SIR-Optimized Weighted Linear Parallel Interference Canceller on Fading Channels

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***Abstract*— In this letter, we present a weighted linear parallel interference canceller (LPIC) where the multiple access interference (MAI) estimate in a stage is weighted by a factor before cancellation on Rayleigh fading and diversity channels. We obtain exact expressions for the average signal-to-interference ratio (SIR) at the output of the cancellation stages which we maximize to obtain the optimum weights for different stages. We also obtain closed-form expressions for the optimum weights for the different stages. We show that this SIR-optimized weighted LPIC scheme clearly outperforms both the matched filter (MF) detector as well as the conventional LPIC (where the weight is taken to be unity for all stages), in both near-far as well as non-near-far conditions on Rayleigh fading and diversity channels.**

***Index Terms*— Linear parallel interference cancellation, signal-to-interference ratio, fading channels.**

I. Introduction

PARALLEL interference cancellation (PIC) is a multiuser detection scheme where a desired user's decision statistic is obtained by subtracting an estimate of the multiple access interference (MAI) from the received signal [1]. PIC lends itself to a multistage implementation where the decision statistics of the users from the previous stage are used to estimate and cancel the MAI in the current stage, and a final decision statistic is obtained at the last stage. When an estimate of the MAI is obtained from the hard bit decisions from the previous stage, it is termed as 'hard-decision PIC' (non-linear PIC). The multistage PIC scheme originally proposed by Varanasi and Aazhang in [2] and several other schemes considered in the literature (e.g., [3]) are of this type. On the other hand, MAI estimates can be obtained using the soft values of the decision statistics from the previous stage, in which case the PIC is termed as linear PIC (LPIC) [4],[5]. LPICs have the advantages of implementation simplicity, analytical tractability, and good performance under certain conditions.

In a conventional LPIC, an estimate of the MAI for a desired user in a stage is obtained using all the other users' soft outputs from the previous stage. It is likely that these MAI estimates are inaccurate due to poor channel conditions (e.g., high interference, deep fades, etc.). Under such conditions, the cancellation can become ineffective to an extent that it may be better not to do cancellation. In fact, it has been known that the conventional LPIC performs worse than the MF detector (where no cancellation is done) at low SNRs, due to the poor accuracy of the MAI estimates at low SNRs. This can be alleviated by properly weighing the MAI estimates before cancellation [5]. A key question is how to choose the weights for different cancellation stages. An intuitive approach is to keep the value of the weight low at the early stages and large at the later stages, as done in [5], because the MAI estimates can be more reliable in the later stages since much of the interference would have been cancelled by then. A more formal approach, which we adopt in this letter, is to obtain appropriate functions (e.g., expressions for interference variance or SIR) which when optimized will give the optimum weights.

The issue of the choice of weights in LPIC has been addressed in [6]-[8], but only for AWGN channels. A new contribution in this letter is that we derive closed-form expressions for optimum weights for different stages in an LPIC on Rayleigh fading and diversity channels. Also, our approach to obtain the optimum weights is that we derive exact expressions for the average SIR at the output of the cancellation stages of the weighted LPIC, and maximize these SIR expressions to obtain the optimum weights for the different stages. In fact, we obtain exact closed-form expressions for the optimum weights for each stage of the LPIC. We show that the proposed SIR-optimized weighted LPIC clearly outperforms the MF detector and the conventional LPIC, in both near-far as well as non-near-far scenarios on Rayleigh and diversity channels.

II. System Model

Consider a K-user synchronous CDMA system where the received signal is given by

$$y(t)=\sum_{k=1}^{K} A_k h_k b_k s_k(t)+n(t), \qquad t\in[0,T], \qquad (1)$$

where $b_k \in \{+1,-1\}$ is the bit transmitted by the kth user, T is one bit duration, A_k is the transmit amplitude of the kth user's signal, h_k is the complex channel fade coefficient corresponding to the kth user, $s_k(t)$ is the unit energy spreading waveform of the kth user defined in the interval $[0,T]$, i.e., $\int_0^T s_k^2(t)dt = 1$, and $n(t)$ is the white Gaussian noise with zero mean and variance σ^2. The fade coefficients h_k's are assumed to be i.i.d complex Gaussian r.v's (i.e., fade amplitudes are Rayleigh distributed) with zero mean and $E[h_{kI}^2]=E[h_{kQ}^2]=1$, where h_{kI} and h_{kQ} are the

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real and imaginary parts of h_k. The channel fade is assumed to remain constant over one bit interval.

We consider a multistage LPIC at the receiver. The first stage is a conventional MF, which is a bank of K correlators, each matched to a different user's spreading waveform. The received vector $\mathbf{y}^{(1)}$ at the output of the MF stage (the superscript (1) in $\mathbf{y}^{(1)}$ denotes the first stage) is given by

$$\mathbf{y}^{(1)} = \left[y_1^{(1)}, y_2^{(1)}, \cdots, y_K^{(1)}\right], \quad (2)$$

where $y_k^{(1)}$ is the kth user's MF output, given by

$$y_k^{(1)} = A_k h_k b_k + \sum_{j=1,\, j\neq k}^{K} \rho_{jk} A_j h_j b_j + n_k, \quad (3)$$

where ρ_{jk} is the cross-correlation coefficient between the jth and kth users' spreading waveforms, given by $\rho_{jk} = \int_0^T s_j(t)s_k(t)dt$, $|\rho_{jk}| \leq 1$, and n_k's are complex Gaussian with zero mean and $E[n_j n_k^*] = 2\sigma^2$ when $j = k$ and $E[n_j n_k^*] = 2\sigma^2 \rho_{jk}$ when $j \neq k$. The received vector $\mathbf{y}^{(1)}$ (without hard decision) is used for MAI estimation and cancellation in the second stage.

A. Conventional LPIC

In a conventional LPIC, an estimate of the MAI for a desired user in the current stage is obtained using all the other users' soft outputs from the previous stage for cancellation in the current stage. More specifically, the MAI estimate for the desired user k in stage m, $m > 1$, is obtained by multiplying $y_j^{(m-1)}$ with ρ_{jk} for all $j \neq k$ and summing them up, i.e., $\sum_{j\neq k} \rho_{jk} y_j^{(m-1)}$ is the MAI estimate for the desired user k in stage m. Accordingly, the bit decision for the kth user after interference cancellation in the mth stage, $\widehat{b}_k^{(m)}$, in conventional LPIC is given by

$$\widehat{b}_k^{(m)} = \mathrm{sgn}\left(\mathrm{Re}\left(h_k^*\left(y_k^{(1)} - \sum_{j=1,\, j\neq k}^{K} \rho_{jk} y_j^{(m-1)}\right)\right)\right). \quad (4)$$

III. Weighted LPIC

In a weighted PIC, the MAI estimate of the desired user k in stage m, $m > 1$, is weighted by a factor $p_k^{(m)}$ before cancellation. In other words, $p_k^{(m)} \sum_{j\neq k} \rho_{jk} y_j^{(m-1)}$ is the weighted MAI estimate for the desired user k in stage m. That is, the mth stage output of the desired user k, $y_k^{(m)}$, is given by

$$y_k^{(m)} = y_k^{(1)} - p_k^{(m)} \sum_{j=1,\, j\neq k}^{K} \rho_{jk} y_j^{(m-1)}. \quad (5)$$

Note that both the conventional LPIC as well as the MF detector become special cases of the weighted LPIC for $p_k^{(m)} = 1, \forall m$ and $p_k^{(m)} = 0, \forall m$, respectively. The bit decision for the desired user k after weighted interference cancellation in stage m is

$$\widehat{b}_k^{(m)} = \mathrm{sgn}\left(\mathrm{Re}\left(h_k^* y_k^{(m)}\right)\right). \quad (6)$$

In the following, we obtain exact expressions for the average SIRs at the output of the different stages of the weighted LPIC, which are then used to obtain closed-form expressions for the optimum weights for the different stages.

A. Average SIR at 2nd Stage Output

The weighted interference cancelled output of the 2nd stage for the desired user k is given by

$$\begin{aligned} y_k^{(2)} &= y_k^{(1)} - p_k^{(2)} \sum_{j=1,\, j\neq k}^{K} \rho_{jk} y_j^{(1)} \\ &= A_k h_k b_k \left(1 - p_k^{(2)} \sum_{j=1,\, j\neq k}^{K} \rho_{jk}^2\right) + I_2 + N_2, \end{aligned} \quad (7)$$

where

$$\begin{aligned} I_2 &= \left(1 - p_k^{(2)}\right) \sum_{j=1,\, j\neq k}^{K} A_j h_j b_j \rho_{jk} \\ &\quad - \sum_{j=1,\, j\neq k}^{K} p_k^{(2)} \rho_{jk} \sum_{\substack{i=1 \\ i\neq j,k}}^{K} \rho_{ij} A_i h_i b_i, \end{aligned} \quad (8)$$

$$N_2 = n_k - p_k^{(2)} \sum_{j=1,\, j\neq k}^{K} \rho_{jk} n_j. \quad (9)$$

The terms I_2 and N_2 in (7) represent the interference and noise terms introduced due to imperfect cancellation in using the soft output values from the MF stage. Since h's are complex Gaussian, both I_2 and N_2 are linear combinations of Gaussian r.v's with zero mean. The variance of I_2, $\sigma_{I_2}^2$, can be obtained as

$$\sigma_{I_2}^2 = \sum_{i=1,\, i\neq k}^{K} 2A_i^2 \left(\left(1 - p_k^{(2)}\right)\rho_{ik} - p_k^{(2)} \sum_{\substack{j=1 \\ j\neq k,i}}^{K} \rho_{jk}\rho_{ij}\right)^2, \quad (10)$$

and the variance of N_2, $\sigma_{N_2}^2$, can be obtained as

$$\sigma_{N_2}^2 = 2\sigma^2 \left(1 - 2p_k^{(2)} \sum_{\substack{j=1 \\ j\neq k}}^{K} \rho_{jk}^2 + \left(p_k^{(2)}\right)^2 \sum_{\substack{i=1 \\ i\neq k}}^{K} \sum_{\substack{j=1 \\ j\neq k}}^{K} \rho_{ik}\rho_{jk}\rho_{ji}\right). \quad (11)$$

The average SIR of the desired user k at the output of the second stage, $\overline{SIR}_k^{(2)}$, is then given by

$$\overline{SIR}_k^{(2)} = \frac{2A_k^2 \left(1 - p_k^{(2)} \sum_{\substack{j=1 \\ j\neq k}}^{K} \rho_{jk}^2\right)^2}{\sigma_{I_2}^2 + \sigma_{N_2}^2}. \quad (12)$$

The optimum weight for the second stage, $p_{k,opt}^{(2)}$, is chosen to be the value of $p_k^{(2)}$ that maximizes the average SIR in (12). In Sec. IV, we present a closed-form expression for $p_{k,opt}^{(2)}$.

B. Average SIR at 3rd Stage Output

The soft values of the interference cancelled outputs of all the other users from the second stage are used to reconstruct (estimate) the MAI for the desired user k in the third stage. The MAI estimate is then weighted by the factor $p_k^{(3)}$ and cancelled. The third stage output of the desired user k, $y_k^{(3)}$, is then given by

$$\begin{aligned} y_k^{(3)} &= y_k^{(1)} - p_k^{(3)} \sum_{j=1,j\neq k}^{K} \rho_{jk} y_j^{(2)} \\ &= A_k h_k b_k X + I_3 + N_3, \end{aligned} \tag{13}$$

where

$$\begin{aligned} X &= 1 - p_k^{(3)} \sum_{\substack{j=1\\j\neq k}}^{K} \rho_{jk}^2 \big(1 - p_j^{(2)}\big) \\ &\quad + p_k^{(3)} \sum_{\substack{j=1\\j\neq k}}^{K} \rho_{jk} p_j^{(2)} \sum_{\substack{i=1\\i\neq j,k}}^{K} \rho_{ij}\rho_{ki}, \end{aligned} \tag{14}$$

$$\begin{aligned} I_3 &= \sum_{\substack{l=1\\l\neq k}}^{K} \rho_{lk} A_l b_l h_l \left(1 - p_k^{(3)}\Big(1 - p_l^{(2)} \sum_{\substack{j=1\\j\neq l}}^{K} \rho_{jl}^2\Big)\right) \\ &\quad + p_k^{(3)} \sum_{\substack{l=1\\l\neq k}}^{K} \rho_{lk} p_l^{(2)} \sum_{\substack{i=1\\i\neq l}}^{K} \rho_{il} \sum_{\substack{j=1\\j\neq l,i,k}}^{K} \rho_{ji} A_j h_j b_j \\ &\quad - p_k^{(3)} \sum_{\substack{l=1\\l\neq k}}^{K} \big(1 - p_l^{(2)}\big)\rho_{lk} \sum_{\substack{j=1\\j\neq l,k}}^{K} \rho_{jl} A_j h_j b_j, \end{aligned} \tag{15}$$

$$N_3 = n_k - p_k^{(3)} \sum_{\substack{l=1\\l\neq k}}^{K} \rho_{lk} \Big(n_l - p_l^{(2)} \sum_{\substack{j=1\\j\neq l}}^{K} \rho_{jl} n_j\Big). \tag{16}$$

The terms N_3 and I_3 are linear combinations of Gaussian r.v's with zero mean and variances $\sigma_{N_3}^2$ and $\sigma_{I_3}^2$, given by

$$\begin{aligned} \sigma_{N_3}^2 &= 2\sigma^2 \Big(1 + p_k^{(3)} \sum_{\substack{l=1\\l\neq k}}^{K} p_l^{(2)} \rho_{lk}^2\Big)^2 \\ &\quad + 2 p_k^{(3)} \sigma^2 \sum_{\substack{i=1\\i\neq k}}^{K} \rho_{ik} \Big(1 + p_k^{(3)} \sum_{\substack{l=1\\l\neq k}}^{K} p_l^{(2)} \rho_{lk}^2\Big) \\ &\qquad \cdot \Big(-\rho_{ik} + \sum_{\substack{j=1\\j\neq k,i}}^{K} p_j^{(2)} \rho_{jk}\rho_{ij}\Big) \\ &\quad + \Big(p_k^{(3)}\Big)^2 \sigma^2 \sum_{\substack{i=1\\i\neq k}}^{K} \sum_{\substack{l=1\\l\neq k}}^{K} \rho_{il} \Big(-\rho_{ik} + \sum_{\substack{j=1\\j\neq k,i}}^{K} p_j^{(2)} \rho_{jk}\rho_{ij}\Big) \\ &\qquad \cdot \Big(-\rho_{lk} + \sum_{\substack{j=1\\j\neq k,l}}^{K} p_j^{(2)} \rho_{jk}\rho_{lj}\Big), \end{aligned} \tag{17}$$

$$\begin{aligned} \sigma_{I_3}^2 &= \sum_{\substack{l=1\\l\neq k}}^{K} 2A_l^2 \Bigg[p_k^{(3)} \sum_{\substack{j=1\\j\neq l,k}}^{K} \rho_{jk} p_j^{(2)} \sum_{\substack{i=1\\i\neq l,j}}^{K} \rho_{ij}\rho_{li} \\ &\quad - p_k^{(3)} \sum_{\substack{j=1\\j\neq k,l}}^{K} \rho_{lj}\rho_{jk}\big(1 - p_j^{(2)}\big) \\ &\quad + \rho_{lk}\left(1 - p_k^{(3)}\Big(1 - p_l^{(2)} \sum_{\substack{j=1\\j\neq l}}^{K} \rho_{jl}^2\Big)\right)\Bigg]^2. \end{aligned} \tag{18}$$

The average SIR of the desired user k at the third stage output, $\overline{SIR}_k^{(3)}$, is then given by

$$\overline{SIR}_k^{(3)} = \frac{2A_k^2 X^2}{\sigma_{I_3}^2 + \sigma_{N_3}^2}. \tag{19}$$

The optimum weight for the third stage, $p_{k,opt}^{(3)}$, is chosen to be the value of $p_k^{(3)}$ that maximizes the average SIR in (19). In the next section, we present a closed-form expression for $p_{k,opt}^{(3)}$.

IV. Optimum Weights in Closed-form

Expressions for the optimum weights $p_{k,opt}^{(2)}$ and $p_{k,opt}^{(3)}$ can be obtained by differentiating (12) and (19) w.r.t. $p_k^{(2)}$ and $p_k^{(3)}$, respectively, and equating to zero. Accordingly, we obtain the expression for $p_{k,opt}^{(2)}$, in closed-form, as

$$p_{k,opt}^{(2)} = \frac{c_1(1-a_1) + e_1}{-a_1(c_1+e_1) + c_1 + d_1 + 2\,e_1 - \sigma^2(a_1^2 - f_1)}, \tag{20}$$

where

$$a_1 = \sum_{\substack{j=1\\j\neq k}}^{K} \rho_{jk}^2, \qquad c_1 = \sum_{\substack{l=1\\l\neq k}}^{K} A_l^2 \rho_{lk}^2,$$

$$d_1 = \sum_{\substack{l=1\\l\neq k}}^{K} A_l^2 \Bigg(\sum_{\substack{j=1\\j\neq k,l}}^{K} \rho_{jk}\rho_{lj}\Bigg)^2,$$

$$e_1 = \sum_{\substack{l=1\\l\neq k}}^{K} A_l^2 \rho_{lk} \sum_{\substack{j=1\\j\neq k,l}}^{K} \rho_{jk}\rho_{lj}, \qquad f_1 = \sum_{\substack{j=1\\j\neq k}}^{K} \rho_{jk} \sum_{\substack{i=1\\i\neq k}}^{K} \rho_{ij}\rho_{ik}.$$

Likewise, the closed-form expression for $p_{k,opt}^{(3)}$ can be obtained as

$$p_{k,opt}^{(3)} = \frac{-2\,a_2\,g_2 - f_2 - \sigma^2(2\,a_2 + t_2)}{a_2 f_2 + 2\,e_2 + \sigma^2(a_2\,t_2 + 2\,v_2)}, \tag{21}$$

where

$$a_2 = \sum_{\substack{j=1\\j\neq k}}^{K} \rho_{jk}^2 \big(1 - p_j^{(2)}\big) - \sum_{\substack{j=1\\j\neq k}}^{K} \rho_{jk} p_j^{(2)} \sum_{\substack{r=1\\r\neq j,k}}^{K} \rho_{rj}\rho_{rk},$$

$$e_2 = \sum_{\substack{l=1\\l\neq k}}^{K} A_l^2 \Big(\rho_{lk}^2 w_2^2 + (c_2 - d_2)^2 + 2\rho_{lk} w_2 (c_2 - d_2)\Big),$$

$$f_2 = \sum_{\substack{l=1\\l\neq k}}^{K} A_l^2 \Big(-2\rho_{lk}^2 w_2 - 2\rho_{lk}(c_2 - d_2)\Big), \quad g_2 = \sum_{\substack{l=1\\l\neq k}}^{K} A_l^2 \rho_{lk}^2,$$

$$w_2 = 1 - p_l^{(2)} \sum_{\substack{q=1\\q\neq l}}^{K} \rho_{ql}^2, \qquad c_2 = \sum_{\substack{j=1\\j\neq k,l}}^{K} \rho_{jk}\rho_{lj}\big(1 - p_j^{(2)}\big),$$

$$d_2 = \sum_{\substack{j=1\\j\neq k,l}}^{K} \rho_{jk} p_j^{(2)} \sum_{\substack{r=1\\r\neq j,l}}^{K} \rho_{rj}\rho_{rl}, \qquad t_2 = 2(u_1 + z_1),$$

$$z_1 = \sum_{\substack{i=1\\i\neq k}}^{K} \rho_{ik} \Big(-\rho_{ik} + \sum_{\substack{j=1\\j\neq k,i}}^{K} p_j^{(2)} \rho_{jk}\rho_{ij}\Big),$$

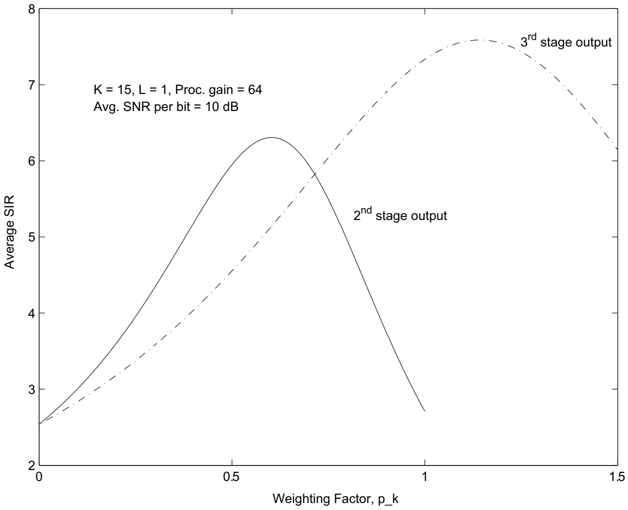

Fig. 1. Average SIR at the 2nd ($m = 2$) and the 3rd stage ($m = 3$) outputs of the desired user ($k = 1$) as a function of the weight, $p_k^{(m)}$. $K = 15$. Processing gain = 64. Random spreading sequences. Average SNR per bit = 10 dB.

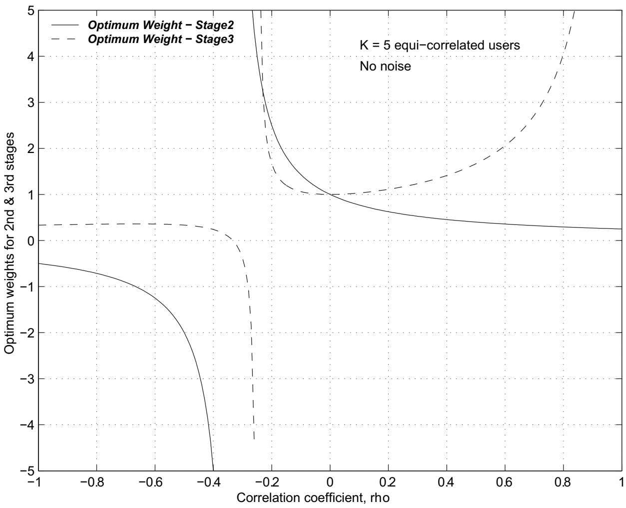

Fig. 2. Optimum weights for the 2nd and 3rd stages, $p_{k,opt}^{(2)}$ and $p_{k,opt}^{(3)}$, for the desired user $k = 1$, as a function of correlation coefficient, ρ. $K = 5$ equi-correlated users ($\rho_{ij} = \rho, \forall\, i, j$) with equal transmit amplitudes ($A_j = A, \forall\, j$) and no noise ($\sigma^2 = 0$).

$$u_1 = \sum_{\substack{l=1\\ l\neq k}} p_l^{(2)} \rho_{lk}^2 \,, \qquad v_2 = u_1^2 + 2u_1 z_1 + t_1 \,,$$

$$t_1 = \sum_{\substack{i=1\\ i\neq k}}^{K} \sum_{\substack{l=1,\\ l\neq k}}^{K} \rho_{il} \Big(-\rho_{ik} + \sum_{\substack{j=1\\ j\neq k,i}}^{K} p_j^{(2)} \rho_{jk}\rho_{ij} \Big) \cdot \Big(-\rho_{lk} + \sum_{\substack{j=1\\ j\neq k,l}}^{K} p_j^{(2)} \rho_{jk}\rho_{lj} \Big) .$$

As can be seen from (20) and (21), the complexity of optimum weights calculation for second and third stages, in terms of number of multiplication/addition operations, is of order K^2 and K^4, respectively. In Fig. 1, using Eqns. (12) and (19), we plot the average SIR at the second ($m = 2$) and third ($m = 3$) stage outputs of the weighted LPIC for the desired user ($k = 1$), as a function of the weight $p_k^{(m)}$ for the case of $K = 15$ users using random spreading sequences with a processing gain of 64 and an average SNR per bit of 10 dB. It is noted that the maximum interference cancelled output SIR increases as the number of stages increases (i.e., $\overline{SIR}_{k,opt}^{(3)} > \overline{SIR}_{k,opt}^{(2)}$), which is intuitively expected. Also, it can be seen from (20) and (21) that the optimum weights depend on the correlation coefficients (ρ_{ij}'s), number of users (K), and the SNRs (A_j's and σ^2). It can be further noted that, since the MAI estimates obtained using the soft outputs are imperfect, the optimum weights can be greater than unity depending on the values of the above parameters. For example, for the case of equi-correlated users ($\rho_{ij} = \rho, \forall\, i, j$) with equal transmit amplitudes ($A_j = A, \forall j$) and ignoring thermal noise ($\sigma^2 = 0$), it can be shown by simplifying (20) that $p_{k,opt}^{(2)} = [1 + (K-2)\rho]^{-1}$. This implies that the optimum weight $p_{k,opt}^{(2)}$ is $i)$ equal to 1 for $\rho = 0$ and also for $K = 2$ users[1], $ii)$ $0 < p_{k,opt}^{(2)} < 1$ for positive correlation (i.e, $0 < \rho \leq 1$), and $iii)$ $p_{k,opt}^{(2)} > 1$ for negative correlation values in the range $-(K-2)^{-1} < \rho < 0$. Likewise, for the same system scenario in the above, it can be shown by simplifying (21) that, for the 3rd stage, $p_{k,opt}^{(3)} = \Big[1 + \rho \Big\{ (1 - p_{k,opt}^{(2)})(K-2) - p_{k,opt}^{(2)}(K^2 - 3K + 3)\rho \Big\} \Big]^{-1}$. From the above expression for $p_{k,opt}^{(3)}$, it can be seen that $p_{k,opt}^{(3)} > 1$ for $0 < \rho \leq 1$. In Fig. 2, we plot the variation of $p_{k,opt}^{(2)}$ and $p_{k,opt}^{(3)}$ as a function of ρ, which shows the regions where the optimum weights can be larger than 1 in the above example. This helps to explain why $p_{k,opt}^{(3)}$ is larger than 1 in Fig. 1.

A. *Probability of Bit Error*

The probability of bit error for the desired user k at the output of the mth stage can be obtained in terms of the optimized SIR as

$$P_k^{(m)} = \frac{1}{2}\left(1 - \sqrt{\frac{\overline{SIR}_{k,opt}^{(m)}}{1 + \overline{SIR}_{k,opt}^{(m)}}}\right). \qquad (22)$$

where $\overline{SIR}_{k,opt}^{(m)}$ is the output SIR when the optimum weight $p_{k,opt}^{(m)}$ is used.

Suppose we consider receive diversity with L equal-energy i.i.d paths. In this case, cancellation is done on each path and the resulting outputs are coherently combined. Accordingly, the bit decision for the desired user k in stage m is given by

$$\hat{b}_k^{(m)} = \operatorname{sgn}\left(\operatorname{Re}\left(\sum_{l=1}^{L} h_k^{l\,*} y_k^{(m)_l}\right)\right), \qquad (23)$$

where h_k^l denotes the kth user's complex channel coefficient on the lth receive antenna path, and $y_k^{(m)_l}$ denotes the kth user's interference cancelled output of the mth stage on the

[1]It is easy to see that for the system with two equi-correlated users and no thermal noise, the other user interference is perfectly cancelled and hence the optimum weight is unity in that case.

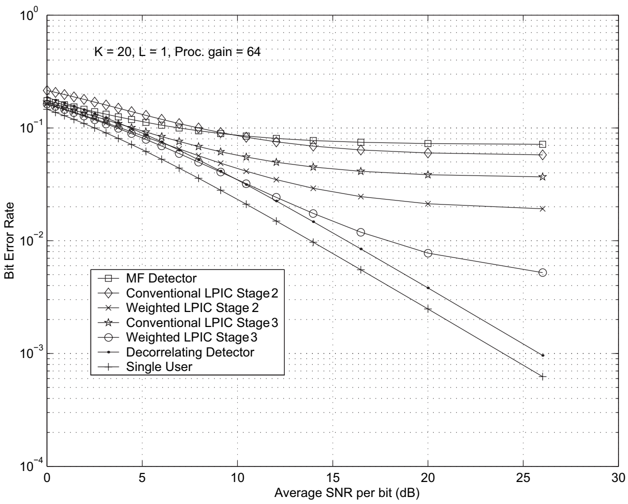

Fig. 3. Comparison of the BER performance of the SIR-optimized weighted LPIC as a function of SNR with that of the MF detector and the conventional LPIC on flat Rayleigh fading channels ($L = 1$). $K = 20$. Processing gain = 64. Random spreading sequences. No near-far effect: $A_1 = A_2 = \cdots = A_K$.

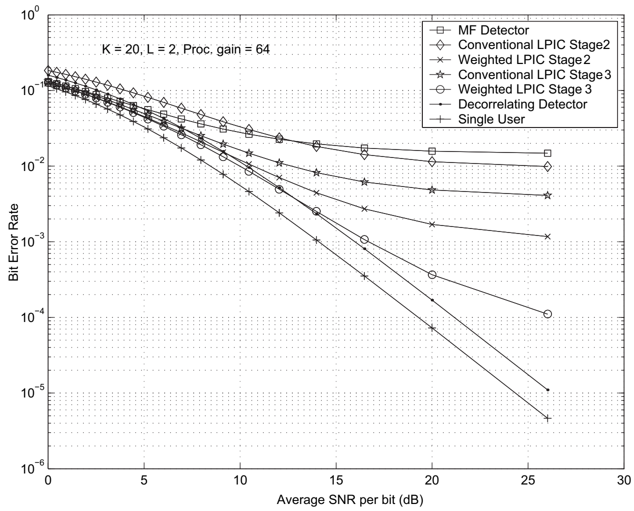

Fig. 4. Comparison of the BER performance of the SIR-optimized weighted LPIC as a function of SNR with that of the MF detector and the conventional LPIC on diversity channels ($L = 2$). $K = 20$. Processing gain = 64. Random spreading sequences. No near-far effect: $A_1 = A_2 = \cdots = A_K$.

lth receive antenna path, given by

$$y_k^{(m)_l} = y_k^{(1)_l} - p_k^{(m)_l} \sum_{j=1,\, j\neq k}^{K} \rho_{jk} y_j^{(m-1)_l}. \qquad (24)$$

On each receive antenna path, the expressions for the variances due to interference, $\sigma^2_{I_m}$, and due to noise, $\sigma^2_{N_m}$, remain the same as those for the flat fading case given before. The probability of bit error for the desired user k with L equal-energy diversity paths can be obtained as (similar to obtaining Eq. 3.140 in [1])

$$P_k^{(m)} = \frac{1}{2}\left[1 - \sqrt{\frac{\overline{SIR}^{(m)}_{k,opt}}{1+\overline{SIR}^{(m)}_{k,opt}}} \left(1 + \sum_{n=1}^{L-1} \frac{1\cdot 3\cdot 5\cdots(2n-1)}{n!2^n\left(1+\overline{SIR}^{(m)}_{k,opt}\right)^n}\right)\right]. \qquad (25)$$

V. Results and Discussions

In this section, we present the BER performance of the SIR-optimized weighted LPIC scheme on Rayleigh fading and diversity channels. In Fig. 3, we illustrate a BER performance comparison of the SIR-optimized weighted LPIC as a function of average SNR per bit with that of the MF detector as well as the conventional LPIC (where the weight is taken to be unity in all stages) on flat Rayleigh fading channels ($L = 1$) for $K = 20$ users with no near-far effect (i.e., $A_1 = A_2 = \cdots = A_K$). In all the numerical results presented here, we assign different random spreading sequences of processing gain 64 to different users, and the cross-correlation coefficients are computed for these random sequences. User 1 is taken to be the desired user. The BER plots for the SIR-optimized weighted LPIC are computed using (22) with the appropriate optimized SIR values. Figure 4 shows a similar performance comparison for diversity channels for $L = 2$, where the BER for the SIR-optimized weighted LPIC is computed using (25).

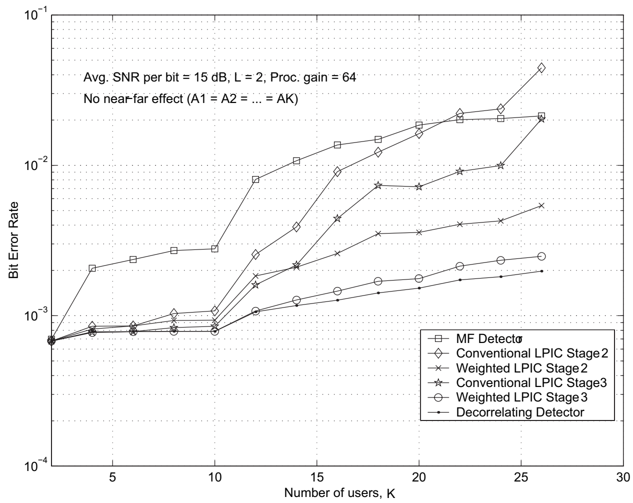

Fig. 5. Comparison of the BER performance of the SIR-optimized weighted LPIC as a function of the number of users, K, with that of the MF detector and the conventional LPIC on diversity channels ($L = 2$). Processing gain = 64. Random spreading sequences. Average SNR per bit = 15 dB. No near-far effect: $A_1 = A_2 = \cdots = A_K$.

We have obtained the BER through simulations as well (we found close match between analysis and simulation results, which is expected since the expressions are exact and there is no approximation involved). The decorrelating detector performance as well as the single user performance are also shown for comparison.

From Figs. 3 and 4 the following observations can be made. At high SNRs, the 2nd stage of the conventional LPIC performs better than the MF detector, due to accurate MAI estimates in good channel conditions (i.e., high SNRs). However, at low SNRs (SNRs < 9 dB in Fig. 3), the 2nd stage of the conventional LPIC performs poorer than the MF detector, which happens due to poor accuracy of the MAI estimates in the early stages of the PIC, particularly when channel conditions are poor (i.e., at low SNRs). This performance behavior improves in the 3rd stage of the conventional LPIC

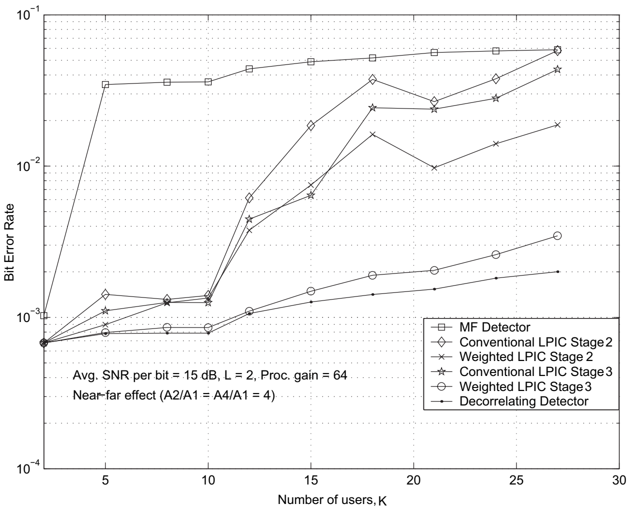

Fig. 6. Comparison of the BER performance of the SIR-optimized weighted LPIC as a function of the number of users, K, with that of the MF detector and the conventional LPIC on diversity channels ($L = 2$). Processing gain = 64. Random spreading sequences. Average SNR per bit = 15 dB. Near-far effect: users 2 & 4 transmit with 4 times more amplitude than other users.

which performs better than the MF detector for all SNRs > 0 dB. However, it is noted that the SIR-optimized weighted LPIC clearly outperforms both the MF detector as well as the conventional LPIC. In fact, even the 2nd stage of the SIR-optimized weighted LPIC outperforms the 3rd stage of the conventional LPIC, and the performance of the 3rd stage of the SIR-optimized LPIC tends closer to that of the decorrelating detector.

Figure 5 illustrates the BER performance comparison of the SIR-optimized weighted LPIC as a function of the number of users K with that of the MF detector as well as the conventional LPIC, for $L = 2$ at an average SNR per bit of 15 dB when there is no near-far effect ($A_1 = A_2 = \cdots = A_K$). Figure 6 presents such a comparison in the presence of near-far effect where users 2 and 4 transmit with 4 times more amplitude than the other users (i.e., $A_2/A_1 = A_4/A_1 = 4$, $A_1 = A_3 = A_5 = \cdots = A_K$). From Figs. 5 and 6, we observe that the SIR-optimized weighted LPIC clearly performs better than the MF detector and conventional LPIC in both in near-far as well as non-near-far conditions. Under these system conditions, the third stage of the SIR-optimized weighted LPIC is found to perform quite close to that of the decorrelating detector.

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