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Performance Analysis of Single-Symbol Maximum Likelihood Decodable Linear STBCs

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***Abstract*— Performance of space-time block codes can be improved using the coordinate interleaving of the input symbols from rotated M-ary phase shift keying (MPSK) and M-ary quadrature amplitude modulation (MQAM) constellations. This paper is on the performance analysis of coordinate-interleaved space-time codes, which are a subset of single-symbol maximum likelihood decodable linear space-time block codes, for wireless multiple antenna terminals. The analytical and simulation results show that full diversity is achievable. Using the equivalent single-input single-output model, simple expressions for the average bit error rates are derived over flat uncorrelated Rayleigh fading channels. Optimum rotation angles are found by finding the minimum of the average bit error rate curves.**

I. Introduction

One of the most powerful techniques to mitigate the performance degradation on fading channels is the use of diversity. Any diversity technique (e.g., space, time, or frequency) tries to provide statistically independent copies of the transmitted sequence at the receiver for reliable detection. Signal space diversity can provide performance improvement over fading channels without using extra bandwidth and power expansion [1–4]. The basic premise of signal space diversity is that multidimensional signal constellations are used and the components of the each signal constellation point are transmitted over independent fading channels. The independence of the fading channels can easily be accomplished by interleaving. In [2], the union bound of the average probability of symbol error for a system employing signal space diversity is calculated for uncorrelated Rayleigh fading channels. The rotation angles are calculated at high signal-to-noise ratio (SNR) to maximize the minimum product distance of the rotated constellations. In [3], the average probability of bit error is approximated by considering only the nearest neighbors. The rotation angles are also chosen based on this approximation. In [5], the closed form analytical expressions of the union bound of bit-error rate (BER) for coordinate interleaved symbols with MPSK signal constellations is presented.

Space-time block coding (STBCs) is a modulation scheme introduced to combat detrimental effects in wireless fading channels. A simple transmit antennas was proposed by Alamouti in [6] and generalized to an arbitrary number of transmit antennas by Tarokh et al. in [7]. STBCs from orthogonal designs (ODs) and coordinate interleaved orthogonal designs (CIOD) have been attracting wider attention due to their amenability for fast (single-symbol) maximum-likelihood (ML) decoding, and full-rate with full-rank over quasi-static fading channels [4].

In this paper, we present analytical expressions for the bit error rate (BER) of the CIOD space-time block codes over frequency-flat Rayleigh fading channels using the equivalent SISO model. The BERs of the CIOD space-time block codes with rotated MPSK and MQAM modulations are expressed in terms of the moment generating function (MGF) of instantaneous SNR for the equivalent SISO model. The analytical and simulation results show that full diversity is achievable. Moreover, the optimum rotation angles are found by minimizing the average bit error rate curves.

II. System Model

Consider a wireless communication scenario with N_s transmit antennas and N_r receive antennas. The channel gains from the ith transmit antenna to the jth receive antenna is denoted by h_{ij}, $i = 1, \ldots, N_s$, $j = 1, \ldots, N_r$. Under the assumption that each link undergoes independent Rayleigh process, h_{ij} are independent complex Gaussian random variables with zero-mean and variances σ_{ij}^2. Since multiple antennas at the source and destination are co-located, and the co-located antennas have the same distances, we skip the i and j such that $\sigma_{ij}^2 = \sigma_\gamma^2$.

Assume that the source wants to send K symbols s_1, s_2, $\ldots$, s_K to the destination during T time slots. T should be less than the coherent interval, that is, the time duration among which channels h_{ij} keep constant. Since T symbol durations are necessary to transmit K symbols, the rate R of the STBC is $R = K/T$. The source should transmit a $T \times N_s$ dimensional space-time code matrix $\boldsymbol{S}$, which consists of linear combinations of T information symbols $s_1, s_2, \ldots, s_T$. Assuming the following normalization:

$$\mathbb{E}\left[\operatorname{tr}\{\boldsymbol{S}^H \boldsymbol{S}\}\right] = \mathbb{E}\left[\operatorname{tr}\left\{\sum_{k=1}^{K} |s_k|^2 \boldsymbol{I}_{N_s}\right\}\right] = N_s, \qquad (1)$$

and the fact that h_{ij} does not vary during T successive intervals, the $T \times N_d$ received signal at the destination can

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be written as

$$\boldsymbol{Y}=\sqrt{\frac{P_0T}{N_s}}\boldsymbol{S}\boldsymbol{H}+\boldsymbol{W}, \tag{2}$$

where $\boldsymbol{H}$ is the $N_s\times N_d$ channel matrix and $\boldsymbol{W}$ is the $T\times N_d$ matrix of the complex zero-mean white Gaussian noise with component-wise variance $\mathcal{N}_0$. Therefore, the source transmits $\sqrt{P_1T/N_s}\boldsymbol{S}$ where P_1T is the average total power used at the source.

Here, we consider an interesting class of full-rank SDD called generalized coordinate interleaved orthogonal designs (GCIOD), which are better than generalized linear complex orthogonal designs (GLCOD) in terms of rate, coding gain, maximum mutual information, and BER [4]. In [4], a GCIOD of size $T\times N_s$ in variables s_i, $i=1,\ldots,K$, (where K is even) is defined as

$$\boldsymbol{S}=\left[\begin{array}{cc}\boldsymbol{\Theta}_1\left(\tilde{s}_1,\ldots,\tilde{s}_{K/2}\right) & \boldsymbol{0}\\ \boldsymbol{0} & \boldsymbol{\Theta}_2\left(\tilde{s}_{K/2+1},\ldots,\tilde{s}_K\right)\end{array}\right], \tag{3}$$

where $\boldsymbol{\Theta}_1\left(s_1,\ldots,s_{K/2}\right)$ and $\boldsymbol{\Theta}_2\left(s_1,\ldots,s_{K/2}\right)$ are GLCODs of size $T_1\times N_1$ and $T_2\times N_2$, respectively [8], [9], where $N_1+N_2=N_s$, $T_1+T_2=T$, $\tilde{s}_i=\mathrm{Re}\{s_i\}+\jmath\,\mathrm{Im}\{s_{(i+K/2)_K}\}$, and $(a)_K$ denotes a mod K. If $\boldsymbol{\Theta}_1=\boldsymbol{\Theta}_2$ then it is called a coordinate interleaved orthogonal design (CIOD).

For example, CIOD of size 4×4 is given by

$$\boldsymbol{S}=\left[\begin{array}{cccc} s_{1I}+j\,s_{3Q} & s_{2I}+j\,s_{4Q} & 0 & 0\\ -s_{2I}+j\,s_{4Q} & s_{1I}-j\,s_{3Q} & 0 & 0\\ 0 & 0 & s_{3I}+js_{1Q} & s_{4I}+j\,s_{2Q}\\ 0 & 0 & -s_{4I}+j\,s_{2Q} & s_{3I}-j\,s_{1Q}\end{array}\right]. \tag{4}$$

III. Equivalent SISO Model

Let us define $\boldsymbol{Y}_1$ of size $\frac{T}{2}\times N_d$, $\boldsymbol{H}_1$ of size $\frac{N_s}{2}\times N_d$, and $\boldsymbol{W}_1$ of size $\frac{T}{2}\times N_d$, as the first half rows of $\boldsymbol{Y}$, $\boldsymbol{H}$, and $\boldsymbol{W}$, respectively. Also, $\boldsymbol{Y}_2$ of size $\frac{T}{2}\times N_d$, $\boldsymbol{H}_2$ of size $\frac{N_s}{2}\times N_d$, and $\boldsymbol{W}_2$ of size $\frac{T}{2}\times N_d$, are defined as the second half rows of $\boldsymbol{Y}$, $\boldsymbol{H}$, and $\boldsymbol{W}$, respectively. Then, from (4), we can decompose the received signal as

$$\boldsymbol{Y}_1=\sqrt{\tfrac{P_0T}{N_s}}\boldsymbol{H}_1\boldsymbol{\Theta}_1+\boldsymbol{W}_1, \tag{5a}$$

$$\boldsymbol{Y}_2=\sqrt{\tfrac{P_0T}{N_s}}\boldsymbol{H}_2\boldsymbol{\Theta}_2+\boldsymbol{W}_2. \tag{5b}$$

The ML decoding finds the codeword that solves the following minimization problem

$$\hat{\boldsymbol{S}}=\arg\min_{\boldsymbol{S}} f(\tilde{s}_1,\tilde{s}_2,\ldots,\tilde{s}_K), \tag{6}$$

where $f(\tilde{s}_1,\tilde{s}_2,\ldots,\tilde{s}_K)=\|\boldsymbol{Y}-\sqrt{\frac{P_0T}{N_s}}\boldsymbol{S}\boldsymbol{H}\|_F$, and $\|\boldsymbol{A}\|_F\triangleq\sqrt{\mathrm{tr}\left(\boldsymbol{A}^H\boldsymbol{A}\right)}$ is the Forbenius norm of matrix $\boldsymbol{A}$.

From (5) and (6), we have

$$\hat{\boldsymbol{S}}=\arg\min_{\boldsymbol{S}}\left\{f_1(\tilde{s}_1,\ldots,\tilde{s}_{K/2})+f_2(\tilde{s}_{K/2+1},\ldots,\tilde{s}_K)\right\}, \tag{7}$$

where $f_1(\tilde{s}_1,\tilde{s}_2,\ldots,\tilde{s}_{K/2})=\|\boldsymbol{Y}_1-\sqrt{\frac{P_0T}{N_s}}\boldsymbol{H}_1\boldsymbol{\Theta}_1\|_F$ and $f_2(\tilde{s}_{K/2+1},\tilde{s}_2,\ldots,\tilde{s}_K)=\|\boldsymbol{Y}_2-\sqrt{\frac{P_0T}{N_s}}\boldsymbol{H}_2\boldsymbol{\Theta}_2\|_F$. Due to the orthogonality of the columns of $\boldsymbol{\Theta}_1$ and $\boldsymbol{\Theta}_2$, the metric in (7) can decompose into K parts which are only a function of $\tilde{s}_k$, $k=1,\ldots,K$ and the optimum detection becomes equivalent to maximum ratio combining (MRC) detection (see, e.g., [10], [11], for the case of GLCOD). Similarly, the equivalent SISO model for the case of GCIOD can be shown as

$$\tilde{r}_k=\begin{cases}\|\boldsymbol{H}_1\|_F\alpha\,\tilde{s}_k+w_k, & \text{if } k\in\{1,\ldots,\frac{K}{2}\},\\ \|\boldsymbol{H}_2\|_F\alpha\,\tilde{s}_k+w_k, & \text{if } k\in\{\frac{K}{2}+1,\ldots,K\},\end{cases} \tag{8}$$

where $\alpha=\sqrt{\frac{P_0T}{N_s}}$ and w_k is an equivalent zero-mean Gaussian noise with variance $\mathcal{N}_0$. Consequently, the minimization of (7) is equivalent to minimizing each decision metric for $\tilde{s}_k$ separately and the ML receiver chooses the optimal $\tilde{s}_k$ as follows:

$$\hat{\boldsymbol{S}}=\arg\sum_{k=1}^{K/2}\min_{\tilde{s}_k} f_{1,k}(\tilde{s}_k)+\sum_{k=K/2+1}^{K}\min_{\tilde{s}_k} f_{2,k}(\tilde{s}_k), \tag{9}$$

where $f_1(\tilde{s}_1,\tilde{s}_2,\ldots,\tilde{s}_{K/2})=\sum_{k=1}^{K/2}f_{1,k}(\tilde{s}_k)$ and $f_2(\tilde{s}_{K/2+1},\tilde{s}_2,\ldots,\tilde{s}_K)=\sum_{k=K/2+1}^{K}f_{2,k}(\tilde{s}_k)$. Therefore, it can be seen from (9) that CIOD can be decoded by single-symbol decodable ML.

Rearranging the in-phase and quadrature-phase components of $\tilde{r}_k$'s in (8), which corresponds to deinterleaving, we have

$$\hat{r}_k=\tilde{r}_{k,I}+j\,\tilde{r}_{k+K/2,Q}=\|\boldsymbol{H}_1\|_F\alpha\,s_{k,I}+\jmath\,\|\boldsymbol{H}_2\|_F\alpha\,s_{k,Q}+v_k, \tag{10}$$

for $k=1,\ldots,\frac{K}{2}$, where it is assumed K is an even number. and

$$\hat{r}_k=\tilde{r}_{k,I}+j\,\tilde{r}_{k-K/2,Q}=\|\boldsymbol{H}_2\|_F\alpha\,s_{k,I}+\jmath\,\|\boldsymbol{H}_1\|_F\alpha\,s_{k,Q}+v_k, \tag{11}$$

for $k=\frac{K}{2}+1,\ldots,K$, where v_k are zero-mean complex Gaussian random variables with variance $\mathcal{N}_0$. Hence, the ML decision rule for selecting the transmitted symbols is given by

$$\hat{s}_k=\begin{cases}\arg\min\limits_{s}\{|\hat{r}_{k,I}-\|\boldsymbol{H}_1\|_F\alpha s_I|^2\\ \quad+|\hat{r}_{k,Q}-\|\boldsymbol{H}_2\|_F\alpha s_Q|^2\}, & \text{if } k\in\{1,\ldots,\frac{K}{2}\},\\ \arg\min\limits_{s}\{|\hat{r}_{k,I}-\|\boldsymbol{H}_2\|_F\alpha s_I|^2\\ \quad+|\hat{r}_{k,Q}-\|\boldsymbol{H}_1\|_F\alpha s_Q|^2\}, & \text{if } k\in\{\frac{K}{2}+1,\ldots,K\}.\end{cases} \tag{12}$$

Assuming the normalization $\mathbb{E}\Big[\mathrm{tr}\{\boldsymbol{S}^H\boldsymbol{S}\}\Big]=N_s$, and from (3), we have

$$\mathbb{E}\Big[\mathrm{tr}\{\boldsymbol{S}^H\boldsymbol{S}\}\Big]=\mathbb{E}\left[2\sum_{k=1}^{K}|s_k|^2\right]. \tag{13}$$

Thus, we have $\mathbb{E}|s_k|^2=\frac{N_s}{2K}$, and the energy per symbol becomes $E_0=\frac{P_0T}{2K}$.

For calculating the SER, we should first find the probability density function (PDF) of the received SNR after detection. The post-detection SNR of the symbols $\tilde{s}_1$ and $\tilde{s}_2$ can be calculated as

$$\chi_1 = \frac{E_0}{\mathcal{N}_0}\sum_{i=1}^{\frac{N_s}{2}}\sum_{j=1}^{N_d}|h_{ij}|^2 = \frac{P_0T}{2\mathcal{N}_0K}\sum_{i=1}^{\frac{N_s}{2}}\sum_{j=1}^{N_d}|h_{ij}|^2. \quad (14)$$

Similarly, the post-detection SNR of the symbols $\tilde{s}_3$ and $\tilde{s}_4$ is given by

$$\chi_2 = \frac{E_0}{\mathcal{N}_0}\sum_{i=\frac{N_s}{2}+1}^{N_s}\sum_{j=1}^{N_d}|h_{ij}|^2 = \frac{P_0T}{2\mathcal{N}_0K}\sum_{i=\frac{N_s}{2}+1}^{N_s}\sum_{j=1}^{N_d}|h_{ij}|^2. \quad (15)$$

IV. Performance Analysis

A. SER Expression for CIOD

Since in (12), the phase-quadrature components can be perfectly separated at the demodulator, the probability of error for QAM can be determined from the probability of error for PAM [12, Ch. 5.2]. Therefore, the conditional probability of a symbol error for the M-ary QAM is

$$\begin{aligned}&P_{\text{QAM}}\left(M|\boldsymbol{H}, s_i\right)\\&= 1-\left[1-P_{\text{PAM}}\left(\sqrt{M}|\boldsymbol{H}_1, s_{i,I}\right)\right]\left[1-P_{\text{PAM}}\left(\sqrt{M}|\boldsymbol{H}_2, s_{i,Q}\right)\right]\\&\approx P_{\text{PAM}}\left(\sqrt{M}|\boldsymbol{H}_1, s_{i,I}\right) + P_{\text{PAM}}\left(\sqrt{M}|\boldsymbol{H}_2, s_{i,Q}\right), \quad (16)\end{aligned}$$

and

$$\begin{aligned}&P_{\text{QAM}}\left(M|\boldsymbol{H}, s_{i+K/2}\right)\\&= 1-\left[1-P_{\text{PAM}}\left(\sqrt{M}|\boldsymbol{H}_2, s_{i+K/2,I}\right)\right]\\&\times\left[1-P_{\text{PAM}}\left(\sqrt{M}|\boldsymbol{H}_1, s_{i+K/2,Q}\right)\right]\\&\approx P_{\text{PAM}}\left(\sqrt{M}|\boldsymbol{H}_2, s_{i+K/2,I}\right) + P_{\text{PAM}}\left(\sqrt{M}|\boldsymbol{H}_1, s_{i+K/2,Q}\right), \quad (17)\end{aligned}$$

for $i = 1, \ldots, K/2$, where the conditional probability of PAM signals can be represented by [13, Eq. (8.3)]

$$\begin{aligned}P_{\text{PAM}}\left(\sqrt{M}|\boldsymbol{H}_1, s_{i,I}\right) &= P_{\text{PAM}}\left(\sqrt{M}|\boldsymbol{H}_1, s_{i+K/2,Q}\right)\\&= c\,Q\left(\sqrt{g\,\chi_1}\right), \quad (18)\end{aligned}$$

$$\begin{aligned}P_{\text{PAM}}\left(\sqrt{M}|\boldsymbol{H}_2, s_{i,Q}\right) &= P_{\text{PAM}}\left(\sqrt{M}|\boldsymbol{H}_2, s_{i+K/2,I}\right)\\&= c\,Q\left(\sqrt{g\,\chi_2}\right), \quad (19)\end{aligned}$$

where $Q(x) = 1/\sqrt{2\pi}\int_x^\infty e^{-u^2/2}\,du$, $c = 2\left(\frac{\sqrt{M}-1}{\sqrt{M}}\right)$, $g = \frac{3}{M-1}$, and χ_1 and χ_2 are found in (14) and (15). We assume the signals s_i are transmitted with equal probability, i.e., $P(s_i) = \frac{1}{K}$, $i = 1, \ldots, K$. Therefore, using (16)-(19), the conditional SER of the CIOD can be given by

$$P_{\text{QAM}}\left(M|\boldsymbol{H}\right) = c\left[Q\left(\sqrt{g\chi_1}\right) + Q\left(\sqrt{g\chi_2}\right)\right]. \quad (20)$$

Averaging over $P_{QAM}\left(M|\boldsymbol{H}\right)$ in (20), and using moment generating function (MGF) method [13], we can have the following expression for the average SER of CIOD:

$$\begin{aligned}P_e(M) =& \int_{0;\ \frac{N_s}{2}-\text{fold}}^{\infty}\frac{c}{\pi}\int_0^{\frac{\pi}{2}}\prod_{i=1}^{N_s/2}\prod_{j=1}^{N_d}e^{-\frac{g\,\gamma_{ij}}{2\sin^2\phi}}\left(p(\gamma_{ij})\,d\gamma_{ij}\right)d\phi\\&+\int_{0;\ \frac{N_s}{2}-\text{fold}}^{\infty}\frac{c}{\pi}\int_0^{\frac{\pi}{2}}\prod_{i=N_s/2+1}^{N_s}\prod_{j=1}^{N_d}e^{-\frac{g\,\gamma_{ij}}{2\sin^2\phi}}\left(p(\gamma_{ij})\,d\gamma_{ij}\right)d\phi,\end{aligned} \quad (21)$$

where $p(\gamma_{ij})$ is the PDF of random variable $\gamma_{ij} = \frac{P_0T|h_{ij}|^2}{2\mathcal{N}_0K}$, which has a exponential distribution with mean $\sigma_{ij}^2 = \frac{P_0T\sigma_{h_{ij}}^2}{2\mathcal{N}_0K}$. Therefore, the average SER can be rewritten as

$$\begin{aligned}P_e(M) =& \int_{0;\ \frac{N_s}{2}-\text{fold}}^{\infty}\frac{c}{\pi}\int_0^{\frac{\pi}{2}}\prod_{i=1}^{N_s/2}\prod_{j=1}^{N_d}\frac{1}{\sigma_{ij}^2}e^{-\frac{g\,\gamma_{ij}}{2\sin^2\phi}-\frac{\gamma_{ij}}{\sigma_{ij}^2}}\,d\phi\,d\gamma_{ij}\\&+\int_{0;\ \frac{N_s}{2}-\text{fold}}^{\infty}\frac{c}{\pi}\int_0^{\frac{\pi}{2}}\prod_{i=N_s/2+1}^{N_s}\prod_{j=1}^{N_d}\frac{1}{\sigma_{ij}^2}e^{-\frac{g\,\gamma_{ij}}{2\sin^2\phi}-\frac{\gamma_{ij}}{\sigma_{ij}^2}}\,d\phi\,d\gamma_{ij}.\end{aligned} \quad (22)$$

If we assume all channels have the same variance $\sigma_{ij}^2 = \sigma_\gamma^2$, due to the similarity of both integrals in (22), the average SER can be rewritten as

$$P_e(M) = \frac{2c}{\pi}\int_0^{\frac{\pi}{2}}\int_{0;\ \frac{N_s}{2}-\text{fold}}^{\infty}\prod_{i=1}^{N_s/2}\prod_{j=1}^{N_d}\frac{1}{\sigma_\gamma^2}e^{-\frac{g\gamma_{ij}}{2\sin^2\phi}-\frac{\gamma_{ij}}{\sigma_\gamma^2}}\,d\phi\,d\gamma_{ij}. \quad (23)$$

By computing the MGF of $\gamma_{i,j}$, i.e., $M_n(s) = \mathbb{E}_\gamma\{e^{s\gamma_{i,j}}\}$, the average SER can be written as

$$\begin{aligned}P_e(M) &= \frac{2c}{\pi}\int_0^{\frac{\pi}{2}}\prod_{n=1}^{\frac{N_sN_d}{2}}M_n\left(-\frac{g}{2\sin^2\phi}\right)d\phi\\&= \frac{2c}{\pi}\int_0^{\frac{\pi}{2}}\prod_{n=1}^{\frac{N_sN_d}{2}}\frac{\sin^2\phi}{\sin^2\phi+\frac{g\sigma_\gamma^2}{2}}\,d\phi.\end{aligned} \quad (24)$$

By making the change of the variable $x = \sin^2\phi$, the average SER can be written as

$$\begin{aligned}P_e(M) =& \frac{c}{\pi}\left(\frac{2}{g\sigma_\gamma^2}\right)^{\frac{N_sN_d}{2}}\\&\int_0^1(1-x)^{-\frac{1}{2}}x^{\frac{N_sN_d-1}{2}}\left(\frac{2}{g\sigma_\gamma^2}x+1\right)^{\frac{-N_sN_d}{2}}dx.\end{aligned} \quad (25)$$

The integral in (25) can be represented in terms of Gauss hypergeometric function ${}_2F_1(a, b; c; x)$ [14, Eq. (2.8)] as

$$\begin{aligned}P_e(M) =& \frac{c\,\Gamma(\frac{N_sN_d+1}{2})\Gamma(\frac{1}{2})}{\pi\,\Gamma(\frac{N_sN_d}{2}+1)}\left(\frac{2}{g\sigma_\gamma^2}\right)^{\frac{N_sN_d}{2}}\\&\times\ {}_2F_1\left(\frac{N_sN_d}{2}, \frac{N_sN_d+1}{2}; \frac{N_sN_d}{2}+1; \frac{-2}{g\sigma_\gamma^2}\right),\end{aligned} \quad (26)$$

where we have used [14, Eq. (2.12)]. The average SER $P_e(M)$ in (26) can be numerically evaluated by either using the Gauss hypergeometric function which is available in MATHEMATICA software or integrating the integral representation of Gauss hypergeometric function.

B. SER Expression for CIOD with Constellation Rotation

In [4, Theorem 33], it is shown that the STBC from CIOD with variables from a signal set achieve full-diversity, if and only if the coordinate product distance (CPD) of that signal set is nonzero. Since regular M-QAM and symmetric M-PSK have CPD of zero, we now study the performance analysis of rotated versions of M-QAM and M-PSK.

For an arbitrary two-dimensional (2-D) signal constellation, a standard approach of evaluating the error probability of a signal set is based on the union bound and the average probability of bit error P_b is thus can be upper bounded as [5]

$$P_b(M) \leq \frac{1}{m\,M} \sum_{s_k \in \mathcal{S}_M^\theta} \sum_{\substack{\hat{s}_k \in \mathcal{S}_M^\theta \\ s_k \neq \hat{s}_k}} a(s_k, \hat{s}_k) P(s_k \to \hat{s}_k), \quad (27)$$

where $\mathcal{S}_M^\theta$ is the signal constellation of size $|\mathcal{S}_M^\theta| = M = 2^m$, $P(s_k \to \hat{s}_k)$ is the unconditional pairwise error probability (PEP) that the receiver estimated $\hat{s}_k$ when s_k was transmitted, and $a(s_k, \hat{s}_k)$ represents the Hamming distance between the sequences of bits of $\hat{s}_k$ and s_k under consideration. For the case of rotated M-PSK, anticlockwise rotation over an angle θ leads to the constellation

$$\mathcal{S}_M^\theta = \{s_k = e^{j(2\pi k/M + \theta)} | k = 0, 1, \ldots, M-1\}. \quad (28)$$

Coordinate interleaving is employed so that the I and the Q channels experience independent fades. Let $p(\chi_1)$ and $p(\chi_2)$ be the PDF of the random variables χ_1 and χ_2 given in (14) and (15), respectively. In order to calculate the average probability of error for a system employing coordinate interleaving, the conditional PEP needs to be averaged over χ_1 and χ_2 given as

$$P(s_k \to \hat{s}_k) = \int_0^\infty \int_0^\infty Q\left(\sqrt{\frac{\chi_1\, d_{k,I}^2 + \chi_2\, d_{k,Q}^2}{2}}\right) \times p(\chi_1) p(\chi_2) d\chi_1 d\chi_2, \quad (29)$$

where $d_{k,I} = |s_{k,I} - \hat{s}_{k,I}|$ and $d_{k,Q} = |s_{k,Q} - \hat{s}_{k,Q}|$. For the case of M-PSK, we have

$$d_{k,I}^2 = (\cos(\phi_k + \theta) - \cos(\hat{\phi}_k + \theta))^2$$
$$d_{k,Q}^2 = (\sin(\phi_k + \theta) - \sin(\hat{\phi}_k + \theta))^2, \quad (30)$$

where ϕ_k and $\hat{\phi}_k$ represent the phase of the two signal constellation points under consideration, respectively. When M-QAM signaling is used, we have

$$d_{k,I}^2 = ((a_k - \hat{a}_k)\cos\theta - (b_k - \hat{b}_k)\sin\theta)^2$$
$$d_{k,Q}^2 = ((b_k - \hat{b}_k)\cos\theta + (a_k - \hat{a}_k)\sin\theta)^2, \quad (31)$$

where $a_k + \jmath\, b_k$ $\hat{a}_k + \jmath\, \hat{b}_k$ are two normalized signal points from un-rotated QAM constellation.

Using MGF method, we can rewrite $P(s_k \to \hat{s}_k)$ in (29) as

$$P(s_k \to \hat{s}_k) = \int_{0;\, N_s-\text{fold}}^\infty \frac{1}{\pi}\int_0^{\frac{\pi}{2}} \prod_{i=1}^{N_s/2} \prod_{j=1}^{N_d} e^{-\frac{d_{k,I}^2 \gamma_{ij}}{4\sin^2\phi}} \left(p(\gamma_{ij})\, d\gamma_{ij}\right) \times \prod_{i=N_s/2+1}^{N_s} \prod_{j=1}^{N_d} e^{-\frac{d_{k,Q}^2 \gamma_{ij}}{4\sin^2\phi}} \left(p(\gamma_{ij})\, d\gamma_{ij}\right) d\phi, \quad (32)$$

where $p(\gamma_{ij})$ is the PDF of random variable $\gamma_{ij} = \frac{P_0 T |h_{ij}|^2}{\mathcal{N}_0 K}$, which has a exponential distribution with mean $\sigma_{ij}^2 = \frac{P_0 T \sigma_{ij}^2}{\mathcal{N}_0 K}$. Therefore, the average PEP can be rewritten as

$$P(s_k \to \hat{s}_k) = \int_{0;\, N_s-\text{fold}}^\infty \frac{1}{\pi}\int_0^{\frac{\pi}{2}} \prod_{i=1}^{N_s/2} \prod_{j=1}^{N_d} \frac{1}{\sigma_{ij}^2} e^{-\frac{d_{k,I}^2 \gamma_{ij}}{4\sin^2\phi} - \frac{\gamma_{ij}}{\sigma_{ij}^2}}\, d\gamma_{ij} \times \prod_{i=N_s/2+1}^{N_s} \prod_{j=1}^{N_d} \frac{1}{\sigma_{ij}^2} e^{-\frac{d_{k,Q}^2 \gamma_{ij}}{4\sin^2\phi} - \frac{\gamma_{ij}}{\sigma_{ij}^2}}\, d\gamma_{ij}\, d\phi. \quad (33)$$

Assuming all channels have the same variance $\sigma_{ij}^2 = \sigma_\gamma^2$, by computing the MGF of $\gamma_{i,j}$, i.e., $M_n(s) = \mathbb{E}_\gamma\{e^{s\gamma_{i,j}}\}$, the average PEP can be written as

$$\begin{aligned} P(s_k \to \hat{s}_k) &= \frac{1}{\pi}\int_0^{\frac{\pi}{2}} \prod_{n=1}^{\frac{N_s N_d}{2}} M_n\!\left(\frac{-d_{k,I}^2}{4\sin^2\phi}\right) M_n\!\left(\frac{-d_{k,Q}^2}{4\sin^2\phi}\right) d\phi \\ &= \frac{1}{\pi}\int_0^{\frac{\pi}{2}} \prod_{n=1}^{\frac{N_s N_d}{2}} \frac{\sin^4\phi}{\left(\sin^2\phi + \frac{d_{k,I}^2\sigma_\gamma^2}{4}\right)\left(\sin^2\phi + \frac{d_{k,Q}^2\sigma_\gamma^2}{4}\right)} d\phi \\ &= \frac{1}{\pi}\int_0^{\frac{\pi}{2}} \frac{\sin^{2N_s N_d}\phi}{\left(\sin^2\phi + \frac{d_{k,I}^2\sigma_\gamma^2}{4}\right)^{\frac{N_s N_d}{2}} \left(\sin^2\phi + \frac{d_{k,Q}^2\sigma_\gamma^2}{4}\right)^{\frac{N_s N_d}{2}}} d\phi. \end{aligned} \quad (34)$$

From (30), $d_{k,I}^2$ and $d_{k,I}^2$ for BPSK signals can be computed as

$$d_{k,I}^2 = 4\cos^2\theta,\; d_{k,Q}^2 = 4\sin^2\theta, \quad (35)$$

and thus, using (27), the average BER of the system with BPSK modulation can be computed as

$$P_b(2) = \frac{1}{\pi}\int_0^{\frac{\pi}{2}} \frac{\sin^{2N_s N_d}\phi}{\left(\sin^2\phi + \cos^2\theta\sigma_\gamma^2\right)^{\frac{N_s N_d}{2}} \left(\sin^2\phi + \sin^2\theta\sigma_\gamma^2\right)^{\frac{N_s N_d}{2}}} d\phi. \quad (36)$$

Next, for illustrative purpose we consider QPSK (4-QAM). Let $\mathcal{S}_4^\theta = \{s_A, s_B, s_C, s_D\}$ where $s_A = e^{\jmath\theta}$, $s_B = \jmath\, e^{\jmath\theta}$, $s_C = -s_A$, and $s_D = -s_B$. Then, for the events $\{s_A \to s_B\}$ and $\{s_A \to s_D\}$, the Euclidean distances $d_{k,I}^2$ and $d_{k,Q}^2$ are given by

$$d_{k,I}^2 = 1 + \sin(2\theta),\; d_{k,Q}^2 = 1 - \sin(2\theta), \quad (37)$$

and for the event $\{s_A \to s_C\}$, we have

$$d_{k,I}^2 = 4\cos^2\theta,\; d_{k,Q}^2 = 4\sin^2\theta. \quad (38)$$

Hence, for the case of QPSK signaling, the average BER can be written as

$$P_b(4) = \frac{1}{2}\left[P(s_A \to s_B) + P(s_A \to s_C) + P(s_A \to s_D)\right], \quad (39)$$

where

$$P(s_A \to s_B) = P(s_A \to s_C) = \frac{1}{\pi}\int_0^{\frac{\pi}{2}} \frac{\sin^{2N_s N_d}\phi}{\left(\sin^2\phi + (1+\sin 2\theta)\sigma_\gamma^2\right)^{\frac{N_s N_d}{2}} \left(\sin^2\phi + (1-\sin 2\theta)\sigma_\gamma^2\right)^{\frac{N_s N_d}{2}}} d\phi, \quad (40)$$

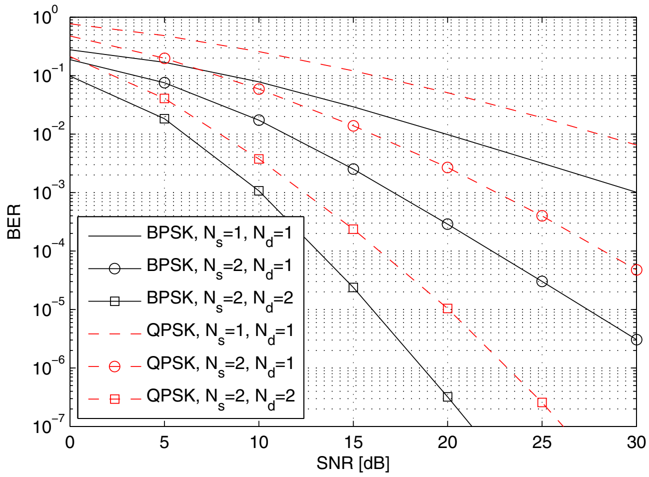

Fig. 1. The average BER curves of SSD system employing CIOD space-time codes with BPSK and QPSK signals, over Rayleigh fading channels with $\theta = 50°$.

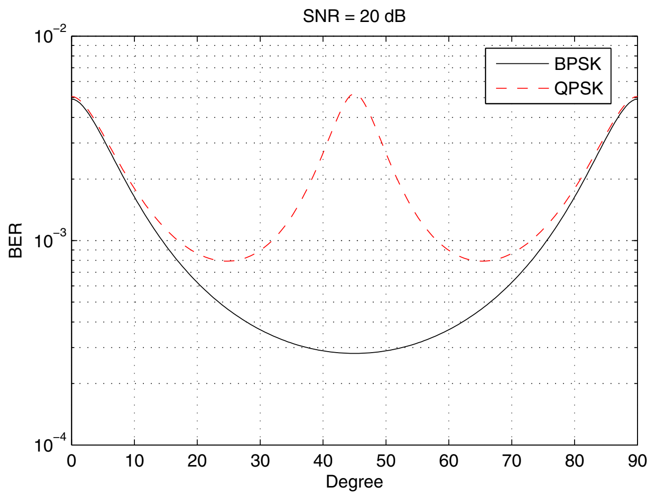

Fig. 2. Average probability of error for QPSK signal constellation over Rayleigh fading channel with perfect CSI at SNR = 20 dB.

and

$$P(s_A \to s_D) = \frac{1}{\pi}\int_0^{\frac{\pi}{2}} \frac{\sin^{2N_sN_d}\phi}{\left(\sin^2\phi + \cos^2\theta\sigma_\gamma^2\right)^{\frac{N_sN_d}{2}}\left(\sin^2\phi + \sin^2\theta\sigma_\gamma^2\right)^{\frac{N_sN_d}{2}}}\, d\phi. \quad (41)$$

V. Numerical Analysis

In this section, the performances of CIOD space-time codes are studied through simulations. The error event is bit error rate (BER). The signal symbols are modulated as BPSK and QPSK. Assume the received antennas have the same value of noise power and all the links have unit-variance Rayleigh flat fading.

Fig. 1 shows the bit error rates of a MIMO system employing CIOD space-time block codes for transmission of BPSK with the rotation of 50 degrees. Observing the curves at high SNR, it can be shown the full diversity order of N_sN_d is obtainable.

In Fig. 2, the BER performance of CIOD space-time coded system at various rotation angles is shown. It is assumed that the transmit SNR is 20 dB and two transmit antennas and one receive antenna are employed. It is evident from (34) that the BER of a system employing coordinate interleaving is dependent upon the constellation rotation (θ). It shows that if the I and Q channels are exposed to independent fades, the system performance varies with constellation rotation. One can observe from Fig. 2 that for BPSK modulation, the 45 degrees rotation is optimal and leads to the minimum BER. For the case of QPSK, the constellation rotations of $\theta = 24.6482, 65.1256$ are optimal.

VI. Conclusions

In this paper, we have derived analytical expressions for the BER of MIMO systems with CIOD space-time coded transmitters using the equivalent SISO model. It is shown that using the rotated M-PSK and M-QAM moduations, the CIOD space-time block codes can achieve full spatial diversity. The BERs are expressed in terms of MGF of the instantaneous SNR per symbol for the equivalent model.

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