End-to-End BER Analysis of Space Shift Keying in Decode-and-Forward Cooperative Relaying

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***Abstract*— Space shift keying (SSK) is a special case of spatial modulation (SM), which is a relatively new modulation technique that is getting recognized to be attractive in multi-antenna communications. Our new contribution in this paper is an analytical derivation of exact closed-form expression for the end-to-end bit error rate (BER) performance of *SSK in decode-and-forward (DF) cooperative relaying*. An incremental relaying (IR) scheme with selection combining (SC) at the destination is considered. In SSK, since the information is carried by the transmit antenna index, traditional selection combining methods based on instantaneous SNRs can not be directly used. To overcome this problem, we propose to do selection between direct and relayed paths based on the Euclidean distance between columns of the channel matrix. With this selection metric, an exact analytical expression for the end-to-end BER is derived in closed-form. Analytical results are shown to match with simulation results.**

Keywords: *Space shift keying, decode-and-forward, cooperative relaying, incremental relaying, selection combining, BER analysis.*

I. INTRODUCTION

Multi-antenna communication techniques are increasingly getting adopted in emerging wireless standards and systems. An issue with multi-antenna communication is the need to have multiple radio frequency (RF) chains in communication terminals, which results in increased complexity/cost and issues related to inter-antenna synchronization. Spatial modulation (SM) is a relatively a new modulation technique which allows the use of less number of transmit RF chains than the number of transmit antennas, while achieving high spectral efficiencies [1]. Space shift keying (SSK) is a special case of SM. In SSK, $n_t = 2^m$ transmit antennas and one transmit RF chain are used for signaling. In each channel use, a group of m information bits are used to choose one among $n_t = 2^m$ transmit antennas. A signal, say +1, which is known to the receiver, is transmitted on this chosen antenna. The remaining $n_t - 1$ antennas remain silent. By doing so, the problem of detection at the receiver becomes one of merely finding out which antenna is transmitting. Also, the spectral efficiency is m bpcu. Analytical evaluation of the performance of SSK is of interest. In this paper, our focus is on the bit error rate (BER) analysis of SSK in cooperative relaying.

Works in the recent past have analyzed the performance of SSK and SM on fading channels [2]-[8]. Several of these performance analysis works consider point-to-point fading channels, and only few of them consider relay channels. For example, BER analysis of SSK in various point-to-point fading scenarios have been reported in [2]-[6]; analysis for i.i.d. Rayleigh fading MIMO in [2], Nakagami-m fading MISO in [3],[4], Rician fading MIMO in [5], and Rayleigh fading MIMO with imperfect CSI in [6]. Likewise, BER analyses of SM in point-to-point fading have been reported in [7],[8]; analysis for i.i.d. Rayleigh fading MIMO in [7], and correlated Rayleigh and Nakagami-m fading MIMO in [8]. We note that the reported BER analysis works on relay channels are fewer compared to those on point-to-point channels. For example, BER analysis of SSK in dual-hop amplify-and-forward (AF) relaying is reported in [9]. Our contribution in this paper makes a new addition to the BER analysis literature of SSK with relaying. In particular, we derive exact closed-form expression for the end-to-end BER of *SSK in decode-and-forward (DF) cooperative relaying*, which, to our knowledge, has not been reported so far.

In dual-hop relaying (like the scheme considered in [9]), there are two links, one from source to relay (S-to-R) and another from relay to destination (R-to-D). In *cooperative relaying* [10], on the other hand, a direct link from source to destination (S-to-D) is present, in addition to the S-to-R and R-to-D links. We consider cooperative relaying, which is more general than dual-hop relaying. In cooperative relaying, the combining of S-to-D and R-to-D signals at the destination can be done using one of maximal-ratio combining (MRC), equal-gain-combining (EGC), and selection combining (SC). We consider decode-and-forward incremental relaying [10] with selection combining at the destination in this paper. A difficulty that arises in doing selection combining with SSK is that traditional selection combining methods based on instantaneous SNRs [11] can not be directly used in SSK. This is because, unlike in traditional modulation methods, information is carried in the antenna index in SSK. We overcome this problem by doing selection based on the Euclidean distance between columns of the channel matrix of the R-to-D and the S-to-D links. We use a similar metric for incremental relaying as well. For this system, we derive exact closed-form expression for the end-to-end BER and validate the analytical results through simulations.

II. SYSTEM MODEL

Consider a cooperative relaying system consisting of a source, relay, and destination as shown Fig. 1. The source and relay are equipped with $n_s = 2$ transmit antennas. Transmissions from the source and relay use SSK. The mapping from data bits to SSK signals for transmission is shown in Table I. The relay and destination are equipped n_r and n_d receive antennas, respectively, $n_r, n_d \geq 1$. The communication happens in two phases. In the first phase, the source (S) transmits us-

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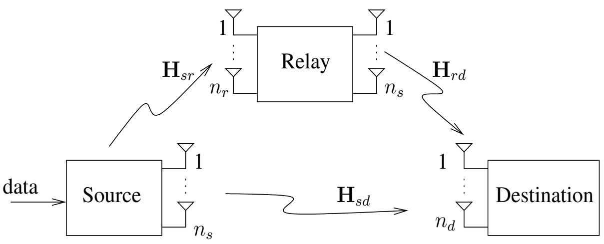

Fig. 1. Cooperative relaying with SSK.

ing SSK, which is heard by both the relay (R) and destination (D). The received signal vectors at R and D, respectively, are

Data bit	SSK tx. signal vector, $\mathbf{x}$	Antenna status
0	$[1, 0]^T$	Ant. #1: +1, Ant. #2: OFF
1	$[0, 1]^T$	Ant. #1: OFF, Ant. #2: +1

TABLE I

DATA BIT TO SSK SIGNAL MAPPING FOR $n_s = 2$.

$$\mathbf{y}_{sr} = \mathbf{H}_{sr}\,\mathbf{x} + \mathbf{w}_{sr} \quad (1)$$

$$\mathbf{y}_{sd} = \mathbf{H}_{sd}\,\mathbf{x} + \mathbf{w}_{sd}, \quad (2)$$

where $\mathbf{x}$ is the SSK signal vector transmitted from S (defined in Table I), $\mathbf{H}_{sr}$ and $\mathbf{H}_{rd}$ are the $n_r \times n_s$ S-to-R channel matrix and $n_d \times n_s$ S-to-D channel matrix, respectively. Each element of $\mathbf{H}_{sr}$ and $\mathbf{H}_{rd}$ are modeled as i.i.d. $\mathcal{CN}(0, \sigma_{sr}^2)$ and $\mathcal{CN}(0, \sigma_{sd}^2)$, respectively. $\mathbf{w}_{sr} \in \mathbb{C}^{n_r \times 1}$ and $\mathbf{w}_{sd} \in \mathbb{C}^{n_d \times 1}$ are additive noise vectors at R and D, respectively.

Incremental Relaying (IR): In the second phase, the relay checks whether a certain metric (η_{sd}), which signifies the quality of the S-to-D channel, is below a pre-determined threshold (μ). If $\eta_{sd} < \mu$, the relay decodes the SSK symbol transmitted by the source in the first phase, and transmits the decoded SSK symbol to the destination. Let $\mathbf{x}_r$ denote the SSK signal vector transmitted by the relay. If $\eta_{sd} \geq \mu$, R does not transmit in the second phase. The signal received at D from R is

$$\mathbf{y}_{rd} = \mathbf{H}_{rd}\,\mathbf{x}_r + \mathbf{w}_{rd}, \quad (3)$$

where $\mathbf{H}_{rd} \in \mathbb{C}^{n_d \times n_s}$ is the R-to-D channel matrix whose entries are modeled as i.i.d. $\mathcal{CN}(0, \sigma_{rd}^2)$. $\mathbf{w}_{rd} \in \mathbb{C}^{n_d \times 1}$ is the additive noise vector. We assume that the relay uses maximum likelihood (ML) detection rule [2]:

$$\mathbf{x}_r = \underset{\boldsymbol{\epsilon} \in \mathbb{S}_{n_s}}{\arg\min} \; \|\mathbf{y}_{sr} - \mathbf{H}_{sr}\boldsymbol{\epsilon}\|^2, \quad (4)$$

where $\mathbb{S}_{n_s}$ is the SSK symbol alphabet corresponding to n_s transmit antennas.

Selection Combining at the Destination: At the end of the two phases of transmission, D processes either the received samples $\mathbf{y}_{sd}$ and $\mathbf{y}_{rd}$ (if $\eta_{sd} < \mu$), or only $\mathbf{y}_{sd}$ (if $\eta_{sd} \geqslant \mu$). In the later case, D performs ML detection on $\mathbf{y}_{sd}$. In the former case, selection combining is performed among the two received vectors $\mathbf{y}_{sd}$ and $\mathbf{y}_{rd}$. This combining is done based on two metrics, η_{sd} and η_{rd}. The metric η_{rd} signifies the quality of the R-to-D channel. In this selection combining, only one among $\mathbf{y}_{sd}$ and $\mathbf{y}_{rd}$ is selected for receiver processing, i.e., D processes $\mathbf{y}_{sd}$ for ML detection if $\eta_{sd} > \eta_{rd}$, and processes $\mathbf{y}_{rd}$ otherwise.

The elements of the additive noise vectors in all the three channels are modeled as i.i.d. $\mathcal{CN}(0, \sigma^2)$. To take into account path loss, we define $\sigma_{sr}^2 \triangleq d_{sr}^{-\alpha}$, $\sigma_{sd}^2 \triangleq d_{sd}^{-\alpha}$, and $\sigma_{rd}^2 \triangleq d_{rd}^{-\alpha}$, where d_{sr}, d_{sr} and d_{sr} are the distances between S-to-R, S-to-D and R-to-D, respectively, and α is the path loss exponent.

Selection Metrics: For the purpose of incremental relaying and selection combining in SSK, we define the following metrics: $\eta_{sd} \triangleq \frac{||\mathbf{h}_{sd}^2 - \mathbf{h}_{sd}^1||^2}{2\sigma^2}$, and $\eta_{rd} \triangleq \frac{||\mathbf{h}_{rd}^2 - \mathbf{h}_{rd}^1||^2}{2\sigma^2}$. Here, $\mathbf{h}_{sd}^i$ and $\mathbf{h}_{rd}^i$ denote the ith columns of $\mathbf{H}_{sd}$ and $\mathbf{H}_{rd}$, respectively. The intuition behind choosing the parameter η_{sd} in such a way is driven by the fact that the BER of the S-to-D channel conditioned on $\mathbf{H}_{sd}$ is $Q\Big(\sqrt{\frac{||\mathbf{h}_{sd}^2 - \mathbf{h}_{sd}^1||^2}{2\sigma^2}}\Big)$ [2], and likewise for choosing η_{rd}.

III. END-TO-END BER ANALYSIS

The average end-to-end bit error probability is given by

$$P(E) = P(\eta_{sd} \geqslant \mu)P(E|\eta_{sd} \geqslant \mu) + P(\eta_{sd} < \mu)\Big[P(E_{sr}|\eta_{sd} < \mu) \\ P(E|E_{sr}, \eta_{sd} < \mu) + P(E_{sr}^c|\eta_{sd} < \mu)P(E|E_{sr}^c, \eta_{sd} < \mu)\Big], \quad (5)$$

where E is the end-to-end error event, E_{sr} is the error event in the S-to-R link, E_{sr}^c is the complement of the event E_{sr}, $P(\eta_{sd} < \mu) = 1 - P(\eta_{sd} \geqslant \mu)$, and $P(E_{sr}^c|\eta_{sd} < \mu) = 1 - P(E_{sr}|\eta_{sd} < \mu)$. In the following, we derive closed-form expressions for the various probabilities in (5).

$P(\eta_{sd} \geqslant \mu)$: The event '$\eta_{sd} \geqslant \mu$' can be written as '$\eta'_{sd} \geqslant \mu/\rho$', where $\rho \triangleq \frac{1}{2\sigma^2}$ and $\eta'_{sd} \triangleq ||\mathbf{h}_{sd}^2 - \mathbf{h}_{sd}^1||^2$. Hence, $P(\eta_{sd} \geqslant \mu) = P(\eta'_{sd} \geqslant \mu/\rho)$. η'_{sd} is a gamma random variable with parameters $\{n_d, 2\sigma_{sd}^2\}$. Therefore,

$$P(\eta_{sd} \geqslant \mu) = \int_{\mu/\rho}^{\infty} \frac{s^{n_d - 1}\exp(-\frac{s}{2\sigma_{sd}^2})}{(2\sigma_{sd}^2)^{n_d}(n_d - 1)!} ds \\ = \sum_{j=0}^{n_d - 1} \frac{1}{j!}\Big(\frac{\mu}{2\sigma_{sd}^2\rho}\Big)^j \exp(-\frac{\mu}{2\sigma_{sd}^2\rho}), \quad (6)$$

where (6) follows from the gamma distribution CDF.

$P(E|\eta_{sd} \geqslant \mu)$: When $\eta_{sd} \geqslant \mu$, the relay does not transmit, and an error occurs due to event E_{sd} (the error event in the S-to-D link). Therefore, $P(E|\eta_{sd} \geqslant \mu) = P(E_{sd}|\eta'_{sd} \geqslant \frac{\mu}{\rho})$. The conditional PDF of η'_{sd} given $\eta'_{sd} \geqslant \frac{\mu}{\rho}$ is given by

$$f_{\eta'_{sd}}(s|\eta'_{sd} \geqslant \frac{\mu}{\rho}) = \frac{s^{n_d - 1}\exp(-\frac{s}{2\sigma_{sd}^2})}{(2\sigma_{sd}^2)^{n_d}(n_d - 1)!P(\eta_{sd} \geqslant \mu)}, \quad \text{if } s \geqslant \frac{\mu}{\rho} \\ = 0, \quad \text{otherwise}. \quad (7)$$

The conditional error probability on S-to-D link is [2]

$$P(E_{sd}|\mathbf{H}_{sd}) = Q\big(\sqrt{\rho\eta'_{sd}}\big). \quad (8)$$

Using (7) and (8), we can write

$$P(E|\eta_{sd} \geqslant \mu) \;=\; \int_{\frac{\mu}{\rho}}^{\infty} Q(\sqrt{\rho s}) f_{\eta'_{sd}}\big(s|\eta'_{sd} \geqslant \frac{\mu}{\rho}\big)\, ds$$

$$= Q(\sqrt{\mu}) - \frac{1}{P(\eta_{sd} \geqslant \mu)} \sum_{j=0}^{n_d-1} \int_{\sqrt{\mu}}^{\infty} \frac{u^{2j}\exp(-\frac{u^2}{2\omega})}{(2\rho\sigma_{sd}^2)^j j! \sqrt{2\pi}} du, \quad (9)$$

where $\omega = \frac{\rho\sigma_{sd}^2}{\rho\sigma_{sd}^2+1}$, and (9) follows after few steps involving changing order of integral and using the CDF of gamma distribution.

Substituting u with $z\sqrt{\omega}$ in the integral in (9), we can write

$$\int_{\sqrt{\mu}}^{\infty} u^{2j} \exp\big(-\frac{u^2}{2\omega}\big) du = \omega^{j+\frac{1}{2}} I^{(j)}\Big(\sqrt{\frac{\mu}{\omega}}\Big), \quad (10)$$

where $I^{(j)}(\alpha) \triangleq \int_{\alpha}^{\infty} z^{2j}\exp(-\frac{z^2}{2})dz$. For $j = 0$, $I^{(0)}(\alpha) = \sqrt{2\pi}Q(\alpha)$. When j is any positive integer, $I^{(j)}(\alpha)$ can be computed using the following lemma.

Lemma 1: If $I^{(j)}(\alpha) \triangleq \int_{\alpha}^{\infty} z^{2j} e^{-\frac{z^2}{2}} dz$, then for any positive integer j, the following relation holds

$$I^{(j)}(\alpha) = \sum_{k=1}^{j} \alpha^{2k-1} \frac{(2j-1)!!}{(2k-1)!!} e^{-\frac{\alpha^2}{2}} + (2j-1)!!\, I^{(0)}(\alpha), \quad (11)$$

where $(2j-1)!! = (2j-1).(2j-3)...3.1$.

Proof: Proof can be shown by induction, which is omitted due to page limit.

Using Lemma 1 in (10), (9) can be simplified as

$$P(E|\eta_{sd} \geqslant \mu) \;=\; Q(\sqrt{\mu}) - \frac{1}{P(\eta_{sd} \geqslant \mu)}\Big\{\sqrt{\omega}Q\Big(\sqrt{\frac{\mu}{\omega}}\Big) - \sum_{j=1}^{n_d-1} \frac{\omega^{j+\frac{1}{2}}(2j-1)!!}{(2\rho\sigma_{sd}^2)^j j!}\Big[\sum_{k=1}^{j}\Big(\frac{\mu}{\omega}\Big)^{\frac{2k-1}{2}} \frac{e^{-\frac{\mu}{2\omega}}}{\sqrt{2\pi}(2k-1)!!} + Q\Big(\sqrt{\frac{\mu}{\omega}}\Big)\Big]\Big\}. \quad (12)$$

$P(E_{sr}|\eta_{sd} < \mu)$: Since E_{sr} is independent of η_{sd}, we have $P(E_{sr}|\eta_{sd} < \mu) = P(E_{sr})$, and $P(E_{sr})$ can be derived as

$$P(E_{sr}) = \frac{1}{2}\Big[1 - \sqrt{\delta} \sum_{j=0}^{n_r-1} \frac{\delta^j (2j)!}{(4\rho\sigma_{sr}^2)^j (j!)^2}\Big], \quad (13)$$

where $\delta = \frac{\rho\sigma_{sr}^2}{1+\rho\sigma_{sr}^2}$. See Appendix I for the derivation of (13).

$P(E|E_{sr}, \eta_{sd} < \mu)$: $P(E|E_{sr}, \eta_{sd} < \mu)$ is the average probability of the bit error at D, given R transmits the incorrectly decoded $\mathbf{x}$, i.e., $\mathbf{x}_r \neq \mathbf{x}$. In selection combining, the received vector $\mathbf{y}_{rd}$ is chosen for processing at the destination if $\eta_{rd} > \eta_{sd}$, i.e., $\eta'_{rd} > \eta'_{sd}$, where $\eta'_{rd} \triangleq ||\mathbf{h}_{rd}^2 - \mathbf{h}_{rd}^1||^2$. In this case, the error at the destination is due to the error event in R-to-D link (denoted by E_{rd}). Therefore, we can write

$$P(E|E_{sr}, \eta_{sd} < \mu, \mathbf{H}_{sd}, \mathbf{H}_{rd}) \;=\; P(E_{sd}|\mathbf{H}_{sd}), \text{ for } \eta'_{sd} > \eta'_{rd}$$
$$= P(E_{rd}|E_{sr}, \mathbf{H}_{rd}), \text{ for } \eta'_{sd} \leqslant \eta'_{rd} < \infty, \quad (14)$$

where $P(E_{rd}|E_{sr}, \mathbf{H}_{rd})$ can be obtained as

$$P(E_{rd}|E_{sr}, \mathbf{H}_{rd}) = 1 - Q\left(\sqrt{\rho\eta'_{rd}}\right). \quad (15)$$

See Appendix II for the derivation of (15).

Substituting (8), (15) in (14), and subsequently on averaging over the densities of η'_{sd} and η'_{rd}, $P(E|E_{sr}, \eta_{sd} < \mu, \mathbf{H}_{sd}, \mathbf{H}_{rd})$ can be written as

$$P(E|E_{sr}, \eta_{sd} < \mu) = \frac{1}{P(\eta_{sd} < \mu)}\Big\{\underbrace{\int_0^{\frac{\mu}{\rho}}\int_s^{\infty} f_{\eta'_{rd}}(r) f_{\eta'_{sd}}(s) dr ds}_{A_{srd}} + \underbrace{\int_0^{\frac{\mu}{\rho}}\int_0^{s} Q\left(\sqrt{\rho s}\right) f_{\eta'_{rd}}(r) f_{\eta'_{sd}}(s) dr ds}_{A_1} - \underbrace{\int_0^{\frac{\mu}{\rho}}\int_s^{\infty} Q\left(\sqrt{\rho r}\right) f_{\eta'_{rd}}(r) f_{\eta'_{sd}}(s) dr ds}_{A_2}\Big\}. \quad (16)$$

Expressions for the terms A_1 and A_2 in (16) are derived in Appendices III and IV, respectively. Expression for the A_{srd} term is derived as follows.

$$A_{srd} = \int_0^{\frac{\mu}{\rho}}\int_s^{\infty} \frac{r^{n_d-1}\exp(-\frac{r}{2\sigma_{rd}^2})}{(2\sigma_{rd}^2)^{n_d}(n_d-1)!} dr f_{\eta'_{sd}}(s) ds$$

$$= \int_0^{\frac{\mu}{\rho}} \Big[\sum_{j=0}^{n_d-1} \frac{s^j e^{-\frac{s}{2\sigma_{rd}^2}}}{(2\sigma_{rd}^2)^j j!}\Big] \frac{s^{n_d-1} e^{-\frac{s}{2\sigma_{sd}^2}} ds}{(2\sigma_{sd}^2)^{n_d}(n_d-1)!} \quad (17)$$

$$= \sum_{j=0}^{n_d-1} \frac{\lambda^{j+n_d}\binom{j+n_d-1}{j}}{\sigma_{rd}^{2j}\sigma_{sd}^{2n_d}} \int_0^{\frac{\mu}{\rho}} \frac{s^{j+n_d-1} e^{-\frac{s}{2\lambda}} ds}{(2\lambda)^{j+n_d}(j+n_d-1)!}$$

$$= \sum_{j=0}^{n_d-1} \frac{\lambda^{j+n_d}\binom{j+n_d-1}{j}}{\sigma_{rd}^{2j}\sigma_{sd}^{2n_d}} \Big[1 - \sum_{i=0}^{j+n_d-1} \frac{\mu^i e^{-\frac{\mu}{2\rho\lambda}}}{(2\rho\lambda)^i i!}\Big], \quad (18)$$

where $\lambda = \frac{\sigma_{sd}^2\sigma_{rd}^2}{\sigma_{sd}^2+\sigma_{rd}^2}$, and (17) and (18) follow from the CDF of gamma distribution. Substituting the expressions of A_1, A_2 and A_{srd} from (38), (44) and (18) respectively in (16), we get $P(E|E_{sr}, \eta_{sd} < \mu)$.

$P(E|E_{sr}^c, \eta_{sd} < \mu)$: $P(E|E_{sr}^c, \eta_{sd} < \mu)$ is the conditional probability of end-to-end bit error, given that the relay transmits the correctly decoded $\mathbf{x}$, i.e., $\mathbf{x}_r = \mathbf{x}$. Therefore,

$$P(E|E_{sr}^c, \eta_{sd} < \mu, \mathbf{H}_{sd}, \mathbf{H}_{rd}) \;=\; Q\left(\sqrt{\rho\eta'_{sd}}\right), \text{ for } \eta'_{sd} > \eta'_{rd}$$
$$= Q\left(\sqrt{\rho\eta'_{rd}}\right), \text{ for } \eta'_{sd} \leqslant \eta'_{rd} < \infty, \quad (19)$$

which, on averaging over the densities of η'_{sd} and η'_{rd}, can be written as

$$P(E|E_{sr}^c, \eta_{sd} < \mu) = \frac{1}{P(\eta'_{sd} < \frac{\mu}{\rho})}\Big\{\underbrace{\int_0^{\frac{\mu}{\rho}}\int_0^{s} Q\left(\sqrt{\rho s}\right) f_{\eta'_{rd}}(r) f_{\eta'_{sd}}(s) dr ds}_{A_1} + \underbrace{\int_0^{\frac{\mu}{\rho}}\int_s^{\infty} Q\left(\sqrt{\rho r}\right) f_{\eta'_{rd}}(r) f_{\eta'_{sd}}(s) dr ds}_{A_2}\Big\}. \quad (20)$$

Substituting the expressions of A_1 and A_2 from (38) and (44), respectively, in (20), we get $P(E|E_{sr}^c, \eta_{sd} < \mu)$.

Finally, substituting the expressions for the probabilities in (5), we get a closed-form exact expression for the end-to-end BER.

IV. Numerical Results

In this section, we present numerical results of the BER performance obtained by computing the analytical BER expressions derived in the previous section. We also present the

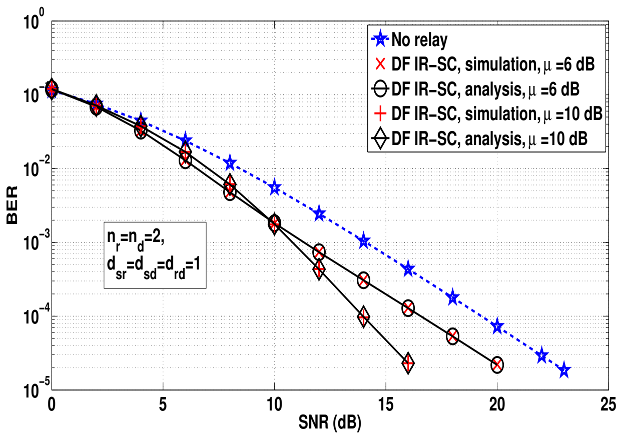

Fig. 2. BER of SSK in DF IR-SC cooperative relaying scheme for $n_r = n_d = 2$, $d_{sr} = d_{sd} = d_{rd} = 1$ for different threshold values (μ = 6 dB, 10 dB). Analysis and simulation.

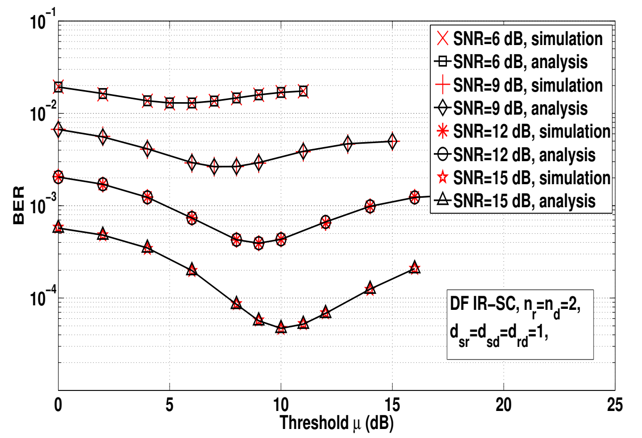

Fig. 3. BER versus μ plots for SSK in DF IR-SC cooperative relaying scheme for $n_r = n_d = 2$, $d_{sr} = d_{sd} = d_{rd} = 1$ for different SNR values (SNR = 6 dB, 9 dB, 12 dB, and 15 dB). Analysis and simulation.

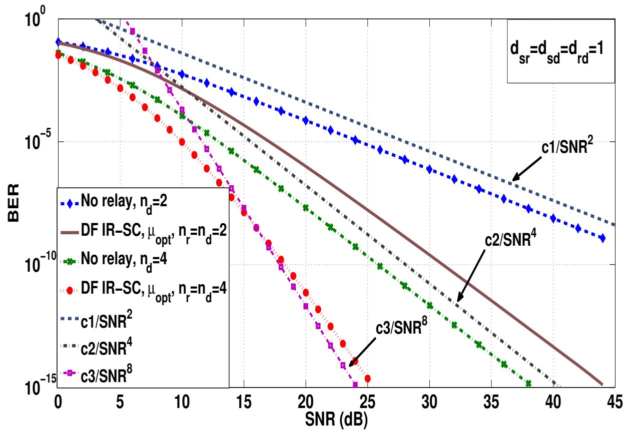

Fig. 4. BER versus SNR plots for SSK in DF IR-SC at optimum threshold values (μ_{opt}) for $n_r = n_d = 2, 4$; $d_{sr} = d_{sd} = d_{rd} = 1$. Analysis.

simulated BER performance to validate the analysis. We refer to the considered system as DF IR-SC (decode-and-forward, incremental relaying-selection combining) cooperative relaying scheme. The path loss exponent α is taken to be 3 in all the plots. In Fig. 2, we plot the BER of SSK in DF IR-SC scheme as a function of SNR, obtained through both analysis and simulation, for $n_s = n_r = n_d = 2$, inter-node distances $d_{sr} = d_{sd} = d_{rd} = 1$, and threshold values μ = 6 dB, 10 dB. The performance with no relaying (system with S-to-D link only) is also plotted for comparison. It can be seen that i) the relaying scheme achieves better performance compared to the no-relaying scheme, and ii) the comparison between the analytical and simulation results show an exact match, thus validating the analysis.

Figure 3 shows the BER plots as a function of the threshold μ, for different values of SNRs (6 dB, 9 dB, 12 dB, and 15 dB) and $n_s = n_r = n_d = 2, d_{sr} = d_{sd} = d_{rd} = 1$. The plots show that i) the optimum threshold values that minimize BER vary for varying SNRs, and ii) analysis and simulation results match in this figure as well. In Fig. 4, we plot the analytical BER versus SNR plots for DF IR-SC at optimum threshold values, i,e., at μ_{opt} (optimum μ) for $n_r = n_d = 2, 4$, and $d_{sr} = d_{sd} = d_{rd} = 1$. μ_{opt} values and the corresponding BERs for a given SNR are obtained by numerically evaluating the BER expressions at the given SNR by varying the threshold values in 0.1 dB steps. Also, c_1/SNR^2, c_2/SNR^4, c_3/SNR^8 lines are plotted (c_1, c_2, c_3 are constants). It can be seen that the BER plots for no relaying with $n_d = 2$ and 4 run parallel to the c_1/SNR^2 and c_2/SNR^4 lines at high SNRs indicating 2nd and 4th order diversity. With relaying, on the other hand, the corresponding BER plots with $n_r = n_d = 2$ and 4 run parallel to the c_2/SNR^4 and c_3/SNR^8 lines at high SNRs indicating 4th and 8th order diversity. This shows the cooperative diversity advantage of DF IR-SC in SSK.

V. Conclusions

We analyzed the end-to-end BER performance of SSK in DF incremental relaying with selection combining. Since the information in SSK is carried in the antenna index, we proposed to do selection between direct and relayed paths based on the Euclidean distance between columns of the channel matrix. We used a similar metric for incremental relaying. We derived exact expression for the end-to-end BER in closed-form. Analytical results were validated through simulations.

Appendix I
Derivation of (13)

Expression for (13) is derived as follows.

$$P(E_{sr}) = \int_0^\infty \int_{\sqrt{\rho s}}^\infty \frac{e^{-\frac{u^2}{2}}}{\sqrt{2\pi}} du \frac{s^{n_r-1}\exp(-\frac{s}{2\sigma_{sr}^2})}{(2\sigma_{sr}^2)^{n_r}(n_r-1)!} ds$$
$$= \int_0^\infty \int_0^{u^2/\rho} \frac{s^{n_d-1}\exp(-\frac{s}{2\sigma_{sr}^2})}{(2\sigma_{sr}^2)^{n_r}(n_r-1)!} ds \frac{e^{-\frac{u^2}{2}}}{\sqrt{2\pi}} du$$
$$= \int_0^\infty \left[1 - \sum_{j=0}^{n_r-1} \frac{u^{2j}\exp(-\frac{u^2}{2\rho\sigma_{sr}^2})}{j!(2\rho\sigma_{sr}^2)^j}\right] \frac{e^{-\frac{u^2}{2}}}{\sqrt{2\pi}} du \quad (21)$$
$$= \frac{1}{2} - \sum_{j=0}^{n_r-1} \int_0^\infty \frac{u^{2j}\exp(-\frac{u^2}{2\delta})}{j!(2\rho\sigma_{sr}^2)^j\sqrt{2\pi}} du$$
$$= \frac{1}{2} - \sum_{j=0}^{n_r-1} \frac{1}{j!(2\rho\sigma_{sr}^2)^j} \frac{\sqrt{\delta}}{2} \int_{-\infty}^\infty \frac{u^{2j}\exp(-\frac{u^2}{2\delta})}{\sqrt{2\pi\delta}} du. \quad (22)$$

(21) follows from the definition of the CDF of gamma distribution. Using the expression of the $2j$th central moment of normal r.v. (i.e., $\delta^j \frac{(2j)!}{j!2^j}$) in (22), we get (13).

Appendix II
Derivation of (15)

We have

$$P(E_{rd}|E_{sr},\mathbf{H}_{rd}) = 1 - P(E_{rd}^c|E_{sr},\mathbf{H}_{rd}), \tag{23}$$

where E_{rd}^c is the event of correct detection at D. Denoting $\mathbf{x}_1 = [1,0]^T$, and $\mathbf{x}_2 = [0,1]^T$, we can write by the law of total probability

$$P(E_{rd}^c|E_{sr},\mathbf{H}_{rd}) = \sum_{i=1}^{2}\frac{1}{2}P(E_{rd}^c|E_{sr},\mathbf{H}_{rd},\mathbf{x}_r=\mathbf{x}_i). \tag{24}$$

We first compute $P(E_{rd}^c|E_{sr},\mathbf{H}_{rd},\mathbf{x}_r=\mathbf{x}_1)$ as follows.

$$\begin{aligned} &P(E_{rd}^c|E_{sr},\mathbf{H}_{rd},\mathbf{x}_r=\mathbf{x}_1)\\ &=P(\|\mathbf{y}_{rd}-\mathbf{H}_{rd}\mathbf{x}_1\|^2>\|\mathbf{y}_{rd}-\mathbf{H}_{rd}\mathbf{x}_2\|^2|\mathbf{x}_r=\mathbf{x}_1)\\ &=P\Big(\Re\{(\mathbf{h}_{rd}^2-\mathbf{h}_{rd}^1)^H\mathbf{w}_{rd}\} > \frac{\|\mathbf{h}_{rd}^2-\mathbf{h}_{rd}^1\|^2}{2}\Big). \end{aligned} \tag{25}$$

$\Re\{(\mathbf{h}_{rd}^2-\mathbf{h}_{rd}^1)^H\mathbf{w}_{rd}\}$ is distributed as $\mathcal{CN}(0,\|\mathbf{h}_{rd}^2-\mathbf{h}_{rd}^1\|^2\sigma^2)$. Hence, using (25), we can write

$$P(E_{rd}^c|E_{sr},\mathbf{H}_{rd},\mathbf{x}_r=\mathbf{x}_1) = Q\left(\sqrt{\frac{||\mathbf{h}_{rd}^2-\mathbf{h}_{rd}^1||^2}{2\sigma^2}}\right). \tag{26}$$

Proceeding similarly, we can derive

$$P(E_{rd}^c|E_{sr},\mathbf{H}_{rd},\mathbf{x}_r=\mathbf{x}_2) = Q\left(\sqrt{\frac{||\mathbf{h}_{rd}^2-\mathbf{h}_{rd}^1||^2}{2\sigma^2}}\right). \tag{27}$$

Hence, using (26), (27) and (24), we get

$$P(E_{rd}^c|E_{sr},\mathbf{H}_{rd}) = Q\left(\sqrt{\frac{||\mathbf{h}_{rd}^2-\mathbf{h}_{rd}^1||^2}{2\sigma^2}}\right). \tag{28}$$

Using (23) and (28), we arrive at (15).

Appendix III
Derivation of A_1

Expression for A_1 is derived as follows.

$$\begin{aligned} A_1 &= \int_0^{\frac{\mu}{\rho}} Q\left(\sqrt{\rho s}\right)\int_0^s \frac{r^{n_d-1}\exp(-\frac{r}{2\sigma_{rd}^2})}{(2\sigma_{rd}^2)^{n_d}(n_d-1)!}dr f_{\eta'_{sd}}(s)ds\\ &= \int_0^{\frac{\mu}{\rho}} Q\left(\sqrt{\rho s}\right)\left[1-\sum_{j=0}^{n_d-1}\frac{s^j e^{-\frac{s}{2\sigma_{rd}^2}}}{(2\sigma_{rd}^2)^j j!}\right]f_{\eta'_{sd}}(s)ds \qquad (29)\\ &= \underbrace{\int_0^{\frac{\mu}{\rho}} Q\left(\sqrt{\rho s}\right)\frac{s^{n_d-1}e^{-\frac{s}{2\sigma_{sd}^2}}}{(2\sigma_{sd}^2)^{n_d}(n_d-1)!}ds}_{A_{11}}\\ &\quad-\sum_{j=0}^{n_d-1}\underbrace{\int_0^{\frac{\mu}{\rho}}\frac{Q\left(\sqrt{\rho s}\right)s^{j+n_d-1}e^{-\frac{s}{2\lambda}}}{(2\sigma_{rd}^2)^j(2\sigma_{sd}^2)^{n_d}j!(n_d-1)!}ds}_{A_{12j}}. \qquad (30) \end{aligned}$$

where $\lambda \triangleq \frac{\sigma_{sd}^2\sigma_{rd}^2}{\sigma_{sd}^2+\sigma_{rd}^2}$. We first derive A_{12j} as follows.

$$\begin{aligned} A_{12j} &= B_j\int_0^{\frac{\mu}{\rho}}\int_{\sqrt{\rho s}}^{\infty}\frac{e^{-\frac{u^2}{2}}}{\sqrt{2\pi}}du\frac{s^{j+n_d-1}e^{-\frac{s}{2\lambda}}}{(2\lambda)^{j+n_d}(j+n_d-1)!}ds\\ &= B_j\Big[\underbrace{\int_0^{\sqrt{\mu}}\int_0^{\frac{u^2}{\rho}}\frac{s^{j+n_d-1}e^{-\frac{s}{2\lambda}}}{(2\lambda)^{j+n_d}(j+n_d-1)!}ds\frac{e^{-\frac{u^2}{2}}}{\sqrt{2\pi}}du}_{A_{12j,a}}\\ &\quad\underbrace{-\int_{\sqrt{\mu}}^{\infty}\int_0^{\frac{\mu}{\rho}}\frac{s^{j+n_d-1}e^{-\frac{s}{2\lambda}}}{(2\lambda)^{j+n_d}(j+n_d-1)!}ds\frac{e^{-\frac{u^2}{2}}}{\sqrt{2\pi}}du}_{A_{12j,b}}\Big], \end{aligned} \tag{31}$$

where $B_j = \frac{\lambda^{j+n_d}\binom{j+n_d-1}{j}}{\sigma_{rd}^{2j}\sigma_{sd}^{2n_d}}$. Using the the CDF of gamma distribution in the inner integral of $A_{12j,a}$ in (31) as before, we can write

$$\begin{aligned} A_{12j,a} &= \int_0^{\sqrt{\mu}}\frac{e^{-\frac{u^2}{2}}}{\sqrt{2\pi}}du - \int_0^{\sqrt{\mu}}\sum_{i=0}^{j+n_d-1}\frac{u^{2i}e^{-\frac{u^2}{2\tau}}}{(2\rho\lambda)^i i!\sqrt{2\pi}}du\\ &= \Big[\int_0^{\infty}\frac{e^{-\frac{u^2}{2}}}{\sqrt{2\pi}}du - \int_{\sqrt{\mu}}^{\infty}\frac{e^{-\frac{u^2}{2}}}{\sqrt{2\pi}}du\Big] - \sum_{i=0}^{j+n_d-1}\frac{1}{(2\rho\lambda)^i i!}\\ &\quad\Big[\int_0^{\infty}\frac{u^{2i}e^{-\frac{u^2}{2\tau}}du}{\sqrt{2\pi}} - \int_{\sqrt{\mu}}^{\infty}\frac{u^{2i}e^{-\frac{u^2}{2\tau}}du}{\sqrt{2\pi}}\Big], \end{aligned} \tag{32}$$

where $\tau \triangleq \frac{\rho\lambda}{\rho\lambda+1}$. Using the expression of $2i$th central moment of normal r.v. $\big($i.e., $\tau^i\frac{(2i)!}{i!2^i}$ $\big)$, we can write

$$\begin{aligned} \int_0^{\infty}\frac{u^{2i}e^{-\frac{u^2}{2\tau}}du}{\sqrt{2\pi}} &= \frac{\sqrt{\tau}}{2}\int_{-\infty}^{\infty}\frac{u^{2i}e^{-\frac{u^2}{2\tau}}du}{\sqrt{2\pi\tau}}\\ &= \tau^{i+\frac{1}{2}}\frac{(2i)!}{i!2^{i+1}}. \end{aligned} \tag{33}$$

Substituting u with $z=\frac{u}{\tau}$ in the integral $\int_{\sqrt{\mu}}^{\infty}u^{2i}e^{-\frac{u^2}{2\tau}}du$ in (31), we get

$$\begin{aligned} \int_{\sqrt{\mu}}^{\infty}u^{2i}e^{-\frac{u^2}{2\tau}}du &= \tau^{i+\frac{1}{2}}\int_{\sqrt{\frac{\mu}{\tau}}}^{\infty}z^{2i}e^{-\frac{z^2}{2}}dz\\ &= \tau^{i+\frac{1}{2}}I^{(i)}\Big(\sqrt{\frac{\mu}{\tau}}\Big). \end{aligned} \tag{34}$$

Using the derivations of (33) and (34) in (32), we get the expression of $A_{12j,a}$ as

$$A_{12j,a} = \frac{1}{2} - Q(\sqrt{\mu}) - \sum_{i=0}^{j+n_d-1}\frac{\tau^{i+\frac{1}{2}}}{(2\rho\lambda)^i i!}\left\{\frac{(2i)!}{i!2^{i+1}} - \frac{I^{(i)}\big(\sqrt{\frac{\mu}{\tau}}\big)}{\sqrt{2\pi}}\right\}. \tag{35}$$

Next, from the gamma distribution CDF, we can write $A_{12j,b}$ as

$$A_{12j,b} = Q(\sqrt{\mu})\Big[1-\sum_{l=0}^{j+n_d-1}\frac{\mu^l e^{-\frac{\mu}{2\rho\lambda}}}{(2\rho\lambda)^l l!}\Big]. \tag{36}$$

The integral A_{11} can be derived similarly as A_{12j}, and the corresponding final expression is

$$A_{11} = \frac{1}{2} - Q(\sqrt{\mu}) - \sum_{t=0}^{n_d-1} \frac{\omega^{t+\frac{1}{2}}}{(2\rho\sigma_{sd}^2)^t t!}\left[\frac{(2t)!}{t!2^{t+1}} - \frac{I^{(t)}\left(\sqrt{\frac{\mu}{\omega}}\right)}{\sqrt{2\pi}}\right] + Q(\sqrt{\mu})\left[1 - \sum_{q=0}^{n_d-1} \frac{\mu^q e^{-\frac{\mu}{2\rho\sigma_{sd}^2}}}{(2\rho\sigma_{sd}^2)^q q!}\right]. \quad (37)$$

Substituting the derivation of $A_{12j,a}$ and $A_{12j,b}$ from (35) and (36), respectively, in (31), we get the expression of A_{12j}. Subsequently, combining the the expression of A_{11} in (37) and the expression of A_{12j} thus obtained, we get A_1 as

$$A_1 = \frac{1}{2} - Q(\sqrt{\mu}) - \sum_{t=0}^{n_d-1} \frac{\omega^{t+\frac{1}{2}}}{(2\rho\sigma_{sd}^2)^t t!}\left[\frac{(2t)!}{t!2^{t+1}} - \frac{I^{(t)}\left(\sqrt{\frac{\mu}{\omega}}\right)}{\sqrt{2\pi}}\right] + Q(\sqrt{\mu})\left[1 - \sum_{q=0}^{n_d-1} \frac{\mu^q e^{-\frac{\mu}{2\rho\sigma_{sd}^2}}}{(2\rho\sigma_{sd}^2)^q q!}\right] - \sum_{j=0}^{n_d-1} B_j \left[\frac{1}{2} - Q(\sqrt{\mu}) - \sum_{i=0}^{j+n_d-1} \frac{\tau^{i+\frac{1}{2}}}{(2\rho\lambda)^i i!}\left\{\frac{(2i)!}{i!2^{i+1}} - \frac{I^{(i)}\left(\sqrt{\frac{\mu}{\tau}}\right)}{\sqrt{2\pi}}\right\} - Q(\sqrt{\mu})\left\{1 - \sum_{l=0}^{j+n_d-1} \frac{\mu^l e^{-\frac{\mu}{2\rho\lambda}}}{(2\rho\lambda)^l l!}\right\}\right]. \quad (38)$$

Appendix IV
Derivation of A_2

We consider the inner integral of A_2, i.e., $\int_s^\infty Q(\sqrt{\rho r}) f_{\eta'_{rd}}(r)dr$. Simplifying the above integral following the steps that involve interchange of the order of the integral and use of CDF of gamma distribution, we can write A_2 as

$$A_2 = \underbrace{\sum_{j=0}^{n_d-1} \int_0^{\frac{\mu}{\rho}} \frac{Q\left(\sqrt{\rho s}\right) s^{j+n_d-1} e^{-\frac{s}{2\lambda}}}{(2\sigma_{rd}^2)^j (2\sigma_{sd}^2)^{n_d} j!(n_d-1)!} ds}_{A_{21}} - \underbrace{\int_0^{\frac{\mu}{\rho}} \sum_{i=0}^{n_d-1} \int_{\sqrt{\rho s}}^\infty \frac{u^{2i}\exp(-\frac{u^2}{2\zeta})du}{(2\rho\sigma_{rd}^2)^i i!\sqrt{2\pi}} f_{\eta'_{sd}}(s)ds}_{A_{22}}, \quad (39)$$

where $\zeta = \frac{\rho\sigma_{rd}^2}{\rho\sigma_{rd}^2+1}$. Comparing (30) and (39), it can be seen that $A_{21} = \sum_{j=0}^{n_d-1} A_{12j}$. Hence, A_{21} can be derived from (35) and (36). To derive A_{22} in (39), we first substitute u with $\sqrt{\zeta}z$. The inner integral of A_{22}can be simplified as

$$\int_{\sqrt{\rho s}}^\infty \frac{u^{2i}\exp(-\frac{u^2}{2\zeta})du}{(2\rho\sigma_{rd}^2)^i i!\sqrt{2\pi}} = \frac{\zeta^{i+\frac{1}{2}}}{(2\rho\sigma_{rd}^2)^i i!\sqrt{2\pi}} I^{(i)}\left(\sqrt{\frac{\rho s}{\zeta}}\right). \quad (40)$$

Expanding $I^{(i)}(\sqrt{\frac{\rho s}{\zeta}})$ using lemma 1 in (40), A_{22} becomes

$$A_{22} = H\underbrace{\int_0^{\frac{\mu}{\rho}} Q\left(\sqrt{\frac{\rho s}{\zeta}}\right) f_{\eta'_{sd}}(s)ds}_{A_{221}} + \sum_{p=1}^{n_d-1} E_p \sum_{k=1}^{p} J_k \underbrace{\int_0^{\frac{\mu}{\rho}} s^{k-\frac{1}{2}} e^{-\frac{\rho s}{2\zeta}} f_{\eta'_{sd}}(s)ds}_{A_{22,k}}, \quad (41)$$

where $H = \sum_{i=0}^{n_d-1} \frac{\zeta^{i+\frac{1}{2}}(2i)!}{(4\rho\sigma_{rd}^2)^i (i!)^2}$, $E_p = \frac{\zeta^{p+\frac{1}{2}}\binom{2p}{p}}{(4\rho\sigma_{rd}^2)^i\sqrt{2\pi}}$, $J_k = \frac{\rho^{k-\frac{1}{2}}}{\zeta^{k-\frac{1}{2}}(2k-1)!!}$. A_{221} in (41) can be derived in a similar way as A_{12j}, and can be written as

$$A_{221} = \frac{1}{2} - Q\left(\sqrt{\frac{\mu}{\zeta}}\right) - \sum_{q_1=0}^{n_d-1} \frac{\zeta^{q_1}\chi^{q_1+\frac{1}{2}}}{(2\rho\sigma_{sd}^2)^{q_1} q_1!}\left\{\frac{(2q_1)!}{q_1!2^{q_1+1}} - \frac{I^{(q_1)}\left(\sqrt{\frac{\mu}{\zeta\chi}}\right)}{\sqrt{2\pi}}\right\} + Q\left(\sqrt{\frac{\mu}{\zeta}}\right)\left[1 - \sum_{q_2=0}^{n_d-1} \frac{\mu^{q_2} e^{-\frac{\mu}{2\rho\sigma_{sd}^2}}}{(2\rho\sigma_{sd}^2)^{q_2} q_2!}\right], \quad (42)$$

where $\chi = \frac{\rho\sigma_{sd}^2}{\rho\sigma_{sd}^2+\zeta}$. The substitution of s with $\frac{z}{v}$ in $A_{22,k}$ in (41), where $v = \frac{\rho\sigma_{sd}^2+\zeta}{2\zeta\sigma_{sd}^2}$, gives

$$A_{22,k} = \frac{\Gamma_i(\frac{v\mu}{\rho}, n_d+k-\frac{1}{2})}{(2\sigma_{sd}^2)^{n_d}(n_d-1)! v^{n_d+k-\frac{1}{2}}}, \quad (43)$$

where $\Gamma_i(m,n) = \int_0^m e^{-z} z^{n-1} dz$ is the incomplete gamma function. Substituting the derivation of A_{221} and $A_{22,k}$ from (42) and (43), respectively, in (41), we get the expression of A_{22}. Subsequently combining the the expression of A_{21} derived from (31), (35) and (36), and the expression of A_{22} thus obtained, and substituting them in (39), we get A_2 as

$$A_2 = \sum_{j=0}^{n_d-1} B_j \left[\frac{1}{2} - Q(\sqrt{\mu}) - \sum_{t=0}^{j+n_d-1} \frac{\tau^{t+\frac{1}{2}}}{(2\rho\lambda)^t t!}\left\{\frac{(2t)!}{t!2^{t+1}} - \frac{I^{(t)}\left(\sqrt{\frac{\mu}{\tau}}\right)}{\sqrt{2\pi}}\right\} - Q(\sqrt{\mu})\left\{1 - \sum_{l=0}^{j+n_d-1} \frac{\mu^l e^{-\frac{\mu}{2\rho\lambda}}}{(2\rho\lambda)^l l!}\right\}\right] - HA_{221} + \sum_{i=1}^{n_d-1} E_i \sum_{k=1}^{i} J_k A_{22,k}, \quad (44)$$

where A_{221} and $A_{22,k}$ is given in (42) and (43), respectively.

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