MMSE Receiver for Multiuser Interference Cancellation in Uplink OFDMA

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***Abstract*—In uplink orthogonal frequency division multiple access (OFDMA) systems, multiuser interference (MUI) occurs due to different carrier frequency offsets (CFO) of different users at the receiver. In this paper, we present a minimum mean square error (MMSE) based approach to MUI cancellation in uplink OFDMA. We derive a recursion to approach the MMSE solution. We present a structure-wise and performance-wise comparison of this recursive MMSE solution with a linear PIC receiver as well as other detectors recently proposed in the literature. We show that the proposed recursive MMSE solution encompasses several known detectors in the literature as special cases.**

Keywords – *Uplink OFDMA, carrier frequency offset, multiuser interference, MMSE receiver, circular convolution.*

I. Introduction

Recent research has witnessed increased focus on orthogonal frequency multiple access (OFDMA) on the uplink [1]-[7]. This is because real-time mobile multimedia applications demand high data rates on the uplink. The performance of OFDM/OFDMA systems depend to a large extent on how well the orthogonality among different subcarriers are maintained at the receiver [9]. Factors including carrier frequency offsets (CFO) between the transmitter and receiver induced by Doppler effects and/or poor oscillator alignments, sampling clock frequency discrepancies, and time delay caused by multipath and non-ideal synchronization can destroy the orthogonality among different subcarriers. Among the above factors, the impact of CFO on the performance is the most crucial one because the CFO values are large (typically of the order of several KHz) due to carrier frequencies being of the order of GHz. In uplink OFDMA, correction to one user's CFO would misalign other initially aligned users. Thus, other user CFO will result in significant multiuser interference (MUI) in uplink OFDMA.

There have been a few recent attempts in the literature to address the issue of MUI due to other user CFO in uplink OFDMA [5]-[7]. In [5], the proposed approach is to feedback the estimated CFO values (at the base station receiver) to the mobiles so that the mobile transmitters can adjust their transmit frequencies. This approach needs additional signalling and hence reduces the system throughput. An alternate and interesting approach is to apply interference cancellation (IC) techniques at the base station receiver. Recently, in [7], Huang and Letaief presented an IC approach which performs CFO compensation and MUI cancellation in frequency domain using circular convolution. We refer to this scheme in [7] as Huang-Letaief Circular Convolution (HLCC) scheme. The circular convolution approach was proposed earlier by Choi et al in [8] as an alternative to the direct time-domain method of CFO compensation. Huang and Letaief refer the scheme in [8] as CLJL scheme (CLJL stands for the first letters of the names of the four authors of [8]). The CLJL scheme does not perform MUI cancellation. The scheme by Huang and Letaief in [7] uses circular convolution for both CFO compensation (as in [8]) as well as MUI cancellation.

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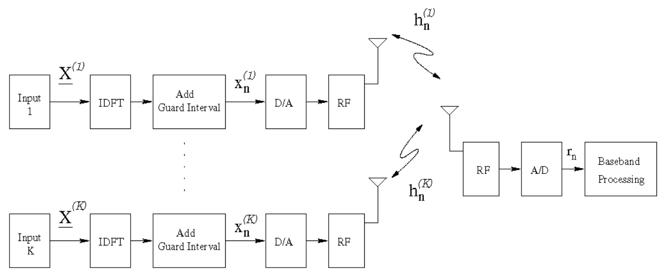

Fig. 1. Uplink OFDMA system model.

Our new contribution in this paper is that we present a minimum mean square error (MMSE) receiver for MUI cancellation in uplink OFDMA. We derive a recursion to approach the MMSE solution. We present a structure-wise and performance-wise comparison of this recursive MMSE solution with a linear parallel interference canceller (PIC) receiver as well as other detectors recently proposed in the literature above. We show that the proposed recursive MMSE solution encompasses several known detectors in the literature as special cases.

The rest of the paper is organized as follows. In Section II, we present the uplink OFDMA system model. In Section III, we present the MMSE detector and derive recursive MMSE solution. A linear PIC and its structural similarity to the recursive MMSE solution is established in Section IV. Results and discussions are presented in Section V. Conclusions are given in Section VI.

II. System Model

We consider an uplink OFDMA system with K users, where each user communicates with a base station (BS) through an independent multipath channel as shown in Fig. 1. We assume that there are N subcarriers in each OFDM symbol and one subcarrier can be allocated to only one user. Define the subcarrier usage of user i by a $N \times N$ diagonal matrix $\mathbf{M}^{(i)}$ given by

$$\mathbf{M}^{(i)}(k,k) = \begin{cases} 1 & \text{if user } i \text{ transmits on the } k\text{th subcarrier} \\ 0 & \text{otherwise.} \end{cases}$$

Let $\mathbf{X}$ denote the OFDM symbol to be transmitted. The information symbol for the ith user is then given by $\mathbf{X}^{(i)} = \mathbf{M}^{(i)}\mathbf{X}$. After IDFT processing, the time domain sequence for the i'th user is given by

$$x_n^{(i)} = \text{IDFT}_N\big(X_k^{(i)}\big), \quad (1)$$

where $X_k^{(i)}$ is the kth element in $\mathbf{X}^{(i)}$. Cyclic prefix of length CP is added to $x_n^{(i)}$ and transmitted. The ith user's signal after passing through the channel is

$$s_n^{(i)} = x_n^{(i)} \star h_n^{(i)} \quad (2)$$

where $\star$ denotes linear convolution and $h_n^{(i)}$ is the ith user's channel impulse response. It is assumed that $h_n^{(i)}$ is non-zero only for $n = 0, \cdots, L-1$, where L is the maximum channel delay spread, and that all users' channels are statistically independent. Note that $CP \geq L$.

The received baseband signal after coarse carrier frequency tracking (leaving some residual carrier frequency offset), and cyclic prefix removal is given by

$$r_n = \sum_{i=1}^{K} s_n^{(i)}\, e^{\frac{j2\pi\epsilon_i n}{N}} + z_n, \quad 0 \leq n \leq N-1, \quad (3)$$

where $\epsilon_i, i = 1, \cdots, K$ denotes the ith users residual carrier frequency offset (CFO) normalized by the subcarrier spacing after coarse frequency tracking, and z_n is the additive white Gaussian noise with zero mean and variance σ^2. We assume all users are time synchronized.

At the receiver, r_n is first fed to the DFT block. The output of the DFT block can be written in the form

$$\mathbf{R} = \sum_{i=1}^{K} \mathbf{C}^{(i)}\mathbf{M}^{(i)}\mathbf{H}\mathbf{X} + \mathbf{Z}, \quad (4)$$

where $\mathbf{C}^{(i)}$ is a $N \times N$ circulant matrix given by

$$\mathbf{C}^{(i)} = \begin{bmatrix} C_0^{(i)} & C_1^{(i)} & \cdots & C_{N-1}^{(i)} \\ C_{N-1}^{(i)} & C_0^{(i)} & \cdots & C_{N-2}^{(i)} \\ \vdots & \vdots & \vdots & \vdots \\ C_1^{(i)} & C_2^{(i)} & \cdots & C_0^{(i)}, \end{bmatrix} \quad (5)$$

where

$$C_k^{(i)} = \text{DFT}_N\left(e^{\frac{j2\pi n\epsilon_i}{N}}\right), \quad (6)$$

$\mathbf{H}$ is the diagonal $N \times N$ channel matrix given by

$$\mathbf{H} = \sum_{i=1}^{K} diag\{\mathbf{M}^{(i)}\mathbf{H}^{(i)}\} \quad (7)$$

where $\mathbf{H}^{(i)} = [H_1^{(i)}, H_2^{(i)}, \cdots, H_N^{(i)}]^T$ and the channel coefficient in frequency domain $H_k^{(i)}$ is given by $H_k^{(i)} = \text{DFT}_N\big(h_n^{(i)}\big)$, and similarly $\mathbf{Z} = [Z_1, Z_2, \cdots, Z_N]^T$ and $Z_k = \text{DFT}_N(z_n)$.

III. MMSE Detector

A linear detector $\mathbf{G}$ is a $N \times N$ linear matrix filter applied to the DFT output vector $\mathbf{R}$ that gives $\mathbf{Y}$, an estimate of $\mathbf{HX}$. That is,

$$\mathbf{Y} = \mathbf{GR}. \quad (8)$$

The corresponding mean square error (MSE) is a scalar defined as

$$\begin{aligned} J &= E\{\parallel \mathbf{GR} - \mathbf{HX} \parallel^2\} \\ &= E\Big\{\mathbf{R}^\dagger\mathbf{G}^\dagger\mathbf{GR} - \mathbf{X}^\dagger\mathbf{H}^\dagger\mathbf{G}^\dagger\mathbf{R} \\ &\qquad - \mathbf{R}^\dagger\mathbf{GHX} + \mathbf{X}^\dagger\mathbf{H}^\dagger\mathbf{HX}\Big\}, \end{aligned} \quad (9)$$

where $(.)^\dagger$ denotes Hermitian operation.

The gradient of J with respect to $\mathbf{G}$ is given by [11]

$$\nabla J = \frac{\partial J}{\partial \mathbf{G}} = \mathbf{G}E\{\mathbf{RR}^\dagger\} - E\{\mathbf{HXR}^\dagger\}. \quad (10)$$

Substituting $\mathbf{R}$ from (4) in (10), and using $E\Big\{\mathbf{M}^{(i)}\,\mathbf{H}\,\mathbf{X}\,\mathbf{X}^\dagger\,\mathbf{H}^\dagger\,\big(\mathbf{M}^{(i)}\big)^\dagger\Big\} = \mathbf{M}^{(i)}$, we have

$$\nabla J = \mathbf{G}\left(\sum_{i=1}^{K}\mathbf{C}^{(i)}\mathbf{M}^{(i)}\big(\mathbf{C}^{(i)}\big)^\dagger + \sigma^2\mathbf{I}\right) - \sum_{i=1}^{K}\mathbf{M}^{(i)}\big(\mathbf{C}^{(i)}\big)^\dagger. \quad (11)$$

The iterative way of reaching the MMSE filter $\mathbf{G}$ by the steepest descent method is given by

$$\begin{aligned} \mathbf{G}_m &= \mathbf{G}_{m-1} - \mu_m\nabla J \\ &= \mathbf{G}_{m-1} - \mu_m\left[\mathbf{G}_{m-1}\left(\sum_{i=1}^{K}\mathbf{C}^{(i)}\mathbf{M}^{(i)}\big(\mathbf{C}^{(i)}\big)^\dagger + \sigma^2\mathbf{I}\right)\right. \\ &\qquad \left. - \sum_{i=1}^{K}\mathbf{M}^{(i)}\big(\mathbf{C}^{(i)}\big)^\dagger\right] \end{aligned} \quad (12)$$

where μ_m is the step size for the mth iteration. Using the results in [12], it can be shown that the sufficient but not necessary condition for convergence is

$$0 < \mu_m < \frac{2}{\lambda_{max} + \sigma^2}, \quad (13)$$

where λ_{max} is the maximum eigen value of the matrix $\sum_{i=1}^{K}\mathbf{C}^{(i)}\mathbf{M}^{(i)}\big(\mathbf{C}^{(i)}\big)^\dagger$. Eqn. (12) can be written in a non-recursive form as

$$\begin{aligned} G_m &= \left(\sum_{i=1}^{K}\mathbf{M}^{(i)}\big(\mathbf{C}^{(i)}\big)^\dagger\right) \\ &\cdot \sum_{l=1}^{m}\prod_{p=l+1}^{m}\left[\mathbf{I} - \mu_p\left(\sum_{i=1}^{K}\mathbf{C}^{(i)}\mathbf{M}^{(i)}\big(\mathbf{C}^{(i)}\big)^\dagger + \sigma^2\mathbf{I}\right)\right]. \end{aligned} \quad (14)$$

Let $\mathbf{Y}_m^{(i)} = \mathbf{M}^{(i)}\mathbf{G}_m\mathbf{R}$ be the mth iteration estimate for the ith user. Then, the mth iteration matrix filter for the ith user $\mathbf{G}_m^{(i)} = \mathbf{M}^{(i)}\mathbf{G}_m$ is given by

$$\begin{aligned} \mathbf{G}_m^{(i)} &= \mathbf{M}^{(i)}\big(\mathbf{C}^{(i)}\big)^\dagger \\ &\cdot \sum_{l=1}^{m}\prod_{p=l+1}^{m}\left[\mathbf{I} - \mu_p\left(\sum_{j=1}^{K}\mathbf{C}^{(j)}\mathbf{M}^{(j)}\big(\mathbf{C}^{(j)}\big)^\dagger + \sigma^2\mathbf{I}\right)\right]. \end{aligned} \quad (15)$$

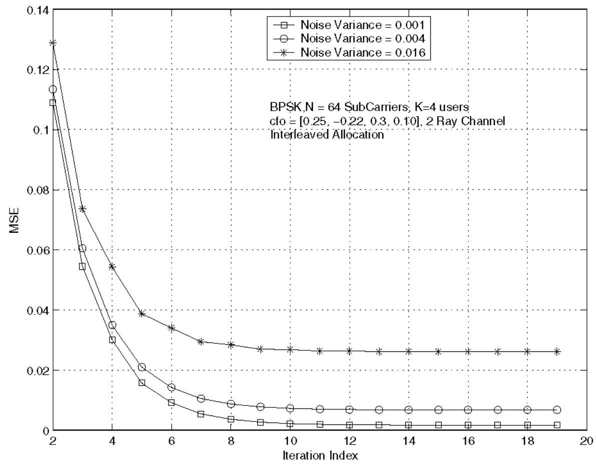

Fig. 2. Mean square error as a function of iteration index m for $N = 64$, $K = 4$, [CFO] $= [\epsilon_1, \epsilon_2, \epsilon_3, \epsilon_4] = [0.25, -0.22, 0.3, 0.1]$, interleaved allocation of subcarriers, BPSK modulation, 2-ray equal gain Rayleigh fading channel model, and $\sigma^2 = 0.001, 0.004, 0.016$.

For the case of $\mu_m = 1, \forall\, m$, the MSE as a function of iteration index m is shown in Fig. 2 for $N = 64$, $K = 4$, [CFO] $= [\epsilon_1, \epsilon_2, \epsilon_3, \epsilon_4] = [0.25, -0.22, 0.3, 0.1]$, interleaved allocation of subcarriers, BPSK modulation, 2-ray equal gain Rayleigh fading channel model, and $\sigma^2 = 0.001, 0.004, 0.016$.

From (15), it is seen that knowledge of the noise variance σ^2 is needed to obtain the matrix filter $\mathbf{G}_m^{(i)}$. In the absence of this knowledge, making $\sigma^2 = 0$ in (15), the mth iteration matrix filter for $\mu_m = 1, \forall\, m$ is given by

$$\mathbf{G}_m^{(i)} = \mathbf{M}^{(i)} \big(\mathbf{C}^{(i)}\big)^\dagger \sum_{l=1}^{m} \left(\mathbf{I} - \sum_{j=1}^{K} \mathbf{C}^{(j)} \mathbf{M}^{(j)} \big(\mathbf{C}^{(j)}\big)^\dagger \right)^{l-1}. \quad (16)$$

IV. A Linear PIC

In this section, we present a linear PIC receiver for the considered uplink OFDMA system and show its structural similarity to the recursive MMSE solution derived in the previous section. Define $\mathbf{D}^{(i)} = \big(\mathbf{C}^{(i)}\big)^\dagger, \forall\, i$ so that $\mathbf{D}^{(i)}\mathbf{C}^{(i)} = \mathbf{I}$, where $\mathbf{C}^{(i)}$ is given by (5). In other words,

$$D_k^{(i)} = \mathrm{DFT}_N \left(e^{\frac{-j2\pi n \epsilon_i}{N}} \right). \quad (17)$$

Using the DFT output $\mathbf{R}$, a multistage linear PIC receiver can be formulated as follows. Let $\mathbf{Y}_m^{(i)}$ denote the mth stage estimate for the ith user. The linear PIC algorithm works as follows:

Linear PIC Algorithm:

Initialization: Set $m = 1$.

$$\mathbf{Y}_m^{(i)} = \mathbf{M}^{(i)}\mathbf{D}^{(i)}\mathbf{R}, \quad \forall i,\ i = 1, 2, \cdots, K. \quad (18)$$

Loop: $m = m + 1$

$$\widehat{\mathbf{Y}}_m^{(i)} = \mathbf{R} - \sum_{j=1, j\neq i}^{K} \mathbf{C}^{(j)} \mathbf{Y}_{m-1}^{(j)}, \ \forall i,\ i = 1, 2, \cdots, K \quad (19)$$

$$\mathbf{Y}_m^{(i)} = \mathbf{M}^{(i)}\mathbf{D}^{(i)}\widehat{\mathbf{Y}}_{m-1}^{(i)}, \quad \forall i,\ i = 1, 2, \cdots, K \quad (20)$$

goto *Loop*.

We now investigate the structural similarity of the above linear PIC algorithm with the recursive MMSE solution in Sec. II as follows. The recursion in (20) can be expanded as

$$\mathbf{Y}_m^{(i)} = \mathbf{M}^{(i)}\mathbf{D}^{(i)} \left(\mathbf{I} - \sum_{j_1=1, j_1\neq i}^{K} \mathbf{C}^{(j_1)}\mathbf{M}^{(j_1)}\mathbf{D}^{(j_1)} \cdot \left(\mathbf{I} - \sum_{j_2=1, j_2\neq j_1}^{K} \mathbf{C}^{(j_2)}\mathbf{M}^{(j_2)}\mathbf{D}^{(j_2)} \cdot \left(\cdots m-1 \text{ times} \cdots \right) \right) \right) \mathbf{R}. \quad (21)$$

Define

$$\mathbf{S}_1^{(i)} = \mathbf{I}, \text{ and} \quad (22)$$

$$\mathbf{S}_m^{(i)} = \sum_{j=1, j\neq i}^{K} \mathbf{C}^{(j)}\, \mathbf{M}^{(j)}\, \mathbf{D}^{(j)}\, \mathbf{S}_{m-1}^{(j)}, \quad m > 1. \quad (23)$$

The estimate in (21) can then be written in a form similar to (8), as

$$\mathbf{Y}_m^{(i)} = \mathbf{B}_m^{(i)}\, \mathbf{R}, \quad (24)$$

where the matrix filter $\mathbf{B}_m^{(i)}$ is

$$\mathbf{B}_m^{(i)} = \mathbf{M}^{(i)}\mathbf{D}^{(i)} \sum_{l=1}^{m} (-1)^{l-1}\, \mathbf{S}_l^{(i)}. \quad (25)$$

We prove by mathematical induction that the MMSE matrix filter in (25) and the linear PIC matrix filter (16) for equal. Let

$$\mathbf{b}_l^{(i)} = (-1)^{l-1}\mathbf{M}^{(i)}\mathbf{D}^{(i)}\mathbf{S}_l^{(i)}, \quad (26)$$

$$\mathbf{g}_l^{(i)} = \mathbf{M}^{(i)}\mathbf{D}^{(i)} \left(\mathbf{I} - \sum_{j=1}^{K} \mathbf{C}^{(j)}\mathbf{M}^{(j)}\mathbf{D}^{(j)} \right)^{l-1}. \quad (27)$$

We show that $\mathbf{b}_l^{(i)} = \mathbf{g}_l^{(i)}, \ \forall\, l,\ l = 1, 2, \cdots, m$, and hence $\mathbf{B}_m^{(i)} = \mathbf{G}_m^{(i)}$. The proof is given in Appendix A.

The above proof shows that the linear LPIC in (18), (19), and (20) is the same as the recursive MMSE solution without the knowledge of the noise variance, i.e., $\sigma^2 = 0$ in the estimate in (15).

Comparison with HLCC Scheme:

We point out that the PIC scheme presented in [7][1] is similar to the one in Eqns. (18), (19), and (20). The difference is that, in [7], masked versions of $\mathbf{R}$ and $\widehat{\mathbf{Y}}_m^{(i)}$ vectors are used in the cancellation instead of using unmasked versions as in (18) and (20). In other words, the HLCC scheme can be viewed as a special case of the linear PIC in Eqns. (18), (19), and (20) by i) replacing $\mathbf{R}$ with $\mathbf{M}^{(i)}\mathbf{R}$ in (18), and ii) replacing $\widehat{\mathbf{Y}}_m^{(i)}$ with $\mathbf{M}^{(i)}\widehat{\mathbf{Y}}_m^{(i)}$ in (20). The effects of the above masking of vectors in HLCC are highlighted in the following section.

[1]We refer to the PIC scheme in [7] as the Huang-Letaief Circular Convolution (HLCC) scheme.

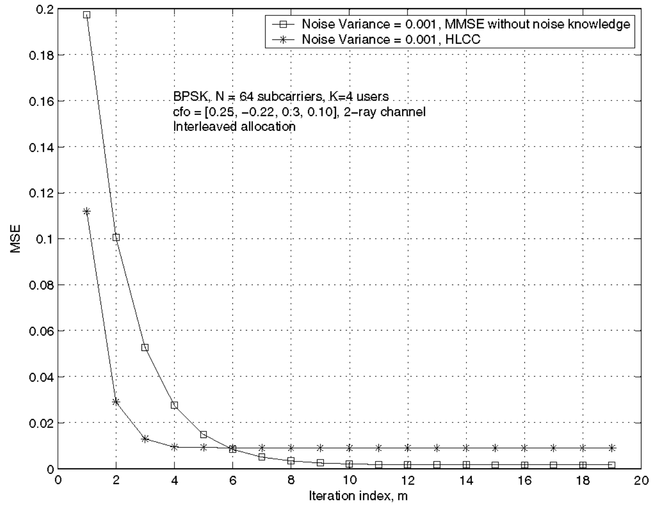

Fig. 3. Mean squared error as function iteration index m for MMSE and HLCC schemes for $N = 64$, $K = 4$, [CFO] $= [\epsilon_1, \epsilon_2, \epsilon_3, \epsilon_4] = [0.25, -0.22, 0.3, 0.1]$, interleaved allocation of subcarriers, BPSK modulation, 2-ray equal gain Rayleigh fading channel model, and $\sigma^2 = 0.001$.

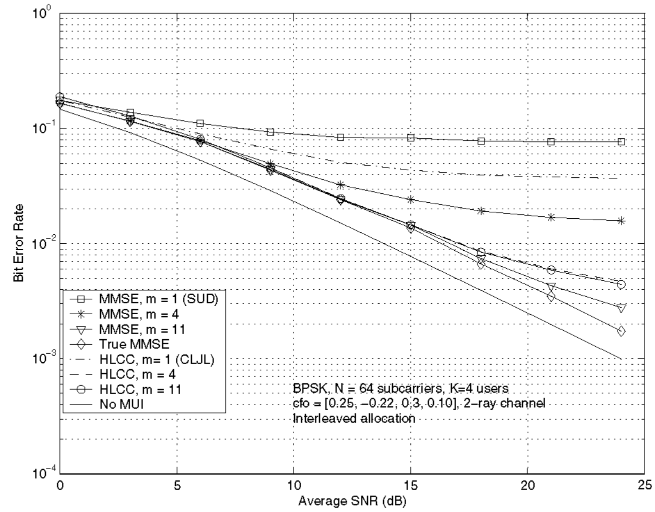

Fig. 4. BER performance comparison at various iteration indices m for MMSE and HLCC schemes for $N = 64$, $K = 4$, [CFO] $= [\epsilon_1, \epsilon_2, \epsilon_3, \epsilon_4] = [0.25, -0.22, 0.3, 0.1]$, interleaved allocation of subcarriers, BPSK modulation, 2-ray equal gain Rayleigh fading channel model.

V. Results and Discussion

The following observations on the structural similarities between the various detectors can be made:

- It can be noted that the first iteration of the MMSE solution (i.e., $m = 1$) is indeed the single user detector (SUD) given in [8], where CFO compensation is done for all $\epsilon_i, i = 1, 2, \cdots, K$, but no MUI cancellation is done.
- As we showed in Sec. IV, making the noise variance as zero and the step size as one in the MMSE solution gives the linear PIC.
- Usage of masked vectors in the linear PIC gives the HLCC scheme.
- The first iteration of the HLCC scheme is nothing but the CLJL scheme given in [8].

As can be seen, the proposed recursive MMSE solution encompasses several known detectors in the literature as special cases.

Performance Comparison:

First, we present a comparison of the mean square error performance of the recursive MMSE solution and the HLCC scheme. In Fig. 3, we plot the MSE for the MMSE detector as well as the HLCC scheme as a function of the iteration index m, for $N = 64$, $K = 4$, $[\epsilon_1, \epsilon_2, \epsilon_3, \epsilon_4] = [0.25, -0.22, 0.3, 0.1]$, BPSK modulation, interleaved allocation of subcarriers, 2-ray equal gain Rayleigh fading channel model. It can be observed that when the iteration index is small, HLCC results in a smaller MSE than the MMSE. This observation is in consistence with that in [7], where HLCC with $m = 1$ (which is the same as CLJL scheme in [8]) performs better than the SUD (which we have now shown to be the same as MMSE with $m = 1$). This is because, as mentioned in [7], the masking by $M^{(i)}$ acts as a filter which keeps most of the ith user's received signal power and eliminates most of other user's power. However, when m gets larger, the MMSE solution approaches the true MMSE performance, whereas the HLCC scheme saturates at a larger MSE than that of the true MMSE as seen in Fig. 3. This saturation happens in HLCC (for $m > 4$ in Fig. 3) because the masking operation prohibits the ith user's signal present in the other users' subcarriers to be retrieved. The MMSE filter essentially could retrieve this signal power also and hence gives better MSE. By the same argument, it can be seen that the linear PIC will result in better performance than HLCC for large m. In terms of complexity, both HLCC and MMSE will have a complexity of $2N^2$ per iteration.

For the same set of system parameters in Fig. 3, a bit error rate (BER) performance comparison of the various detectors are presented in Fig. 4. The true MMSE performance as well as No MUI performance are also shown for comparison. The various iteration indices considered for MMSE and HLCC are $m = 1, 4, 11$. It can be seen that as in the case of MSE, the BER performance is better for HLCC for $m = 1$ and $m = 4$. However, increasing m from 4 to 11 does not improve the BER performance of HLCC (as expected from the observation of saturated MSE beyond $m = 4$). Whereas, the MMSE performance improves significantly when m is increased to 11 from 4. In fact, $m = 11$ performs quite close to the true MMSE.

VI. Conclusions

We presented a MMSE based approach to multiuser interference cancellation in uplink OFDMA. We derived a recursion to approach the MMSE solution. We present a structure-wise and performance-wise comparison of this recursive MMSE solution with a linear PIC receiver as well as other detectors recently proposed in the literature. We showed that the proposed recursive MMSE solution encompasses several known detectors in the literature as special cases.

APPENDIX A

In this Appendix, we show that $\mathbf{b}_l^{(i)} = \mathbf{g}_l^{(i)},\ \forall l,\ l = 1, 2, \cdots, m,$ where $\mathbf{b}_l^{(i)}$ and $\mathbf{g}_l^{(i)}$ are as defined in (26) and (27).

For $l = 1$, it is see that

$$\mathbf{g}_1^{(i)} = \mathbf{b}_1^{(i)} = \mathbf{M}^{(i)}\mathbf{D}^{(i)}. \tag{28}$$

For $l = 2$,

$$\begin{aligned}\mathbf{g}_2^{(i)} &= \mathbf{M}^{(i)}\mathbf{D}^{(i)}\left(\mathbf{I} - \sum_{j=1}^{K}\mathbf{C}^{(j)}\mathbf{M}^{(j)}\mathbf{D}^{(j)}\right)\\ &= \mathbf{M}^{(i)}\mathbf{D}^{(i)}\left(\mathbf{I} - \mathbf{C}^{(i)}\mathbf{M}^{(i)}\mathbf{D}^{(i)}\right.\\ &\qquad \left. - \sum_{j=1, j\neq i}^{K}\mathbf{C}^{(j)}\mathbf{M}^{(j)}\mathbf{D}^{(j)}\right)\\ &= (-1)\mathbf{M}^{(i)}\mathbf{D}^{(i)}\left(\sum_{j=1, j\neq i}^{K}\mathbf{C}^{(j)}\mathbf{M}^{(j)}\mathbf{D}^{(j)}\right)\\ &= (-1)\mathbf{M}^{(i)}\mathbf{D}^{(i)}\mathbf{S}_2^{(i)}.\end{aligned} \tag{29}$$

Hence the equality holds for $l = 2$ as well. Let the result be true for $l = q$; we show that the equality holds for $l = q + 1$ as well.

$$\begin{aligned}\mathbf{g}_{q+1}^{(i)} &= \mathbf{M}^{(i)}\mathbf{D}^{(i)}\left(\mathbf{I} - \sum_{j=1}^{K}\mathbf{C}^{(j)}\mathbf{M}^{(j)}\mathbf{D}^{(j)}\right)^{q}\\ &= \mathbf{M}^{(i)}\mathbf{D}^{(i)}\left(\mathbf{I} - \sum_{j=1}^{K}\mathbf{C}^{(j)}\mathbf{M}^{(j)}\mathbf{D}^{(j)}\right)^{q-1}\\ &\quad \cdot\left(\mathbf{I} - \sum_{j=1}^{K}\mathbf{C}^{(j)}\mathbf{M}^{(j)}\mathbf{D}^{(j)}\right)\\ &=(-1)^{q-1}\mathbf{M}^{(i)}\mathbf{D}^{(i)}\mathbf{S}_q^{(i)}\left(\mathbf{I} - \sum_{j=1}^{K}\mathbf{C}^{(j)}\mathbf{M}^{(j)}\mathbf{D}^{(j)}\right).\end{aligned} \tag{30}$$

Now from (29) we have,

$$\mathbf{M}^{(i)}\mathbf{D}^{(i)}\left(\mathbf{I} - \sum_{j=1}^{K}\mathbf{C}^{(j)}\mathbf{M}^{(j)}\mathbf{D}^{(j)}\right) = (-1)\mathbf{M}^{(i)}\mathbf{D}^{(i)}\mathbf{S}_2^{(i)}. \tag{31}$$

Multiplying both sides by $\mathbf{C}^{(i)}$ and summing $\forall i \neq p$, we have

$$\mathbf{S}_2^{(p)}\left(\mathbf{I} - \sum_{j=1}^{K}\mathbf{C}^{(j)}\mathbf{M}^{(j)}\mathbf{D}^{(j)}\right) = (-1)\,\mathbf{S}_3^{(p)} \tag{32}$$

The same process can be repeated to obtain

$$\mathbf{S}_q^{(p)}\left(\mathbf{I} - \sum_{j=1}^{K}\mathbf{C}^{(j)}\mathbf{M}^{(j)}\mathbf{D}^{(j)}\right) = (-1)\mathbf{S}_{q+1}^{(p)} \tag{33}$$

Substituting this in (30), we get

$$\mathbf{g}_{q+1}^{(i)} = \mathbf{b}_{q+1}^{(i)}. \tag{34}$$

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