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Precoder Designs for MIMO Broadcast Channels with Imperfect CSI

P. Ubaidulla and A. Chockalingam
Department of ECE, Indian Institute of Science, Bangalore 560012, INDIA

***Abstract*—In this paper, we propose two robust precoder designs for multiple-input multiple-output (MIMO) broadcast channels *with imperfect channel state information (CSI)*. First, we consider a precoder design for a multiuser multiple-input single-output (MISO) downlink, where a base station (BS) equipped with multiple transmit antennas communicates with user terminals which are equipped with a single receive antenna. For this scenario, we propose a robust precoder design based on the minimization of the total BS transmit power under constraints on individual user signal-to-interference-plus-noise ratio (SINR). We show that this problem can be formulated as a second order cone program (SOCP) that can be solved efficiently. Next, we consider a precoder design for a multiuser MIMO downlink, where the user terminals are equipped with multiple receive antennas. For this scenario, we propose a robust joint precoder/receive filter design based on the minimization of a stochastic function of the sum mean square error (SMSE), under a constraint on the total BS transmit power. We solve this problem through an iterative algorithm, wherein the optimization is performed with respect to the transmit precoder and the receive filter in an alternating fashion.**

I. INTRODUCTION

There has been considerable interest in multiuser multiple-input multiple-output (MIMO) wireless communication systems in view of their potential to offer the benefits of transmit diversity and increased data rates [1], [2]. Because of the difficulty in providing mobile user terminals with several antennas due to space constraints, multiuser multiple-input single-output (MISO) wireless communication on the downlink, where the base station (BS) is equipped with multiple transmit antennas and each user terminal is equipped with a single receive antenna, is of interest [3]. Emerging wireless systems consider the use of multiple receive antennas (e.g., two receive antennas) at the user terminals. Such multiple receive antenna systems for multiuser downlink communication have also attracted recent investigations [3].

Transmit-side processing in the form of precoding has been investigated in both MISO and MIMO downlink as a means to mitigate multiuser interference effects [2]. Linear and non-linear precoder designs for MISO downlink and their performance have been investigated in the literature [4]–[8]. These precoder designs are based on the assumption of perfect knowledge of the channel state information (CSI) at the transmitter. However, in practice, CSI at the transmitter suffers from inaccuracies caused by errors in channel estimation and/or limited, delayed or erroneous feedback. The performance of precoding schemes is sensitive to such inaccuracies [9]. Hence, it is important to design precoding schemes which are robust in the presence of inaccuracies in CSI. Robust non-linear and linear precoder designs for MISO downlink based on ZF and MMSE criteria are reported in [10]. Robust precoder designs with QoS constraints are reported in [11]–[13]. Our first contribution in this paper is in the context of robust precoder design for multiuser MISO downlink. We propose a robust design for precoder with SINR constraints, which we formulate as a second order cone program (SOCP) that can be solved efficiently. We note that the proposed precoder design is computationally less complex compared to the SDP formulations presented in [11].

As mentioned earlier, MIMO downlink communication systems, where user terminals have more than one receive antenna are being increasingly considered. For such MIMO downlink systems, precoder designs have been investigated in the literature [14]–[18]. Recently, a robust precoder design for multiuser MIMO downlink based on total BS transmit power minimization under individual user mean square error (MSE) constraints has been reported in [19]. Our second contribution in this paper is a robust precoder design for multiuser MIMO downlink, based on minimizing a stochastic function of sum-MSE (SMSE) under a total BS transmit power constraint. We present an iterative optimization algorithm, wherein, the joint optimization with respect to the precoder matrix and receive filter is replaced by an optimization over the precoder and the receive filter in an alternating fashion. The performance of the proposed design is illustrated through simulations.

The rest of the paper is organized as follows. The system model is presented in Section II . The proposed robust precoder for multiuser MISO downlink is presented in Section III. The proposed robust precoder/receive filter design for multiuser MIMO downlink is presented in Section IV. Conclusions are presented in Section V.

II. SYSTEM MODEL

We consider a multiuser MIMO downlink, where a base station (BS) communicates with M users on the downlink. The BS employs N_t transmit antennas and the kth user is equipped with N_{r_k} receive antennas, $1 \leq k \leq M$. Let $\mathbf{u}_k$ denote[1] the

[1] Vectors are denoted by boldface lowercase letters, and matrices are denoted by boldface uppercase letters. $[.]^T$, $[.]^H$, and $[.]^\dagger$, denote transpose, Hermitian, and pseudo-inverse operations, respectively. $[\mathbf{A}]_{ij}$ denotes the element on the ith row and jth column of the matrix $\mathbf{A}$. vec(.) operator stacks the columns of the input matrix into one column-vector.

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$L_k \times 1$ unit-norm data symbol vector for the kth user, where L_k, $k = 1, 2, \cdots, M$, is the number of data streams for the kth user. Stacking the data vectors for all the users, we get the global data vector $\mathbf{u} = [\mathbf{u}_1^T, \cdots, \mathbf{u}_M^T]^T$. Let $\mathbf{B}_k \in \mathcal{C}^{N_t \times L_k}$ represent the precoding matrix for the kth user. The global precoding matrix $\mathbf{B} = [\mathbf{B}_1, \mathbf{B}_2, \cdots, \mathbf{B}_M]$. The transmit vector is given by

$$\mathbf{x} = \mathbf{B}\mathbf{u}. \tag{1}$$

The kth component of the transmit vector $\mathbf{x}$ is transmitted from the kth transmit antenna. The total BS transmit power can be represented as

$$P = \mathbb{E}\{\|\mathbf{x}\|^2\} = \text{Tr}(\mathbf{B}^H\mathbf{B}), \tag{2}$$

where $\mathbb{E}\{\cdot\}$ denotes the expectation operator. The global channel matrix is

$$\mathbf{H} = [\mathbf{H}_1^T \quad \mathbf{H}_2^T \cdots \mathbf{H}_M^T]^T, \tag{3}$$

where $\mathbf{H}_k$ is the $N_{r_k} \times N_t$ channel matrix of the kth user. The entries of the channel matrices are assumed to be zero-mean, unit-variance complex Gaussian random variables. The received signal vectors are given by

$$\mathbf{y}_k = \mathbf{H}_k\mathbf{B}\mathbf{u} + \mathbf{n}_k, \quad 1 \leq k \leq M, \tag{4}$$

where $\mathbf{n}_k \in \mathcal{CN}(0, \sigma_n^2\mathbf{I})$ is the noise vector at the kth user. The users estimate data vector meant for them as

$$\begin{aligned} \widehat{\mathbf{u}}_k &= \mathbf{C}_k\mathbf{y}_k = \mathbf{C}_k\mathbf{H}_k\mathbf{B}\mathbf{u} + \mathbf{C}_k\mathbf{n}_k \\ &= \mathbf{C}_k\mathbf{H}_k\Big(\sum_{j=1}^{M}\mathbf{B}_j\mathbf{u}_j\Big) + \mathbf{C}_k\mathbf{n}_k, \ 1 \leq k \leq M, \end{aligned} \tag{5}$$

where $\mathbf{C}_k$ is the $L_k \times N_{r_k}$ dimensional receive filter of the kth user. Stacking the estimated vectors of all users, the global estimate can be written as

$$\widehat{\mathbf{u}} = \mathbf{C}\mathbf{H}\mathbf{B}\mathbf{u} + \mathbf{C}\mathbf{n}, \tag{6}$$

where $\mathbf{C}$ is a block diagonal matrix with $\mathbf{C}_k, 1 \leq k \leq M$ on the diagonal, and $\mathbf{n} = [\mathbf{n}_1^T, \cdots, \mathbf{n}_M^T]^T$. As the receivers are non-cooperative, the global receive matrix $\mathbf{C}$ has a block diagonal structure.

A. CSI Error Models

We consider the following models for the CSI error. Consider that the transmitter CSI $\widehat{\mathbf{H}}$ is related to the true channel $\mathbf{H}$ as

$$\mathbf{H} = \widehat{\mathbf{H}} + \mathbf{E}, \tag{7}$$

where $\widehat{\mathbf{H}} = [\widehat{\mathbf{H}}_1^T \quad \widehat{\mathbf{H}}_2^T \cdots \widehat{\mathbf{H}}_M^T]^T$, and $\mathbf{E} = [\mathbf{E}_1^T \quad \mathbf{E}_2^T \cdots \mathbf{E}_M^T]^T$ denotes the CSI error matrix. In a norm-bounded error (NBE) model,

$$\|\mathbf{E}_k\|_F \leq \delta_k, \quad 1 \leq k \leq M, \tag{8}$$

or, equivalently, the true channel $\mathbf{H}_k$ belongs to the uncertainty set $\mathcal{R}_k$ given by

$$\mathcal{R}_k = \{\boldsymbol{\zeta} \big| \boldsymbol{\zeta} = \widehat{\mathbf{H}}_k + \mathbf{E}_k, \|\mathbf{E}_k\|_F \leq \delta_k\}, \ 1 \leq k \leq M. \tag{9}$$

We use this NBE model in Section III. An alternate error model is a stochastic error (SE) model, where $\widehat{\mathbf{H}}_k$ is the estimated channel matrix of the kth user, and $\mathbf{E}_k$ is the estimation error matrix. The error matrix $\mathbf{E}_k$ is assumed to be Gaussian distributed with zero mean and $\mathbb{E}\{\mathbf{E}_k\mathbf{E}_k^h\} = \sigma_E^2\mathbf{I}_{Nr_k,Nr_k}$. We use this SE model in Section IV.

III. Robust Linear Precoder Design for Multiuser MISO Downlink

In this section, we consider the design of a robust precoder for multiuser MISO downlink. The precoder design is based on the minimization of total BS transmit power needed to satisfy the SINR requirements of each user. We assume the NBE model for the imperfections in the CSI. The SINR at ith user terminal is given by

$$\text{SINR}_i = \frac{|\mathbf{h}_i^H\mathbf{b}_i|^2}{\sum_{j=1,j\neq i}^{M}|\mathbf{h}_i^H\mathbf{b}_j|^2 + \sigma_n^2}. \tag{10}$$

When the transmitter has perfect knowledge of CSI, the problem of designing a precoder which transmits minimum power while ensuring the required SINR at each user can be posed as

$$\begin{aligned} \min_{\mathbf{B}} \quad & \text{trace}\left(\mathbf{B}^H\mathbf{B}\right) \\ \text{subject to} \quad & \text{SINR}_i \geq \eta_i, \quad 1 \leq i \leq M, \end{aligned} \tag{11}$$

where η_i is the SINR required at the ith user. It is shown in [8] that this problem can be formulated as the following SOCP:

$$\begin{aligned} \min_{\mathbf{B}} \quad & \tau \\ \text{subject to} \quad & \|\mathbf{B}\| - \tau \leq 0, \\ & \big\|[\mathbf{h}_i^H\mathbf{B} \ \sigma_n]\big\| - a_i\mathbf{h}_i^H\mathbf{b}_i \leq 0, \quad 1 \leq i \leq M, \end{aligned} \tag{12}$$

where $a_i = \sqrt{\frac{1}{1+\eta_i}}$. Here, we have assumed that the imaginary part of $\mathbf{h}_i^H\mathbf{b}_i$ is zero. This is possible because we can add arbitrary phase rotation to the columns of $\mathbf{B}$ without affecting the SINR.

A. Proposed Robust Linear Precoder with SINR Constraints

When the CSI at the transmitter is imperfect, the precoder designed based on (12) assuming perfect CSI fails to meet the SINR requirements. Here, we consider a precoder design that is robust in the presence of CSI imperfections, which can be modeled by the NBE model. For this error model, the robustness requirement of the precoder can be represented, in terms of the SOCP formulation, as

$$\begin{aligned} \min_{\mathbf{B}} \quad & \tau \\ \text{subject to} \quad & \|\mathbf{b}\| - \tau \leq 0, \\ & \max_{\mathbf{h}_i \in \mathcal{R}_i}\left(\big\|[\mathbf{h}_i^H\mathbf{B} \ \sigma_n]\big\| - a_i\mathbf{h}_i^H\mathbf{b}_i\right) \leq 0, \\ & 1 \leq i \leq M. \end{aligned} \tag{13}$$

This problem is akin to the Robust Optimization (RO) [20], which is one of the methodologies for solving optimization problems under parameter uncertainties. The general problem of optimization under parameter uncertainties has the following form:

$$\min \quad f_0(\boldsymbol{\zeta}) \tag{14}$$
$$\text{subject to} \quad f_i(\boldsymbol{\zeta}, \mathbf{d}) \leq 0, \quad \forall \mathbf{d} \in \mathcal{Z}, 1 \leq i \leq M,$$

where $\boldsymbol{\zeta} \in \mathbb{R}^n$ is the vector of decision variables, $\mathbf{d} \in \mathbb{R}^k$ is the data vector, f_i are the constraints, and $\mathcal{Z}$ is the uncertainty set. This problem is computationally intractable, in general. Recent results have shown that it is possible to have robust counterparts which preserve the structure of the nominal problem [21]. For the precoder design, this means that the robust design problem can be formulated as a SOCP, which results in a significant reduction in complexity compared to the SDP formulation in [11]. In this context, consider the data perturbation model

$$\mathbf{d} \;=\; \mathbf{d}^0 + \sum_{j \in N} \boldsymbol{\Delta}\mathbf{d}^j z_j, \tag{15}$$

where $\mathbf{d}^0$ is the nominal data vector, $\Delta\mathbf{d}^j$ are the directions of data perturbations, and $\{z_j, 1 \leq j \leq N\}$ are the zero mean i.i.d random variables. Robust optimization aims at finding a robust optimal $\mathbf{x}$ which will meet the following constraint:

$$\max_{\mathbf{d} \in \mathcal{Z}_\Omega} f(\boldsymbol{\zeta}, \mathbf{d}) \leq 0, \tag{16}$$

where
$$\mathcal{Z}_\Omega \;=\; \left\{ \mathbf{d}_0 + \sum_{j \in N} \boldsymbol{\Delta}\mathbf{d}^j u_j \middle| \|\mathbf{u}\| \leq \Omega \right\}. \tag{17}$$

The following linearized version of the constraint (16) is considered in [21]:

$$\max_{(\mathbf{v},\mathbf{w}) \in \mathcal{V}_\Omega} f(\boldsymbol{\zeta}, \mathbf{d}^0) + \sum_{j \in N} \{ f(\boldsymbol{\zeta}, \boldsymbol{\Delta}\mathbf{d}) v_j + f(\boldsymbol{\zeta}, -\boldsymbol{\Delta}\mathbf{d}) w_j \} \leq 0 \tag{18}$$

where $\mathcal{V}_\Omega \;=\; \left\{ (\mathbf{v}, \mathbf{w}) \in \mathbb{R}_+^{2|N|} \middle| \|\mathbf{v} + \mathbf{w}\| \leq \Omega \right\}$.

It is shown in [21] that, for an SOC constraint, $\boldsymbol{\zeta}$ is feasible in (16) if $\boldsymbol{\zeta}$ is feasible in (18). We state the following theorem for the specific case of SOCP constraints.

Theorem 1: (Bertsimas-Sim [21])
a) Constraint (18) is equivalent to

$$f(\boldsymbol{\zeta}, \mathbf{d}^0) + \Omega \|\mathbf{s}\| \leq 0, \tag{19}$$

where $s_j = \max\left\{ f(\boldsymbol{\zeta}, \boldsymbol{\Delta}\mathbf{d}^j), f(\boldsymbol{\zeta}, -\boldsymbol{\Delta}\mathbf{d}^j) \right\}$.
b) Equation (19) can be written as $\exists (y, \mathbf{t}) \in \mathbb{R}^{|N|+1}$

$$f(\boldsymbol{\zeta}, \mathbf{d}^0) \;\leq\; -\Omega y \tag{20a}$$
$$f(\boldsymbol{\zeta}, \boldsymbol{\Delta}\mathbf{d}) \;\leq\; t_j \quad \forall j \in N \tag{20b}$$
$$f(\boldsymbol{\zeta}, -\boldsymbol{\Delta}\mathbf{d}) \;\leq\; t_j \quad \forall j \in N \tag{20c}$$
$$\|\mathbf{t}\| \;\leq\; y. \tag{20d}$$

The data perturbation model (15) for the second order cone constraint of the precoder design problem in (13) takes the form[2]

$$\mathbf{d}_i \;=\; \mathbf{d}_i^0 + \sum_{M} \boldsymbol{\Delta}\mathbf{d}_i^j \overline{\mathbf{e}}_{i,j}, 1 \leq i \leq M, 1 \leq j \leq 2N_t. \tag{21}$$

where $\mathbf{d}_i = \text{vec}\big([\overline{\mathbf{h}}_i \quad \overline{\mathbf{h}}_i]\big)$, $\mathbf{d}^0 = \text{vec}\Big([\overline{\widehat{\mathbf{h}}}_i \quad \overline{\widehat{\mathbf{h}}}_i]\Big)$, $\boldsymbol{\Delta}\mathbf{d}_i^j = [\mathbf{i}_j \quad \mathbf{i}_j]^T$, and $\mathbf{i}_j$ is the jth row of $\mathbf{I}_{2N_t \times 2N_t}$. $\mathbf{d}$ is the vector of all data in the problem and has the structure given above as $\overline{\mathbf{h}}_i$ appears twice in the constraint in (13). $\boldsymbol{\Delta}\mathbf{d}_i^j$ indicates how the error in the jth component of $\overline{\mathbf{h}}_i$ affects $\mathbf{d}$. Based on this data perturbation model, it is obvious that the channel uncertainty region $\mathcal{R}_i$ of the robust precoder design problem of (13) corresponds to the uncertainty region in (17), with $\Omega = \delta_i$.

Using the data perturbation model in (21) and applying Theorem-1 to (13), we obtain the following SOCP formulation of the proposed robust precoder design:

$$\min_{\mathbf{B}} \quad \tau \tag{22a}$$
$$\text{subject to} \quad \left\|\overline{\mathbf{b}}_k\right\| \leq \tau \tag{22b}$$
$$\left\|[\overline{\mathbf{h}}_k^T \overline{\mathbf{B}} \quad \sigma_n]\right\| - a_k \overline{\mathbf{h}}_k^T \overline{\mathbf{b}}_k \leq -\delta_k y_k \tag{22c}$$
$$\left\|[\overline{\mathbf{b}}_i^T \quad \sigma_n]\right\| - a_k \overline{\mathbf{B}}_{i,k} \leq t_{k,i} \tag{22d}$$
$$\left\|[\overline{\mathbf{b}}_i^T \quad \sigma_n]\right\| + a_k \overline{\mathbf{B}}_{i,k} \leq t_{k,i} \tag{22e}$$
$$\|\mathbf{t}_k\| \leq y_k \tag{22f}$$
$$1 \leq k \leq M, \quad 1 \leq i \leq 2N_t, \tag{22g}$$

where $\mathbf{b}_i$ is the ith row of $\mathbf{B}$. In this formulation of the robust precoder design, the constraints are of the same type as the nominal problem (12). Hence, the computational complexity is of the same order as the nominal problem.

The robustness constraint in (18) is a relaxation of the constraint in (16). By selecting appropriate value of $\kappa, 0 \leq \kappa \leq 1$, and replacing δ_k in (22) by $\kappa\delta_k$, it is possible to get a robust precoder which transmits less power while achieving the required SINR constraints. Through simulations, it is possible to find values of κ which provide good balance between the achieved SINR and the transmit power. For example, it is found that $\kappa = 0.25$ is an appropriate value for $N_t = 3$ and $M = 3$.

B. Simulation Results

In this section, we present the performance of the proposed robust precoder design (22) through simulations. We compare this performance with other robust designs available in the literature. The components of the estimated channel vectors $\widehat{\mathbf{h}}_k$, $1 \leq k \leq M$ are i.i.d zero mean unit variance proper complex Gaussian random variables. We compare the performance of the proposed design with the robust SOCP design (denoted here by SDP-1) and the unstructured SDP design (denoted by SDP-2) in [11], and the robust power control (denoted by RPC) in [13].

First, we compare the CDF of the achieved SINR of SDP-1, SDP-2, and RPC with the CDF of SINR of the proposed robust design. Figure 1 shows the CDF for various methods. In this experiment, we consider a system with a base station having $N_t = 3$ transmit antennas and $M = 3$ single antenna receivers. The uncertainty size of CSI at the transmitter is assumed to be same for all users and is $\delta = 0.015$. The target SINR for all users is $\gamma = 5\,$dB. In case of SDP-1 and RPC, it is evident

[2] For a matrix $\mathbf{A}$

$$\overline{\mathbf{A}} \;=\; \begin{bmatrix} \Re(\mathbf{A}) & \Im(\mathbf{A}) \\ -\Im(\mathbf{A}) & \Re(\mathbf{A}) \end{bmatrix},$$

and for a column vector $\mathbf{z}$, $\overline{\mathbf{z}} = [\Re\{\mathbf{z}\}^T \quad -\Im\{\mathbf{z}\}^T]^T$.

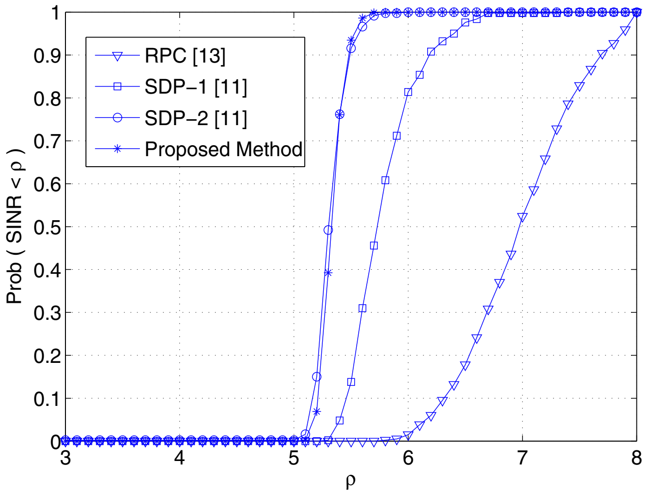

Fig. 1. CDF of achieved SINR at the downlink users. Minimum required SINR $\gamma_1 = \gamma_2 = \gamma_3 = 5$ dB. $N_t = M = 3$, uncertainty size, $\delta_1 = \delta_2 = \delta_3 = 0.015$.

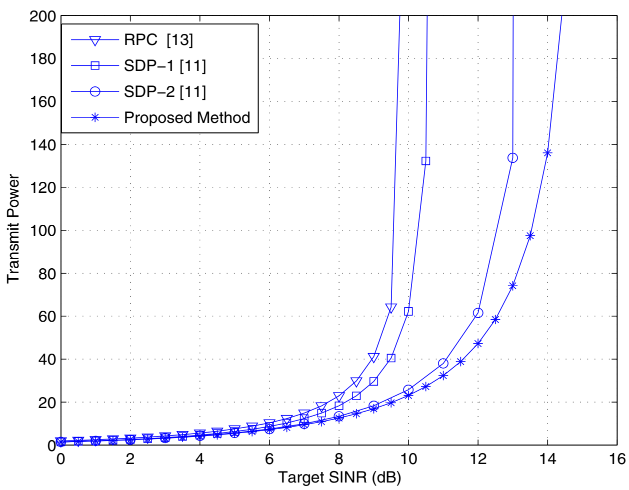

Fig. 2. Transmit power versus SINR requirement of the users. Uncertainty size $\delta = \delta_1 = \delta_2 = \delta_3 = 0.02$, $N_t = M = 3$.

that, most of the time, the users get SINR much higher than the target SINR. This implies that these algorithms result in much higher transmit power than required. The SDP-2 and the proposed design have almost same CDF, and is very near to the required SINR. That is, performance-wise, the proposed design achieves almost the same performance as SDP-2 in [11] but with reduced complexity.

Figure 2 shows the transmit power $\text{Tr}\{\mathbf{B}^H\mathbf{B}\}$ for various robust designs in order to achieve different target SINRs. This experiment also has the same setting as in Fig. 1, except for the target SINR which is varied from 0 dB to 10 dB. The SDP-1 and RPC methods transmits more power compared to SDP-2 and the proposed method. This higher transmit power results in the higher SINRs at the users.

Figure 3 shows the transmit power for the different robust designs for different values of the size of channel uncertainty. The SINR requirement for all users is 5 dB. The SDP-1 and

TABLE I
COMPARISON OF RUN-TIME IN SECONDS FOR DIFFERENT PRECODING METHODS

Method	M,N_t=3	M,N_t=4	M,N_t=5	M,N_t=6
Proposed	0.10	0.2	0.44	0.6
SDP-1 [11]	0.2	0.3	0.6	1
SDP-2 [11]	4.5	16	61	121

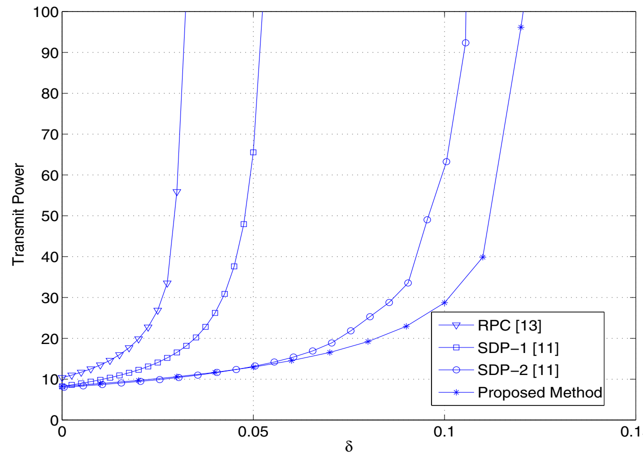

Fig. 3. Transmit power versus channel uncertainty size, $N_t = M = 3$, SINR requirement of the users $\gamma_1 = \gamma_2 = \gamma_3 = 5$ dB.

RPC methods end up in higher transmit power compared to SDP-2 and the proposed method. This higher transmit power results in the higher SINRs at the users. Also the range of δ for which the proposed method is feasible is larger than other methods.

Table-1 shows the comparison of computation time in seconds required for solving the robust precoder using different methods on a 2.66 GHz machine using the solver SeDuMi. Computation time for SDP-2 is the highest. Computation time for SDP-1 and the proposed method are comparable. Thus, the proposed method is able to achieve the performance comparable to SDP-2 at the computational cost of SDP-1. In summary, the proposed robust design achieves better performance than the other methods compared while being computationally less intensive.

IV. ROBUST TRANSCEIVER DESIGN FOR MULTIUSER MIMO DOWNLINK

In this section, we consider a robust joint design of precoder/receive filter for a multiuser MIMO downlink with imperfect CSI at the transmitter. We assume that the CSI imperfections can be modeled by the SE model. The precoder design is based on the minimization of the SMSE under a constraint on the total BS transmit power. When the transmitter has perfect knowledge of CSI, the problem of designing the transmit precoder $\mathbf{B}$ and receive filter $\mathbf{C}$ which minimizes the SMSE under a transmit power constraint can be written as

$$\min_{\mathbf{B},\mathbf{C}} \quad \texttt{smse} \qquad (23)$$
$$\text{subject to} \quad \text{Tr}(\mathbf{B}^H\mathbf{B}) \le P_T,$$

where P_T is the maximum allowed transmit power, Tr(.) is the trace operator, and $\texttt{smse} = \mathbb{E}\{\|\hat{\mathbf{u}} - \mathbf{u}\|^2\}$.

A. Proposed Robust Design with imperfect CSI

In order to make the precoder design robust in the presence of CSI imperfections modeled by the SE model, we consider the SMSE averaged over the CSI error $\mathbf{E}$ as the objective function. Following this approach, the robust precoder design problem can written as

$$\begin{aligned} \min_{\mathbf{B},\mathbf{C}} \quad & \mu \\ \text{subject to} \quad & \mathrm{Tr}\left(\mathbf{B}\mathbf{B}^H\right) \leq P_T, \end{aligned} \tag{24}$$

where

$$\begin{aligned} \mu &\triangleq \mathbb{E}_{\mathbf{E}}\{\mathtt{smse}\} \\ &= \mathrm{Tr}\Big(\mathbf{C}_k\widehat{\mathbf{H}}_k\mathbf{B}\mathbf{B}^H\widehat{\mathbf{H}}_k^H\mathbf{C}_k^H - 2\Re(\mathbf{C}_k\widehat{\mathbf{H}}_k\mathbf{B}_k) + \mathbf{I}\Big) \\ &\quad + \mathrm{Tr}\Big(\mathbf{C}_k(\sigma_E^2\mathrm{Tr}(\mathbf{B}\mathbf{B}^H) + \sigma_n^2\mathbf{I})\mathbf{C}_k^H\Big). \end{aligned} \tag{25}$$

The new objective function is convex in $\mathbf{B}_k$ for a fixed value of $\mathbf{C}_k$ and vice versa, but not jointly convex in $\mathbf{B}_k$ and $\mathbf{C}_k$. Hence, we solve the optimization problem in (24) by optimizing over $\mathbf{B}_k$ and $\mathbf{C}_k$ alternately. In [22], we had proposed an iterative solution, wherein the robust receiver design for a given precoder $\mathbf{B}$ has a closed-form solution, whereas the robust precoder design for a given receive filter $\mathbf{C}$ is formulated as an SOCP. In the iterative algorithm proposed here, we provide a semi-analytic solution to the precoder design problem.

1) Robust Receiver Filter and Precoder Design: The optimum robust receiver $\mathbf{C}_k^{\mathbf{B}}$ for the kth user for a given precoder matrix $\mathbf{B}$ is the value of $\mathbf{C}_k$ that minimizes μ. Differentiating $\mu(\mathbf{B},\mathbf{C})$ with respect to $\mathbf{C}_k$ and equating the result to zero, we get [22]

$$\mathbf{C}_k^{\mathbf{B}} = \mathbf{B}_k^H\mathbf{H}_k^H\left(\mathbf{H}_k\mathbf{B}\mathbf{B}_k^H + (\sigma_E^2\mathrm{Tr}\left(\mathbf{B}\mathbf{B}^H\right) + \sigma_n^2)\mathbf{I}\right)^{-1}, \quad 1 \leq k \leq M. \tag{26}$$

For a given global precoder matrix $\mathbf{B}$, the global receive filter matrix $\mathbf{C}^{\mathbf{B}}$ is obtained as a block-diagonal matrix with $\mathbf{C}_k^{\mathbf{B}},\ 1 \leq k \leq M$ on the diagonal. For a given $\mathbf{C}$, the problem designing a robust precoder $\mathbf{B}^{\mathbf{C}}$ can be written as

$$\begin{aligned} \min_{\mathbf{B}} \quad & \mu \\ \text{subject to} \quad & \|\mathbf{b}\|^2 \leq P_T. \end{aligned} \tag{27}$$

Introducing $\widehat{\mathbf{D}} = \mathbf{I} \otimes (\mathbf{C}\widehat{\mathbf{H}})$, $\widetilde{\mathbf{D}} = \mathbf{I} \otimes (\mathbf{C}\widehat{\mathbf{E}})$, $\mathbf{b} = \mathrm{vec}(\mathbf{B})$, $\mathbf{c} = \mathrm{vec}(\mathbf{C})$, and $\mathbf{f} = \mathrm{vec}(\mathbf{I})$ and after some manipulations, (25) can be written as

$$\mu(\mathbf{b},\mathbf{C}) = \left\|\mathbf{D}\mathbf{b} - \mathbf{f}\right\|^2 + \left\|\mathbf{c}\right\|^2\left(\sigma_E^2\left\|\mathbf{b}\right\|^2 + \sigma_n^2\right). \tag{28}$$

The Lagrangian associated with the problem in (27) is

$$\left\|\mathbf{D}\mathbf{b} - \mathbf{f}\right\|^2 + \left\|\mathbf{b}\right\|^2(\sigma_E^2\left\|\mathbf{c}\right\|^2 - \lambda) + \sigma_n^2\left\|\mathbf{c}\right\|^2, \tag{29}$$

where λ is the Lagrange multiplier associated with the power constraint. Using the expression for μ as given in (28), the KKT conditions [23] associated with the optimization problem in (27) are given by

$$\mathbf{b}^H\mathbf{b} - P_T \leq 0, \tag{30}$$

$$\lambda \geq 0, \tag{31}$$

$$\lambda(\mathbf{b}^H\mathbf{b} - P_T) = 0, \tag{32}$$

$$\mathbf{D}^H\mathbf{D}\mathbf{b} - \mathbf{D}^H\mathbf{f} + (\sigma_E^2 + \lambda)\mathbf{b} = 0. \tag{33}$$

Based on the conditions above, we can compute the optimum precoder as

$$\mathbf{b} = \begin{cases} \left(\mathbf{D}^H\mathbf{D} + (\sigma_E^2 + \lambda)\mathbf{I}\right)^{-1}\mathbf{D}^H\mathbf{f}, & \lambda > 0 \quad (34) \\ \left(\mathbf{D}^H\mathbf{D} + \sigma_E^2\mathbf{I}\right)^{-1}\mathbf{D}^H\mathbf{f}, & \text{otherwise.} \quad (35) \end{cases}$$

In order to compute λ in (34), consider the singular value decomposition (SVD) $\mathbf{D} = \mathbf{U}\mathbf{\Sigma}\mathbf{V}^H$. The singular value matrix $\mathbf{\Sigma}$ is a diagonal matrix with $\sigma_i,\ 1 \leq i \leq r$, where r is the rank of $\mathbf{D}$, as the elements. In terms of the SVD, (34) and (35) can be equivalently represented as

$$\mathbf{b} = \begin{cases} \mathbf{V}\left(\mathbf{\Sigma}^2 + (\sigma_E^2 + \lambda)\mathbf{I}\right)^{-1}\mathbf{\Sigma}\bar{\mathbf{f}}, & \lambda > 0 \quad (36) \\ \mathbf{V}\left(\mathbf{\Sigma}^2 + \sigma_E^2\mathbf{I}\right)^{-1}\mathbf{\Sigma}\bar{\mathbf{f}}, & \text{otherwise.} \quad (37) \end{cases}$$

When $\lambda > 0$, from (31) and (32), we find that, $\mathbf{b}^H\mathbf{b} = P_T$. Based on (36), this relation can be expressed as

$$\bar{\mathbf{f}}^H\mathbf{\Sigma}\mathbf{V}^H\left(\mathbf{\Sigma}^2 + (\sigma^2 + \lambda)\mathbf{I}\right)^{-2}\mathbf{V}\mathbf{\Sigma}\bar{\mathbf{f}} = P_T. \tag{38}$$

The above equation can be written as the following *secular equation* [24]

$$\sum_{j=1}^{r} g_j \frac{\sigma_j^2}{(\sigma_j^2 + \lambda + \sigma_E)^2} - P_T = 0, \tag{39}$$

where g_i are the diagonal elements of $\bar{\mathbf{f}}\bar{\mathbf{f}}^H$. If any $\lambda > 0$ satisfies (38), then the optimum precoder can be computed using (34), otherwise the optimum precoder can be computed using (35). In the latter case, as we can find from the complimentary slackness condition (32), the power constraint is not active at the optimal solution.

2) Iterative Algorithm for Solving (24): In this subsection, we present the proposed alternating optimization approach for the minimization of the SMSE averaged over the CSI error under a constraint on the total BS transmit power. At the $(n+1)$th iteration, the value of $\mathbf{B}$, denoted by $\mathbf{B}^{n+1}$, is the solution to the following problem

$$\mathbf{B}^{n+1} = \underset{\mathbf{B}:\mathrm{Tr}(\mathbf{B}\mathbf{B}^H)\leq P_T}{\mathrm{argmin}} \ \mu(\mathbf{B},\mathbf{C}^n), \tag{40}$$

which is solved in the previous subsection. Having computed $\mathbf{B}^{n+1}$, $\mathbf{C}^{n+1}$ is the solution to the following problem:

$$\mathbf{C}^{n+1} = \underset{\mathbf{C}}{\mathrm{argmin}}\ \mu(\mathbf{B}^{n+1},\mathbf{C}), \tag{41}$$

and its solution is given in (26). This alternating optimization over $\{\mathbf{B}\}$ and $\{\mathbf{C}\}$ can be repeated till convergence of the optimization variables. As the objective in (28) is monotonically decreasing after each iteration and is lower bounded, convergence is guaranteed. The iteration is terminated when the norm of the difference in the results of consecutive iterations are below a threshold or when the maximum number of iterations is reached. We note that proposed algorithm is not guaranteed to converge to the global minimum.

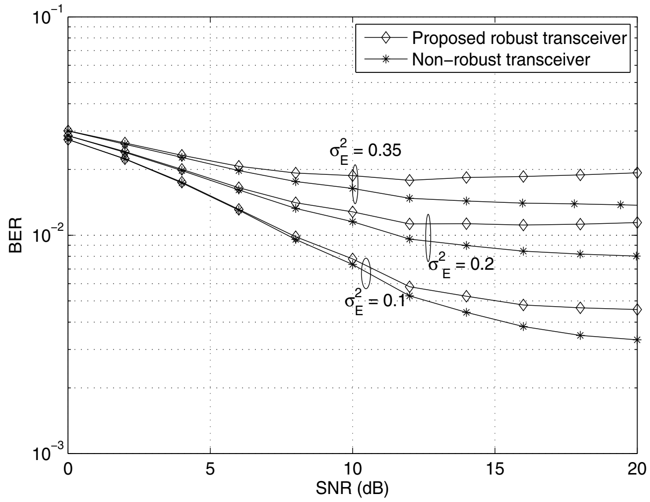

Fig. 4. Bit error rate performance of the proposed robust transceiver design for $N_t = 8$, $M = 3$, $N_r = 2$, $L = 2$, $\sigma_E^2 = 0.1,\ 0.2,\ 0.35,$ QPSK.

B. Simulation Results

In this section, we illustrate the performance of the proposed robust precoder design for multiuser MIMO downlink evaluated through simulations. We compare the performance of the proposed robust design with that of the non-robust design in [17], which has better performance compared to others in [14], [15]. The comparison is based on the bit error rate (BER) averaged over all users versus the SNR defined as P_{Tr}/σ_n^2, where P_{Tr} is the total transmit power.

The channel fading is modeled as Rayleigh, with the channel matrices $\mathbf{H}_k,\ 1 \leq k \leq M$, comprising of i.i.d samples of a complex Gaussian process with zero mean and unit variance. The elements of the channel error matrices $\mathbf{E}_k,\ 1 \leq k \leq M$ are zero-mean complex Gaussian random variables with variance σ_E^2. QPSK modulation is employed on each data stream. We consider system with the BS equipped with $N_t = 8$ transmit antennas, transmitting $L = 2$ data streams to each user. There are $M = 3$ users, each equipped with $N_r = 2$ receive antennas. The simulation results are shown in fig. 4. BER performances of the proposed robust design and the non-robust design proposed in [17] for $\sigma_E^2 = 0.1,\ 0.2,$ and 0.35 are compared. The proposed robust design is found to outperform the non-robust design. It is found that the difference between the performance of these algorithms increase as the SNR increases. This is observable in (28), where the second term shows the effect of the channel error variance amplified by the transmit power.

V. Conclusions

We considered two robust precoder designs for broadcast channels with imperfect CSI at the transmitter. The first design, for a multiuser MISO downlink, was based on the minimization of total BS transmit power under constraints the SINR of individual users. We formulated this problem as a SOCP that can be solved efficiently. The second design, for a multiuser MIMO downlink, was based on the minimization of a stochastic function of the SMSE under a constraint on the total BS transmit power. We proposed an iterative algorithm to solve this problem. Through simulation results we showed that the proposed robust designs outperform other robust designs in the literature.

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