Sparsity-Exploiting Detection of Large-Scale Multiuser GSM-MIMO Signals Using FOCUSS

Sandeep Bhat and A. Chockalingam
Department of ECE, Indian Institute of Science, Bangalore

***Abstract*—In this paper, we investigate the detection problem in large-scale multiuser *generalized spatial modulation MIMO* (GSM-MIMO) systems on the uplink. The inherent sparsity present in multiuser GSM-MIMO signals enables us to view the signal detection problem as one of sparse signal recovery in underdetermined systems using compressed sensing techniques. Accordingly, we propose a FOCUSS (FOCal Underdetermined System Solver) based algorithm for GSM-MIMO signal detection. Viewing the FOCUSS based detection as a sequence of weighted l_2-norm minimizations, we compare the performance of the FOCUSS based detection with that of the classical l_2-norm minimizer, the MMSE detection. Simulation results show that the FOCUSS based detection beats the MMSE detection by about 10 dB at 10^{-3} BER at a system loading factor of 1.1. This SNR gain of FOCUSS based detection over MMSE detection is found to increase with increasing system loading factors. Also, the FOCUSS based detection is shown to achieve better performance compared to OMP and CoSaMP based detection.**

Keywords – *Generalized spatial modulation, large-scale multiuser MIMO, underdetermined systems, sparsity, FOCUSS.*

I. Introduction

Large-scale MIMO systems (e.g., massive MIMO systems) have attracted increased research attention because of their increased spectral and power efficiencies [1],[2],[3]. Multiuser MIMO has several advantages over single-user (point-to-point) MIMO [4]. Generalized spatial modulation (GSM) is an attractive modulation scheme for multi-antenna communications [5]-[7]. In GSM, there are n_t antennas and n_{rf} radio frequency (RF) chains at the transmitter, $1 \leq n_{rf} \leq n_t$. So, the need to have a large number of RF chains at the transmitter can be alleviated in GSM. In a given channel use, n_{rf} out of n_t transmit antennas are chosen and activated. The remaining $n_t - n_{rf}$ antennas remain silent. On the chosen antennas, n_{rf} modulation symbols (one on each chosen antenna) from an alphabet $\mathbb{A}$ (e.g., QAM) are transmitted. The indices of the n_{rf} active antennas out of n_t available antennas convey $\lfloor \log_2 \binom{n_t}{n_{rf}} \rfloor$ information bits. This is in addition to the $n_{rf} \lfloor \log_2 |\mathbb{A}| \rfloor$ information bits conveyed by the n_{rf} modulation symbols. Spatial modulation (SM) is a special case of GSM with $n_{rf} = 1$. Spatial multiplexing (SMP) is another special case of GSM with $n_{rf} = n_t$. GSM in multiuser MIMO systems on the uplink has been studied in [8],[9], and it is shown to be a promising modulation scheme for large-scale multiuser MIMO systems [9].

This work was supported in part by the J. C. Bose National Fellowship, Department of Science and Technology, Government of India, and by the Indo-French Centre for Applied Mathematics.

The inactive antennas in GSM naturally result in a sparse transmitted signal. The GSM signal detection problem, therefore, can be viewed in the compressed sensing (CS) framework. Also, GSM systems can be underdetermined; i.e., if K is the number of uplink users, n_t is the number of transmit antennas in each user, and N is the number of receive antennas at the BS, then the GSM system becomes underdetermined if $\frac{Kn_t}{N} > 1$ (i.e., if number of BS receive antennas is less than the total number of transmit antennas in all the users). From an efficient system utilization point of view, it is desired to increase the number of users K in the system. But, for a given n_t and N, increasing K can lead to underdetermined condition. Therefore, efficient GSM signal detection in underdetermined settings is of interest. Again, CS framework comes as a natural choice as the mathematics at the foundation of CS makes it possible to reconstruct signals even when the number of available measurements is smaller than the signal dimension (i.e., underdetermined systems) [10].

CS based detection schemes for space shift keying (SSK) and generalized SSK (GSSK) in single-user (point-to-point) settings have been studied in [11], [12]. These schemes used orthogonal matching pursuit (OMP) in the CS literature and modifications thereof to detect the active antenna indices (called the support of the transmitted signal vector). However, we found that OMP [13] did not provide good bit error rate (BER) performance for the multiuser GSM-MIMO system model under consideration (see Fig. 2). Compressive sampling matching pursuit (CoSaMP) [14] is a robust variant of OMP. Though CoSaMP performed better than OMP, CoSaMP still witnessed high error rates (see Fig. 2). We speculate that this poor performance in OMP and CoSaMP could be due to the following reasons: i) the sparsity level in the multiuser GSM-MIMO signal is significantly lower than the sparsity levels considered in [11], [12], ii) the sparsity in the overall multiuser signal is constrained by the sparsity profiles of the individual users, which disallows some of the sparse vectors as solutions, and iii) the symbols from the modulation alphabet also have to be detected in addition to detecting the active antenna indices. The need thus arises for robust CS techniques to recover both the support and signal components of the transmitted signal vector in multiuser GSM-MIMO systems.

The FOCUSS(**FOC**al **U**nderdetermined **S**ystem **S**olver) algorithm in the sparse signal recovery literature is a promising approach [15]-[17]. The strength of the FOCUSS technique lies in it being formulated as a general estimation tool using classical optimization methods usable across several application domains (e.g., biomedical signal processing, image

978-1-5090-1698-3/16/$31.00 ©2016 IEEE

reconstruction and restoration, dictionary learning, and MIMO radar imaging [18]-[20]). To the best of our knowledge, use of FOCUSS in communication theory/systems domain has not been reported in the literature so far.

In this paper, we consider the uplink of a multiuser MIMO system, where each user equipment employs GSM with n_t transmit antennas and n_{rf} transmit RF chains. The underlying signal sparsity and underdetermined nature of the system motivate us to consider a FOCUSS based approach. Accordingly, our new contribution in this paper is that we propose a FOCUSS based algorithm for large-scale multiuser GSM-MIMO signal detection. Viewing the FOCUSS based detection as a sequence of weighted l_2-norm minimizations, we compare the performance of the FOCUSS based detection, with that of the classical l_2-norm minimizer, the MMSE detection. Simulation results show that the proposed FOCUSS based detection performs significantly better than MMSE, OMP, and CoSaMP detectors in GSM-MIMO and SM-MIMO systems. The SNR gain of FOCUSS detection over MMSE detection is found to increase with increasing system loading factors.

II. System Model

Consider a multiuser system with K users communicating on the uplink with a BS having N receive antennas (see Fig. 1). N is in the order of tens to hundreds. Each user has n_t transmit antennas and employs GSM for transmission using n_{rf} RF chains, $1 \leq n_{rf} \leq n_t$. The ratio $\frac{Kn_t}{N}$ is the system loading factor. An $n_{rf} \times n_t$ switch connects the RF chains to the transmit antennas. In a given channel use, each user selects n_{rf} of its n_t antennas and transmits n_{rf} symbols from a modulation alphabet $\mathbb{A}$ on the selected antennas. The remaining $n_t - n_{rf}$ antennas remain silent (they could be viewed as transmitting a zero). The first $\lfloor \log_2 \binom{n_t}{n_{rf}} \rfloor$ information bits are used to select the antennas. The next $n_{rf}\lfloor \log_2 |\mathbb{A}| \rfloor$ bits are used to form n_{rf} modulation symbols to be transmitted on the selected antennas. So, the total number of bits conveyed by each user per channel use is $\lfloor \log_2 \binom{n_t}{n_{rf}} \rfloor + n_{rf}\lfloor \log_2 |\mathbb{A}| \rfloor$. For example, a GSM transmitter with $n_t = 4$, $n_{rf} = 2$, and 4-QAM conveys 6 bpcu. When $n_{rf} = 1$, GSM specializes to SM, giving the total number of bits conveyed by each user per channel use as $\lfloor \log_2 n_t \rfloor + \lfloor \log_2 |\mathbb{A}| \rfloor$. An SM transmitter with $n_t = 4$ and 16-QAM also conveys 6 bpcu.

The mapping of information bits to active antenna indices is described next. Define an 'antenna activation pattern' to be an $n_t \times 1$ vector consisting of 0's and 1's, where a 1 in a coordinate position indicates that the antenna corresponding to that coordinate is active and a 0 indicates that the corresponding antenna is silent. There are $\binom{n_t}{n_{rf}}$ such activation patterns possible. For instance, with $n_t = 4$ and $n_{rf} = 2$ the following six activation patterns are possible:

$$[1\,1\,0\,0]^T, [0\,1\,1\,0]^T, [0\,0\,1\,1]^T, [1\,0\,0\,1]^T, [1\,0\,1\,0]^T, [0\,1\,0\,1]^T.$$

It is noted that the antennas to be activated in a given channel use are chosen solely based on information bits, and not based on the channel matrix. Channel state information at the transmitter (CSIT) is not needed in GSM.

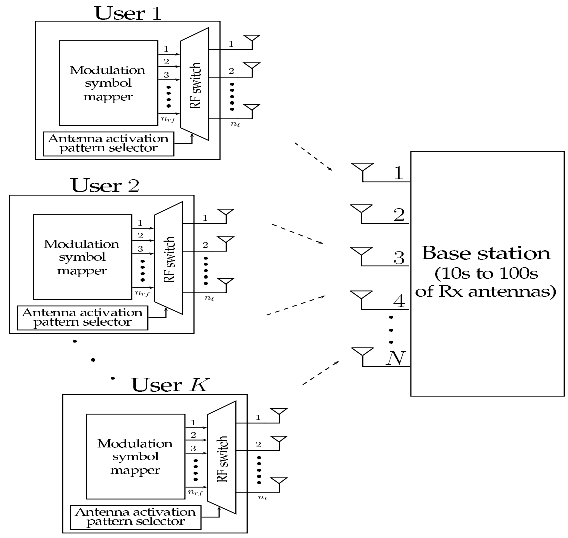


Fig. 1. Multiuser GSM-MIMO system

Out of the $\binom{n_t}{n_{rf}}$ possible activation patterns, only $2^{\lfloor \log_2 \binom{n_t}{n_{rf}} \rfloor}$ activation patterns are needed for signaling. Let $\mathcal{S}$ denote the set of these $2^{\lfloor \log_2 \binom{n_t}{n_{rf}} \rfloor}$ activation patterns chosen from the set of all possible patterns. Continuing the above example, let the set of chosen activation patterns be

$$\mathcal{S} = \{[1\ 1\ 0\ 0]^T, [0\ 1\ 1\ 0]^T, [0\ 0\ 1\ 1]^T, [1\ 0\ 0\ 1]^T\}.$$

$2^{\lfloor \log_2 \binom{n_t}{n_{rf}} \rfloor}$ possible combinations of $\lfloor \log_2 \binom{n_t}{n_{rf}} \rfloor$ information bits are then mapped to the elements of $\mathcal{S}$.

GSM signal set: Let $\mathbb{S}^{n_{rf}}_{n_t,\mathbb{A}}$ denote the GSM signal set, which is the set of GSM signal vectors that can be transmitted. Then,

$$\mathbb{S}^{n_{rf}}_{n_t,\mathbb{A}} = \{\mathbf{s} : s_j \in \mathbb{A} \cup \{0\}, \|\mathbf{s}\|_0 = n_{rf}, \mathcal{I}(\mathbf{s}) \in \mathcal{S}\},$$

where $\mathbf{s}$ is the $n_t \times 1$ transmit vector, s_j is the jth entry of $\mathbf{s}$, $j = 1, \ldots, n_t$, $\|\mathbf{s}\|_0$ is the l_0-norm of the vector $\mathbf{s}$, and $\mathcal{I}(\cdot)$ is a function that gives the activation pattern of its (vector) argument; for example $\mathcal{I}([+1\ 0\ -1\ +1]^T) = [1\ 0\ 1\ 1]^T$.

Example: Let $n_t = 4$, $n_{rf} = 2$, BPSK, and $\mathcal{S} = \{[1\ 1\ 0\ 0]^T, [0\ 1\ 1\ 0]^T, [0\ 0\ 1\ 1]^T, [1\ 0\ 0\ 1]^T\}$. The GSM signal set for this scheme will be

$$\mathbb{S}^2_{4,\text{bpsk}} = \left\{ \begin{bmatrix}+1\\+1\\0\\0\end{bmatrix}, \begin{bmatrix}+1\\-1\\0\\0\end{bmatrix}, \begin{bmatrix}-1\\+1\\0\\0\end{bmatrix}, \begin{bmatrix}-1\\-1\\0\\0\end{bmatrix}, \begin{bmatrix}0\\+1\\+1\\0\end{bmatrix}, \begin{bmatrix}0\\+1\\-1\\0\end{bmatrix}, \begin{bmatrix}0\\-1\\+1\\0\end{bmatrix}, \begin{bmatrix}0\\-1\\-1\\0\end{bmatrix}, \right.$$
$$\left. \begin{bmatrix}0\\0\\+1\\+1\end{bmatrix}, \begin{bmatrix}0\\0\\+1\\-1\end{bmatrix}, \begin{bmatrix}0\\0\\-1\\+1\end{bmatrix}, \begin{bmatrix}0\\0\\-1\\-1\end{bmatrix}, \begin{bmatrix}+1\\0\\0\\+1\end{bmatrix}, \begin{bmatrix}+1\\0\\0\\-1\end{bmatrix}, \begin{bmatrix}-1\\0\\0\\+1\end{bmatrix}, \begin{bmatrix}-1\\0\\0\\-1\end{bmatrix} \right\}.$$

If $\mathbf{x}_k \in \mathbb{S}^{n_{rf}}_{n_t,\mathbb{A}}$ denotes the transmit vector from the kth user, $\mathbf{x} = [\mathbf{x}_1{}^T, \ldots, \mathbf{x}_K{}^T]^T$ represents the K-user GSM transmitted signal. Note that $\mathbf{x} \in (\mathbb{S}^{n_{rf}}_{n_t,\mathbb{A}})^K$. Let $\mathbf{H} \in \mathbb{C}^{N \times Kn_t}$ denote the channel gain matrix, where $H_{i,(k-1)n_t+j}$ denotes the complex channel gain from the jth transmit antenna of the kth user to the ith BS receive antenna. The channel gains are assumed to be independent Gaussian with zero mean and variance σ^2_κ, such that $\sum_{\kappa=1}^{Kn_t} \sigma^2_\kappa = Kn_t$. The σ^2_κ models the imbalance in the received power from the κth antenna, $\kappa \in \{1, \cdots, Kn_t\}$, due to path loss etc., and $\sigma^2_\kappa = 1$ corresponds to the case of perfect power control. Assuming perfect synchronization, the received signal at the ith antenna of the BS is given by

$$y_i = \sum_{k=1}^{K} \mathbf{h}_{i,[k]}\mathbf{x}_k + n_i, \quad (1)$$

where $\mathbf{h}_{i,[k]}$ is a $1 \times n_t$ vector obtained from the ith row and $(k-1)n_t+1$ to kn_t columns of $\mathbf{H}$ and n_i is the noise modeled as a complex Gaussian r. v. with zero mean and variance σ^2. The received signal at the BS can be written as

$$\mathbf{y} = \mathbf{H}\mathbf{x} + \mathbf{n}, \quad (2)$$

where $\mathbf{y} = [y_1, y_2, \cdots, y_N]^T$, $\mathbf{n} = [n_1, n_2, \cdots, n_N]^T$, and the average received SNR is given by $\frac{\mathbb{E}[\mathbf{x}^H\mathbf{x}]}{\sigma^2}$.

III. FOCUSS BASED GSM-MIMO DETECTION

In this section, we present the development of the FOCUSS based detection algorithm for multiuser GSM-MIMO. The aim is to get an estimate of the multiuser transmitted vector $\mathbf{x}$ in (2). The maximum a posteriori estimate of $\mathbf{x}$ is given by

$$\begin{aligned} \hat{\mathbf{x}} &= \underset{\mathbf{x}\in(\mathbb{S}^{n_{rf}}_{n_t,\mathbb{A}})^K}{\text{argmax}} \ \Pr(\mathbf{x}|\mathbf{y},\mathbf{H}) \\ &= \underset{\mathbf{x}\in(\mathbb{S}^{n_{rf}}_{n_t,\mathbb{A}})^K}{\text{argmax}} \ [\log\Pr(\mathbf{y}|\mathbf{x},\mathbf{H}) + \log\Pr(\mathbf{x})] \\ &= \underset{\mathbf{x}\in(\mathbb{S}^{n_{rf}}_{n_t,\mathbb{A}})^K}{\text{argmin}} \ [\|\mathbf{y}-\mathbf{H}\mathbf{x}\|^2 - \log\Pr(\mathbf{x})], \end{aligned} \quad (3)$$

where the term $\|\mathbf{y}-\mathbf{H}\mathbf{x}\|^2$ follows from the Gaussian distribution of the noise. Since the elements of $\mathbf{x}$ come from the set $\mathbb{A}\cup\{0\}$, (3) is a constrained optimization problem (constrained by the multiuser signal set $(\mathbb{S}^{n_{rf}}_{n_t,\mathbb{A}})^K$), the solution to which is NP-hard. So, for the development of the proposed algorithm, we relax this constraint by assuming that the real and imaginary parts of each of the elements of $\mathbf{x}$ in (3) are drawn from a generalized Gaussian distribution, the pdf of which is given by [21]

$$f(t,p,\beta) = \frac{p}{2\sqrt[p]{2}\beta\Gamma(\frac{1}{p})} e^{-(|t|^p/2\beta^p)}, \quad (4)$$

where $\Gamma(\cdot)$ is the gamma function. The parameter p governs the shape of the distribution and β is the variance. The motivation for choosing this distribution comes from the fact that we desire sparse solutions to (3) because of the sparse nature of the transmitted vector. This can be seen from the fact that the generalized Gaussian distribution moves toward a peaky distribution at $t = 0$ as $p \to 0$. Because of the above relaxation, the vector $\mathbf{x}$ in (3) now belongs to $\mathbb{C}^{Kn_t}$, containing i.i.d generalized Gaussian distributed entries x_i, $i = 1, \ldots, Kn_t$, i.e.,

$$\begin{aligned} \Pr(\mathbf{x}) &= \Pr(x_1,\ldots,x_{Kn_t}) \\ &= \Big(\frac{p}{2\sqrt[p]{2}\beta\Gamma(\frac{1}{p})}\Big)^{2Kn_t} \exp\Big(-\frac{1}{\beta^p}\sum_{k=1}^{Kn_t}|x_k|^p\Big). \end{aligned} \quad (5)$$

Therefore, a relaxed estimate of $\mathbf{x}$ in (3) is given by

$$\hat{\mathbf{x}} = \underset{\mathbf{x}\in\mathbb{C}^{Kn_t}}{\text{argmin}} \ [\|\mathbf{y}-\mathbf{H}\mathbf{x}\|^2 + \gamma D^{(p)}(\mathbf{x})], \quad (6)$$

Algorithm 1: FOCUSS algorithm based GSM detection

Input: $\mathbf{H}$, $\mathbf{y}$, $\mathbf{x}^{(0)}$, *max_iter*, *p*
Output: $\hat{\mathbf{x}}$
Set $t = 0$
while $t < max\_iter$ **do**
  $\mathbf{W}^{(t+1)} = \text{diag}(|x_1^{(t)}|^{1-\frac{p}{2}}, \ldots, |x_{Kn_t}^{(t)}|^{1-\frac{p}{2}})$
  $\mathbf{H}^{(t+1)} = \mathbf{H}\mathbf{W}^{(t+1)}$
  $\mathbf{x}^{(t+1)} = \mathbf{W}^{(t+1)}\mathbf{H}^{(t+1)^H}(\mathbf{H}^{(t+1)}\mathbf{H}^{(t+1)^H} + \lambda\mathbf{I})^{-1}\mathbf{y}$
  $t = t+1$
end
$\hat{\mathbf{x}} = f(\mathbf{x}^{(max\_iter)})$

where $\gamma = \frac{\sigma^2}{\beta^p}$ is a regularization parameter, and $D^{(p)}(\mathbf{x})$ is the l_p-diversity measure [17] defined by

$$D^{(p)}(\mathbf{x}) = \sum_{k=1}^{Kn_t}|x_k|^p. \quad (7)$$

The choice of p in the range $p \le 1$ ensures sparse solutions [17]. The regularization parameter γ controls the tradeoff between the quality of the solution (in terms of $\|\mathbf{y}-\mathbf{H}\mathbf{x}\|$) and the amount of sparsity. Large values of γ give sparse solutions and small values give a lower $\|\mathbf{y}-\mathbf{H}\mathbf{x}\|$. Solving (6) using the factored gradient approach in [16], the optimum solution $\mathbf{x}_o$ can be obtained as

$$\mathbf{x}_o = \mathbf{W}(\mathbf{x}_o)(\mathbf{H}\mathbf{W}(\mathbf{x}_o))^H(\mathbf{H}\mathbf{W}(\mathbf{x}_o)+\lambda\mathbf{I})^{-1}(\mathbf{H}\mathbf{W}(\mathbf{x}_o))^H\mathbf{y}, \quad (8)$$

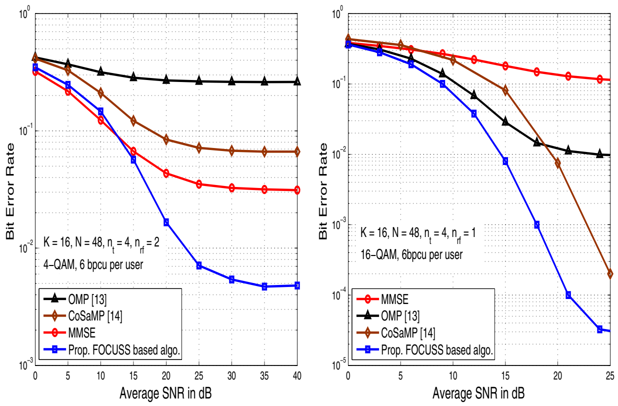
where $\mathbf{W}(\mathbf{x}_o) = \text{diag}(|x_{oi}|^{1-\frac{p}{2}}, i = 1,\ldots,Kn_t)$ and $\lambda = \frac{p}{2}\frac{\sigma^2}{\beta^p}$. Equation (8) suggests that $\mathbf{x}_o$ can be obtained iteratively. The vector $\mathbf{x}_o$ is then mapped to the nearest vector in $\mathbb{S}^{n_{rf}}_{n_t,\mathbb{A}}$ in the Euclidean distance sense to obtain the final solution $\hat{\mathbf{x}}$. The proposed FOCUSS based iterative algorithm to obtain $\hat{\mathbf{x}}$ is listed in Algorithm 1.

In Algorithm 1, *max_iter* is the maximum number of iterations, $\mathbf{x}^{(0)}$ is the unconstrained initial solution vector (e.g., $\mathbf{x}^{(0)}$ can be the output from matched filter or zero-forcing/MMSE filters), and $f(\cdot)$ is a function which maps each $n_t \times 1$ vector in its $Kn_t \times 1$-sized vector argument to the nearest $n_t \times 1$ vector in the GSM signal set. Choosing the parameter λ is nontrivial and there appears to be no rule in general for the same. However, driven by the requirement to have $\|\mathbf{y}-\mathbf{H}\mathbf{x}\| \to 0$ as noise variance $\sigma^2 \to 0$ (high SNRs) and observing that $\sigma^2 \to 0$ implies $\lambda \to 0$, we set this parameter equal to the noise variance σ^2. The Algorithm 1 has a complexity of order $O(Kn_tN^2)$ for $Kn_t > N$, which is mainly due to the update of the $\mathbf{x}^{(t)}$ vector, i.e., computation of $\mathbf{x}^{(t+1)}$.

IV. RESULTS AND DISCUSSIONS

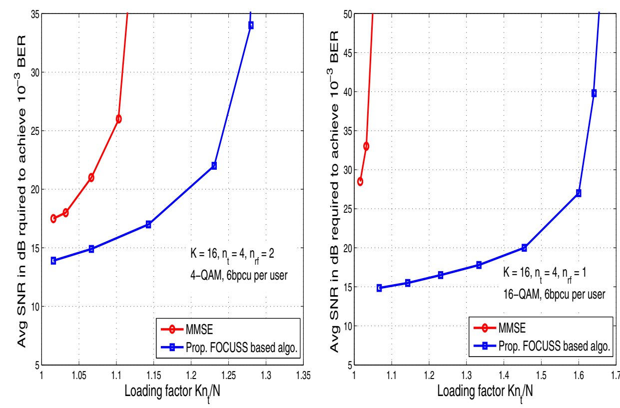
In this section, we present the simulation results of the BER performance of the proposed FOCUSS based detection algorithm for various configurations of large-scale multiuser GSM-MIMO. We compare the performance of the FOCUSS based detection with that of the conventional MMSE detection.

In Fig. 2, we compare the BER vs SNR performance of the FOCUSS based detector with that of the MMSE, OMP, and



(a) GSM: $n_{rf} = 2$, 4-QAM (b) SM: $n_{rf} = 1$, 16-QAM

Fig. 2. BER performance comparison between FOCUSS, MMSE, OMP, and CoSaMP based detectors in GSM-MIMO and SM-MIMO systems. $K = 16, n_t = 4$, $N = 48$, and 6 bpcu per user.



(a) GSM: $n_{rf} = 2$, 4-QAM (b) SM: $n_{rf} = 1$, 16-QAM

Fig. 3. SNR required by FOCUSS based and MMSE detectors to achieve a BER of 10^{-3} as a function of system loading factor $\frac{Kn_t}{N}$ in GSM-MIMO and SM-MIMO systems. $K = 16, n_t = 4$, and 6 bpcu per user.

CoSaMP based detectors for multiuser GSM and SM systems with number of users $K = 16$, transmit antennas per user $n_t = 4$, number of BS antennas $N = 48$, and $\sigma_\kappa^2 = 1$, $\kappa \in \{1, \cdots, Kn_t\}$. The system loading factor is $\frac{Kn_t}{N} = \frac{64}{48} = 1.33$. We compare the performance of GSM with $n_{rf} = 2$ and SM with $n_{rf} = 1$ for the same spectral efficiency of 6 bpcu per user. GSM uses 4-QAM and SM uses 16-QAM to achieve the same 6 bpcu per user. The MMSE solution vector is used as the initial solution in the FOCUSS based algorithm, p is chosen to be 0.5, and the number of iterations used is 8.

The following interesting observations can be made in Fig. 2: $i)$ FOCUSS based detection performs significantly better than MMSE, OMP, and CoSaMP based detection in both GSM and SM (e.g., in SM, the FOCUSS based detector outperforms CoSaMP based detector by about 5 dB at 10^{-3} BER); $ii)$ OMP and CoSaMP based detection perform poorly compared to MMSE detection in GSM owing to the number of observations N being less and half of the elements of the transmitted vector being non-zeros; $iii)$ With MMSE detection, SM performs worse than GSM. This is because of the larger-sized QAM used by SM (i.e., 16-QAM in SM vs 4-QAM in GSM); and $iv)$ With the FOCUSS based detection, SM is found to achieve better performance compared to GSM. This is because, the higher sparsity levels in SM than in GSM are more favorable for the FOCUSS based algorithm to exploit sparsity.

The above observations are further reinforced in Fig. 3, which show the average SNR required to achieve a target BER of 10^{-3} as a function of system loading factor $\frac{Kn_t}{N}$ in the range 1 to 1.7 for the same set of parameters in Fig. 2. The loading factor is increased by keeping K and n_t fixed and progressively decreasing N. Note that system loading factors greater than 1 can be equivalently obtained by fixing n_t and N and increasing the number of users K. From Fig. 3, it is clearly seen that MMSE detector is not robust to increase in system loading factor. On the other hand, with FOCUSS based detection, both GSM and SM are much more robust to increase in loading factor. Among GSM and SM with FOCUSS detection, SM is more robust than GSM because of the higher level of sparsity in SM. The above observations made in Figs. 2 and3 highlight the effectiveness of the proposed algorithm in exploiting sparsity in underdetermined multiuser GSM-MIMO systems.

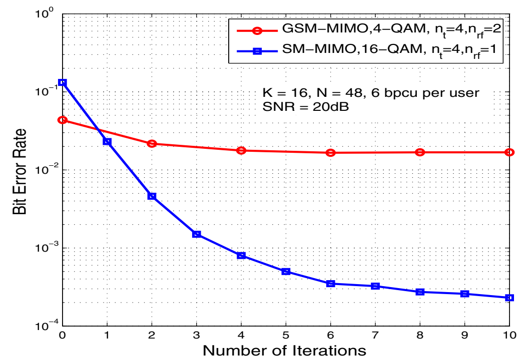


Fig. 4. Variation of BER as a function of number of iterations in FOCUSS based algorithm in GSM-MIMO and SM-MIMO systems. $K = 16, N = 48, n_t = 4$, 6 bpcu per user, and 20 dB SNR

Figure 4 shows the variation in BER as a function of the number of iterations in the FOCUSS based algorithm for GSM-MIMO and SM-MIMO systems at 20 dB SNR. Zero iterations correspond to the initial solution $\mathbf{x}^{(0)}$, which we take to be the MMSE solution. We see that most of the gain in BER performance comes within 4 to 5 iterations, an observation which can be used to limit computation complexity. The interplay between QAM size and sparsity discussed above is evident from the crossover happening from 0 iterations (MMSE detector) to 2 iterations (FOCUSS based detector), after which sparsity gains over QAM size.

Figure 5 shows the SNR required to achieve 10^{-3} BER with increasing loading factors (1 to 1.7) and number of iterations (2, 5, 8) in SM-MIMO with 6 bpcu per user. We see that for loading factors in the range 1.1 to 1.5, about 2 to 5 iterations of the algorithm brings most of the gains in SNR. For loading factors beyond 1.5, more iterations result in better gains.

In order to further validate the sparsity exploiting nature of the FOCUSS based algorithm, we compare its performance

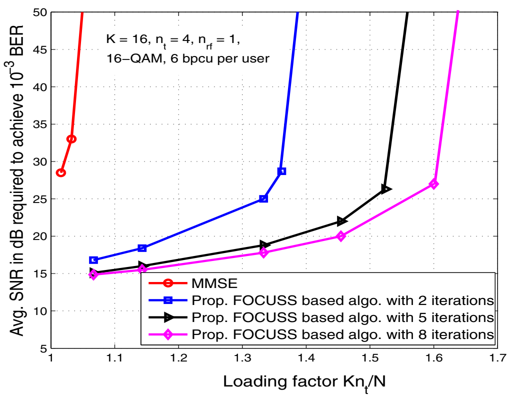
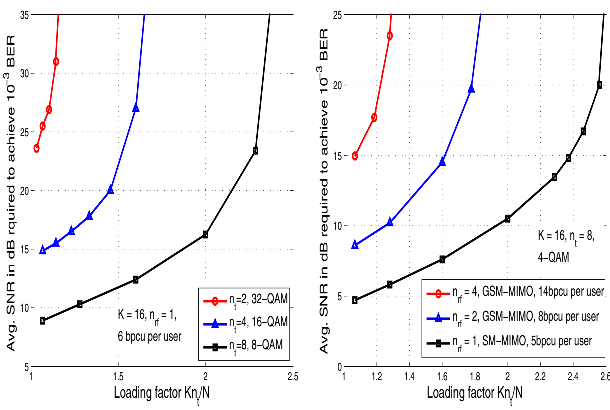


Fig. 5. Variation of SNR required to achieve 10^{-3} BER as a function of number of iterations and loading factor in SM-MIMO system. $K = 16, n_t = 4$, and 6 bpcu per user



(a) SM: 6 bpcu, varying n_t (b) GSM, SM: varying bpcu

Fig. 6. Variation of SNR required to achieve 10^{-3} BER as a function of loading for various sparsity levels and bpcu values in SM-MIMO and GSM-MIMO systems.

for varying sparsity levels (number of zeros in relation to the total number of elements in the vector) in two scenarios. In the first scenario (Fig. 6(a)), we fix the bpcu at 6 per user, and consider three SM-MIMO systems with $i)$ $n_t = 2$, 32-QAM, $ii)$ $n_t = 4$, 16-QAM, and $iii)$ $n_t = 8$, 8QAM. The sparsity levels in these systems are $\frac{1}{2}$, $\frac{3}{4}$, and $\frac{7}{8}$, respectively. We see that the third system with most zeros gains about 6 dB with respect to the second system, and more than about 15 dB compared to the first system at a loading factor of 1.06. These gains become more pronounced for increasing loading factors (Fig. 6(a)). In the second scenario (Fig. 6(b)), we fix the number of transmit antennas n_t and the QAM size, and consider the following systems for comparison $i)$ $n_{rf} = 4$, GSM-MIMO, $ii)$ $n_{rf} = 2$, GSM-MIMO, and $iii)$ $n_{rf} = 1$, SM-MIMO. The sparsity levels in these systems are the same as those considered in the first scenario. This leads to 14 bpcu, 8 bpcu, and 5 bpcu per user, respectively. As before, the third system which is the most sparse achieves the best performance (Fig. 6(b)). Beside the higher energy required for a higher order QAM, the identical shapes of the curves in both the scenarios suggests a lesser sensitivity of the FOCUSS based algorithm to QAM size.

V. Conclusion

We introduced FOCUSS algorithm, a sparsity-exploiting algorithm hitherto unused in communication theory/systems domain, for the purpose of signal detection in large-scale multiuser GSM-MIMO systems. The proposed FOCUSS based detection algorithm used a relaxation that encouraged sparsity in the solution. Simulation results showed that the proposed FOCUSS based detection outperforms MMSE detection by increasingly larger margins for increasingly larger system loading factors. This can help increasing the number of users in multiuser GSM-MIMO systems.

References

[1] A. Chockalingam and B. S. Rajan, *Large MIMO Systems*, Cambridge Univ. Press, Feb. 2014.

[2] E. G. Larsson, O. Edfors, F. Tufvesson, and T. L. Marzetta, "Massive MIMO for next generation wireless systems," *IEEE Commun. Mag.*, vol. 52, no. 2, pp. 186-195, Feb. 2014.

[3] B. Panzner, W. Zirwas, S. Dierks, M. Lauridsen, P. Mogensen, K. Pajukoski, and D. Miao, "Deployment and implementation strategies for massive MIMO in 5G," *IEEE GLOBECOM*, pp. 346-351, Dec. 2014.

[4] D. Gesbert, M. Kountouris, R. W. Heath Jr., C-B. Chae, and T. Sälzer, "Shifting the MIMO paradigm," *IEEE Signal Process. Mag.*, vol. 24, no. 5, pp. 36-46, Sep. 2007.

[5] M. Di Renzo, H. Haas, A. Ghrayeb, S. Suguira, and L. Hanzo, "Spatial modulation for generalized MIMO: challenges, opportunities and implementation," *Proc. of the IEEE*, vol. 102, no. 1, pp. 56-103, Jan. 2014.

[6] J. Wang, S. Jia, and J. Song, "Generalised spatial modulation system with multiple active transmit antennas and low complexity detection scheme," *IEEE Trans. Wireless Commun.*, vol. 11, no. 4, pp. 1605-1615, Apr. 2012.

[7] T. Datta and A. Chockalingam, "On generalized spatial modulation," *IEEE WCNC 2013*, pp. 2716-2721, Apr. 2013.

[8] N. Serafimovski, S. Sinanovic, M. Di Renzo, and H. Haas, "Multiple access spatial modulation," *EURASIP J. Wireless Commun. and Networking 2012*. doi:10.1186/1687-1499-2012-299.

[9] T. L. Narasimhan, P. Raviteja, and A. Chockalingam, "Generalized spatial modulation in large-scale multiuser MIMO systems," *IEEE Trans. Wireless Commun.*, vol. 14, no. 7, pp. 3764-3779, Jul. 2015.

[10] S. Foucart and H. Rauhut, *A Mathematical Introduction to Compressive Sensing*, Birkhäuser/Springer, New York (2013).

[11] C-M. Yu, S-H. Hsieh, H-W. Liang, C-S. Lu, W-H. Chung, S-Y. Kuo, and S-C. Pei, "Compressed sensing detector design for space shift keying in MIMO systems," *IEEE Commun. Lett.*, pp. 1556-1559, Oct. 2012.

[12] C-H. Wu, W-H. Chung, and H-W. Liang, "OMP-based detector design for space shift keying in large MIMO systems," *IEEE GLOBECOM*, pp. 4072-4076, Dec. 2014.

[13] J. A. Tropp and A. C. Gilbert, "Signal recovery from random measurements via orthogonal matching pursuit," *IEEE Trans. Inform. Theory*, vol. 53, no. 12, pp. 4655-4666, Dec. 2007.

[14] D. Needell and A. Tropp, "CoSaMP: Iterative signal recovery from incomplete and inaccurate samples," *Appl. Comput. Harmon. Anal.* 26 (2009) 301-321.

[15] I. F. Gorodnitsky and B. D. Rao, "Sparse signal reconstruction from limited data using FOCUSS: a re-weighted minimum norm algorithm," *IEEE Trans. Signal Process.*, vol. 45, no. 3, pp. 600-616, Mar. 1997.

[16] B. D. Rao and K. Kreutz-Delgado, "An affine scaling methodology for best basis selection," *IEEE Trans. Signal Process.*, pp. 187-200, Jan. 1999.

[17] B. D. Rao, K. Engan, S. F. Cotter, J. Palmer, and K. Kreutz-Delgado, "Subset selection in noise based on diversity measure minimization," *IEEE Trans. Signal Process.*, vol. 51, no. 3, pp. 760-770, Mar. 2003.

[18] K. Kreutz-Delgado, J. F. Murray, B. D. Rao, K. Engan, T-W. Lee, and T. J. Sejnowski, "Dictionary learning algorithms for sparse representation," *Neural Comput.*, vol. 15, no. 2, pp. 349-396, Feb. 2003.

[19] H. Jung, J. C. Ye, and E. Y. Kim, "Improved k-t BLAST and k-t SENSE using FOCUSS," *Phys. Med. Biol.*, pp. 3201-3226, May 2007.

[20] X. Tian, W. Roberts, J. Li, and P. Stoica, "Sparse learning via iterative minimization with application to MIMO radar imaging," *IEEE Trans. Signal Process.*, vol. 59, no. 3, pp. 1088-1101, Mar. 2011.

[21] M. K. Varanasi and B. Aazhang, "Parametric generalized Gaussian density estimation," *J. Acoust. Soc. Amer.*, pp. 1404-1415, Oct. 1989.