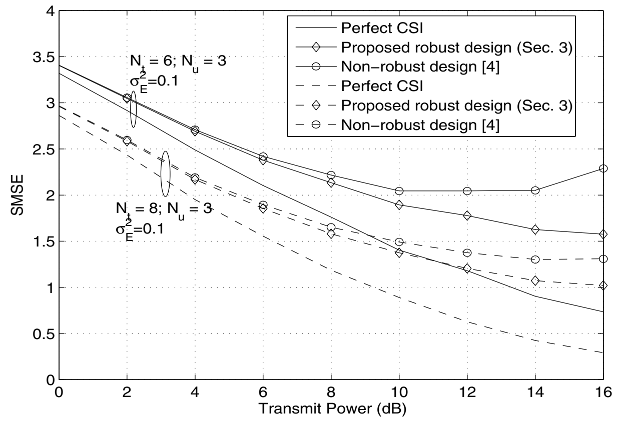

Fig. 1. SMSE versus transmit power $P_{Tr} = \|\mathbf{B}\|_F^2$. $N_t = 8, 6$, $M = 3$, $N_{r_1} = N_{r_2} = N_{r_3} = 2$, $L_1 = L_2 = L_3 = 2$, $\sigma_E^2 = 0.1$

we consider the performance of the robust precoder design with SE model for the CSIT error. We consider a system with the base station transmitting $L = 2$ data streams each to $M = 3$ users, each equipped with $N_r = 2$ receive antennas. The simulation results are shown in Fig. 1[2]. The SMSE performances of the proposed robust design and the non-robust design proposed in [4] for $\sigma_E^2 = 0.1$, and $N_t = 6, 8$ are compared. We have also shown the performance of the designs when the CSIT is perfectly known.The proposed robust design is seen to outperform the non-robust design in [4]. It is found that the difference between the performance of these algorithms increase as the SNR increases. This is observable in (12), where the second term shows the effect of the CSIT error variance amplified by the transmit power. Next, we consider the performance of the robust precoder design with NBE model for CSIT. In this experiment, we consider a system with $N_t = 4$ transmit antennas and $M = 2$ users, each equipped with $N_r = 2$ receive antennas. The BS transmits $L = 2$ data streams to each user. The simulation results in terms of SMSE versus the CSIT uncertainty size δ for $P_T = 15$ dB and 25 dB are shown in Fig. 2[2]. We observe improved performance of the proposed robust design compared to that of the non-robust design in [4]. Like in the case of the robust transceiver design with SE model, here also we observe that the difference in SMSE between the robust design and the non-robust design is larger for larger transmit power.

6. CONCLUSIONS

We presented two robust joint designs of linear precoder and receive filter for multiuser MIMO downlink with imperfect CSIT. Both the designs were based on the minimization of SMSE under a constraint on the total BS transmit power. The first design is for the scenario where the CSIT error can be modeled by the SE model. For this case, we showed that the MSE duality holds between the downlink and the corresponding uplink. We proposed an iterative algorithm for the robust joint precoder/receiver design by using this duality. The second design is for the scenario where the CSIT error can be modeled by the NBE model. We proposed an iterative algorithm to solve this problem. Through simulation results, we illustrated the superior performance of the proposed robust designs compared to the non-robust designs in the presence of CSIT imperfections.

[2]Similar performance is observed for other settings of the parameters also. Corresponding results are not included due to space constraint.

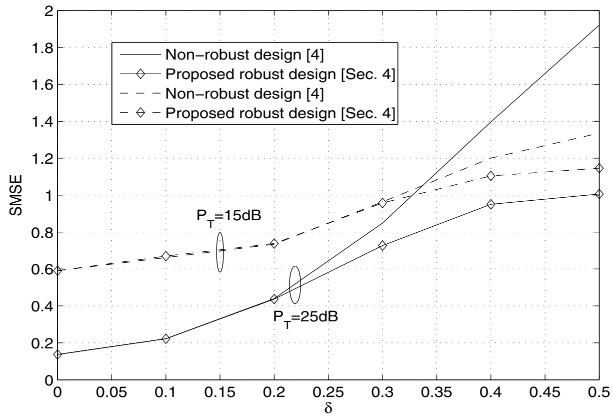

Fig. 2. SMSE versus CSIT uncertainty size $\delta = \delta_1 = \delta_2$. $N_t = 4$, $M = 2$, $N_{r_1} = N_{r_2} = 2$, $L_1 = L_2 = 2$, $P_T = 15\,\text{dB}, 25\,\text{dB}$.

7. REFERENCES

[1] D. Tse and P. Viswanath,*Fundamentals of Wireless Communication*, Cambridge University Press, 2006.

[2] H. Bolcskei, D. Gesbert, C.B. Papadias, and All-Jan van der Veen, *Space-time Wireless Systems: From Array Processing to MIMO Communications*, Cambridge University Press, 2006.

[3] S. Shi, M. Schubert, and H. Boche, k "Downlink MMSE transceiver optimization for multiuser MIMO systems: Duality and sum-MSE minimization," *IEEE Trans. Signal Process.*, vol. 55, pp. 5436–5446, Nov. 2007.

[4] A. Mezghani, M. Joham, R. Hunger, and W. Utschick, "Transceiver design for multi-user MIMO systems," in *Proc. WSA 2006*, Mar. 2006.

[5] N. Jindal, "MIMO broadcast channels with finite rate feed-back," in *Proc. IEEE GLOBECOM'2005*, Nov. 2005.

[6] A. Ben-Tal and A. Nemirovsky, "Robust convex optimization," *Mathematics of Operations Research*, vol. 23, no. 4, pp. 769–805, Nov. 1998.

[7] M. B. Shenouda and T. N. Davidson, "Linear matrix inequality formulations of robust QoS precoding for broadcast channels," in *Proc. CCECE'2007*, Apr. 2007, pp. 324–328.

[8] M. Payaro, A. Pascual-Iserte, and M. A. Lagunas, "Robust power allocation designs for multiuser and multiantenna downlink communication systems through convex optimization," *IEEE J. Sel. Areas Commun.*, vol. 25, pp. 1392–1401, Sept. 2007.

[9] N. Vucic, H. Boche, and S. Shi, "Robust transceiver optimization in downlink multiuser MIMO systems with channel uncertainty," in *Proc. IEEE ICC'2008*, May 2008.

[10] S. G. Nash and A. Sofer, *Linear and Nonlinear Programming*, McGraw-Hill, 1996.

[11] R. A. Horn and C. R. Johnson, *Matrix Analysis*, Cambridge University Press, 1985.

[12] Y. C. Eldar, A. Ben-Tal, and A.Nemirovski, "Robust mean-squared error estimation in the presence of model uncertainties," *IEEE Trans. Signal Process.*, vol. 53, pp. 161–176, Jan. 2005.