Distributed Space-Time Block Codes for Two-Hop Wireless Relay Networks

J. Harshan
Dept of ECE, Indian Institute of Science,
Bangalore 560012, India.
Email:harshan@ece.iisc.ernet.in

B. Sundar Rajan
Dept of ECE, Indian Institute of Science,
Bangalore 560012, India.
Email:bsrajan@ece.iisc.ernet.in

(*Invited Paper*)

Abstract— **Recently, the idea of space-time coding has been applied to wireless relay networks wherein a set of geographically separated relay nodes cooperate to process the received signal from the source and forward them to the destination such that the signal received at the destination appears like a Space-Time Block Code (STBC). Such STBCs (referred to as Distributed Space-Time Block Codes (DSTBCs)) when appropriately designed are known to offer spatial diversity. It is known that different classes of DSTBCs can be designed primarily depending on (i) whether the Amplify and Forward (AF) protocol or the Decode and Forward (DF) protocol is employed at the relays and (ii) whether the relay nodes are synchronized or not. In this paper, we present a survey on the problems and results associated with the design of DSTBCs for the following classes of two-hop wireless relay networks: (i) synchronous relay networks with AF protocols, (ii) asynchronous relay networks with AF protocols (iii) synchronous relay networks with DF protocols and (iv) asynchronous relay networks with DF protocols.**

Index Terms— **Cooperative diversity, distributed space-time block codes, relay channel, space-time coding.**

I. Introduction and Preliminaries

For point to point communication in co-located Multiple-Input Multiple-Output (MIMO) channels (shown in Fig. I)), space-time coding has been an effective technique to combat the degrading effects of multi-path fading. In particular, with such techniques, a spatial diversity order equal to the product of the number of transmit antennas and receive antennas can potentially be obtained in slow-fading scenarios [1]. It is also known that the capacity of a co-located MIMO channel scales linearly with the minimum of the number of transmit and receive antennas for high receive Signal to Noise Ratio (SNR) values when perfect estimates of the channel are available at the receiver [2], [3].

In a wireless network, if the source terminal and the destination terminal are engaged in a point to point communication and are precluded from using multiple antennas (wherein all the terminals are assumed to be small devices, examples include mobile units or wireless nodes in sensor network applications [4]), then it is well known that spatial diversity cannot be obtained. Recently, a promising technique called cooperative communication has attracted a lot of attention in the research community wherein several users in the network which are geographically separated support the source in transmitting information to the destination (see Fig. 2). Since, the destination receives the source signal through several independent paths, a potential spatial diversity order of at most the number of relays (including the source terminal) is promised. Such a method of obtaining spatial diversity is termed as cooperative diversity [4]-[9]. Moving one step further, the idea of space-time coding which was originally devised for a Multiple-Input Multiple-Output (MIMO) channel has been applied to wireless networks under the frame-work of cooperative communication [6], [9]. In this scenario, the source broadcasts its signal to several terminals in the network referred to as relays excluding/including the destination terminal. The half duplex constrained relay nodes cooperate to process the received source signal and forward them to the destination such that the signal received at the destination

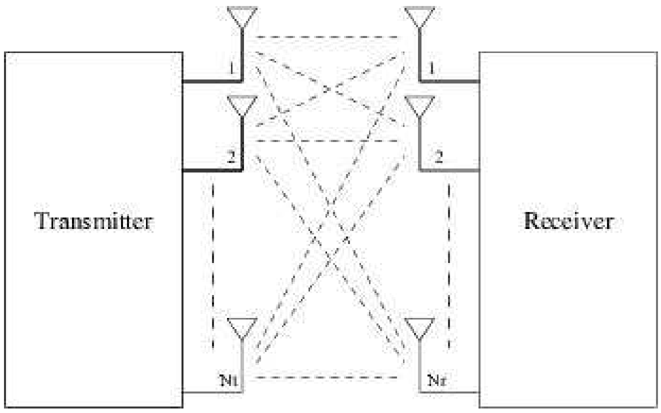

Fig. 1. Co-located MIMO channel model

doi:10.4304/jcm.5.4.282-296

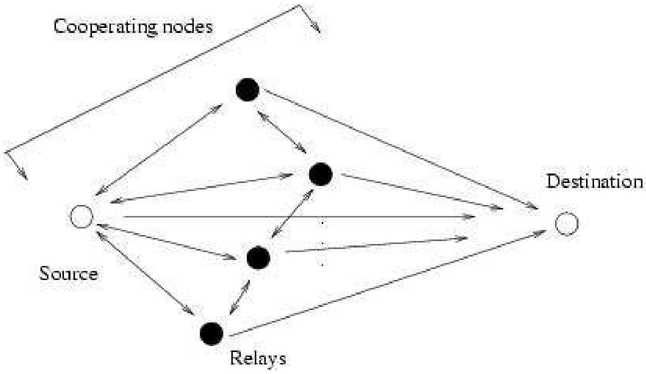

Fig. 2. A cooperative communication model

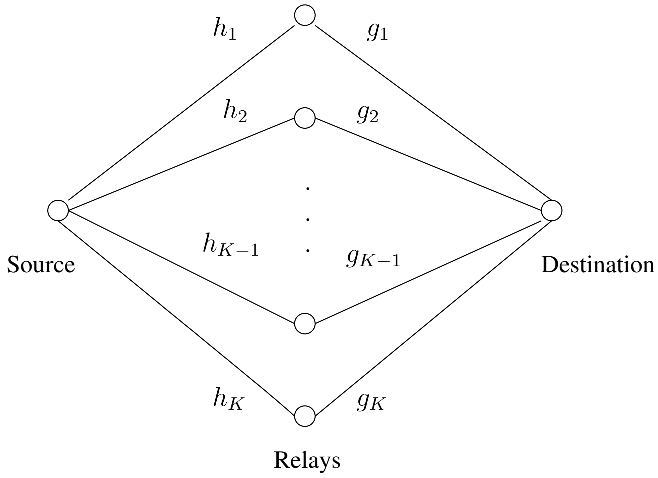

Fig. 3. Two-hop wireless relay network model.

appears like a Space-Time Block Code (STBC) [1] (in general, a finite set of complex matrices constitute an STBC. A design [18] in complex variables can be used to construct an STBC by making the variables take values from an underlying complex signal set). Such a cooperative scenario is referred to as a two-hop cooperative communication, the signaling scheme is referred to as Distributed Space-Time Coding (DSTC) and the corresponding codes are called Distributed Space-Time Block Codes (DSTBC). Apart from using DSTBCs, there are several other techniques of obtaining spatial diversity in a wireless network. One such method is to select a single best relay among the several users in the network based on some criterion to support the communication between the source and the destination [10]. For more details on relay selection, we refer the readers to [11] and the references within. Such schemes are also shown to provide spatial diversity at the cost of using additional resources (such as power and bandwidth) on the training sequences in selecting the best relay. In this paper, we only focus on the design of *DSTBCs* to obtain full diversity in two-hop relay networks.

Several protocols such as Amplify and Forward (AF), Decode and Forward (DF), Compress and Forward (CF) and Demodulate and Forward (DF) [12], [13] protocols are available for user cooperation. For various other protocols, we refer the reader to [7]. Among them, the prominent ones such as AF and DF protocols are amenable for the construction of DSTBCs [7]. Diversity Multiplexing Tradeoff (DMT) [14] analysis of AF and DF protocols in co-operative networks can also be found in [15]. Henceforth, throughout the paper we consider only AF and DF protocols at the relays. With the DF protocol, it is difficult to achieve full diversity because of the possible erroneous decoding at some of the relays and forwarding of the same. To mitigate such scenarios, use of relay selection strategies or cyclic redundancy check (CRC) codes or fountain codes [16] becomes essential which in turn increases the overhead on the system and/or requires more resources such as power and bandwidth. The former protocol is attractive considering the simplicity of the underlying technique wherein, every relay node normalizes its received signal and forwards an appropriately scaled version to the destination. A generalized protocol to AF is Linear-Process and Forward (LPF) protocol wherein every relay linearly processes the received signal before transmitting to the destination [9]. The choice of the linear processing at every relay will determine the error performance of the overall protocol. Henceforth, throughout the paper, we consider only LPF protocol. Further, we assume an orthogonal LPF protocol where the source and the relays do not transmit simultaneously in the same frequency band.

In this survey paper, cooperative communications in *two-hop wireless networks* is studied wherein the source signal reaches the destination terminal through two sets of channels. The first set consists of the channels from the source to the relays and the second set consists of the channels from the relays to the destination (see Fig. 3 for a two-hop model). In DSTC, the signals transmitted from all the relays may or may not reach the destination coherently, i.e. all the relay nodes may or may not be time synchronized. Depending on whether the relay nodes are time synchronized or not, DSTC schemes can be broadly classified in to two classes namely (i) synchronous DSTC schemes and (ii) asynchronous DSTC schemes. For synchronous DSTC schemes, it is assumed that each relay knows the timing offset from itself to the destination. As a result, the signals transmitted from the relays can be apriori time shifted such that all the signals are received at the destination with symbol level synchronisation. However, there is an overhead in conveying the timing offset (timing offset from the relay to the destination) to each relay. To avoid the overhead or the difficulty in conveying the timing offsets to the relays, one can design DSTBCs when only the maximum of the timing delays is known to all the relays. In this paper, we present a survey on DSTBCs for synchronous and asynchronous relay networks based on both AF and DF protocols (see Fig. 4 for the classification of DSTBCs in two-hop relay networks) wherein, we address the problems and solutions involving the design, construction and performance analysis (in terms of PEP) of DSTBCs.

Organization of the paper: In Section II, a short review on space-time coding for co-located MIMO channels is provided. In Section III, we present an overview on distributed space-time coding techniques in wireless networks. Section IV,

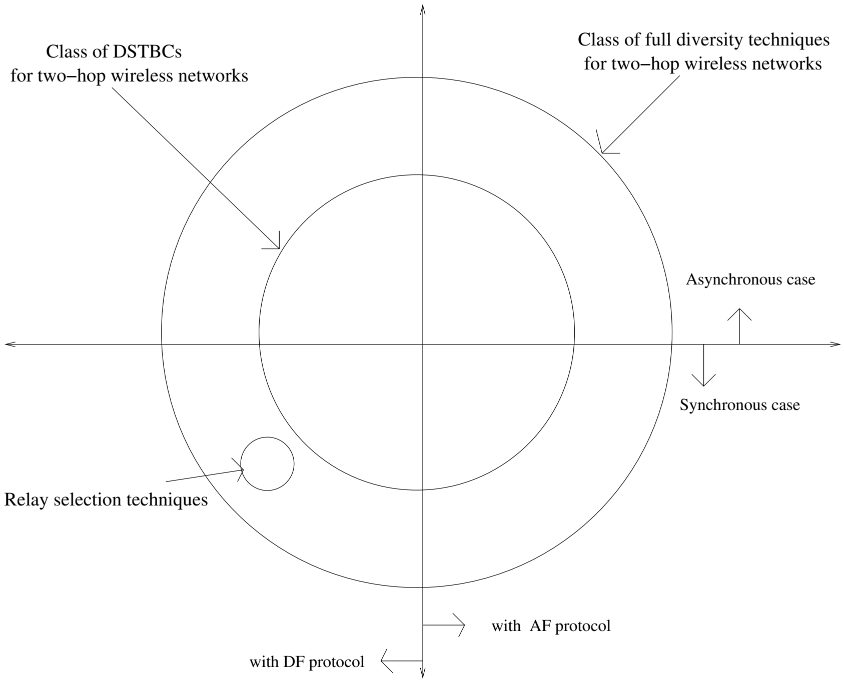

Fig. 4. Classification of DSTBCs for two-hop wireless relay networks

Section V, Section VI and Section VII respectively deals with designing DSTBCs for (i) synchronous relay networks with AF protocols, (ii) asynchronous relay networks with AF protocols (iii) synchronous relay networks with DF protocols and (iv) asynchronous relay networks with DF protocols. Finally, possible directions of future work and concluding remarks constitute Section VIII.

Notations: Throughout the paper, lower case boldface letters and capital boldface letters are used to represent vectors and matrices respectively. For a complex matrix $\mathbf{X}$, the matrices $\mathbf{X}^*$, $\mathbf{X}^T$, $\mathbf{X}^H$, $|\mathbf{X}|$, Re $\mathbf{X}$ and Im $\mathbf{X}$ denote, respectively, the conjugate, transpose, conjugate transpose, determinant, real part and imaginary part of $\mathbf{X}$. The element in the r_1-th row and the r_2-th column of the matrix $\mathbf{X}$ is denoted by $[\mathbf{X}]_{r_1,r_2}$. The $T \times T$ identity matrix and the $T \times T$ zero matrix are respectively denoted by $\mathbf{I}_T$ and $\mathbf{0}_T$. The magnitude of a complex number x, is denoted by $|x|$ and $E[x]$ is used to denote the expectation of the random variable x. A circularly symmetric complex Gaussian random vector $\mathbf{x}$, with mean $\boldsymbol{\mu}$ and covariance matrix $\mathbf{\Gamma}$ is denoted by $\mathbf{x} \sim \mathcal{CSCG}(\boldsymbol{\mu}, \mathbf{\Gamma})$. The set of all integers, the real numbers and the complex numbers are respectively, denoted by $\mathbb{Z}$, $\mathbb{R}$ and $\mathbb{C}$, and $\mathbf{j}$ is used to represent $\sqrt{-1}$.

II. Design of STBCs for co-located MIMO systems

A MIMO channel with N_t antennas at the source (transmitter) and N_r antennas at the destination (receiver) as shown in Fig. I is considered. The channel between every pair of the source and the destination antennas is assumed to be i.i.d. flat fading. The channels are also assumed to be constant over a block of T complex channel uses and take independent realizations in every block. The MIMO channel equation for one block (one block corresponds to T complex channel uses) is given by,

$$\mathbf{Y} = \sqrt{\frac{\rho}{N_t}}\mathbf{X}_l\mathbf{H} + \mathbf{N}, \tag{1}$$

where $\mathbf{Y}$ is the $T \times N_r$ received matrix, $\mathbf{X}_l$ is the $T \times N_t$ transmitted codeword matrix, $[\mathbf{H}]_{i,j}$, is the fade coefficient between the i-th transmit antenna and the j-th receive antenna which is distributed as $\mathcal{CSCG}(0,1)$ for all $i = 1, 2, \cdots N_t$ and $j = 1, 2, \cdots N_r$. The additive Gaussian noise matrix at the destination is $\mathbf{N}$ whose components are also distributed as $\mathcal{CSCG}(0,1)$ and ρ is the receive SNR at each of the destination antenna. The collection $\mathcal{C}$ of matrices

$$\mathcal{C} = \{\mathbf{X}_l \mid l = 1, 2, \cdots L\}$$

is called a Space-Time Block code (STBC). When the perfect estimate of $\mathbf{H}$ is available at the destination (referred to as the Channel State Information (CSI)), the channel is referred to as a coherent co-located MIMO channel. However, the overhead and/or the difficulty involved in obtaining these perfect estimates at the receiver leads to the need for designing signaling schemes assuming that the receiver does not have the knowledge of the channel. Such a channel is referred to as non-coherent co-located MIMO channel and the corresponding signaling schemes are called non-coherent signaling schemes. For a coherent MIMO channel, the Maximum Likelihood (ML) decoder decodes for a codeword $\hat{\mathbf{X}}$ given by,

$$\hat{\mathbf{X}} = \arg \min_{\mathbf{X} \in \mathcal{C}} ||\mathbf{Y} - \sqrt{\frac{\rho}{N_t}}\mathbf{X}\mathbf{H}||^2. \tag{2}$$

For a non-coherent MIMO channel with the class of unitary STBCs, the Maximum Likelihood (ML) decoder (referred to as non-coherent ML decoder) decodes for a codeword $\hat{\mathbf{X}}$ given by

$$\hat{\mathbf{X}} = \arg \max_{\mathbf{X} \in \mathcal{C}} \text{ Trace } ||\mathbf{Y}^H \mathbf{X}\mathbf{X}^H \mathbf{Y}||^2. \quad (3)$$

Note that the above decoder does not need the knowledge of $\mathbf{H}$ for decoding $\mathbf{X}$. The codebook $\mathcal{C}$ can be appropriately designed to obtain a diversity order of $N_t N_r$. However, the design criteria for full diversity depends on whether $\mathbf{H}$ is known at the destination or not. Signal designs for both coherent [18]-[25] and non-coherent co-located MIMO channels [65] have been fairly well developed.

III. Introduction to Cooperative Communications for Wireless Networks

In a wireless relay network, if the source and the destination terminals (each equipped with only single antenna) are engaged in a point to point communication, then spatial diversity is forbidden unlike that of a MIMO system as described in Section II. For such a model, other users in the network which are geographically separated can act as relays and assist the source in forwarding the information to the destination [4], [5]. In such a scenario, the source broadcasts its signal to all the relays and all the relays forward the source signal to the destination. Since the source signal reaches the destination through independent paths, a diversity order equal to the number of relays can potentially be obtained [6]. Therefore, some users in the network can act as virtual transmit antennas of the source and help the source to provide transmit diversity. As a result, existing space-time schemes for co-located MIMO channels can potentially be implemented in a distributed way and such schemes are referred to as distributed space-time coding (DSTC) schemes [6], [7]. However, the performance benefits of DSTC schemes depend on the type of processing employed at the relays [7]. Also, since the users are separated geographically, signals transmitted by all the users may not reach the destination at the same time, i.e. the relays may not be synchronized. In the rest of this paper, we present a survey of DSTBCs for wireless relay networks (i) when different protocols such as AF and DF protocols are employed at the relays and (ii) when the relays in the network are synchronized or not [51].

IV. DSTBCs for Synchronous Networks with AF Protocol

In this section, we discuss the design of DSTBCs for synchronous cooperative networks based on AF protocols. In particular, we consider DSTBC design based on LPF protocol (a generalization of the AF protocol). We introduce the network model and provide various ingredients required in the network to construct STBCs in a distributed way.

A. Signal model

The wireless network considered as shown in Fig. 3 consists of $K+2$ nodes, each having a single antenna. There is one source node and one destination node. All the other K nodes are relays. We denote the channel from the source node to the k-th relay as h_k and the channel from the k-th relay to the destination node as g_k for $k = 1, 2, \cdots, K$. The following assumptions are made in the channel model: (i) All the nodes are half duplex constrained, (ii) Fading coefficients h_k and g_k are i.i.d. $\mathcal{CSCG}(0,1)$ with a coherence time interval of at least N and T channel uses respectively, (iii) All the nodes are synchronized at the symbol level, (iv) Relay nodes do not have the knowledge of the fade coefficients h_k, (v) Destination knows all the fade coefficients g_k, h_k for $k = 1, 2, \cdots K$.

The source is equipped with a codebook $\mathcal{S} = \{\mathbf{x}_1, \mathbf{x}_2, \mathbf{x}_3, \cdots, \mathbf{x}_L\}$ consisting of information vectors $\mathbf{x}_l \in \mathbb{C}^{N\times 1}$ such that $E\left[\mathbf{x}_l^H \mathbf{x}_l\right] = 1$. Every transmission from the source to the destination comprises of two phases. When the source needs to transmit an information vector $\mathbf{x} \in \mathcal{S}$ to the destination, it broadcasts the vector $\mathbf{x}$ to all the K relays (but not to the destination since it is assumed to be far from the source). The received vector at the k-th relay is given by $\mathbf{r}_k = \sqrt{P_1 N} h_k \mathbf{x} + \mathbf{n}_k$, for all $k = 1, 2, \cdots, K$ where $\mathbf{n}_k \sim \mathcal{CSCG}(\mathbf{0}_{N\times 1}, \mathbf{I}_N)$ is the additive noise at the k-th relay and P_1 is the total power used at the source node for every channel use. In the second phase, all the relay nodes are scheduled to transmit T length vectors to the destination simultaneously. Each relay is equipped with a fixed $T \times N$ matrix $\mathbf{A}_k$ called the relay matrix[1] and is allowed to linearly process the received vector. The k-th relay is scheduled to transmit

$$\mathbf{t}_k = \sqrt{\frac{P_2 T}{(1+P_1)N}} \mathbf{A}_k \bar{\mathbf{r}}_k$$

where $\bar{\mathbf{r}}_k$ can either be $\mathbf{r}_k$ or $\mathbf{r}_k^*$. Note that P_2 is the total power used at each relay for every channel use in the second phase. The vector received at the destination is given by

$$\mathbf{y} = \sum_{k=1}^{K} g_k \mathbf{t}_k + \mathbf{w}$$

where $\mathbf{w} \sim \mathcal{CSCG}(\mathbf{0}_{T\times 1}, \mathbf{I}_T)$ is the additive noise at the destination. Substituting for $\mathbf{t}_k$, $\mathbf{y}$ can be written as

$$\mathbf{y} = \sqrt{\frac{P_1 P_2 T}{(1+P_1)}} \mathbf{X}(\mathbf{x}) \mathbf{f} + \mathbf{n} \quad (7)$$

where

- $\mathbf{n} = \sqrt{\frac{P_2 T}{(1+P_1)N}} \left[\sum_{k=1}^{K} g_k \{\mathbf{A}_k \mathbf{n}_k\}\right] + \mathbf{w}$.
- The equivalent channel $\mathbf{f}$ is given by $[g_1 \bar{h}_1 \ \ g_2 \bar{h}_2 \ \cdots \ g_K \bar{h}_K] \in \mathbb{C}^{K\times 1}$ wherein $\bar{h}_i$ is either h_i or h_i^* for all $i = 1$ to K.
- Every codeword $\mathbf{X}(\mathbf{x}) \in \mathbb{C}^{T\times K}$ which is of the form (4) (shown at the top of next page) is a function of the information vector $\mathbf{x}$.

The covariance matrix $\mathbf{R} \in \mathbb{C}^{T\times T}$ of the noise vector $\mathbf{n}$ is given in (5). The Maximum Likelihood (ML) decoder decodes for a vector $\hat{\mathbf{x}}$ where $\hat{\mathbf{x}}$ is given in (6) (shown at the top of this page).

Definition 1: The collection $\mathcal{C}$ of $T \times K$ codeword matrices shown below,

[1] In general, each relay can be equipped with a fixed pair of $T \times N$ rectangular matrices $\mathbf{A}_k$, $\mathbf{B}_k$ for more complex linear processing. For example, the relays can also perform conjugation operation on the received vector as $\mathbf{A}_k \bar{\mathbf{r}}_k + \mathbf{B}_k \bar{\mathbf{r}}_k^*$.

$$\mathbf{X}\left(\mathbf{x}\right)=\left[\mathbf{A}_1\mathbf{x}\ \ \mathbf{A}_2\mathbf{x}\ \ \cdots\ \ \mathbf{A}_K\mathbf{x}\right]. \tag{4}$$

$$\mathbf{R}=\frac{P_2T}{(1+P_1)N}\left[\sum_{k=1}^{K}|g_k|^2\left\{\mathbf{A}_k\mathbf{A}_k^H\right\}\right]+\mathbf{I}_T. \tag{5}$$

$$\hat{\mathbf{x}}=\ \arg\ \min_{\mathbf{x}\in\mathcal{S}}\left[-2Re\left(\sqrt{\frac{P_1P_2T}{(1+P_1)}}\mathbf{g}\mathbf{X}\mathbf{R}^{-1}\mathbf{y}^H\right)+\frac{P_1P_2T}{(1+P_1)}\mathbf{g}\mathbf{X}\mathbf{R}^{-1}\mathbf{X}^H\mathbf{g}^H\right]. \tag{6}$$

$$\mathcal{C}=\{\mathbf{X}(\mathbf{x})\mid\forall\ \mathbf{x}\in\mathcal{S}\} \tag{8}$$

is called a Distributed Space-Time Block Code (DSTBC) which is determined by the set of relay matrices $\{\mathbf{A}_k\}$ and, $\mathcal{S}$.

The channel equation in (7) looks similar to the MIMO channel except that every component of **f** is a product of two Gaussian random variables instead of a single Gaussian random variable. The vector **f** can be written as the component wise product of two vectors **h**, the set of source to relay channels and **g**, the set of relay to destination channels. Recall that for a MIMO channel, STBCs are constructed based on different design criteria which depends on the availability of the CSI at the destination. Similarly, in a cooperative channel, DSTBCs are constructed based on different design criteria which depends on the availability of **f** at the destination through the knowledge of the vectors **h** and **g**. The performance of a DSTBC in terms of the Pairwise Error Probability (PEP) is determined by the set of relay matrices $\mathcal{A}=\{\mathbf{A}_1,\mathbf{A}_2\cdots\mathbf{A}_K\}$ and the set of information bearing vectors $\mathcal{S}$. From the results of [9], for the set $\mathcal{C}$ to be fully diverse, for any $\mathbf{X}_1,\mathbf{X}_2\in\mathcal{C}$, the matrix $\mathbf{X}_1-\mathbf{X}_2$ should have full rank. Since the design criteria for full diversity is same as that for coherent MIMO channels [1], all STBCs well known for coherent MIMO channels are potentially applicable for coherent DSTC.

In particular, for a given extent of the channel knowledge at the destination, the sets $\mathcal{A}$ and $\mathcal{S}$ must be chosen such that the DSTBC provides a diversity order equal to the number of relays. Towards that direction, DSTBCs are classified into different groups depending on the channel knowledge at the destination. The following definition partitions DSTBCs into two classes based on the knowledge of h_k's and g_k's at the destination.

Definition 2: A DSTBC is referred to as coherent DSTBC, when the destination has the knowledge of both h_k's and g_k's. Otherwise, it is called non-coherent DSTC. In a non-coherent DSTBC, if the destination has the knowledge of only h_k's but not g_k's, then it is called partially coherent DSTBC. When the destination has no knowledge of both h_k's and g_k's, then it is called fully non-coherent DSTBC.

B. Distributed STBCs with low ML decoding complexity for relay networks

Since the work of [4]-[9], lot of efforts have been made to generalize various aspects of space-time coding which were originally proposed for co-located MIMO systems to the distributed framework. One such important aspect is the design of low-complexity ML decodable DSTBCs. In the following subsections, we provide a survey on DSTBCs with low-ML decoding complexity.

1) Single-Symbol ML Decodable Distributed STBCs for relay networks: For a co-located MIMO channel, an STBC is said to be Single-Symbol Maximum Likelihood (ML) Decodable (SSD) if the ML decoding metric splits as a sum of several terms, with each term being a function of only one of the information symbols. A DSTBC is said to be SSD if the STBC seen by the destination from the set of relays is SSD. For a background on SSD STBCs for co-located MIMO channels, we refer the reader to [18] - [24]. In this subsection, we discuss the issues related to the design of SSD DSTBCs only.

Recently, in [26], Distributed Orthogonal Space-Time Codes (DOSTBCs) achieving single-symbol decodability have been introduced for cooperative networks. The authors considered a special class of DOSTBCs which makes the covariance matrix **R**, a diagonal matrix and such a class of codes has been referred to as row monomial DOSTBCs. The maximum symbol-rate (in complex symbols per channel use in the second phase) of row monomial DOSTBCs have been derived and is shown to be upper-bounded by $\frac{2}{K}$ where K denotes the number of relays in the network. A systematic construction of such codes has also been proposed. The constructed codes are shown to meet the upper bound for even number of relays. In [27], the same authors have also derived an upper bound on the symbol-rate of DOSTBCs when the additive noise at the destination is correlated and have shown that the improvement in the rate is not significant when compared to the case when the noise at the destination is uncorrelated [26]. An example of DOSTBC for $K=4$ is presented in (9) (shown at the top of the next page).

In [27] and [42], SSD DSTBCs have been proposed when every relay node is assumed to have the perfect knowledge of the phase component of the channel from the source to the relay. An upper bound on the symbol rate for such a set up is shown to be $\frac{1}{2}$ which is independent of the number of relays. The codes proposed in [27] and [42] are derived from Real Orthogonal Designs (RODs). Hence, such designs are referred

$$\mathbf{X}'(4,4) = \begin{bmatrix} h_1x_1 & -h_2^*x_2^* & 0 & 0 \\ h_1x_2 & h_2^*x_1^* & 0 & 0 \\ h_1x_3 & -h_2^*x_4^* & 0 & 0 \\ h_1x_4 & h_2^*x_3^* & 0 & 0 \\ 0 & 0 & h_3x_1 & -h_4^*x_2^* \\ 0 & 0 & h_3x_2 & h_4^*x_1^* \\ 0 & 0 & h_3x_3 & -h_4^*x_4^* \\ 0 & 0 & h_3x_4 & h_4^*x_3^* \end{bmatrix}. \quad (9)$$

to as "CODs from RODs" in this paper. An example for the codes proposed in [27] for $K = 4$ is given below.

$$\mathbf{X}_{\text{COD from ROD}} = \begin{bmatrix} x_1 & x_2 & x_3 & x_4 \\ -x_2 & x_1 & -x_4 & x_3 \\ -x_3 & x_4 & x_1 & -x_2 \\ -x_4 & -x_3 & x_2 & x_1 \\ x_1^* & x_2^* & x_3^* & x_4^* \\ -x_2^* & x_1^* & -x_4^* & x_3^* \\ -x_3^* & x_4^* & x_1^* & -x_2^* \\ -x_4^* & -x_3^* & x_2^* & x_1^* \end{bmatrix}. \quad (10)$$

In [26], [27] and [42], the source node transmits the information symbols to all the relays without any processing. Using the framework proposed in [26], the authors in [30] propose SSD DSTBCs aided by linear precoding of the information vector at the source. In such a set-up, the relay nodes do not have the knowledge of the channel from the source to itself. A new class of DSTBCs called Precoded DSTBCs (PDSTBCs) is introduced where the source performs linear precoding of information symbols appropriately before transmitting it to all the relays. Within this class, the authors identify codes that are SSD PDSTBCs referred to as Semi-orthogonal SSD-PDSTBCs (Semi-SSD-PDSTBC). An upper bound on the maximal symbol-rate of row monomial Semi-SSD-PDSTBCs is derived. It is shown that, the symbol rate of such codes is upper bounded by $\frac{4}{K}$. Construction of row monomial Semi-SSD-PDSTBCs is also presented when $K \geq 4$. The proposed construction provides codes achieving the upper bound on the symbol rate when K is 0 or 3 modulo 4. For other values of K, the constructed codes do not achieve the upper bound, but have higher rates than DOSTBCs. An example of a Semi-SSD-PDSTBC for 4 relays is given below,

$$\mathbf{X}(4,4) = \begin{bmatrix} h_1\tilde{x}_1 & -h_2^*\tilde{x}_2^* & h_3\tilde{x}_3 & -h_4^*\tilde{x}_4^* \\ h_1\tilde{x}_2 & h_2^*\tilde{x}_1^* & h_3\tilde{x}_4 & h_4^*\tilde{x}_3^* \\ h_1\tilde{x}_3 & -h_2^*\tilde{x}_4^* & h_3\tilde{x}_1 & -h_4^*\tilde{x}_2^* \\ h_1\tilde{x}_4 & h_2^*\tilde{x}_3^* & h_3\tilde{x}_2 & h_4^*\tilde{x}_1^* \end{bmatrix}, \quad (11)$$

where $\tilde{x}_1 = x_{1I} + \mathbf{j}x_{4Q}$; $\tilde{x}_2 = x_{2I} + \mathbf{j}x_{3Q}$; $\tilde{x}_3 = x_{1Q} + \mathbf{j}x_{4I}$ and $\tilde{x}_4 = x_{2Q} + \mathbf{j}x_{3I}$. The variables $\tilde{x}_1, \tilde{x}_2, \cdots \tilde{x}_4$ are obtained using the precoding matrices at the source.

2) Training embedded Single-Symbol ML Decodable Distributed STBCs for relay networks: For point to point co-located MIMO channels, complex orthogonal designs (CODs) [18], [20], coordinate interleaved orthogonal designs (CIODs) [23] and Clifford unitary weight designs (CUWDs) [24] are well known for their SSD property when used to generate STBCs. Note that, with the assumption of the knowledge of the phase component of the source-relay channel at the relays, all CODs can be constructed as DSTBCs. The extensions of CODs such as CIODs and CUWDs can also be distributively constructed. However, CODs (other than the Alamouti design), CIODs and CUWDs (other than that for 4 antennas) do not retain the SSD property.

In [31], the authors propose high rate, training embedded SSD DSTBCs. The proposed codes include the training symbols in the structure of the code which is shown to be the key point to obtain high rate as well as the SSD property. In [31], the number of channel uses spent on transmitting training signals from the source to the relays are accounted in computing the rate of the proposed DSTBCs. When all the zero entries of a COD (square or non-square) are replaced by a non-zero constant, the resulting design is called a Training-Embedded-COD (TE-COD). These are shown to generate SSD DSTBCs. This essentially enables all CODs to be usable as SSD DSTBCs with full-diversity for arbitrary complex constellations. Compared to the existing SSD codes of [42], the class of non-square TE-CODs are shown to provide higher rates for two-hop networks with number of relays less than 10. Example of a square TE-COD for 4 relays is shown below. For the well known 4×4 COD [18] of rate $\frac{3}{4}$, with x_1, x_2 and x_3 being the complex variables, the corresponding TE-COD is given by,

$$\mathbf{X}_{\text{TE-COD}} = \begin{bmatrix} x_3 & \alpha & x_2 & x_1 \\ \alpha & x_3 & x_1^* & -x_2^* \\ x_2^* & x_1 & -x_3^* & \alpha \\ x_1^* & -x_2 & \alpha & -x_3^* \end{bmatrix}. \quad (12)$$

To implement the above design, the number of channel uses required in the first phase is 4 (3 channel uses for the variables and the rest for transmitting α). The number of channel uses in the second phase is also 4. Hence, the rate of this scheme is $\frac{3}{8}$.

Note that, the known codes in [27], [42] and [31] (non-square TE-CODs) have exponential decoding delay and hence, in [32] the authors focus on constructing SSD DSTBCs with *low delay*. Also, the number of complex symbols that a TE-COD for 2^a relays can accommodate is only $a+1$ (which is same as that of a COD for 2^a antennas). Therefore, the rate of TE-DSTBCs from TE-CODs (in symbols per channel use) when employed as in [31] is given by

$$R_{\text{TE-COD}} = \frac{a+1}{a+1+\lceil \frac{2^a-a-1}{2} \rceil + 2^a}. \quad (13)$$

In [32], the authors propose training embedded SSD DSTBCs for relay networks with rates higher than that of DSTBCs from TE-CODs (given in (13)). In particular, the authors employ linear precoding of information symbols at the source [30] and use CIODs of [23] instead of CODs to obtain a class of high-rate SSD DSTBCs. Using square TE-CODs as ingredients, TE-CIODs are constructed using the coordinate interleaved variables. Unlike TE-CODs, not all the entries of a TE-CIOD are non-zero. In particular, TE-CIODs have a block diagonal structure. Exploiting the block diagonal structure of TE-CIODs, it is shown that the rate of TE-DSTBCs from TE-CIODs is

$$R_{\text{TE-CIODs}} = \frac{2a}{2a + \lceil \frac{2^{a-1}-a}{2} \rceil + 2^a}, \tag{14}$$

which is larger than $R_{\text{TE-COD}}$ given by (13), while retaining the SSD and full-diversity property. Example of a TE-CIOD for 8 relays is given below.

Example 1: The 8×8 TE-CIOD is given by

$$\begin{bmatrix} \tilde{x}_3 & \alpha & \tilde{x}_2 & \tilde{x}_1 & 0 & 0 & 0 & 0 \\ \alpha & \tilde{x}_3 & \tilde{x}_1^* & -\tilde{x}_2^* & 0 & 0 & 0 & 0 \\ \tilde{x}_2^* & \tilde{x}_1 & -\tilde{x}_3^* & \alpha & 0 & 0 & 0 & 0 \\ \tilde{x}_1^* & -\tilde{x}_2 & \alpha & -\tilde{x}_3^* & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \tilde{x}_6 & \alpha & \tilde{x}_5 & \tilde{x}_4 \\ 0 & 0 & 0 & 0 & \alpha & \tilde{x}_6 & \tilde{x}_4^* & -\tilde{x}_5^* \\ 0 & 0 & 0 & 0 & \tilde{x}_5^* & \tilde{x}_4 & -\tilde{x}_6^* & \alpha \\ 0 & 0 & 0 & 0 & \tilde{x}_4^* & -\tilde{x}_5 & \alpha & -\tilde{x}_6^* \end{bmatrix}, \tag{15}$$

where $\tilde{x}_m = x_{mI} + \mathbf{j}x_{(m+3)Q}$ and $\tilde{x}_{m+3} = x_{(m+3)I} + \mathbf{j}x_{mQ}$ for $m = 1$ to 3. The number of channel uses in the first phase and second phase are 7 and 8, respectively. Therefore, the rate of the scheme is $\frac{6}{15}$ complex symbols per channel use.

3) Distributed STBCs from Real Orthogonal Designs: A scheme to apply the rate-1 real orthogonal designs (RODs) [18] in relay networks with single real-symbol decodability of the symbols at the destination for any arbitrary number of relays is proposed in [33]. In the case where the relays do not have any information about the channel gains from the source to themselves, the best known distributed space-time block codes (DSTBCs) for K relays with single real-symbol decodability offer an overall rate of $\frac{2}{2+K}$ complex symbols per channel use [26]. The scheme proposed in [33] offers an overall rate of $\frac{1}{4}$ complex symbol per channel use, which is independent of the number of relays. Furthermore, in the scenario where the relays have partial channel information in the form of channel phase knowledge, the best known DSTBCs with single real-symbol decodability offer an overall rate of $\frac{1}{3}$ complex symbols per channel use [42]. In [33], making use of RODs, a scheme which achieves the same overall rate of $\frac{1}{3}$ complex symbols per channel use but with a decoding delay that is 50 percent of that of the best known DSTBCs, is presented.

4) Multi-group ML Decodable Distributed STBCs for relay networks: In a co-located MIMO channel, an STBC is said to be multi-group Maximum Likelihood (ML) Decodable if the ML decoding metric splits as a sum of several terms, with each term being a function of a disjoint subset of the information symbols. In [38], DSTBCs which admit four-group maximum likelihood (ML) decoding are studied. In particular, the Jing-Hassibi protocol [9] is generalized to allow non-unitary matrices at the relays. The necessary and sufficient conditions needed for DSTBCs to be multi-group ML decodable are identified and three new classes of four group ML decodable DSTBCs which achieve full cooperative diversity for any number of relays are also provided. The proposed DSTBCs achieve the least possible ML decoding complexity compared to all other DSTBC constructions, having the same transmission rate in complex symbols per channel use in the literature. Two-group decodable DSTBCs have been proposed in [64] and [41]. A thorough survey on STBCs with low ML decoding complexity can be found in [39].

C. DSTBCs for wireless relay networks with multiple antenna nodes

In [9], the idea of space-time coding devised for point to point co-located multiple antenna systems has been applied for a wireless relay network with single antenna nodes and PEP (Pairwise Error Probability) of such a scheme was derived. It is shown that in a relay network with a single source, a single destination with K single antenna relays, DSTC achieves the diversity of a co-located multiple antenna system with K transmit antennas and one receive antenna, asymptotically.

Subsequently, in [17], the idea of [9] is extended to relay networks where the source, the destination and the relays have multiple antennas. But, co-operation between the multiple antennas of each relay is not used, i.e., the co-locatedness of the antennas is not exploited. Hence, a total of K relays each with a single antenna is assumed in the network instead of a total of K antennas in a smaller number of relays. With this set up, for a network with M antennas at the source, N antennas at the destination and a total of K antennas at K relays, for large values of P (where P is the total power used by all the nodes per channel use), the PEP of the network, varies with P as

$$\begin{aligned} &\left(\frac{1}{P}\right)^{min(M,N)K} && \text{if} \quad M \neq N \text{ and} \\ &\left(\frac{(\log_e^{1/M} P)}{P}\right)^{MK} && \text{if} \quad M = N. \end{aligned}$$

In particular, the PEP of the scheme in [17] for large P when specialized to $M = N = 1$ with $2K$ antennas at relays is upper-bounded by

$$\left[\frac{32R}{T(\rho')^2}\right]^{2R} \left[\frac{(\log_e(P))^{2R}}{P^{2R}}\right], \tag{16}$$

where $(\rho')^2$ is the minimum singular value of $(\mathbf{X}-\mathbf{X}')^H(\mathbf{X}-\mathbf{X}')$, where $\mathbf{X}$ and $\mathbf{X}'$ are the two distinct codewords of a distributed space-time block code.

In [29], it is shown that multiple antennas at the relays can be exploited to improve the performance of the network. Towards this end, a single antenna source and a single antenna destination with two antennas at each of the K relays is considered. Also, the two phase protocol as in [17] is assumed where the first phase consists of transmission of a T length complex vector from the source to all the relays (not to the destination) and the second phase consists of transmission of a T length complex vector from each of the antennas of the relays to the destination. The modification in the protocol introduced in [29] is that the two received vectors at the two antennas of a relay during the first phase is coordinate interleaved as defined below.

Definition 3: Given two complex vectors $\mathbf{y}_1, \mathbf{y}_2 \in \mathbb{C}^T$, we define a Coordinate Interleaved Vector Pair of $\mathbf{y}_1, \mathbf{y}_2$, denoted as CIVP $\{\mathbf{y}_1, \mathbf{y}_2\}$ to be the pair of complex vectors $\{\mathbf{y}_1', \mathbf{y}_2'\}$, where $\mathbf{y}_1', \mathbf{y}_2' \in \mathbb{C}^T$, given by

$$\mathbf{y}_1' = \mathrm{Re}\,\mathbf{y}_1 + \mathbf{j}\,\mathrm{Im}\,\mathbf{y}_2,\ \mathbf{y}_2' = \mathrm{Re}\,\mathbf{y}_2 + \mathbf{j}\,\mathrm{Im}\,\mathbf{y}_1,$$

or equivalently,

$$\mathbf{y}_1' = (\mathbf{y}_1 + \mathbf{y}_1^* + \mathbf{y}_2 - \mathbf{y}_2^*)/2, \tag{17}$$

$$\mathbf{y}_2' = (\mathbf{y}_2 + \mathbf{y}_2^* + \mathbf{y}_1 - \mathbf{y}_1^*)/2. \tag{18}$$

Then, multiplying the coordinate interleaved vector with the predecided antenna specific $T \times T$ unitary matrices, each antenna produces a T length vector that is transmitted to the destination in the second phase. The collection of all such vectors, as columns of a $T \times 2K$ matrix constitutes a codeword matrix and collection of all such codeword matrices is referred to as coordinate interleaved distributed space-time code (CIDSTC). For $T \geq 4K$, an upper bound on the PEP with fully diverse CIDSTC, at large values of the total power P has been derived. For $T \geq 2K$, the PEP derived in [17] with fully diverse DSTBC is upper bounded by the expression given in (16). Comparing this bound, with the one in [29], for equal number of $2K$ antennas, CIDSTC scheme is shown to provide asymptotic coding gain compared to the scheme in [17]. Note that the improvement in the PEP comes just by *vector co-ordinate interleaving* at every relay whose complexity is negligible. Following the work of [17], constructions of DSTBCs for networks with multiple antenna nodes are presented in [61], [62]. In [62], a full rate strategy is proposed which relies on division algebras, an algebraic object which allows the design of fully diverse matrices. The code construction is shown to be applicable to systems with any number of transmit/receive antennas and relays, and has better performance than random code constructions, with much less encoding complexity. The robustness of the proposed distributed space-time codes to node failures is also exhibited.

D. Non-coherent distributed space-time codes

From Definition 2, a DSTBC is referred to as fully non-coherent DSTBC, when the destination has no knowledge of both h_k's and g_k's. For such a scenario, the overhead or the difficulty in conveying all the channel information in the network to the destination is avoided. In the rest of this section, we consider designing DSTBCs based on differential schemes for fully non-coherent, synchronous relay networks. Several code construction procedures based on differential techniques are also provided. A thorough survey on non-coherent space-time codes for co-located MIMO systems and distributed relay networks can be found in [65] and [35] respectively.

1) Differential coding technique for non-coherent cooperative communications: In the differential encoding scheme for the MIMO channel, the source conveys information to the destination by transmitting one of the several unitary matrices using two successive codeword uses. However, in a cooperative set-up, the source node is restricted to transmit only vectors (this is because the source node is assumed to have single antenna) and hence the techniques for co-located MIMO channel cannot be trivially extended. In this subsection, a differential coding scheme is briefly presented for the co-operative channel which is a generalized version of the scheme for the MIMO channel. With reference to channel model in Section IV-A, we assume $N = T$ throughout this section. i.e the number of channel uses in the first phase and the second phase are equal.

For the differential encoding scheme, the source node is equipped with a code $\{\mathbf{x}, \mathcal{G}\}$, where $\mathbf{x}$ is a fixed $T \times 1$ complex vector and $\mathcal{G}$ is a finite set of $T \times T$ unitary matrices such that $\mathbf{G}_i\mathbf{s} \neq \mathbf{G}_j\mathbf{s}$ for all $\mathbf{G}_i \neq \mathbf{G}_j$. The two sets $\mathcal{A}$ and $\mathcal{G}$ should satisfy the following conditions

$$\mathbf{G}_i\mathbf{A}_k = \mathbf{A}_k\mathbf{G}_j \text{ for } 1 \leq j,k \leq |\mathcal{G}|$$

and

$$\mathbf{X}^H\mathbf{X} = \mathbf{I}_K$$

where $\mathbf{X}(\mathbf{x}) = [\mathbf{A}_1\mathbf{x}\ \ \mathbf{A}_2\mathbf{x}\ \ \cdots\ \ \mathbf{A}_K\mathbf{x}]$ is the initial matrix corresponding to the vector $\mathbf{x}$ used in the code. The transmission follows the two stages per block model with differential encoding performed at the source node. If $\mathbf{x}(t-1)$ denotes the transmitted vector by the source in the $(t-1)$-block, the vector transmitted in the t-block is of the form $\mathbf{x}(t) = \mathbf{G}\mathbf{x}(t-1)$ where $\mathbf{G}(t) \in \mathcal{G}$ is chosen based on the information to be transmitted. The vector $\mathbf{x}$ is the initial vector used by the source, i.e. $\mathbf{x}(0) = \mathbf{x}$. After linear processing from the relays, the received vector at the destination in the t-th block is $\mathbf{y}(t) = \mathbf{X}(t)\mathbf{f} + \mathbf{n}(t)$ (excluding the scaling factors) where

$$\mathbf{X}(t) = [\mathbf{A}_1\mathbf{x}(t)\ \mathbf{A}_2\mathbf{x}(t)\ \ \cdots \mathbf{A}_K\mathbf{x}(t)]\,.$$

If the code $\{\mathbf{x}, \mathcal{G}\}$ and the set of relay matrices $\mathcal{A}$ satisfy the condition $\mathbf{G}_i\mathbf{A}_k = \mathbf{A}_k\mathbf{G}_j$ for all i, j, then

$$\mathbf{X}(t) = \mathbf{G}(t)\,[\mathbf{A}_1\mathbf{x}(t-1)\ \ \mathbf{A}_2\mathbf{x}(t-1)\ \ \cdots\ \ \mathbf{A}_K\mathbf{x}(t-1)]$$

for some $\mathbf{G}(t) \in \mathcal{G}$. Therefore,

$$\mathbf{y}(t) = \mathbf{G}(t)\mathbf{X}(t-1)\mathbf{f} + \mathbf{n}(t) = \mathbf{G}(t)\mathbf{y}(t-1) + \mathbf{n}'(t)$$

where $\mathbf{n}'(t) = \mathbf{n}(t) - \mathbf{G}(t)\mathbf{n}(t-1)$. Thus the distributed STBC is given by,

$$\mathcal{C} = \left\{ \mathbf{G}\mathbf{X} \mid \mathbf{G} = \mathbf{I}_K \text{ or } \prod_k \mathbf{G}_k \text{ such that } \mathbf{G}_k \in \mathcal{G} \right\}.$$

The following decoder has been commonly used (referred to as ML 1-lag detector) for decoding the codewords of $\mathcal{G}$,

$$\hat{\mathbf{G}}(t) = \arg\min_{\mathbf{G}(t)\in\mathcal{G}} ||\mathbf{y}(t) - \mathbf{G}(t)\mathbf{y}(t-1)||^2. \tag{19}$$

Notice that the decoder in (19) is fully non-coherent since it does not use the knowledge of the channel vector $\mathbf{f}$. In [63] a Chernoff bound on the PEP is derived for the above decoder and it is shown that, for full diversity, the design criterion on the codebook is the same as that of the codebook for coherent DSTC [9]. One of the key requirements in the distributed construction of differential codes is to generate two commuting sets of unitary matrices. Even though several full diversity achieving unitary codes are available for the MIMO channel, such codes cannot be directly applied in the distributed set-up since the codewords need to commute with the relay matrices. In the following subsection, various techniques of constructing the sets $\mathcal{A}$ and $\mathcal{G}$ are presented.

Different tools and methods to generate the necessary ingredients, $\mathcal{A}$ and $\mathcal{G}$ are provided in the following subsections. The methods presented include algebraic tools such as division algebras, Cayley Algebras, and cyclic groups.

2) Constructing commuting set of matrices from division algebras: The authors in [43] have introduced commuting sets of unitary matrices from division algebra for the sets $\mathcal{A}$ and $\mathcal{G}$. We give only a flavor on how to choose such matrices and refer the readers to the literature for a detailed treatment on space-time code constructions based on division algebra. A division algebra $\mathcal{D}$ is a vector space over a field, $\mathbb{F}$ which is also an associative ring, not necessarily commutative and every element $d \in \mathcal{D}$ has a multiplicative inverse. Let n denote the dimension of the vector space $\mathcal{D}$ over $\mathbb{F}$. For an arbitrary element, $d \in \mathcal{D}$ the map $L_d(x) = dx$ is a $\mathbb{F}$ linear isomorphism from $\mathcal{D}$ to $\mathcal{D}$. This defines a one-one map from d to the set of all $\mathbb{F}$ linear isomorphism of $\mathcal{D}$ which associates d to L_d. With a fixed basis of $\mathcal{D}$, every d can be associated with an $n \times n$ invertible matrix over $\mathbb{F}$ denoted as L_d. This is referred to as the left regular representation of $\mathcal{D}$. Similarly, right regular representation of $\mathcal{D}$ is denoted by R_d. The most important fact about these two representations is that these two commute. Thus, the commuting sets of $n \times n$ matrices each of them isomorphic to the division algebra have been identified. The set $\mathcal{G}$ is chosen as a subset of unitary matrices from $\mathcal{L}$ (set of matrices from left regular representation of $\mathcal{D}$) and $\mathcal{A}$ as a subset from $\mathcal{R}$ (set of matrices from right regular representations of $\mathcal{D}$). This ensures that any matrix from $\mathcal{G}$ commutes with any matrix from $\mathcal{A}$. Furthermore since $\mathcal{D}$ is a division algebra, full diversity requirement is also guaranteed.

3) Codes from cyclic groups: A distributed differential coding scheme is proposed in [62] where diagonal unitary matrices have been used for the sets $\mathcal{G}$ and $\mathcal{A}$. Since the matrices in each set are diagonal, the two sets will trivially satisfy the commutative property. Additionally, $\mathcal{G}$ is given the structure of a cyclic group. It is shown that full diversity can be obtained by choosing the generator of the group with an appropriate structure. The relay matrix set $\mathcal{A}$ is constructed from a Generalized Butson Hadamard (GBH) matrix $M \in \mathbb{C}^{K \times K}$ (see the following definition).

Definition 4: [58] A Generalized Butson-Hadamard (GBH) matrix is a $T \times T$ matrix $\mathbf{M}$ with entries such that $\mathbf{M}^H\mathbf{M} = \mathbf{M}\mathbf{M}^H = T\mathbf{I}_T$ and the conjugate of every entry $[\mathbf{M}]_{ij}$ is its inverse.

The matrix $\mathbf{M}$ is used to construct $\mathcal{A}$ where the elements of the i-th column of $\mathbf{M}$ are used as the diagonal elements of $\mathbf{A}_i$. An example for such a code is provided below for a network with $K = 3$ and $|G| = 63$.

Example 2:

$$\mathcal{G} = \left\{ \mathbf{D}^i, \mathbf{D} = \begin{bmatrix} w_{63} & 0 & 0 \\ 0 & w_{63}^{17} & 0 \\ 0 & 0 & w_{63}^{26} \end{bmatrix} \right\}$$

for $i = 1, \cdots 63$ and $\mathbf{x} = \frac{1}{\sqrt{K}}[11 \cdots 1]^T \in \mathbb{C}^K$ where $w_{63} = exp(\frac{2\pi\mathbf{j}}{63})$. The corresponding GBH matrix is

$$\mathbf{M} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & w_3 & w_3^2 \\ 1 & w_3^2 & w_3 \end{bmatrix}$$

where $w_3 = exp(\frac{2\pi\mathbf{j}}{3})$.

4) Non-differential codes from cyclic groups: Recently in [34] STBCs are constructed based on non-intersecting subspaces for partially coherent channel. The proposed codes also make use of unitary matrix groups at the source and diagonal matrices at the relays. When the group is cyclic, a necessary and sufficient condition on the generator of the cyclic group to achieve full diversity and to minimize the PEP is proved. Certain conditions on the choice of a generator of the cyclic group are provided to reduce the decoding complexity at the destination as well.

5) Codes from Cayley algebra: Recently, differential DSTBCs have been proposed using unitary Cayley transforms [60] using which the set $\mathcal{G}$ can be constructed. For a Hermitian matrix $\mathbf{X}$, the unitary Cayley transform $\mathbf{U}$ is defined by $\mathbf{U} = (\mathbf{I} + \mathbf{jX})^{-1}(\mathbf{I} - \mathbf{jX})$. A Cayley code $\mathcal{G}$ is a family of unitary matrices given by

$$\mathcal{G} = \left\{ \mathbf{U}_j = (\mathbf{I} + \mathbf{jX}_j)^{-1}(\mathbf{I} - \mathbf{jX}_j) \mid j = 1, 2, \cdots L \right\}$$

for a family $\mathcal{X} = \{\mathbf{X}_j\}$ of Hermitian matrices. It is proved that the set $\mathcal{G}$ is fully diverse if the set $\mathcal{X}$ is designed using the multiplication matrices from the number fields. Note that the above construction is available for any number of relays.

Differential DSTBCs have also been proposed using circulant matrices with M-PSK signal sets in [59]. More details on this can be found in [59] and [35].

6) Codes with low encoding and decoding complexity: In [37], unitary DSTBCs from linear designs having low encoding and decoding complexity is proposed for distributed differential coding. The readers can refer [25], [18] for details on linear designs. Towards constructing unitary STBCs from linear designs, codebooks $\mathcal{G}$ consisting of unitary matrices are generalized to allow codebooks consisting of scaled unitary matrices. It is shown that the use of scaled unitary matrices in $\mathcal{G}$ provides opportunity to use linear designs in the differential set-up. Motivated by the low decoding complexity of the Alamouti design, four group decodable codes were proposed recently in [37] for networks with number of relays of the form 2^a for any positive integer a.

Notice that the 4-group decodability is the property of the linear design. Linear designs with four-group decodable property are constructed using extended Clifford algebras. However, the STBCs from the proposed designs are not fully diverse for an arbitrary signal set and moreover, all the columns of the design are not orthogonal. Hence, such designs cannot be directly used to construct scaled unitary matrices using arbitrary signal set. Therefore, the authors explicitly construct signal sets for the proposed linear designs such that the resulting linear STBCs have scaled unitary matrices meeting the power constraint and provide full diversity as well.

Example 3: For $K = 4$, the unitary STBC $\mathcal{G}$ is obtained using the following design,

$$\mathbf{X} = \frac{1}{\sqrt{4}} \begin{bmatrix} x_1 & x_2 & -x_3^* & -x_4^* \\ x_2 & x_1 & -x_4^* & -x_3^* \\ x_3 & x_4 & x_1^* & x_2^* \\ x_4 & x_3 & x_2^* & x_1^* \end{bmatrix}.$$

Note that the first column of the design is not orthogonal to the second column. The appropriate signal set for the above design can be obtained using Construction 4.4 in [37]. This code is 4-group decodable or single complex symbol decodable (since the number of real symbols is 8).

V. DSTBCs for Asynchronous Networks with AF Protocol

A. A distributed space-time coding in asynchronous wireless relay networks

The authors in [9] recently presented a detailed analysis on the problem of distributed space-time coding for a synchronized two-hop wireless relay network. In [47], the authors extend the work of [9] to asynchronous wireless networks by employing orthogonal frequency division multiplexing (OFDM) to combat timing errors from relay nodes. Based on a layered structure, a distributed space-time code design is presented which achieves full spatial diversity for an asynchronous wireless relay network. In the rest of this subsection, we provide the channel model and the code constructions presented in [47] along with an example.

The wireless network considered in this section is similar to the one considered in Section III. The broadcast phase remains same as in Section III, however, there are changes in the second phase, and hence we only point out the changes in the (i) processing and (ii) the ingredients at the relay nodes to handle the asynchronous condition. The k-th relay is scheduled to transmit

$$\mathbf{t}_k = \sqrt{\frac{P_2 T}{P_r}} \mathbf{A}_k \mathbf{r}_k \in \mathbb{C}^{T \times 1} \tag{20}$$

where P_r is the average norm of the vector $\mathbf{r}_k$. It is assumed that the transmission delay in the path between the k-th relay and the destination is δ_k. Each relay is assumed to know l_p, the maximum of δ_k for all $k = 1, 2, \cdots K$. The k-th relay applies a N-point IDFT ($N \geq T$) on $\mathbf{t}_k$ and adds the cyclic prefix of length l_p before transmitting it to the destination.

At the destination node, after the CP removal and N-Point FFT transformation, the received signal can be written as

$$\mathbf{y} = T\sqrt{\frac{P_1 P_2}{P_r}} \mathbf{X}\mathbf{f} + \mathbf{n} \in \mathbb{C}^{T \times 1},$$

where $\mathbf{w} \sim \mathcal{CSCG}(\mathbf{0}_{T\times 1}, \mathbf{I}_T)$ is the additive noise at the destination. The equivalent channel $\mathbf{f}$ is given by $[h_1 g_1 \ h_2 g_2 \ \cdots \ h_K g_K]^T \in \mathbb{C}^{K \times 1}$. Every codeword $\mathbf{X} \in \mathbb{C}^{T \times K}$ which is of the form,

$$\mathbf{X} = [\mathbf{A}_1 \mathbf{x} \circ \mathbf{d}^{\tau_1} \quad x\mathbf{A}_2 \mathbf{x} \circ \mathbf{d}^{\tau_2} \quad \cdots \quad \mathbf{A}_K \mathbf{x} \circ \mathbf{d}^{\tau_K}] \in \mathbb{C}^{T \times K}, \tag{21}$$

where the elements of $\mathbf{d}^{\tau_k}$ are some powers of $\exp(\frac{\mathbf{j}2\pi\tau_k}{N})$. From the results of [9], for the set $\mathcal{C}$ to be fully diverse, for any $\mathbf{X}_1, \mathbf{X}_2 \in \mathcal{C}$, the matrix $\mathbf{X}_1 - \mathbf{X}_2$ should have full rank. For full diversity, the codewords have to satisfy the full rank property in the presence of the vectors $\mathbf{d}^{\tau_k}$. In the same paper, the authors present a method of constructing full diversity codes which are based on layered structure (Section III, [47]). For example, a DSTBC for $K = 5$ and $T = 5$ is presented below.

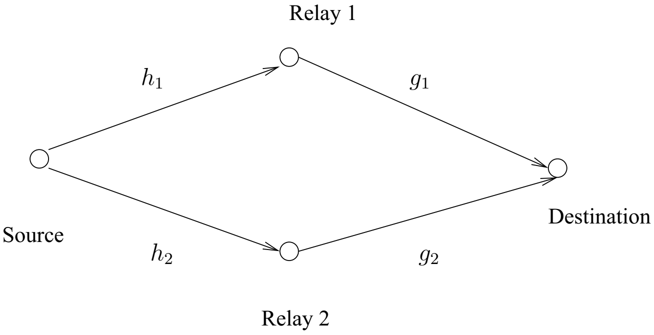

Fig. 5. Wireless relay network model with two relays.

$$\mathbf{X} = \begin{bmatrix} x_1 & x_5\gamma^4 & x_4\gamma^3 & x_3\gamma^2 & x_2\gamma \\ x_2\gamma & x_1 & x_5\gamma^4 & x_4\gamma^3 & x_3\gamma^2 \\ x_3\gamma^2 & x_2\gamma & x_1 & x_5\gamma^4 & x_4\gamma^3 \\ x_4\gamma^3 & x_3\gamma^2 & x_2\gamma & x_1 & x_5\gamma^4 \\ x_5\gamma^4 & x_4\gamma^3 & x_2\gamma^2 & x_2\gamma & x_1 \end{bmatrix},$$

where the variables x_i's take values from a QAM constellation. A 64-tone ($N = 64$) OFDM is employed at each relay. The presented code is fully diverse for arbitrary timing errors when $\gamma = \exp(\frac{\mathbf{j}2\pi}{37})$.

B. A Simple Alamouti Space-Time Transmission Scheme for Asynchronous Cooperative Systems

In [46], the authors propose DSTBCs based on orthogonal frequency-division multiplexing (OFDM) for an asynchronous cooperative system with two relays (shown in Fig. 5). In such a scheme, OFDM is implemented at the source node wherein the OFDM symbol is preceded with a cyclic prefix (CP) of length l_p before broadcasting it to all the relays. The cyclic prefix length l_p corresponds to the maximum of the possible relative timing errors of the signals arrived at the destination from the relay nodes. At the relay nodes, the OFDM symbols undergo a time-reversion and complex conjugation processing before getting forwarded to the destination. It is emphasized that the relay nodes only need to implement the time-reversion, some sign changes from plus to minus, and/or the complex conjugation to the received signals, and hence the processing complexity at the relays is low. In this scheme, the received signals at the destination node have the Alamouti code structure on each sub-carrier, and thus, it has the fast symbol-wise ML decoding. The authors show that this simple scheme achieves second-order diversity gain without the synchronization requirement at the relay nodes.

When there are more than two relay nodes, the authors use the idea of clustered nodes wherein all the relay nodes can be split in to two clusters of nodes. The nodes in the same cluster implement the same space-time processing. The first cluster serves as one relay and the second cluster serves as the other relay. However, the diversity offered by such an implementation is only 2 irrespective of the number of relays.

In [48], the same authors extend their work in [46] and propose Alamouti coded orthogonal frequency-division multiplexing (OFDM) scheme for cooperative communication system based on decode and forward protocol. The proposed scheme is shown to be robust to both timing errors and frequency offsets. In order to mitigate the inter-carrier interference (ICI) caused by multiple frequency offsets in the cooperative system, an ICI-self cancellation scheme is constructed, which can suppress ICI effectively. Moreover, in the proposed scheme, if the channels are real-valued fading channels, the received signals at the destination node have the Alamouti code structure on each sub-carrier and thus it has the fast symbol-wise ML decoding and when frequency offsets are not large, the new scheme can achieve diversity order 2.

C. OFDM based Distributed Space-Time Coding for Asynchronous Relay Networks

Motivated by the simplicity of the scheme proposed in [46], the authors in [40] have proposed more general transmission schemes that can achieve full cooperative diversity for any number of relays. The conditions on the distributed space-time block code (DSTBC) structure that admit its application in the proposed transmission scheme are identified and it is pointed out that the recently proposed full diversity four group decodable DSTBCs [36] from precoded co-ordinate interleaved orthogonal designs and extended Clifford algebras satisfy these conditions. It is then shown how differential encoding at the source can be combined with the proposed transmission scheme to arrive at a new transmission scheme that can achieve full cooperative diversity in asynchronous wireless relay networks with no channel information and also no timing error knowledge at the destination node. Finally, four group decodable distributed differential space-time block codes [37] applicable in this new transmission scheme for power of two number of relays are also provided.

VI. DSTBCs for Synchronous Networks with DF Protocol

A. Signal model for decode and forward protocol

The wireless network considered in this section is similar to the one considered in Section III. With reference to the signal model in Section III, we only point out the changes in the (i) processing and (ii) the ingredients at the various nodes required for the DF protocol.

As against one of the assumptions listed in Section III, in this set-up, we assume that the relay nodes have the knowledge of all the fade coefficients h_k's. Note that if the above assumption is not made, then each relay has to use a non-coherent decoder (for decoding the information symbols) which is sub-optimal in terms of error performance compared to the coherent decoder. The broadcast phase remains same as that for the AF protocol, however, in the second phase, the k-th relay node decodes the information vector $\mathbf{x}$ using the received vector, $\mathbf{r}_k$. Using the decoded information symbols, all the relays distributively construct a chosen STBC by transmitting the predecided column of the codeword. For example, if the relay nodes decide to distributively transmit the following STBC

$$\mathbf{X}(\mathbf{x}) = [\mathbf{A}_1\mathbf{x} \;\; \mathbf{A}_2\mathbf{x} \;\; \cdots \;\; \mathbf{A}_K\mathbf{x}]$$

then, the k-th relay will transmit the vector $\mathbf{A}_k\mathbf{x}$. The vector received at the destination is given by

$$\mathbf{y} = \sqrt{P_2}\mathbf{X}(\mathbf{x})\,\mathbf{g} + \mathbf{w}$$

where $\mathbf{w} \sim \mathcal{CSCG}(\mathbf{0}_{T\times 1}, \mathbf{I}_T)$ is the additive noise at the destination. The channel $\mathbf{g}$ is given by $[g_1\ g_2\ \cdots\ g_K] \in \mathbb{C}^{K\times 1}$. The above channel equation looks similar to that of a MIMO channel with single receive antenna. As a result, all the existing STBCs for a MISO channel are applicable in this set-up.

Note that DSTBCs based on the DF protocol provide a diversity order of K only if all the K relays decode the information symbols of the source without error for every codeword use. However, in practice, all the K relays need not decode the information symbols without error and hence, for a relay network with K relays using the DF protocol, a diversity order of K is not guaranteed. To mitigate such scenarios, use of relay selection strategies or cyclic redundancy check (CRC) codes or fountain codes [16] become essential which in turn increases the overhead on the system and/or requires more resources such as power and bandwidth. With such schemes, nodes which decode the information without error participates in relaying. Considering the above factors, the diversity order obtained with above selection based DF protocol is a random number upper-bounded by K. Also, note that the computational complexity at the relays is assumed to be high for the purpose of (i) channel estimation and (ii) decoding the information symbols.

Now, we present the DSTBC construction proposed in [44] for the DF protocol. The DSTBCs in [44] are designed for networks which have a large set of single-antenna relay nodes N. However, because of the DF protocol, at any given time only a small, apriori unknown subset of nodes can be active (as exemplified in Fig. 6). In their proposed scheme, each relay is assigned a unique node signature vector of length N_c. The signal transmitted by an active relay node is the product of a predecided STBC and the signature vector. Unlike the earlier explained processing at the relays, the relays do not transmit a column of a STBC. It is shown that existing STBCs designed for $N_c > 2$ co-located antennas are favorable choices for the code matrix, guaranteeing a diversity order of $d = \min(N_S, N_c)$ if N_S nodes are active. For the case, $N_S > N_c$, the performance loss entailed by the distributed implementation is analytically characterized. Furthermore, the authors provide efficient methods for the optimization of the set of signature vectors. Depending on the chosen design, the proposed DSTBCs allow for low-complexity coherent, differential, and non-coherent detection, respectively.

VII. DSTBCs for Asynchronous Networks with DF Protocol

In this section, we discuss the design of DSTBCs for asynchronous relay networks when all the relays employ the

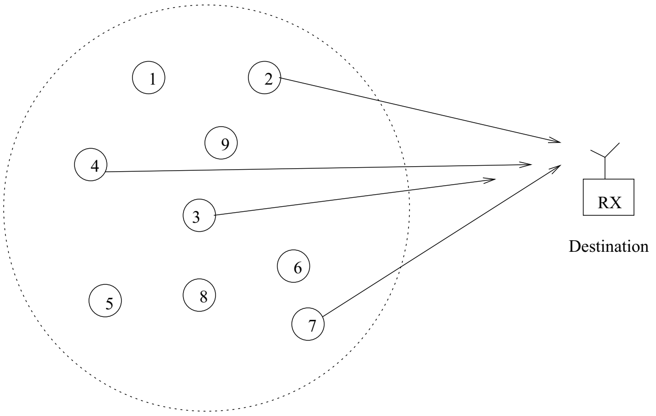

Fig. 6. Example for the communication network considered in [44] with a total of 9 relay nodes. Only the nodes $\{4, 2, 3, 7\}$ are active and transmit to the destination.

DF protocol. To focus on the problems involved in the code design for asynchronous networks, we assume that all the K relay nodes that participate in the cooperation have decoded the information symbols without error.

Let $\mathcal{C}$ be a $T \times K$ STBC (designed for synchronous relay networks) to be distributively constructed by all the relays. In asynchronous networks, the columns of a codeword may reach the destination with different delays. Let all the timing delays in the channel be captured by the delay profile $\Delta = (\delta_1, \delta_2, \cdots, \delta_K)$ where δ_k denotes the relative delay of the signal received from the k-th relay as referenced to the earliest received relay signal. Let δ_{max} denote the maximum component of Δ. When a codeword $\mathbf{X} \in \mathcal{C}$ is distributively transmitted from all the relays, the received vector (excluding the scaling factors) at the destination is given by $\mathbf{y} = \bar{\mathbf{X}}\mathbf{g} + \mathbf{n}$, where $\mathbf{y} \in \mathbb{C}^{(T+\delta_{max})}$ and, $\bar{\mathbf{X}} \in \mathbb{C}^{(T+\delta_{max})\times K}$ is given by

$$\bar{\mathbf{X}} = \left[\begin{array}{cccc} \mathbf{0}_{\delta_1} & \mathbf{0}_{\delta_2} & & \mathbf{0}_{\delta_K} \\ \mathbf{c}_1 & \mathbf{c}_2 & \cdots & \mathbf{c}_K \\ \mathbf{0}_{\delta_{max}-\delta_1} & \mathbf{0}_{\delta_{max}-\delta_2} & & \mathbf{0}_{\delta_{max}-\delta_K} \end{array} \right]$$

such that $\mathbf{c}_k$ denotes the k-th column of $\mathbf{X}$ and $\mathbf{0}_{\delta_k}$ denotes a zero vector of length δ_k. Therefore, $\bar{\mathcal{C}}$ is the equivalent STBC seen by the destination which contains matrices of the form $\bar{\mathbf{X}}$. Hence, for full diversity, all the pairwise difference matrices between the codewords of $\bar{\mathcal{C}}$ must have full rank. An STBC $\mathcal{C}$ is said to have a delay tolerance of τ for an asynchronous relay network if the equivalent STBC $\bar{\mathcal{C}}$ is fully diverse for arbitrary delays of duration up to τ symbols.

Through an example, we show that STBCs that are fully diverse for a synchronous relay network need not be fully diverse when employed in an asynchronous relay network. We consider the example of Alamouti code

$$\mathbf{X} = \left[\begin{array}{cc} x & -y^* \\ y & x^* \end{array} \right],$$

where the complex variables x and y take values from a complex signal set. Note that the columns correspond to relay indices and the rows correspond to complex channel uses. The first relay transmits the symbols x and y in consecutive time slots. whereas the second relay transmits $-y^*$ and x^* in consecutive time slots. For the synchronous case, it is well known that the Alamouti code provides full diversity. However, in an asynchronous network, when the delay profile is $\Delta = (0, 1)$ (when the second column is delayed by one symbol compared to the first column), the difference matrix between two codewords is of the form

$$\Delta\bar{\mathbf{X}} = \left[\begin{array}{cc} x_1 & 0 \\ y_1 & -y_1^* \\ 0 & x_1^* \end{array} \right] - \left[\begin{array}{cc} x_2 & 0 \\ y_2 & -y_2^* \\ 0 & x_2^* \end{array} \right] = \left[\begin{array}{cc} \Delta x & 0 \\ \Delta y & -\Delta y^* \\ 0 & \Delta x* \end{array} \right],$$

where x_i and y_i correspond to the realizations of x and y respectively. When $x_1 = x_2$ and $y_1 \neq y_2$, the difference matrix will be

$$\Delta\mathbf{X} = \left[\begin{array}{cc} 0 & 0 \\ \Delta y & -\Delta y^* \\ 0 & 0 \end{array} \right]$$

which does not have rank two. Hence, the Alamouti code is not fully diverse in asynchronous relay networks. On the similar lines, it can be shown that the well known, high rate STBCs from cyclic division algebras (CDA) [66] are not fully diverse in asynchronous networks [56].

In the rest of this section, we present some well known constructions of full diversity DSTBCs for asynchronous relay networks based on the DF protocol.

In [49], Hammons and El Gamal developed binary rank criteria that allowed algebraic design of STBCs with full diversity for any number of antennas. From such binary rank criteria, the same authors developed the general stacking construction for full diversity STBCs, examples of which include STBCs derived from Galois rings/fields and space-time trellis codes (STTC) corresponding to rate $\frac{1}{K}$ convolutional codes.

Li and Xia [51] later investigated the use of binary trellis codes derived from the Hammons-El Gamal stacking constructions for asynchronous relay networks. The authors showed that when these codes are used in the multilevel Lu-Kumar construction [50] for PSK and QAM modulation, the resulting STBCs also achieve full diversity in asynchronous networks. Some diversity product properties of space-time trellis codes are also studied and simplified decoding methods are discussed. However, the memory sizes of the STTCs in [51] grow exponentially in terms of the number of relays which may cause a high decoding complexity when the number of relays is not small. To avoid such a problem, Shang and Xia [52] studied a systematic method of generating delay tolerant STTCs with smallest possible constraint length.

In [53], Hammons has shown that various generalizations of Lu-Kumar multilevel construction to more general AM-PSK constellations also preserve delay tolerance. In particular, the first delay tolerant DSTBC was proposed in [53]. The author shows that for small constellation sizes, the proposed codes can be decoded by brute-force and for larger constellations one can use lattice decoders. In the sequel, [54] discusses how lattice decoding techniques can be adapted to perform decoding of such STBCs for asynchronous diversity.

For point-point MIMO channels, there has been substantial work on the design STBCs that provide full diversity and admit near ML lattice based decoders with reasonable

$$\mathbf{X}_1 = \begin{bmatrix} 0 & x \\ x^* & 0 \\ x^* & 0 \end{bmatrix} + \mathbf{j} \begin{bmatrix} y & 0 \\ 0 & y^* \\ 0 & y^* \end{bmatrix}. \tag{22}$$

$$\mathbf{X}_2 = \frac{1}{\sqrt{2(1+r^2)}} \begin{bmatrix} x_2 - rx_3 & x_1 + \mathbf{j}rx_4 \\ \mathbf{j}rx_1 + x_4 & rx_2 + x_3 \\ \mathbf{j}rx_1 + x_4 & rx_2 + x_3 \end{bmatrix} \text{ where } r = \frac{-1+\sqrt{5}}{2}. \tag{23}$$

complexity. Specifically, CDA based STBCs and Threaded Algebraic Space-Time (TAST codes) [55] provide high rate (in symbol per channel use) and full diversity order. However, both TAST codes and CDA based STBCs are not delay tolerant. In particular, all STBCs with minimum delay (i.e. $K = T$) have been shown not to be delay tolerant since they contain diagonal matrices as one of their threads and diagonal matrices are not delay tolerant [56]. As a result, non-square DSTBCs (such that $T > K$) have to be constructed for asynchronous networks. Towards that direction the authors in [56] propose distributed TAST codes that are delay tolerant. Instead of presenting a detailed construction of distributed TAST codes, we present two DSTBCs as examples in (22) and (23) for a network with 2 relays. The presented codes in (22) and (23) are variants of the well known Alamouti code and the Golden code respectively. More details on this can be found in [56]. Similar to TAST codes, the new delay tolerant codes can also be decoded by lattice decoding algorithms such as sphere or sequential decoders. The readers can also refer to [57] for more details on delay tolerant DSTBCs for asynchronous networks.

VIII. Conclusion and Future Trends

A survey of distributed space-time coding was presented for two-hop wireless relay networks. In particular, DSTC schemes for asynchronous networks (both with AF and DF protocols) were also covered. We point out that DSTC schemes based on DF protocol are not favourable because of (i) high complexity needed at the relay nodes and (ii) moreover, a fixed diversity order is not guaranteed. On the other hand, DSTC schemes based on AF protocol are favourable since complexity is required at the relays and a fixed diversity order is guaranteed. To summarize, out of the four classes of DSTBCs studied, we believe that DSTC schemes with AF protocol for asynchronous networks are of at most practical importance. Some possible directions for future work in this area are as follows:

- DSTBCs with low ML decoding complexity have been well studied for synchronous networks with AF protocol (see Section IV). An interesting direction is to design low ML decoding complexity DSTBCs for asynchronous networks. In particular, it is interesting to design SSD DSTBCs for asynchronous, AF based protocols.
- In Subsection IV-B, a training based DSTC technique is presented to construct the variants of the well known class of CODs in two-hop relay networks using the amplify and forward protocol. The inclusion of training symbols into the structure of the code has been shown to provide high rate along with the SSD property for the constructed codes. This idea can be extended to construct all the multi-group decodable codes [67] existing for point-to-point co-located MIMO channels in two-hop wireless networks.
- We are not aware of constructions of non-coherent DSTBCs which are delay tolerant in DF protocol based two-hop wireless networks. This is also an interesting direction for further research.

Acknowledgment

This work was supported partly through grants to B. Sundar Rajan by the DRDO-IISc program on Advanced Research in Mathematical Engineering.

References

[1] V. Tarokh, N. Seshadri, A.R. Calderbank, ”Space-Time Codes for High Data Rate Wireless communication: Performance Criterion and Code Construction”, *IEEE Trans. Information theory*, vol. 44, no. 02, pp. 744-765, 1998, DOI:10.1109/18.661517.

[2] G. J. Foschini, ”Layered space-time architecture for wireless communication in a fading environment when using multi element antennas”, *Bell labs technical journal*, vol. 01, no. 2, pp. 41-59, 1996, DOI:10.1002/bltj.2015.

[3] I.E. Teletar, ”Capacity of multi-antenna Gaussian channels” in *European transactions on Tele-communications*. vol. 10, no. 06, pp. 585-595, 1999, DOI:10.1002/ett.4460100604.

[4] A. Sendonaris, E. Erkip and B. Aazhang, ”User cooperation diversity-Part 1: Systems description,” *IEEE Trans. communications*, vol. 51, pp, 1927-1938, Nov 2003, DOI:10.1109/TCOMM.2003.818096.

[5] A. Sendonaris, E. Erkip and B. Aazhang, ”User cooperation diversity-Part 2: implementation aspects and performance analysis,” *IEEE Trans. communications*, vol. 51, pp. 1939-1948, Nov 2003, DOI:10.1109/TCOMM.2003.819238.

[6] J. M. Laneman and G. W. Wornell, ”Distributed space-time coded protocols for exploiting cooperative diversity in wireless network” *IEEE Trans. Information theory*, vol. 49, no. 10, pp. 2415-2425, Oct. 2003, DOI:10.1109/TIT.2003.817829.

[7] J. L. Laneman, David N. C. Tse, and G. W. Wornell, ”Cooperative diversity in wireless networks: efficient protocols and outage behavior”, *IEEE Trans. Information theory*, vol. 50, no. 12, 3062-3080, Dec. 2004, DOI:10.1109/TIT.2004.838089.

[8] R. U. Nabar, H. Bolcskei and F. W. Kneubuhler, ”Fading relay channels: performance limits and space-time signal design,” *IEEE Journal on Selected Areas in Communication*., vol. 22, no. 6, pp. 1099-1109, Aug. 2004, DOI:10.1109/JSAC.2004.830922.

[9] Yindi Jing and Babak Hassibi, ”Distributed space-time coding in wireless relay networks” *IEEE Trans. Wireless communication*, vol. 5, no 12, pp. 3524-3536, Dec. 2006, DOI:10.1109/TWC.2006.256975.

[10] A. Bletsas, A. Khisti, D. Reed and A Lippman, ”A simple cooperative diversity method based on network path selection”. *IEEE Journal on Selected Areas in Communications*, vol. 24, no 3, pp. 659-672, 2006, DOI:10.1109/JSAC.2005.862417.

[11] Chu, J.P.K.; Adve, R.S.; Eckford, A.W., ”Relay Selection for Low-Complexity Coded Cooperation,” in the proceedings of *IEEE Global Telecommunications Conference, 2007 (GLOBECOM ’07)* , pp.3932-3936, 26-30 Nov. 2007, DOI:0.1109/GLOCOM.2007.747.

[12] D. Chen, Laneman J. Nicholas, ”Non-coherent demodulation for co-operative diversity in wireless systems”. in the proceedings of *IEEE GLOBECOM 2004*, pp. 31-35.

[13] D. Chen, Laneman J. Nicholas, "Modulation and Demodulation for Cooperative diversity in wireless systems", *IEEE Transactions on wireless communication*, vol. 5, no. 07, pp. 1785-1794, 2006, DOI:10.1109/TWC.2006.1673090.

[14] L. Zheng and D. Tse, "Diversity and Multiplexing: A fundamental tradeoff in multiple-antenna channels", *IEEE Trans. Information theory*, vol. 49, no. 05, pp. 1073-1096. 2003, DOI:10.1109/TIT.2003.810646.

[15] Petros Elia, K. Vinodh , M. Anand, and P. Vijay Kumar, D-MG Tradeoff and Optimal Codes for a Class of AF and DF Cooperative Communication Protocols, *IEEE Trans. Information theory*, vol. 55, no. 7, July 2009, DOI:10.1109/TIT.2009.2021314.

[16] M. Luby, "LT codes", in proceedings of *43rd Annual IEEE Symposium on Foundations of Computer Science (FOCS)*, pp. 271-282, 2002, DOI:10.1109/SFCS.2002.1181950.

[17] Y. Jing and B. Hassibi, "Diversity analysis of distributed space-time codes in relay networks with multiple transmit/receive antennas," *EURASIP Jour. on Advances in Signal Processing*, vol. 2008, Article ID 254573, 17 pages, 2008, DOI:10.1155/2008/254573.

[18] V. Tarokh, H. Jafarkhani and A. R. Calderbank, "Space-time block codes from orthogonal designs", *IEEE Trans. Information theory*, vol. 45, no.05, pp.1456-1467, July 1999, DOI:10.1109/18.771146.

[19] O.Tirkkonen and A. Hottinen, "Square matrix embeddable space-time block codes from complex signal constellations", *IEEE Trans. Information theory*, vol. 48, pp.384-395, Feb 2002, DOI:10.1109/18.978740.

[20] Xue-Bin Liang, "Orthogonal Designs with Maximal rates," *IEEE Trans. Information theory*, vol. 49, No.10, pp.2468 - 2503, Oct. 2003, DOI:10.1109/TIT.2003.817426.

[21] H. Wang, D.Wang, X-G.Xia, "On Optimal QOSTBC with minimal decoding complexity," submitted to *IEEE Trans. Information theory*, vol.55, no. 03, pp.1104-1130, March 2009, DOI:10.1109/TIT.2008.2011521.

[22] Xue-Bin Liang, X.-G. Xia, "On the non existence of rate one generalised complex orthogonal designs," *IEEE Trans. Information theory*, vol. 49, no. 11, pp.2984-2989, Nov. 2003, DOI:10.1109/TIT.2003.818396.

[23] Zafar Ali Khan, Md., and B. Sundar Rajan, "Single Symbol Maximum Likelihood Decodable Linear STBCs", *IEEE Trans. Information theory*, vol. 52, No. 5, pp.2062-2091, May 2006, DOI:10.1109/TIT.2006.872970.

[24] Sanjay Karmakar and B. Sundar Rajan, "Minimum-decoding-complexity maximum-rate space-time block codes from Clifford algebras," in the proceedings of *IEEE ISIT*, Seattle, USA, July 09-14, 2006, pp.788-792, DOI:10.1109/ISIT.2006.261721.

[25] B. Hassibi and B.M. Hochwald, "High-rate codes that are linear in space and time", *IEEE Trans. Information theory*, vol.48, no.7, Jul. 2002, pages 1804-1824, DOI:10.1109/TIT.2002.1013127.

[26] Zhihang Yi and Il-Min Kim "Single symbol ML decodable distributed STBCs for cooperative networks", *IEEE Trans. Information theory*, vol.53, no.8, 2007, pp. 2977-2985, DOI:10.1109/TIT.2007.901177.

[27] Zhihang Yi and Il-Min Kim, "The impact of Noise Correlation and Channel Phase Information on the Data-Rate of the Single-Symbol ML Decodable Distributed STBCs," Unpublished. Submitted to *IEEE Trans. Information theory*, Aug 2007. Available online at http://arxiv.org/abs/0708.3387.

[28] Zhihang Yi and Il-Min Kim, "Distributed space-time coding for amplify and forward cooperative networks," *Cooperative Communications for Improved Wireless Network Transmission: Frameworks for Virtual Antenna Array Applications*, (Chapter 8), pp. 238-259, 2010, IGI Global .

[29] J. Harshan and B. Sundar Rajan, "Co-ordinate Interleaved Distributed Space-Time Coding for Two-Antenna-Relays Networks," *IEEE Transactions on Wireless Communications*, vol. 08, no. 04, pp. 1783-1791, April 2009, DOI:10.1109/TWC.2009.071303.

[30] J. Harshan and B. Sundar Rajan, "High-Rate, Single-Symbol ML Decodable Precoded DSTBCs for Cooperative Networks," *IEEE Trans. Information theory*, vol. 55, no. 05, May. 2009, pp. 2004-2015, DOI:10.1109/TIT.2009.2016013.

[31] J. Harshan, B. Sundar Rajan, and Are Hjørunjnes "High-Rate, Distributed Training-Embedded Complex Orthogonal Designs for Relay Networks,". Unpublished. Available online at http://arxiv.org/abs/0908.0051.

[32] J. Harshan, B. Sundar Rajan, and Are Hjørunjnes "Training-Embedded, Single-Symbol ML-Decodable, Distributed STBCs for Relay Networks,". In Press. To appear in *IEEE International Conference on Communications (ICC) 2010* to be held at Cape Town, South Africa. Available online at http://arxiv.org/abs/0909.1525.

[33] Pavan Srinath and B. Sundar Rajan, "Single Real-Symbol Decodable, High-Rate, Distributed Space-Time Block Codes,". In Press. To appear in *IEEE Information theory workshop (ITW) 2010* to be held at Cairo, Egypt.

[34] J. Harshan and B. Sundar Rajan, "A Non-differential Distributed Space-Time Coding for Partially-Coherent Cooperative Communication," *IEEE Transactions on Wireless Communications*, vol. 07, No. 11, Nov. 2008, pp.4076-4081, DOI:10.1109/T-WC.2008.070526.

[35] J. Harshan, G. Susinder Rajan and B. Sundar Rajan, "Space-Time Coding for Non-coherent Cooperative Communications," *Cooperative Communications for Improved Wireless Network Transmission: Frameworks for Virtual Antenna Array Applications*, (Chapter 13), pp. 352-380, 2010, IGI Global .

[36] G. Susinder Rajan and B. Sundar Rajan, "Leveraging Coherent Distributed Space-Time Codes for Noncoherent Communication in Relay Networks via Training," *IEEE Transactions on Wireless Communications*, vol. 08, no. 02, Feb. 2009, pp. 683-688, DOI:10.1109/TWC.2009.080545.

[37] G. Susinder Rajan and B. Sundar Rajan, "Algebraic Distributed Differential Space-Time Codes with Low Decoding Complexity," *IEEE Transactions on Wireless Communications*, vol. 07, no. 10, October 2008, pp.3962-3971, DOI:10.1109/T-WC.2008.070530.

[38] G. Susinder Rajan and B. Sundar Rajan, "Multi-group ML Decodable colocated and Distributed Space-Time Block Codes,". In Press. Accepted for publication in *IEEE Trans. Information theory*. Also available online at http://arxiv.org/abs/0712.2384.

[39] G. Susinder Rajan and B. Sundar Rajan, "Distributed Space-Time Codes with Low ML Decoding Complexity,". In Press. Selected Topics in Information and Coding Theory, World Scientific Publishing Company.

[40] G. Susinder Rajan and B. Sundar Rajan, "OFDM based Distributed Space-Time Coding for Asynchronous Relay Networks" *IEEE International Conference on Communications*, Beijing, China, May 19-23, 2008, DOI:10.1109/ICC.2008.218.

[41] G. Susinder Rajan and B. Sundar Rajan, "Low Complexity Distributed STBCs with Unitary Relay Matrices for any Number of Relays," Proceedings of *IEEE International Symposium on Information Theory, (ISIT 2009)*, Seoul, South Korea, June 28- July 03, 2009, 80-84, DOI:10.1109/ISIT.2009.5205649.

[42] D. Sreedhar, A. Chockalingam, and B. Sundar Rajan, "Single-Symbol ML Decodable Distributed STBCs for Partially-Coherent Cooperative Networks" *IEEE Transactions on Wireless Communications*, vol. 08, no. 5, pp. 2672-2681, May 2009, DOI:10.1109/TWC.2009.080796.

[43] Kiran T. and B. Sundar Rajan, "Partially-coherent distributed space-time codes with differential encoder and decoder," *IEEE Journal on Selected Areas in Communications: Special issue on Cooperative Communications and Networking*, vol. 25, no.2, Feb. 2007, pp.426-433, DOI:10.1109/JSAC.2007.070217.

[44] Simon Yiu, Robert Schober and Lutz Lampe, "Distributed space-time block coding," *IEEE transactions on communications*, vol. 54, no. 07, pp. 1195-1206, July 2006, DOI:10.1109/TCOMM.2006.877947.

[45] Simon Yiu, Robert Schober and Lutz Lampe, " Distributed space-time block coding for cooperative networks with multiple-antenna nodes," *IEEE International Workshop on Computational Advances in Multi-Sensor Adaptive Processing*, pp. 52-55, 13-15 Dec. 2005, DOI:10.1109/CAMAP.2005.1574181 .

[46] Zheng Li and Xiang-Gen Xia, "A simple Alamouti space-time transmission scheme for asynchronous cooperative system," *IEEE signal processing letters*, vol. 14, no. 11, pp. 804-807, November 2007, DOI:10.1109/LSP.2007.900224.

[47] Xiaoyong Guo; Xiang-Gen Xia,"A distributed space-time coding in asynchronous wireless relay networks" *IEEE Transactions on Wireless Communications*, vol. 07, no. 05, Part 2, pp. 1812 - 1816, May 2008, DOI:10.1109/TWC.2008.070042.

[48] Zheng Li and Xiang-Gen Xia, "An Alamouti coded OFDM transmission for cooperative systems robust to both timing errors and frequency offsets" *IEEE transactions on wireless communications*, vol. 07, no. 05, pp. 1839-1844, May 2008, DOI:10.1109/TWC.2008.070443.

[49] A. R. Hammons Jr. and H. El. Gamal, "On the theory of space-time codes for PSK modulation", *IEEE Trans. Information theory*, vol. 46, no. 02, pp. 524-542, March 2000, DOI:10.1109/18.825816.

[50] H.F. Lu. and P.V. Kumar, "A unified construction of space-time codes with optimal rate-diversity tradeoff", *IEEE Trans. Information theory*, vol. 51, no. 05, pp. 1709-1730, May 2005, DOI:10.1109/TIT.2005.846403.

[51] Yabo Li and Xiang-Gen Xia, "A family of distributed space-time trellis codes with asynchronous cooperative diversity," *IEEE transactions on communications*, vol. 55, no. 04, pp. 790-800, April 2007, DOI:10.1109/TCOMM.2007.894117.

[52] Y. Shang and Xiang-Gen Xia, "Shift full rank matrices and applications in space-time trellis codes for relay networks with asynchronous cooperative diversity," *IEEE Trans. Information theory*, vol. 52, no. 07, pp. 3153-3167, July 2006, DOI:10.1109/TIT.2006.876222.

[53] Hammons, A.R., "Algebraic space-time codes for quasi-synchronous cooperative diversity," *IEEE International conference on wireless net-*

works, Communications, and Mobile Computing, Maui, Hawaii, June 13-16, 2005, DOI:10.1109/WIRLES.2005.1549377.

[54] Hammons, A.R., and Damen, M.O., "On lattice decoding of space-time block codes for asynchronous cooperative relaying," *IEEE Sarnoff Symposium, 2007* , pp.1-5, April-30, 2007-May 2 2007, Princeton, NJ, USA, DOI:10.1109/SARNOF.2007.4567358.

[55] H. El. Gamal and M.O. Damen, "Universal space-time coding", *IEEE Trans. Information theory*, vol. 49, no. 05, pp. 1097-1110, May 2003, DOI:10.1109/TIT.2003.810644.

[56] M. O. Damen and A. R. Hammons Jr., "Delay tolerant distributed TAST Codes for cooperative diversity." *IEEE Trans. Information theory*, vol. 53, no. 10, pp. 3755-3773, October 2007, DOI:10.1109/TIT.2007.904773.

[57] M. O. Damen and A. R. Hammons Jr., "On delay tolerant distributed space-time block codes," *IEEE Military Communication Conference 2007*, pp. 1-6, 29th-31st October 2007, 10.1109/MILCOM.2007.4454944.

[58] K. J. Horadam, "A Generalized Hadamard Transform", *IEEE International Symposium on Information Theory 2005* held at Adelaide, pp. 1006-1008, DOI:10.1109/ISIT.2005.1523490.

[59] Jing, Y. and Jafarkhani, H. "Distributed Differential Space-Time Coding for Wireless Relay Networks" *IEEE Trans. Communications*, vol. 56, no. 07, pp. 1092-1100, DOI:0.1109/TCOMM.2008.060388.

[60] F. Oggier and Emmanuel Lequeu. "Differential distributed space-time coding based on Cayley codes". in the proceedings of *IEEE International Symposium on Information Theory (ISIT) 2008*, Toronto, Canada, July 06-11, pp. 2548-2552, DOI:10.1109/ISIT.2008.4595451.

[61] Frederique Oggier, Babak Hassibi, "A Coding Scheme for Wireless Networks with Multiple Antenna Nodes and no Channel Information", *IEEE ICASSP 07*, Hawaii, DOI:10.1109/ICASSP.2007.366560.

[62] F. Oggier and Babak Hassibi, "An Algebraic Coding scheme for Wireless Relay Networks with Multiple-Antenna Nodes" *IEEE Transactions on Signal processing*, vol 56, no. 07, July 2008, DOI:10.1109/TSP.2008.917410.

[63] Frdrique E. Oggier and, Babak Hassibi. "A Coding Strategy for Wireless Networks with no Channel Information". Presented at the Forty-Fourth Annual Allerton Conference on Communication, Control and Computing, 2006.

[64] Kiran T. and B. Sundar Rajan, "Distributed space-time codes with reduced decoding complexity," Proceedings of *IEEE International Symposium on Information Theory (ISIT 2006)*, Seattle, USA, July 09-14, 2006, pp.542-546, DOI:10.1109/ISIT.2006.261775.

[65] J. C. Belfiore and A. M. Cipriano, Space-time coding for non-coherent channels, chapter 10 of the book, *Space-time wireless systems: From array processing to MIMO communication*, Editors: D. Gesbert, C.B. Papadias and A. J. Vander Veen. Publisher: Cambridge university press.

[66] B. A. Sethuraman, B. Sundar Rajan and V. Shashidhar, "Full-diversity, High-rate Space-Time Block Codes from Division Algebras," *IEEE Trans. Information theory, Special Issue on Space-Time Transmission, Reception, Coding and Signal Design*, vol.49, no.10, October, 2003, pp.2596-2616, DOI:10.1109/TIT.2003.817831.

[67] Sanjay Karmakar and B. Sundar Rajan, "Multigroup-Decodable STBCs from Clifford Algebras," *IEEE Trans. Information theory*, vol. 55, no. 01, pp. 223-231, Jan. 2009, DOI:10.1109/TIT.2008.2008138.

J. Harshan was born in Karnataka, India. He received the Bachelor of Engineering degree in Electronics and Communications from the National Institute of Engineering, Mysore, India in 2004.

From October 2004 to December 2005, he was with Robert Bosch (India) limited, Bangalore, India. Currently, he is a working towards his Ph.D. in the Department of Electrical Communication Engineering, Indian Institute of Science, Bangalore, India. His primary research interests include wireless communications, space-time coding, coding for wireless relay networks and coding for multiple access channels.

B. Sundar Rajan (S'84-M'91-SM'98) was born in Tamil Nadu, India. He received the B.Sc. degree in mathematics from Madras University, Madras, India, the B.Tech degree in electronics from Madras Institute of Technology, Madras, and the M.Tech and Ph.D. degrees in electrical engineering from the Indian Institute of Technology, Kanpur, India, in 1979, 1982, 1984, and 1989 respectively.

He was a faculty member with the Department of Electrical Engineering at the Indian Institute of Technology in Delhi, India, from 1990 to 1997. Since 1998, he has been a Professor in the Department of Electrical Communication Engineering at the Indian Institute of Science, Bangalore, India. His primary research interests include space-time coding for MIMO channels, distributed space-time coding and cooperative communication, coding for multiple-access, relay channels and network coding with emphasis on algebraic techniques.

He is an Associate Editor of the IEEE Transactions on Information Theory, an Editor of the IEEE Transactions on Wireless Communications, and an Editorial Board Member of International Journal of Information and Coding Theory. He served as Technical Program Co-Chair of the IEEE Information Theory Workshop (ITW'02), held in Bangalore, in 2002. He is a Fellow of Indian National Academy of Engineering, a Fellow of the National Academy of Sciences, India, recipient of Prof. Rustum Choksi award by I.I.Sc for excellence in research in Engineering for the year 2009, and recipient of the IETE Pune Center's S.V.C Aiya Award for Telecom Education in 2004. Also, Dr. Rajan is a Member of the American Mathematical Society.