Robust MMSE Tomlinson-Harashima Precoder for Multiuser MISO Downlink with Imperfect CSI

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***Abstract*—Non-linear precoding for the downlink of a multiuser MISO (multiple-input single-output) communication system in the presence of imperfect channel state information (CSI) is considered. The base station is equipped with multiple transmit antennas and each user terminal is equipped with a single receive antenna. The CSI at the transmitter is assumed to be perturbed by an estimation error. We propose a robust minimum mean square error (MMSE) Tomlinson-Harashima precoder (THP) design, which can be formulated as an optimization problem that can be solved efficiently by the method of *alternating optimization* (AO). In this method of optimization, the entire set of optimization variables is partitioned into non-overlapping subsets, and an iterative sequence of optimizations on these subsets is carried out, which is often simpler compared to simultaneous optimization over all variables. In our problem, the application of the AO method results in a second-order cone program which can be numerically solved efficiently. The proposed precoder is shown to be less sensitive to imperfect channel knowledge. Simulation results illustrate the improvement in performance compared to other robust linear and non-linear precoders in the literature.**

Keywords – *Multiuser MISO downlink, multiuser interference, imperfect CSI, Tomlinson-Harashima precoder, alternating optimization method.*

I. Introduction

There has been considerable interest in multiuser multiple-input multiple-output (MIMO) wireless communications in view of their potential for transmit diversity and increased channel capacity [1],[2]. Since it is difficult to provide mobile user terminals with large number of antennas due to space constraints, multiuser multiple-input single-output (MISO) wireless communications on the downlink, where the base station is equipped with multiple transmit antennas and each user terminal is equipped with a single receive antenna is of significant practical interest. In such multiuser MISO systems, multiuser interference at the receiver is a crucial issue. One way to deal with this interference issue is to use multiuser detection [3] at the receivers, which increases the receiver complexity. As an alternate way, transmit side processing in the form of precoding is being studied widely [2],[4]. Several linear precoders such as transmit zero-forcing (ZF) and minimum mean square error (MMSE) filters, and non-linear precoders including Tomlinson-Harashima precoder (THP) have been proposed and widely investigated in the literature [5],[6]. Non-linear precoding strategies, though more complex than the linear strategies, result in improved performance compared to linear pre-processing. Transmit side precoding techniques, linear or non-linear, can render the receiver side processing at the user terminal simpler. However, transmit side precoding techniques require channel state information (CSI) at the transmitter.

Several studies on transmit precoding assume perfect knowledge of CSI at the transmitter. However, in practice, CSI at the transmitter suffers from inaccuracies caused by errors in channel estimation and/or limited, delayed or erroneous feedback. The performance of precoding schemes is sensitive to such inaccuracies [7]. Several papers in the literature have proposed precoder designs, both linear and non-linear, which are robust in the presence of channel estimation errors [8],[9].

In this paper, we consider non-linear precoding for the downlink of a multiuser MISO wireless communication system in the presence of imperfect CSI. The CSI at the transmitter is assumed to be perturbed by estimation error. We propose a robust MMSE THP design, which can be formulated as an optimization problem that can be solved efficiently by the method of *alternating optimization* (AO) [10]. In this method of optimization, the entire set of optimization parameters is partitioned into non-overlapping subsets, and an iterative sequence of optimizations on these subsets is carried out, which is often simpler compared to simultaneous optimization over all parameters. In our problem, the application of the AO method results in a second-order cone program which can be numerically solved efficiently. The proposed non-linear precoder is shown to be less sensitive to imperfect channel knowledge. Simulation results illustrate the improvement in performance compared to other robust linear and non-linear precoders in the literature.

The rest of the paper is organized as follows. In Sec. II, we present the system model. The proposed robust non-linear precoder design is presented in Sec. III. Performance results and comparisons are presented in Sec. IV. Conclusions are presented in Sec. V.

II. System Model

We consider a multiuser MISO system, where a base station (BS) communicates with N_u users on the downlink. A block diagram of the system considered is shown in Fig. 1. The BS employs N_t transmit antennas and each user is equipped with one receive antenna. Let $\mathbf{u}$ denote[1] the $N_u \times 1$ data symbol vector, where $u_i, i = 1, 2, \cdots, N_u$, denotes the complex valued data symbol meant for user i. Transmit pre-processing is carried out on this vector $\mathbf{u}$. The output of the pre-processing operation is denoted by the $N_t \times 1$ vector $\mathbf{x}$, where x_j, $j = 1, 2, \cdots, N_t$, denotes the complex-valued symbol transmitted on the jth transmit antenna.

[1] Vectors are denoted by boldface lowercase letters, and matrices are denoted by boldface uppercase letters. $[.]^T$, $[.]^H$, and $[.]^\dagger$, denote transpose, Hermitian, and pseudo-inverse operations, respectively. $[\mathbf{A}]_{ij}$ denotes the element on the ith row and jth column of the matrix $\mathbf{A}$. vec$(.)$ operator stacks the columns of the input matrix into one column-vector.

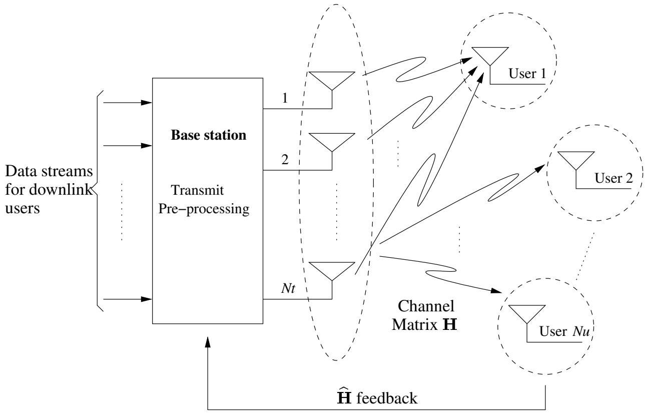

Fig. 1. Multiuser MISO Downlink with imperfect CSI at the transmitter.

The received signal at user i, denoted by y_i, can be written as

$$y_i = \sum_{i=1}^{N_t} h_{ij} x_j + n_i, \tag{1}$$

where h_{ij} denotes the complex channel gain from the transmit antenna j to the receive antenna of user i, and n_i is an i.i.d complex Gaussian random variable with zero mean and variance of σ_n^2 representing the noise at the ith receiver. The channel gains are assumed to be independent zero mean complex Gaussian variables of equal variance $E[|h_{ij}|^2] = 1$. The received signals at all the user nodes can be represented in vector form as

$$\mathbf{y} \quad = \quad \mathbf{H}\mathbf{x} + \mathbf{n}, \tag{2}$$

where $\mathbf{H}$ is the $N_u \times N_t$ channel matrix, given by

$$\mathbf{H} \quad = \quad \left[\begin{array}{cccc} h_{11} & h_{12} & \cdots & h_{1N_t} \\ h_{21} & h_{22} & \cdots & h_{2N_t} \\ \vdots & \vdots & \ddots & \vdots \\ h_{N_u1} & h_{N_u2} & \cdots & h_{N_uN_t} \end{array} \right], \tag{3}$$

$\mathbf{y} = [y_1, y_2, \cdots, y_{N_u}]^T$, $\mathbf{x} = [x_1, x_2, \cdots, x_{N_t}]^T$, and $\mathbf{n} = [n_1, n_2, \cdots, n_{N_u}]^T$.

A. Tomlinson-Harashima Precoder

We consider the well known Tomlinson-Harashima precoder (THP) as the transmit side pre-processor. The block diagram of the THP is shown in Fig. 2. The $\mathbf{I} - \mathbf{G}$ operation in Fig. 2 essentially performs successive interference cancellation at the transmitter, and the $\mod (M)$ operation ensures that the resulting cancelled output values are contained within a certain acceptable range, where M is the cardinality of the modulation alphabet. The matrix $\mathbf{G}$ is upper triangular with unit diagonal.

The THP design involves the choice of the matrices $\mathbf{B}$ and $\mathbf{G}$ using the knowledge of the channel matrix $\mathbf{H}$ at the transmitter. This choice can be made based on the optimization of certain metrics, such as signal-to-interference ratio (SIR), mean square error (MSE), etc. As mentioned earlier, it is of interest

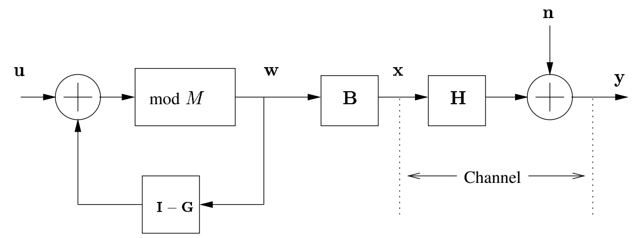

Fig. 2. Matrix form of Tomlinson-Harashima precoder.

to consider imperfect channel knowledge at the transmitter. Consequently, in the following, we address the problem of choosing $\mathbf{B}$ and $\mathbf{G}$ for the case of imperfect CSI at the transmitter. Specifically, we propose a robust precoder design that employs the alternating optimization (AO) method to obtain $\mathbf{B}$ and $\mathbf{G}$ for the case of imperfect CSI.

B. Channel Uncertainty Model

The precoder design in this paper is based on a statistical model for the error in CSI at the transmitter. In this model, the true channel matrix $\mathbf{H}$ is represented as

$$\mathbf{H} = \widehat{\mathbf{H}} + \mathbf{E}, \tag{4}$$

where $\widehat{\mathbf{H}}$ is the estimated channel matrix and $\mathbf{E}$ is the estimation error matrix. The error matrix $\mathbf{E}$ is assumed to be Gaussian distributed with zero mean and $E\{\mathbf{E}\mathbf{E}^H\} = \sigma_{\mathbf{E}}^2 \mathbf{I}_{N_uN_u}$, where $E\{.\}$ denotes the expectation operator. This model is suitable for systems with uplink-downlink reciprocity [8].

III. Robust THP Design with Imperfect CSI

In this section, we present the design of the proposed robust THP for the case of imperfect CSI, under an average transmit power constraint. In this case, the matrices $\mathbf{B}$ and $\mathbf{G}$ are chosen such that the system performance does not fall below a desired level even when the CSI at the transmitter is imperfect. In order to achieve this, the channel error model has to be suitably taken into account in the optimization. One way to do this, in the case of a stochastic model of errors in CSI, is to consider the performance metric averaged over the error as the optimization objective [8]. We adopt this method in our approach to the design of robust THP.

In order to simplify the analysis, we resort to a linear representation of the modulo device [11], as shown in Fig. 3. The signal vectors $\mathbf{a}$ and $\mathbf{d}$ in Fig. 3 are introduced to satisfy the requirement that $\mathbf{w}$ has the same value as in the case of modulo operation. The modulo operation at the transmitter alters the statistics of the precoded symbols. Let $\mathbf{\Phi_w} = E\{\mathbf{w}\mathbf{w}^H\}$ and $\mathbf{\Phi_u} = E\{\mathbf{u}\mathbf{u}^H\}$. $\mathbf{\Phi_w}$ is a diagonal matrix, and $[\mathbf{\Phi_w}]_{ii} = \frac{M}{(M-1)}[\mathbf{\Phi_u}]_{ii}$, $i = 1, \cdots, N_u$.

In the case of perfect CSI at the transmitter, an MMSE THP design can be obtained by minimizing the mean squared error (MSE), ϵ, between the scaled value of the symbol vector $\mathbf{d}$ and the received vector $\mathbf{y}$, given by

$$\begin{aligned} \epsilon \quad &= \quad E\Big\{ \parallel c\,\mathbf{d} - \mathbf{y} \parallel^2 \Big\} \\ &= \quad E_{\mathbf{w},\mathbf{n}}\Big\{ \parallel c\,\mathbf{d} - \mathbf{H}\mathbf{B}\mathbf{w} + \mathbf{n} \parallel^2 \Big\}, \end{aligned} \tag{5}$$

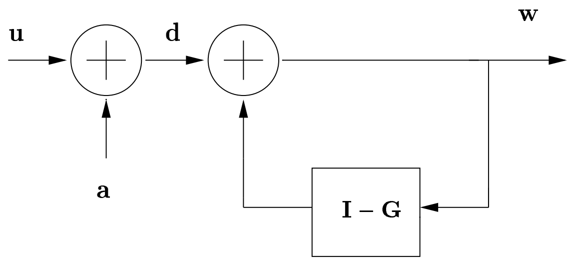

Fig. 3. Linear representation of the modulo device.

where the 2nd step in (5) follows from $\mathbf{x} = \mathbf{B}\mathbf{w}$ (as seen from Fig. 2), and the expectation is over the noise vector, $\mathbf{n}$, and the modified symbol vector, $\mathbf{w}$. Also, from Fig. 3, it can be seen that $\mathbf{d} = \mathbf{G}\mathbf{w}$. Substituting $\mathbf{G}\mathbf{w}$ for $\mathbf{d}$ in (5), ϵ can be written as

$$\epsilon = E_{\mathbf{w}}\left\{\mathbf{w}^H(c\mathbf{G}-\mathbf{H}\mathbf{B})^H(c\mathbf{G}-\mathbf{H}\mathbf{B})\mathbf{w}\right\} + N_u\sigma_n^2. \quad (6)$$

Under imperfect CSI, since $\mathbf{H} = \widehat{\mathbf{H}} + \mathbf{E}$, in order to make the design robust, we consider the MSE in (6) averaged over the error matrix $\mathbf{E}$ (in addition to averaging over $\mathbf{w}$ and $\mathbf{n}$) as the performance metric for the optimization. This optimization is performed subject to an average transmit power constraint. The average transmit power, P_T, is given by

$$\begin{aligned} P_T &= E\left\{\mathbf{x}^H\mathbf{x}\right\} \\ &= E\left\{\mathbf{w}^H\mathbf{B}^H\mathbf{B}\mathbf{w}\right\} \\ &= \text{trace}(\mathbf{B}^H\mathbf{B}\mathbf{\Phi}_{\mathbf{w}}). \end{aligned} \quad (7)$$

Assuming $\Phi_{\mathbf{w}} = \mathbf{I}_{N_u}$, P_T can be expressed as

$$P_T = \text{trace}(\mathbf{B}^H\mathbf{B}). \quad (8)$$

As the last term in (6) is a constant, this term can be dropped from the objective function. Now, taking the expectation of (6) over $\mathbf{E}$, we get

$$\begin{aligned} &E_{\mathbf{E},\mathbf{w}}\left\{\mathbf{w}^H(c\mathbf{G}-\mathbf{H}\mathbf{B})^H(c\mathbf{G}-\mathbf{H}\mathbf{B})\mathbf{w}\right\} \\ &\quad = E_{\mathbf{E}}\{\text{trace}((c\mathbf{G}-\mathbf{H}\mathbf{B})^H(c\mathbf{G}-\mathbf{H}\mathbf{B}))\mathbf{\Phi}_{\mathbf{w}})\} \\ &\quad = E_{\mathbf{E}}\{\text{trace}((c\mathbf{G}-\mathbf{H}\mathbf{B})^H(c\mathbf{G}-\mathbf{H}\mathbf{B})))\}. \end{aligned} \quad (9)$$

Based on the above development, we can formulate the proposed robust MMSE THP design problem as the following constrained optimization program:

$$\begin{aligned} \min_{\mathbf{B},\mathbf{G},c} \quad & E_{\mathbf{E}}\left\{\text{trace}((c\mathbf{G}-\mathbf{H}\mathbf{B})^H(c\mathbf{G}-\mathbf{H}\mathbf{B})))\right\} \\ \text{subject to} \quad & \text{trace}(\mathbf{B}\mathbf{B}^H) \leq P_{th}, \\ & c \geq c_{th}. \end{aligned} \quad (10)$$

The scaling factor c is lower-bounded in order to avoid the trivial solution of $c = 0$ and $\mathbf{B} = 0$.

Let $\mathbf{g} = \text{vec}(\mathbf{G})$, $\mathbf{b} = \text{vec}(\mathbf{B})$, and $\mathbf{A} = \mathbf{I} \otimes (\widehat{\mathbf{H}} + \mathbf{E}) = \widehat{\mathbf{A}} + \tilde{\mathbf{A}}$. Then,

$$\text{vec}(c\mathbf{G} - (\widehat{\mathbf{H}} + \mathbf{E})\mathbf{B}) = c\mathbf{g} - \mathbf{A}\mathbf{b}. \quad (11)$$

Substituting $\mathbf{H} = \widehat{\mathbf{H}} + \mathbf{E}$ and using (11) in (9), the objective in (10) can be rewritten as

$$\begin{aligned} &E_{\mathbf{E}}\left\{\text{trace}((c\mathbf{G} - (\widehat{\mathbf{H}} + \mathbf{E})\mathbf{B})^H(c\mathbf{G} - (\widehat{\mathbf{H}} + \mathbf{E})\mathbf{B}))\right\} \\ &\quad = E_{\mathbf{E}}\left\{(c\mathbf{g} - \mathbf{A}\mathbf{b})^H(c\mathbf{g} - \mathbf{A}\mathbf{b})\right\} \\ &\quad = \|\, \widehat{\mathbf{A}}\mathbf{b} - c\mathbf{g} \,\|^2 + \mathbf{b}^H E_{\mathbf{E}}\left\{\tilde{\mathbf{A}}^H\tilde{\mathbf{A}}\right\}\mathbf{b} \\ &\quad = \|\, \widehat{\mathbf{A}}\mathbf{b} - c\mathbf{g} \,\|^2 + \sigma_{\mathbf{E}}^2 \,\|\, \mathbf{b} \,\|^2 . \end{aligned} \quad (12)$$

In the above, both $\mathbf{b}$ and $\mathbf{g}$ are optimization parameters, whereas in a linear precoder only $\mathbf{b}$ is the optimization parameter. Using (12), and noting that the second constraint in (10) is active at optimality, the optimization problem in (10) can be rewritten as

$$\begin{aligned} \min_{\mathbf{b},\mathbf{g}} \quad & J(\mathbf{b},\mathbf{g}) \stackrel{\triangle}{=} \|\, \widehat{\mathbf{A}}\mathbf{b} - c_{th}\mathbf{g} \,\|^2 + \sigma_{\mathbf{E}}^2 \,\|\, \mathbf{b} \,\|^2 \\ \text{subject to} \quad & \|\, \mathbf{b} \,\|^2 \leq P_{th}. \end{aligned} \quad (13)$$

A. Alternating Optimization

As the optimization has to be performed with respect to both $\mathbf{b}$ and $\mathbf{g}$ in (13), we can use the method of Alternating Optimization (AO) [10], wherein the optimization over an entire set of variables is replaced by a sequence of easier optimizations involving grouped subsets of the variables. In the present problem, we partition the optimization set $\{\mathbf{b},\mathbf{g}\}$ into the non-overlapping subsets $\{\mathbf{b}\}$ and $\{\mathbf{g}\}$ and perform the optimization with respect to these subsets in an alternating fashion.

The algorithmic form of the alternating optimization for the computation of the matrices $\mathbf{G}$ and $\mathbf{B}$ is shown in Table-I. At the $(n+1)$th iteration, the value of $\mathbf{b}$ is the solution to the following problem

$$\mathbf{b}^{n+1} = \underset{\mathbf{b}}{\text{argmin}} \; J(\mathbf{b},\mathbf{g}^n), \quad (14)$$

where $\mathbf{b}$ satisfies the constraint in (13). This problem can be efficiently solved as a second order cone program [12]. Having computed $\mathbf{b}^{n+1}$, $\mathbf{g}^{n+1}$ is the solution to the following problem:

$$\mathbf{g}^{n+1} = \underset{\mathbf{g}}{\text{argmin}} \; J(\mathbf{b}^{n+1},\mathbf{g}), \quad (15)$$

This problem has the following solution:

$$\mathbf{g}^{n+1} = \left(\frac{1}{c_{th}}\right)\text{vec}\left(\text{triu}\left(\text{mat}\left(\mathbf{A}\mathbf{b}^{n+1}\right)\right) + \mathbf{I}\right), \quad (16)$$

where mat(.) operator constructs a square matrix from the input vector and triu(.) operator extracts the upper triangular part of the input matrix. This alternating optimization over $\{\mathbf{b}\}$ and $\{\mathbf{g}\}$ can be repeated till convergence of the optimization variables. As the objective in (13) is monotonically decreasing after each iteration and is lower bounded, convergence is guaranteed. The iteration is terminated when the norm of the difference in the results of consecutive iterations are below a threshold or when the maximum number of iterations is reached.

TABLE–I : Algorithm for computation of precoding matrices

Select N_{max} (Maximum Number iterations), tth (Termination Threshold), $\mathbf{X^0} = [\mathbf{b^0 g^0}]$

1) n = 0
2) **While** $n \leq N_{max}$
3) compute $\mathbf{b^{n+1}}$ using $\mathbf{g^n}$
6) compute $\mathbf{g^{n+1}}$ using $\mathbf{b^{n+1}}$
7) $\mathbf{x^{n+1}} = [\mathbf{b^{n+1} g^{n+1}}]$
8) if $\parallel \mathbf{x^{n+1}} - \mathbf{x^n} \parallel \leq tth$ then
9) **break**
10) **end if**
11) n $\longleftarrow$ n+1
12) **end while**

IV. Simulation Results

In this section, we illustrate the performance of the proposed algorithm evaluated through simulations. The channel fading is modeled as Rayleigh, with the channel matrix $\mathbf{H}$ comprising of independent and identically distributed (i.i.d) samples of a complex Gaussian process with zero mean and unit variance. We compare this performance with other precoders in the literature. The comparison is based on the average uncoded bit error rate (BER) versus the average signal-to-noise ratio (SNR), which is defined as $\frac{P_T}{N_u \sigma_n^2}$ [8]. The modulation scheme used is QPSK. The elements of the estimation error matrix, $\mathbf{E}$, are generated independently from zero-mean Gaussian distribution of variance $\sigma_{\mathbf{E}}^2$. We compare the BER performance of the proposed robust MMSE-THP with that of $i)$ the robust linear MMSE precoder in [8], and $ii)$ the robust ZF-THP in [9]. Figure 4 shows the BER performance of the various precoders in a system with four transmit antennas ($N_t = 4$) at the BS, four users ($N_u = 4$) with one receive antenna each, and channel estimation error with variance $\sigma_{\mathbf{E}}^2 = 0.05$. For the same system parameters setting, Fig. 5 presents the results for $\sigma_{\mathbf{E}}^2 = 0.2$. From Figs. 4 and 5, it can be observed that the proposed robust MMSE-THP performs better than the robust linear MMSE precoder in [8] as well as the robust ZF-THP in [9]. The performance cross-overs between the THP precoders and the linear MMSE precoder at low SNRs are due to the power enhancement effect of the modulo operation in the THP.

V. Conclusions

We addressed the problem of designing a robust non-linear precoder for MISO systems with imperfect channel state information. It was shown that the precoder design problem can be formulated as an optimization problem, which can be solved using the method of Alternating Optimization. This robust non-linear precoder outperforms the robust zero-forcing THP and the robust MMSE linear precoder reported in the literature before. So far in this study, we have assumed that all users are homogeneous in terms of their required QoS (i.e.,

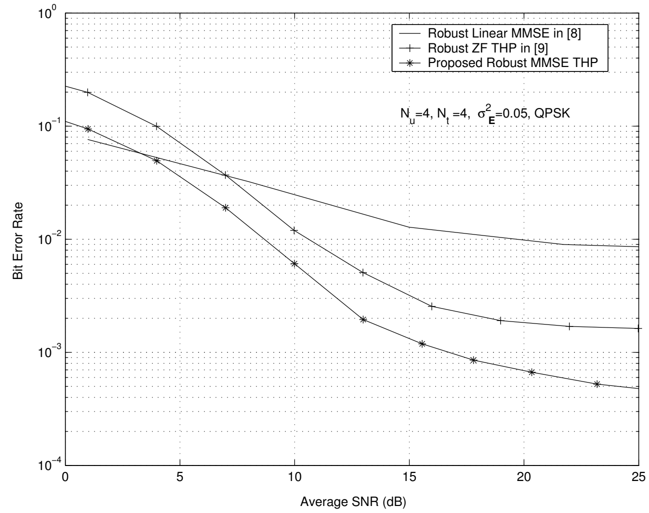

Fig. 4. Uncoded BER versus average SNR performance for different precoders with imperfect CSI at the transmitter: $i)$ robust linear MMSE precoder in [8],, $ii)$ robust ZF THP in [9], and $iii)$ proposed robust MMSE THP. $N_t = 4$, $N_u = 4$, QPSK, $\sigma_{\mathbf{E}}^2 = 0.05$.

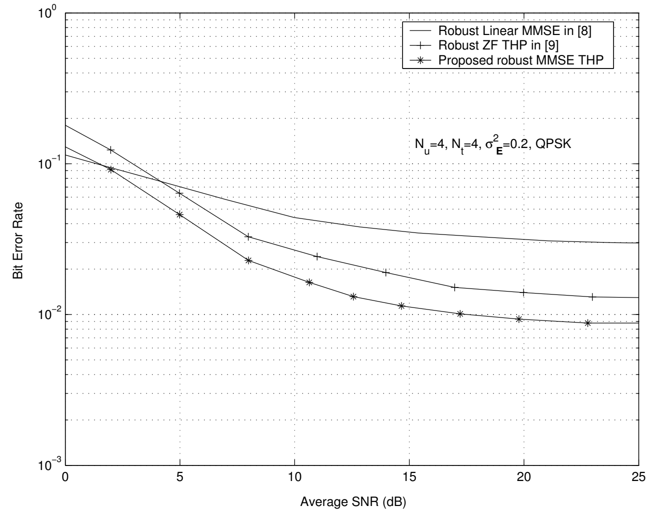

Fig. 5. Uncoded BER versus average SNR performance for different precoders with imperfect CSI at the transmitter: $i)$ robust linear MMSE precoder in [8], $ii)$ robust ZF THP in [9], and $iii)$ proposed robust MMSE THP. $N_t = 4$, $N_u = 4$, QPSK, $\sigma_{\mathbf{E}}^2 = 0.2$.

required SIR or BER). We are extending the proposed optimization approach to the case of different QoS requirements for different users.

References

[1] D. Tse and P. Viswanath, *Fundamentals of Wireless Communication*, Cambridge University Press, 2006.

[2] H. Bolcskei, D. Gesbert, C. B. Papadias, and All-Jan van der Veen, *Space-time wireless systems: From Array Processing to MIMO Communications*, Cambridge University Press, 2006.

[3] S. Verdu, *Multiuser Detection,* Cambridge University Press, 1998.

[4] R. Fischer, C. Windpassinger, A. Lampe, and J. Huber, "MIMO precoding for decentralized receivers," *Proc. IEEE ISIT'2002*, p. 496, 2002.

[5] Q. H. Spencer, A. L. Swindlehurst, and M. Haardt, "Zero-forcing methods for downlink spatial multiplexing in multiuser MIMO channels," *IEEE Trans. on Signal Processing,* vol. 52, pp. 461-471, February 2004.

[6] K. Kusume, M. Joham, W. Utschick, and G Bauch, "Efficient Tomlinson-Harashima Precoding for spatial multiplexing on flat

MIMO channel," *Proc. IEEE ICC'2005,* vol. 3, pp. 20212025, May 2005,

[7] N. Jindal, "MIMO broadcast channels with finite rate feed-back, *Proc. IEEE GLOBECOM'2005*, November 2005.

[8] M. B. Shenouda and T. N. Davidson, "Robust linear precoding for uncertain MISO broadcast Channels," *Proc. IEEE ICASSP'2006*, vol. 4, pp. 37-40, 2006.

[9] R. Hunger, F. Dietrich, M. Joham, and W. Utschick, "Robust transmit zero-forcing filters," *Proc. ITG Workshop on Smart Antennas*, Munich, pp. 130-137, March 2004.

[10] J. C. Bezdek and R. J. Hathaway, "Some notes on Alternating Optimization," *AFSS2002, LNAI 2275*, pp.288-300, 2002, Springer-Verlag Berlin Heidelberg 2002.

[11] R. Fischer, *Precoding and Signal Shaping for Digital Transmission*, John-Wiley and Sons, New York, 2002.

[12] J. Sturm, "Using SeDuMi 1.02, a MATLAB toolbox for optimization over symmetric cones," *Optimization Methods and Software,* vol. 11, pp. 625-653, 1999.