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A Rate-One Full-Diversity Quasi-Orthogonal Design for Eight Tx Antennas

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by

B. Sundar Rajan, Moon Ho Lee†, Md. Zafar Ali Khan
ECE, IISc, Bangalore.
emails: bsrajan@ece.iisc.ernet.in,
zafar@protocol.ece.iisc.ernet.in.
† Institute for Information and Communication,
Chonbuk National University, Chonju, Korea,
E-mail:moonho@chonbuk.ac.kr.

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Indian Institute of Science
Bangalore 560 012

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ABSTRACT

Space-Time block codes (STBC) from orthogonal designs (OD) have been attracting much attention due to their fast decoding (single-symbol decoding), minimal-delay and full-diversity. ODs offering full-diversity, rate-one, minimal delay and single-symbol decoding exist only for two transmit antennas. For more than two antennas some of these advantages need to be sacrificed. Recently, for the case of four transmit antennas, Quasi-Orthogonal designs (QOD) have been proposed by Su and Xia which retain all these advantages except instead of single-symbol decoding double-symbol decoding is required. Their construction does not give rate one for more than four transmit antennas since it is based on the existence of rate-one, size n square ODs to get size $2n$ QODs and rate-one square ODs do not exist for four antennas. In this correspondence, we present a non-orthogonal design using which one obtains rate-one, full-diversity, double-symbol decoding STBCs for eight transmit antennas.

Keywords: Orthogonal designs, space-time block codes, coordinate interleaving, wireless communication, fading channels.

1 Introduction

Transmit and receive diversity provided by multiple transmit and multiple receive antennas is a very promising candidate to combat the detrimental effects of fading channels in wireless communications. Teletar [1] and Foschini and Gan [2] have shown that using systems with multiple transmit and receive antennas very high capacities can be achieved: if the number of transmit and receive antennas are same the capacity increases linearly with the number of antennas. A design criterion for designing codes for multiple transmit and receive antennas suitable for fading channels was first proposed in [3] and the implementation of multi-antenna systems in the context of transmit diversity was extensively investigated in [4] with space-time trellis codes (STTC).

A brief review of the model [4] used in this correspondence for a space-time coded system follows. Let the number of transmit antennas be N and the number of receive antennas be M. At each time slot t, complex signal points, $s_{it},\ i = 0, 1, \cdots, N-1$ are transmitted from the N transmit antennas simultaneously. Let $h_{ij} = \alpha_{ij} e^{\mathbf{j}\theta_{ij}}$ denote the path gain from the transmit antenna i to the receive antenna j, where $\mathbf{j} = \sqrt{-1}$. Assuming that the path gains are constant over a frame length $L \geq N$, the received signal v_{jt} at the antenna j at time t, is given by

$$v_{jt} = \sum_{i=0}^{N-1} h_{ij} s_{it} + n_{jt}, \quad j = 0, \cdots, M-1; \quad t = 0, \cdots, L-1. \tag{1}$$

In matrix notation,

$$\mathbf{V} = \mathbf{SH} + \mathbf{N} \tag{2}$$

where $\mathbf{V} \in \mathbb{C}^{L \times M}$ ($\mathbb{C}$ denotes the complex field) is the received signal matrix, $\mathbf{S} \in \mathbb{C}^{L \times N}$ is the transmission matrix (codeword matrix), $\mathbf{H} \in \mathbb{C}^{N \times M}$ denotes the channel matrix and $\mathbf{N} \in \mathbb{C}^{L \times M}$ has entries that are Gaussian distributed with zero mean and unit variance and also are temporally and spatially white. In $\mathbf{V}, \mathbf{S}$ and $\mathbf{N}$ time runs vertically and space runs horizontally. The channel matrix $\mathbf{H}$ and the transmitted codeword $\mathbf{S}$ are assumed to have unit variance entries. (Through out the paper, for a matrix $\mathbf{A}$, $\mathbf{A}^{\mathcal{H}}$ represents the Hermitian (conjugate transpose), $\mathbf{A}^T$ the transpose and $|\mathbf{A}|$ the determinant of $\mathbf{A}$). If the transmission power constraint is $E\ tr\{\mathbf{SS}^H\} = LN$ then

$$\mathbf{V} = \sqrt{\frac{\rho}{N}} \mathbf{SH} + \mathbf{N} \tag{3}$$

where ρ is the SNR at each receiver. Assuming that perfect channel state information (CSI) is available at the receiver, the decision rule for ML decoding is to minimize the metric

$$\sum_{t=0}^{L-1} \sum_{j=0}^{M-1} \left| v_{jt} - \sum_{i=0}^{N-1} h_{ij} s_{it} \right|^2 \tag{4}$$

over all codeword matrices $\mathbf{S}$. This results in exponential decoding complexity, because of the joint decision on all the symbols s_{it} in the matrix $\mathbf{S}$. If the throughput rate of such a scheme is R in bits/sec/Hz, then 2^{RL} metric calculations are required; one for each possible transmission matrix $\mathbf{S}$. Even for modest antenna configurations and rates this could be very large resulting in search for codes that admit a simple decoding while providing full diversity gain. Space-Time Block Codes (STBCs) obtained from Orthogonal Designs (ODs) [5] achieve these. A Generalized Complex Linear Processing Orthogonal Design (GCLPOD) in k complex indeterminates $x_1, x_2, \cdots, x_k$ of size n and rate $\mathcal{R} = k/p$ symbols/channel use, $p \geq n$ is a $p \times n$ matrix Θ, such that

- the entries of Θ are complex linear combinations of $0, \pm x_i, \quad i = 1, \cdots, k$ and their conjugates.
- $\Theta^H \Theta = \mathbf{D}$, where $\mathbf{D}$ is a diagonal matrix whose entries are a linear combination of $|x_i|^2, i = 1, \cdots, k$ with all strictly positive real coefficients.

When k=n=p and no entry is zero, then Θ is called a Complex Linear Processing Orthogonal Design (CLPOD). Furthermore, when the entries are only from $\{\pm x_1, \pm x_2, \cdots, \pm x_k\}$, their conjugates and multiples of $\mathbf{j}$ then Θ is called a Complex Orthogonal Design (COD). STBCs from ODs are obtained by replacing x_i by s_i and allowing s_i to take all values from a signal set $\mathcal{A}$. Note that if the signal set is of size 2^b the throughput rate R in bits per second per Hertz is related to the rate of the design $\mathcal{R}$ as $R = \mathcal{R}b$. A GCLPOD is said to be of minimal-delay if $n = p$. The Alamouti scheme [6], which is of minimal-delay, full-rank and full-rate is basically the STBC arising from the size 2 COD $\begin{bmatrix} x_0 & x_1 \\ -x_1^* & x_0^* \end{bmatrix}$. For STBCs from ODs, the decoding metric (4) can be written as sum of several square terms, one for each variable of the design [5] which reduces the decoding complexity enormously. We will refer to this as "single-symbol decoding". If the decoding metric is writtenable as sum of several terms each involving two variables and a variable appearing in one term does not appear in any other term, we refer to the decoding as "double-symbol decoding".

Unfortunately, orthogonal designs for complex constellations giving full-diversity and full-rate (symbol rate one per channel use) with single-symbol decoding exist only for 2 transmit antennas. For larger number of antennas, either rate needs to be sacrificed or the delay increases and moreover the theory is not fully developed to get CODs for arbitrary number of antennas.

For the case of four antennas, several authors have reported the performance of STBCs obtained from different versions of orthogonal and quasi-orthogonal designs (QOD) [5, 9, 7, 8, 10, 11, 12]. In [11] the authors introduce coordinate interleaving among the complex indeterminates resulting in a non-orthogonal design called coordinate interleaved design (CID), and achieve single-symbol decoding along with full diversity and full-rate for four Tx Antennas. We shall elaborate this approach below since the basic ingredient in this correspondence for eight antennas is this CID for four antennas. Table 1 summarizes the performance of various schemes for four antennas including that of the STBC from CID. Let $x_0, x_1, x_2, x_3,$ be four complex

Table 1: Comparison of known ODs, QODs and CID for four transmit antennas

	Symbol Rate	Diversity	Decoding
Orthogonal Designs[5, 9]	3/4	4	single-symbol
QOD; Jhafarkhani [7]	1	2	double-symbol
QOD; Su and Xia [8]	1	4	double-symbol
QOD; Sharma and Papadias[10]	1	4	double-symbol
CID; Khan and Rajan[11]	1	4	single-symbol

indeterminates that take values from a complex signal constellation $\mathcal{A}$ where $x_i = x_{iI} + \mathbf{j}x_{iQ}$. Let

$$\begin{aligned} \tilde{x_0} &= x_{0I} + \mathbf{j}x_{2Q} \\ \tilde{x_1} &= x_{1I} + \mathbf{j}x_{3Q} \\ \tilde{x_2} &= x_{2I} + \mathbf{j}x_{0Q} \\ \tilde{x_3} &= x_{3I} + \mathbf{j}x_{1Q} \end{aligned} \tag{5}$$

be the new indeterminates obtained by coordinate interleaving the in-phase and quadrature-phase components of the original indeterminates x_0, x_1, x_2 and x_3. Then, the coordinate interleaved design (CID) for four antennas in variables x_0, x_1, x_2, x_3 is defined to be

$$\mathcal{S}_4(x_0, x_1, x_2, x_3) = \begin{bmatrix} \tilde{x}_o & \tilde{x}_1 & 0 & 0 \\ -\tilde{x}_1^* & \tilde{x}_0^* & 0 & 0 \\ 0 & 0 & \tilde{x}_2 & \tilde{x}_3 \\ 0 & 0 & -\tilde{x}_3^* & \tilde{x}_2^* \end{bmatrix}. \tag{6}$$

Notice that when x_0, x_1, x_2, x_3 take values from a signal set $\mathcal{A}$, the coordinate interleaved symbols are not from $\mathcal{A}$ but from an expanded version of $\mathcal{A}$ and the decoding will be for the original symbols after deinterleaving the received symbols appropriately [11]. For notational simplicity, we denote the above matrix by $\mathcal{S}_4$. The matrix $\mathcal{S}_4$ has the property that

$$\mathcal{S}_4^{\mathcal{H}}\mathcal{S}_4 = \begin{bmatrix} a & 0 & 0 & 0 \\ 0 & a & 0 & 0 \\ 0 & 0 & c & 0 \\ 0 & 0 & 0 & c \end{bmatrix} \tag{7}$$

where

$$\begin{aligned} a &= \tilde{x}_0\tilde{x}_0^* + \tilde{x}_1\tilde{x}_1^* = |\tilde{x}_0|^2 + |\tilde{x}_1|^2, \\ c &= \tilde{x}_2\tilde{x}_2^* + \tilde{x}_3\tilde{x}_3^* = |\tilde{x}_2|^2 + |\tilde{x}_3|^2. \end{aligned}$$

In [11], The above structure is exploited to achieve a single-symbol decoding with the modification from the conventional single-symbol decoding being that the inphase and quadrature components of the original symbols see different fading. For further details the readers are referred to [11].

For the case of eight antennas, several authors have studied STBCs using ODs and QODs [5, 7, 8]. Table 2 summarizes the performance of these schemes in terms of rate, diversity and decoding and also shows in the last row the performance achieved by the design of this correspondence.

Table 2: Comparison of known designs for eight transmit antennas with our design

	Symbol Rate	Diversity	Decoding
Orthogonal Design [5]	1/2	8	single-symbol
Jhafarkhani [7]	3/4	4	double-symbol
Su and Xia [8]	3/4	8	double-symbol
Proposed scheme	1	8	double-symbol

The content of this correspondence is organized as follows: In Section 2, we introduce a non-orthogonal design , called coordinate interleaved design (CID) for eight transmit antennas, and describe the STBC obtained from this. The double-symbol decoding for the proposed STBC scheme is described in Section 3. The scheme of this correspondence is shown to give diversity order 8 for any signal constellation simply by rotating the constellation appropriately for four of the eight indeterminates, in Section 4. Also, the parameters that determine the coding gain are identified. Simulation results are presented in Section 5 along with that of the other known schemes for 8 antennas. A summary of the results and concluding remarks constitute Section 6.

2 Coordinate-Interleaved Design for Eight Tx Antennas

Let $x_0, x_1, x_2, x_3, x_4, x_5, x_6$ and x_7 denote eight complex indeterminates, where $x_i = x_{iI} + \mathbf{j}x_{iQ}$. We define,

$$\begin{aligned}
\tilde{x}_0 &= x_{0I} + \mathbf{j}x_{2Q} \\
\tilde{x}_1 &= x_{1I} + \mathbf{j}x_{3Q} \\
\tilde{x}_2 &= x_{2I} + \mathbf{j}x_{0Q} \\
\tilde{x}_3 &= x_{3I} + \mathbf{j}x_{1Q} \\
\tilde{x}_4 &= x_{4I} + \mathbf{j}x_{6Q} \\
\tilde{x}_5 &= x_{5I} + \mathbf{j}x_{7Q} \\
\tilde{x}_6 &= x_{6I} + \mathbf{j}x_{4Q} \\
\tilde{x}_7 &= x_{7I} + \mathbf{j}x_{5Q}
\end{aligned} \tag{8}$$

to be the new complex indeterminates, called interleaved indeterminates henceforth, obtained by coordinate interleaving the in-phase and quadrature-phase components of the original indeterminates. The design proposed for eight transmit antennas is

$$\mathcal{S}_8(x_0,x_1,x_2,x_3,x_4,x_5,x_6,x_7) = \begin{bmatrix}
\tilde{x}_o & \tilde{x}_1 & 0 & 0 & \tilde{x}_4 & \tilde{x}_5 & 0 & 0 \\
-\tilde{x}_1^* & \tilde{x}_0^* & 0 & 0 & -\tilde{x}_5^* & \tilde{x}_4^* & 0 & 0 \\
0 & 0 & \tilde{x}_2 & \tilde{x}_3 & 0 & 0 & \tilde{x}_6 & \tilde{x}_7 \\
0 & 0 & -\tilde{x}_3^* & \tilde{x}_2^* & 0 & 0 & -\tilde{x}_7^* & \tilde{x}_6^* \\
\tilde{x}_4 & \tilde{x}_5 & 0 & 0 & \tilde{x}_0 & \tilde{x}_1 & 0 & 0 \\
-\tilde{x}_5^* & \tilde{x}_4^* & 0 & 0 & -\tilde{x}_1^* & \tilde{x}_0^* & 0 & 0 \\
0 & 0 & \tilde{x}_6 & \tilde{x}_7 & 0 & 0 & \tilde{x}_2 & \tilde{x}_3 \\
0 & 0 & -\tilde{x}_7^* & \tilde{x}_6^* & 0 & 0 & -\tilde{x}_3^* & \tilde{x}_2^*
\end{bmatrix} \tag{9}$$

which is obtained by simply using $\begin{bmatrix} A & B \\ B & A \end{bmatrix}$ construction proposed in [9] with the matrix S_4 given by (6) for A with indeterminates $\tilde{x}_0, \tilde{x}_1, \tilde{x}_2, \tilde{x}_3$ and for B with indeterminates $\tilde{x}_4, \tilde{x}_5, \tilde{x}_6$ and $\tilde{x}_7$. We will use only S_8 for the matrix above, dropping the arguments. It is easily verified that

$$\mathcal{S}_8^{\mathcal{H}}\mathcal{S}_8 = \begin{bmatrix} a & 0 & 0 & 0 & b & 0 & 0 & 0 \\ 0 & a & 0 & 0 & 0 & b & 0 & 0 \\ 0 & 0 & c & 0 & 0 & 0 & d & 0 \\ 0 & 0 & 0 & c & 0 & 0 & 0 & d \\ b & 0 & 0 & 0 & a & 0 & 0 & 0 \\ 0 & b & 0 & 0 & 0 & a & 0 & 0 \\ 0 & 0 & d & 0 & 0 & 0 & c & 0 \\ 0 & 0 & 0 & d & 0 & 0 & 0 & c \end{bmatrix} \tag{10}$$

where

$$\begin{aligned} a &= \tilde{x}_0\tilde{x}_0^* + \tilde{x}_1\tilde{x}_1^* + \tilde{x}_4\tilde{x}_4^* + \tilde{x}_5\tilde{x}_5^* = |\tilde{x}_0|^2 + |\tilde{x}_1|^2 + |\tilde{x}_4|^2 + |\tilde{x}_5|^2 \\ b &= \tilde{x}_0\tilde{x}_4^* + \tilde{x}_4\tilde{x}_0^* + \tilde{x}_1\tilde{x}_5^* + \tilde{x}_5\tilde{x}_1^* = 2(\tilde{x}_{0I}\tilde{x}_{4I} + \tilde{x}_{0Q}\tilde{x}_{4Q} + \tilde{x}_{1I}\tilde{x}_{5I} + \tilde{x}_{1I}\tilde{x}_{5I}) \\ c &= \tilde{x}_2\tilde{x}_2^* + \tilde{x}_3\tilde{x}_3^* + \tilde{x}_6\tilde{x}_6^* + \tilde{x}_7\tilde{x}_7^* = |\tilde{x}_2|^2 + |\tilde{x}_3|^2 + |\tilde{x}_6|^2 + |\tilde{x}_7|^2 \\ d &= \tilde{x}_2\tilde{x}_6^* + \tilde{x}_6\tilde{x}_2^* + \tilde{x}_3\tilde{x}_7^* + \tilde{x}_7\tilde{x}_3^* = 2(\tilde{x}_{2I}\tilde{x}_{6I} + \tilde{x}_{2Q}\tilde{x}_{6Q} + \tilde{x}_{3I}\tilde{x}_{7I} + \tilde{x}_{3I}\tilde{x}_{7I}) \end{aligned} \tag{11}$$

The STBC based on the design $\mathcal{S}_8$ transmits signal matrices obtained by replacing the indeterminates x_i by s_i in S_8 and allowing $s_i,\ i = 0,1,2,3$ to take values from a signal set $\mathcal{A}$ and the remaining four variables $s_i,\ i = 4,5,6,7$ to take values from a rotated version $\mathcal{A}e^{\mathbf{j}\theta}$ of $\mathcal{A}$. This rotation is needed to achieve full-diversity as elaborated in Section 4. It is important to note that when the variables $s_i,\ i = 0,1,2,3,$ take values from $\mathcal{A}$ the transmission matrix have entries which are coordinate interleaved versions of the variables and hence the actual signal points transmitted are not from $\mathcal{A}$ but from an *expanded version of* $\mathcal{A}$ which we denote by $\tilde{\mathcal{A}}$. Figure 1(a) shows $\tilde{\mathcal{A}}$ when $\mathcal{A} = \{1, -1, \mathbf{j}, -\mathbf{j}\}$ which is shown in Figure 1(c). Notice that $\tilde{\mathcal{A}}$ has 8 signal points whereas $\mathcal{A}$ has 4. Figure 1(b) shows $\tilde{\mathcal{A}}'$ where $\mathcal{A}'$ is the four point signal set obtained by rotating $\mathcal{A}$ by 13.2825 degrees counter clockwise i.e., $\mathcal{A}' = \{e^{\mathbf{j}\theta}, -e^{\mathbf{j}\theta}, \mathbf{j}e^{\mathbf{j}\theta}, -\mathbf{j}e^{\mathbf{j}\theta}\}$ where $\theta = 13.2825$ degrees as shown in Figure 1(d). Notice that now the expanded signal set has 16 signal points (The choice of the value $\theta = 13.2825$ has no particular significance).

Another important aspect to notice in the signal transmission matrix (9) is that at any time only four antennas transmit signals. However, at any time the symbols that are transmitted by four antennas carry a component (in- or quadrature-) of all the eight indeterminates $s_i,\ \ i = 0,1,...,7$. Moreover, as we shall see in the section ahead, the decoding will be for the variables s_i's and not for the interleaved versions of these.

3 Decoding

In this section, we show that the STBC based on the design discussed in the previous section admits double-symbol decoding. We consider only the case of one receive antenna, the extension to multiple receive antenna being straight forward. Equation (2) in this case is

$$\begin{bmatrix} v_0 \\ v_1 \\ v_2 \\ v_3 \\ v_4 \\ v_5 \\ v_6 \\ v_7 \end{bmatrix} = \begin{bmatrix} \tilde{s}_o & \tilde{s}_1 & 0 & 0 & \tilde{s}_4 & \tilde{s}_5 & 0 & 0 \\ -\tilde{s}_1^* & \tilde{s}_0^* & 0 & 0 & -\tilde{s}_5^* & \tilde{s}_4^* & 0 & 0 \\ 0 & 0 & \tilde{s}_2 & \tilde{s}_3 & 0 & 0 & \tilde{s}_6 & \tilde{s}_7 \\ 0 & 0 & -\tilde{s}_3^* & \tilde{s}_2^* & 0 & 0 & -\tilde{s}_7^* & \tilde{s}_6^* \\ \tilde{s}_4 & \tilde{s}_5 & 0 & 0 & \tilde{s}_0 & \tilde{s}_1 & 0 & 0 \\ -\tilde{s}_5^* & \tilde{s}_4^* & 0 & 0 & -\tilde{s}_1^* & \tilde{s}_0^* & 0 & 0 \\ 0 & 0 & \tilde{s}_6 & \tilde{s}_7 & 0 & 0 & \tilde{s}_2 & \tilde{s}_3 \\ 0 & 0 & -\tilde{s}_7^* & \tilde{s}_6^* & 0 & 0 & -\tilde{s}_3^* & \tilde{s}_2^* \end{bmatrix} \begin{bmatrix} h_0 \\ h_1 \\ h_2 \\ h_3 \\ h_4 \\ h_5 \\ h_6 \\ h_7 \end{bmatrix} + \begin{bmatrix} n_0 \\ n_1 \\ n_2 \\ n_3 \\ n_4 \\ n_5 \\ n_6 \\ n_7 \end{bmatrix} \tag{12}$$

where v_i is the received signal at time instant $i = 0, 1, \cdots, 7$. Taking complex conjugate of v_1, v_3, v_5 and v_7, the (12) can be re-written as

$$\begin{bmatrix} v_0 \\ v_1^* \\ v_2 \\ v_3^* \\ v_4 \\ v_5^* \\ v_6 \\ v_7^* \end{bmatrix} = \begin{bmatrix} h_o & h_1 & 0 & 0 & h_4 & h_5 & 0 & 0 \\ h_1^* & -h_0^* & 0 & 0 & h_5^* & -h_4^* & 0 & 0 \\ 0 & 0 & h_2 & h_3 & 0 & 0 & h_6 & h_7 \\ 0 & 0 & h_3^* & -h_2^* & 0 & 0 & h_7^* & -h_6^* \\ h_4 & h_5 & 0 & 0 & h_0 & h_1 & 0 & 0 \\ h_5^* & -h_4^* & 0 & 0 & h_1^* & -h_0^* & 0 & 0 \\ 0 & 0 & h_6 & h_7 & 0 & 0 & h_2 & h_3 \\ 0 & 0 & h_7^* & -h_6^* & 0 & 0 & h_3^* & -h_2^* \end{bmatrix} \begin{bmatrix} \tilde{s}_0 \\ \tilde{s}_1 \\ \tilde{s}_2 \\ \tilde{s}_3 \\ \tilde{s}_4 \\ \tilde{s}_5 \\ \tilde{s}_6 \\ \tilde{s}_7 \end{bmatrix} + \begin{bmatrix} n_0 \\ n_1^* \\ n_2 \\ n_3^* \\ n_4 \\ n_5^* \\ n_6 \\ n_7^* \end{bmatrix} \tag{13}$$

which we denote in matrix notation as

$$\mathbf{V} = \mathbf{H}\tilde{\mathbf{S}} + \mathbf{N} \tag{14}$$

where

$$\begin{aligned} \mathbf{V} &= [v_0,\ v_1^*,\ v_2,\ v_3^*,\ v_4,\ v_5^*\ , v_6,\ v_7^*]^T, \\ \tilde{\mathbf{S}} &= [\tilde{s}_0,\ \tilde{s}_1,\ \tilde{s}_2,\ \tilde{s}_3,\ \tilde{s}_4,\ \tilde{s}_5,\ \tilde{s}_6,\ \tilde{s}_7]^T, \\ \mathbf{N} &= [n_0,\ n_1^*,\ n_2,\ n_3^*\ , n_4,\ n_5^*,\ n_6,\ n_7^*]^T \end{aligned}$$

and $\mathbf{H}$ is the square matrix appearing in (13), referred as the equivalent channel matrix. This equivalent channel matrix is such that

$$\mathbf{H}^{\mathcal{H}}\mathbf{H} = \left[\begin{array}{cccccccc} a' & 0 & 0 & 0 & b' & 0 & 0 & 0 \\ 0 & a' & 0 & 0 & 0 & b' & 0 & 0 \\ 0 & 0 & c' & 0 & 0 & 0 & d' & 0 \\ 0 & 0 & 0 & c' & 0 & 0 & 0 & d' \\ b' & 0 & 0 & 0 & a' & 0 & 0 & 0 \\ 0 & b' & 0 & 0 & 0 & a' & 0 & 0 \\ 0 & 0 & d' & 0 & 0 & 0 & c' & 0 \\ 0 & 0 & 0 & d' & 0 & 0 & 0 & c' \end{array}\right] \qquad (15)$$

where

$$\begin{aligned} a' &= h_0h_0^* + h_1h_1^* + h_4h_4^* + h_5h_5^* = |h_0|^2 + |h_1|^2 + |h_4|^2 + |h_5|^2 \\ b' &= h_0h_4^* + h_4h_0^* + h_1h_5^* + h_5h_1^* \\ c' &= h_2h_2^* + h_3h_3^* + h_6h_6^* + h_7h_7^* = |h_2|^2 + |h_3|^2 + |h_6|^2 + |h_7|^2 \\ d' &= h_2h_6^* + h_6h_2^* + h_3h_7^* + h_7h_3^*. \end{aligned} \qquad (16)$$

Notice, from (15) that in the equivalent channel, each of the pairs of signal points
$\{\tilde{s}_0, \tilde{s}_4\}, \{\tilde{s}_1, \tilde{s}_5\}, \{\tilde{s}_2, \tilde{s}_6\}, \{\tilde{s}_3, \tilde{s}_7\}$ face fading that is independent of the fading faced by other pairs. This is used to achieve double-symbol decoding as follows.

Let $\mathbf{V}' = [v'_0,\ v'_1,\ v'_2,\ v'_3,\ v'_4,\ v'_5,\ v'_6,\ v'_7]^T = \mathbf{H}^{\mathcal{H}}\mathbf{V}$. Then, we have

$$\mathbf{V}' = \left[\begin{array}{cccc} a'I_2 & O_2 & b'I_2 & O_2 \\ O_2 & c'I_2 & O_2 & d'I_2 \\ b'I_2 & O_2 & a'I_2 & O_2 \\ O_2 & d'I_2 & O_2 & c'I_2 \end{array}\right] \tilde{\mathbf{S}} + \mathbf{H}^{\mathcal{H}}\mathbf{N}. \qquad (17)$$

where I_2 and O_2 denote, respectively, the 2×2 Identity matrix and the zero matrix. Coordinate interleaving the in-phase and quadrature-phase components of v'_i's, $i = 0, 1, \cdots, 7$, as shown below, we have,

$$\begin{aligned} \hat{v}_0 = v'_{0I} + \mathbf{j}v'_{2Q} &= a's_{0I} + b's_{4I} + \mathbf{j}(c's_{0Q} + d's_{4Q}) + \hat{n}_{0I} + \mathbf{j}\hat{n}_{0Q} \qquad (18) \\ \hat{v}_1 = v'_{1I} + \mathbf{j}v'_{3Q} &= a's_{1I} + b's_{5I} + \mathbf{j}(c's_{1Q} + d's_{5Q}) + \hat{n}_{1I} + \mathbf{j}\hat{n}_{1Q} \\ \hat{v}_2 = v'_{2I} + \mathbf{j}v'_{0Q} &= c's_{2I} + d's_{6I} + \mathbf{j}(a's_{2Q} + b's_{6Q}) + \hat{n}_{2I} + \mathbf{j}\hat{n}_{2Q} \\ \hat{v}_3 = v'_{3I} + \mathbf{j}v'_{1Q} &= c's_{3I} + d's_{7I} + \mathbf{j}(a's_{3Q} + b's_{7Q}) + \hat{n}_{3I} + \mathbf{j}\hat{n}_{3Q} \\ \hat{v}_4 = v'_{4I} + \mathbf{j}v'_{6Q} &= b's_{0I} + a's_{4I} + \mathbf{j}(d's_{0Q} + c's_{4Q}) + \hat{n}_{4I} + \mathbf{j}\hat{n}_{4Q} \qquad (19) \\ \hat{v}_5 = v'_{5I} + \mathbf{j}v'_{7Q} &= b's_{1I} + a's_{5I} + \mathbf{j}(d's_{1Q} + c's_{5Q}) + \hat{n}_{5I} + \mathbf{j}\hat{n}_{5Q} \\ \hat{v}_6 = v'_{6I} + \mathbf{j}v'_{4Q} &= d's_{2I} + c's_{6I} + \mathbf{j}(b's_{2Q} + a's_{6Q}) + \hat{n}_{6I} + \mathbf{j}\hat{n}_{6Q} \\ \hat{v}_7 = v'_{7I} + \mathbf{j}v'_{5Q} &= d's_{3I} + c's_{7I} + \mathbf{j}(b's_{3Q} + a's_{7Q}) + \hat{n}_{7I} + \mathbf{j}\hat{n}_{7Q} \end{aligned}$$

where

$$
\begin{aligned}
\hat{n}_{0I} &= h_{0I}n_{0I} + h_{0Q}n_{0Q} + h_{1I}n_{1I} + h_{1Q}n_{1Q} + h_{4I}n_{4I} + h_{4Q}n_{4Q} + h_{5I}n_{5I} + h_{5Q}n_{5Q} && (20)\\
\hat{n}_{0Q} &= h_{2I}n_{2Q} - h_{2Q}n_{2I} - h_{3I}n_{3Q} + h_{3Q}n_{3I} + h_{6I}n_{6Q} - h_{6Q}n_{6I} - h_{7I}n_{7Q} + h_{7Q}n_{7I} && (21)\\
\hat{n}_{1I} &= h_{1I}n_{0I} + h_{1Q}n_{0Q} - h_{0I}n_{1I} - h_{0Q}n_{1Q} + h_{5I}n_{4I} + h_{5Q}n_{4Q} - h_{4I}n_{5I} - h_{4Q}n_{5Q} \\
\hat{n}_{1Q} &= h_{3I}n_{2Q} - h_{3Q}n_{2I} + h_{2I}n_{3Q} - h_{2Q}n_{3I} + h_{7I}n_{6Q} - h_{7Q}n_{6I} + h_{6I}n_{7Q} - h_{6Q}n_{7I} \\
\hat{n}_{2I} &= h_{2I}n_{2I} + h_{2Q}n_{2Q} + h_{3I}n_{3I} + h_{3Q}n_{3Q} + h_{6I}n_{6I} + h_{6Q}n_{6Q} + h_{7I}n_{7I} + h_{7Q}n_{7Q} \\
\hat{n}_{2Q} &= h_{0I}n_{0Q} - h_{0Q}n_{0I} - h_{1I}n_{1Q} + h_{1Q}n_{1I} + h_{4I}n_{4Q} - h_{4Q}n_{4I} - h_{5I}n_{5Q} + h_{5Q}n_{5I} \\
\hat{n}_{3I} &= h_{3I}n_{2I} + h_{3Q}n_{2Q} - h_{2I}n_{3I} - h_{2Q}n_{3Q} + h_{7I}n_{6I} + h_{7Q}n_{6Q} - h_{6I}n_{7I} - h_{6Q}n_{7Q} \\
\hat{n}_{3Q} &= h_{1I}n_{0Q} - h_{1Q}n_{0I} + h_{0I}n_{1Q} - h_{0Q}n_{1I} + h_{5I}n_{4Q} - h_{5Q}n_{4I} + h_{4I}n_{5Q} - h_{4Q}n_{5I} \\
\hat{n}_{4I} &= h_{4I}n_{0I} + h_{4Q}n_{0Q} + h_{5I}n_{1I} + h_{5Q}n_{1Q} + h_{0I}n_{4I} + h_{0Q}n_{4Q} + h_{1I}n_{5I} + h_{1Q}n_{5Q} && (22)\\
\hat{n}_{4Q} &= h_{6I}n_{2Q} - h_{6Q}n_{2I} - h_{7I}n_{3Q} + h_{7Q}n_{3I} + h_{2I}n_{6Q} - h_{2Q}n_{6I} - h_{3I}n_{7Q} + h_{3Q}n_{7I} && (23)\\
\hat{n}_{5I} &= h_{5I}n_{0I} + h_{5Q}n_{0Q} - h_{4I}n_{1I} - h_{4Q}n_{1Q} + h_{1I}n_{4I} + h_{1Q}n_{4Q} - h_{0I}n_{5I} - h_{0Q}n_{5Q} \\
\hat{n}_{5Q} &= h_{7I}n_{2Q} - h_{7Q}n_{2I} + h_{6I}n_{3Q} - h_{6Q}n_{3I} + h_{3I}n_{6Q} - h_{3Q}n_{6I} + h_{2I}n_{7Q} - h_{2Q}n_{7I} \\
\hat{n}_{6I} &= h_{6I}n_{2I} + h_{6Q}n_{2Q} + h_{7I}n_{3I} + h_{7Q}n_{3Q} + h_{2I}n_{6I} + h_{2Q}n_{6Q} + h_{3I}n_{7I} + h_{3Q}n_{7Q} \\
\hat{n}_{6Q} &= h_{4I}n_{0Q} - h_{4Q}n_{0I} - h_{5I}n_{1Q} + h_{5Q}n_{1I} + h_{0I}n_{4Q} - h_{0Q}n_{4I} - h_{1I}n_{5Q} + h_{1Q}n_{5I} \\
\hat{n}_{7I} &= h_{7I}n_{2I} + h_{7Q}n_{2Q} - h_{6I}n_{3I} - h_{6Q}n_{3Q} + h_{3I}n_{6I} + h_{3Q}n_{6Q} - h_{2I}n_{7I} - h_{2Q}n_{7Q} \\
\hat{n}_{7Q} &= h_{5I}n_{0Q} - h_{5Q}n_{0I} + h_{4I}n_{1Q} - h_{4Q}n_{1I} + h_{1I}n_{4Q} - h_{1Q}n_{4I} + h_{0I}n_{5Q} - h_{0Q}n_{5I}.
\end{aligned}
$$

Note that the variance of each of

$$\hat{n}_{0I}, \quad \hat{n}_{1I}, \quad \hat{n}_{2Q}, \quad \hat{n}_{3Q}, \quad \hat{n}_{4I}, \quad \hat{n}_{5I}, \quad \hat{n}_{6Q}, \quad \hat{n}_{7Q}$$

is a' and the variance of each of

$$\hat{n}_{0Q}, \quad \hat{n}_{1Q}, \quad \hat{n}_{2I}, \quad \hat{n}_{3I}, \quad \hat{n}_{4Q}, \quad \hat{n}_{5Q}, \quad \hat{n}_{6I}, \quad \hat{n}_{7I}$$

is c'. The random variables $\hat{n}_{iI}$ and $\hat{n}_{iQ}$ are independent for all $i = 0, 1, \cdots, 7$. The decoding can now be done independently for each of the following pairs

$$\{s_0, s_4\}, \{s_1, s_5\}, \{s_2, s_6\}, \{s_3, s_7\}$$

using the following pairs of interleaved received components

$$\{\hat{v}_0, \hat{v}_4\}, \{\hat{v}_1, \hat{v}_5\}, \{\hat{v}_2, \hat{v}_6\}, \{\hat{v}_3, \hat{v}_7\}.$$

We explicitly give the ML decision rule for the pair $\{s_0, s_4\}$ using equations (18)-(23) : The pair $\{\hat{n}_{0I}, \hat{n}_{4I}\}$ is correlated and the covariance matrix is given by $C_I = \begin{bmatrix} a' & b' \\ b' & a' \end{bmatrix}$ and similarly for the pair $\{\hat{n}_{0Q}, \hat{n}_{4Q}\}$, which is independent of $\{\hat{n}_{0I}, \hat{n}_{4I}\}$, the covariance matrix is $C_Q = \begin{bmatrix} c' & d' \\ d' & c' \end{bmatrix}$. The joint pdf of $\hat{n}_0$ and $\hat{n}_4$ is

$$p_{\hat{n}_0,\hat{n}_4}(\hat{n}_0, \hat{n}_4) = \frac{1}{(2\pi)^2|\sqrt{C_I C_Q}|} e^{-\frac{\hat{n}_{0I}^2 a' - 2\hat{n}_{0I}\hat{n}_{4I}b' + \hat{n}_{4I}a'}{2({a'}^2 - {b'}^2)}} e^{-\frac{\hat{n}_{0Q}^2 c' - 2\hat{n}_{0Q}\hat{n}_{4Q}d' + \hat{n}_{4q}c'}{2({c'}^2 - {d'}^2)}}. \tag{24}$$

Let $y_0^{(i)} \in \mathcal{A}$ denote the signal points for different values of i and similarly $y_4^{(j)} \in \mathcal{A}e^{\mathbf{j}\theta}$ denote the signal points for different values of j. Recall that the variables s_0, s_1, s_2, s_3 take values from $\mathcal{A}$ and s_4, s_5, s_6, s_7 from $\mathcal{A}e^{\mathbf{j}\theta}$. The ML decision rule is: Decide $s_0 = y_0^{(i)},\ s_4 = y_4^{(j)}$ for which

$$p_{\hat{n}_0,\hat{n}_4}(\hat{v}_0, \hat{v}_4 / y_0^{(i)}, y_4^{(j)}) \tag{25}$$

is maximum among all possible values for i and j using (24).

4 Diversity and Coding Gain

In this section we derive the conditions under which the scheme presented in the previous two sections achieves full diversity and also show that this condition can always be met by appropriately rotating any given constellation. Also, we identify the parameters that determine the coding gain.

It is known [4] that maximum diversity is achieved when the codeword difference matrix $B(\mathbf{S}, \mathbf{S}') = \mathbf{S} - \mathbf{S}'$ is of full rank for any two distinct codeword matrices $\mathbf{S}, \mathbf{S}'$. For our case, we have

$$B(\mathbf{S}, \mathbf{S}') = \mathbf{S} - \mathbf{S}' = \begin{bmatrix} \tilde{s}_0 - \tilde{s}'_0 & \tilde{s}_1 - \tilde{s}'_1 & 0 & 0 & \tilde{s}_4 - \tilde{s}'_4 & \tilde{s}_5 - \tilde{s}'_5 & 0 & 0 \\ -\tilde{s}_1^* + \tilde{s}_1'^* & \tilde{s}_0^* - \tilde{s}_0'^* & 0 & 0 & -\tilde{s}_5^* + \tilde{s}_5'^* & \tilde{s}_4^* - \tilde{s}_4'^* & 0 & 0 \\ 0 & 0 & \tilde{s}_2 - \tilde{s}'_2 & \tilde{s}_3 - \tilde{s}'_3 & 0 & 0 & \tilde{s}_6 - \tilde{s}'_6 & \tilde{s}_7 - \tilde{s}'_7 \\ 0 & 0 & -\tilde{s}_3^* + \tilde{s}_3'^* & \tilde{s}_2^* - \tilde{s}_2'^* & 0 & 0 & -\tilde{s}_7^* + \tilde{s}_7'^* & \tilde{s}_6^* - \tilde{s}_6'^* \\ \tilde{s}_4 - \tilde{s}'_4 & \tilde{s}_5 - \tilde{s}'_5 & 0 & 0 & \tilde{s}_0 - \tilde{s}'_0 & \tilde{s}_1 - \tilde{s}'_1 & 0 & 0 \\ -\tilde{s}_5^* + \tilde{s}_5'^* & \tilde{s}_4^* - \tilde{s}_4'^* & 0 & 0 & -\tilde{s}_1^* + \tilde{s}_1'^* & \tilde{s}_0^* - \tilde{s}_0'^* & 0 & 0 \\ 0 & 0 & \tilde{s}_6 - \tilde{s}'_6 & \tilde{s}_7 - \tilde{s}'_7 & 0 & 0 & \tilde{s}_2 - \tilde{s}'_2 & \tilde{s}_3 - \tilde{s}'_3 \\ 0 & 0 & -\tilde{s}_7^* + \tilde{s}_7'^* & \tilde{s}_6^* - \tilde{s}_6'^* & 0 & 0 & -\tilde{s}_3^* + \tilde{s}_3'^* & \tilde{s}_2^* - \tilde{s}_2'^* \end{bmatrix} \tag{26}$$

and

$$B^{\mathcal{H}}(\mathbf{S}, \mathbf{S}') B(\mathbf{S}, \mathbf{S}') = \begin{bmatrix} a_\Delta I_2 & O_2 & b_\Delta I_2 & O_2 \\ O_2 & c_\Delta I_2 & O_2 & d_\Delta I_2 \\ b_\Delta I_2 & O_2 & a_\Delta I_2 & O_2 \\ O_2 & d_\Delta I_2 & O_2 & c_\Delta I_2 \end{bmatrix} \tag{27}$$

where

$$\begin{gathered} a_\Delta = |\tilde{s}_0 - \tilde{s'}_0|^2 + |\tilde{s}_1 - \tilde{s'}_1|^2 + |\tilde{s}_4 - \tilde{s'}_4|^2 + |\tilde{s}_5 - \tilde{s'}_5|^2 \\ b_\Delta = 2((\tilde{s}_{0I} - \tilde{s}'_{0I})(\tilde{s}_{4I} - \tilde{s}'_{4I}) + (\tilde{s}_{0Q} - \tilde{s}'_{0Q})(\tilde{s}_{4Q} - \tilde{s}'_{4Q}) + (\tilde{s}_{1I} - \tilde{s}'_{1I})(\tilde{s}_{5I} - \\ \tilde{s}'_{5I}) + (\tilde{s}_{1I} - \tilde{s}'_{1I})(\tilde{s}_{5I} - \tilde{s}'_{5I})) \\ c_\Delta = |\tilde{s}_2 - \tilde{s'}_2|^2 + |\tilde{s}_3 - \tilde{s'}_3|^2 + |\tilde{s}_6 - \tilde{s'}_6|^2 + |\tilde{s}_7 - \tilde{s'}_7|^2 \\ d_\Delta = 2((\tilde{s}_{2I} - \tilde{s}'_{2I})(\tilde{s}_{6I} - \tilde{s}'_{6I}) + (\tilde{s}_{2Q} - \tilde{s}'_{2I})(\tilde{s}_{6Q} - \tilde{s}'_{6I}) + (\tilde{s}_{3I} - \tilde{s}'_{3I})(\tilde{s}_{7I} - \\ \tilde{s}'_{7I}) + (\tilde{s}_{3I} - \tilde{s}'_{3I})(\tilde{s}_{7I} - \tilde{s}'_{7I})). \end{gathered}$$

Using Schur's determinant formula, it is easily seen that the determinant of the matrix (27) is $(a_\Delta^2 - b_\Delta^2)^2 (c_\Delta^2 - d_\Delta^2)^2$. So, we get full rank iff

$$(a_\Delta - b_\Delta)(a_\Delta + b_\Delta)(c_\Delta - d_\Delta)(c_\Delta + d_\Delta) \neq 0$$

for all possible distinct pairs of signal points s_i and s'_i with each taking values taken from the signal set $\mathcal{A}$ for $i = 0, 1, 2, 3$ and from the signal set $\mathcal{A}e^{\mathbf{j}\theta}$ for $i = 4, 5, 6, 7$. It is easily verified by straight forward manipulation of terms that the above condition translates to the product of the following four terms being non-zero:

$$
\begin{aligned}
&[\{(s_{0I}-s'_{0I})-(s_{4I}-s'_{4I})\}^2+\{(s_{2Q}-s'_{2Q})-(s_{6Q}-s'_{6Q})\}^2+\{(s_{1I}-s'_{1I})-(s_{5I}-s'_{5I})\}^2+\{(s_{3Q}-s'_{3Q})-(s_{7Q}-s'_{7Q})\}^2]\\
&[\{(s_{0I}-s'_{0I})+(s_{4I}-s'_{4I})\}^2+\{(s_{2Q}-s'_{2Q})+(s_{6Q}-s'_{6Q})\}^2+\{(s_{1I}-s'_{1I})+(s_{5I}-s'_{5I})\}^2+\{(s_{3Q}-s'_{3Q})+(s_{7Q}-s'_{7Q})\}^2\\
&[\{(s_{0Q}-s'_{0Q})-(s_{4Q}-s'_{4Q})\}^2+\{(s_{2I}-s'_{2I})-(s_{6I}-s'_{6I})\}^2+\{(s_{1Q}-s'_{1Q})-(s_{5Q}-s'_{5Q})\}^2+\{(s_{3I}-s'_{3I})-(s_{7I}-s'_{7I})\}^2]\\
&[\{(s_{0Q}-s'_{0Q})+(s_{4Q}-s'_{4Q})\}^2+\{(s_{2I}-s'_{2I})+(s_{6I}-s'_{6I})\}^2+\{(s_{1Q}-s'_{1Q})+(s_{5Q}-s'_{5Q})\}^2+\{(s_{3I}-s'_{3I})+(s_{7I}-s'_{7I})\}^2].
\end{aligned}
$$

Now, suppose $s_i = s'_i$ for all values of i except for one, say $i = 0$. Then the condition above requires that $(s_{0I} - s'_{0I})(s_{0Q} - s'_{0Q}) \neq 0$. To put this necessary condition in a neat form we introduce

Definition 4.1 *The Coordinate Product Distance (CPD) between any two signal points $u = u_I + \mathbf{j}u_Q$ and $v = v_I + \mathbf{j}v_Q$, $u \neq v$, in a signal set $\mathcal{A}$ is defined as*

$$CPD(u, v) = |u_I - v_I||u_Q - v_Q| \tag{28}$$

and the minimum of this value among all possible pairs is defined as the CPD of $\mathcal{A}$.

Then we have

Theorem 4.1 *A necessary condition for the STBC obtained from the design S_8 with variables taking values from signal sets $\mathcal{A}$ and $\mathcal{A}e^{\mathbf{j}\theta}$, to give full-rank (equal to 8) is that the CPD of $\mathcal{A}$ as well as that of $\mathcal{A}e^{\mathbf{j}\theta}$ is not zero.*

In simple terms, the condition of the above theorem is that no two signal points in $\mathcal{A}$ as well as in $\mathcal{A}e^{\mathbf{j}\theta}$ lie on a horizontal or vertical line of the complex plane. It follows that signal constellations with $CPD = 0$ like regular M-ary QAM, symmetric M-ary PSK with one signal point on the x-axis, will not achieve full diversity. But the situation gets easily salvaged by simply rotating these signal sets by appropriate angle and get this condition satisfied. Notice that there are only a finite set of angles of rotations for which the CPD is zero and infinitely many for which the condition is fulfilled. Hence forth we assume that the condition of Theorem 4.1 is satisfied.

Even with the condition of Theorem 4.1 satisfied, we may still end up with the product of the four terms discussed above being zero, since the terms of the form

$$\{(s_{iI} - s'_{iI}) \pm (s_{jI} - s'_{jI})\} \text{ or } \{(s_{iQ} - s'_{iQ}) \pm (s_{jQ} - s'_{jQ})\} \; i = 0, 1, 2, 3 \text{ and } j = 4, 5, 6, 7$$

may still be zero. However, we can choose the angle of rotation θ for the signal set $\mathcal{A}e^{\mathbf{j}\theta}$ such that none of these terms become zero as follows: Let

$$\mathcal{A} = \{a^{(1)}, \quad a^{(2)}, \quad \cdots, \quad a^{(M)}\}$$

be the signal set with M points from which the variables s_i, $i = 0, 1, 2, 3$ take values and the CPD of $\mathcal{A}$ is not zero. Also let

$$d_I^{(i_1,i_2)} = a_I^{(i_1)} - a_I^{(i_2)} \quad \text{and} \quad d_Q^{(i_1,i_2)} = a_Q^{(i_1)} - a_Q^{(i_2)}$$

and

$$\mathcal{D}_I = \{d_I^{(i_1,i_2)} | i_1, i_2 = 1, 2, \cdots, M\} \text{ and } \mathcal{D}_Q = \{d_Q^{(i_1,i_2)} | i_1, i_2 = 1, 2, \cdots, M\}.$$

Note that $d_I^{(i_1,i_2)}$ ($d_Q^{(i_1,i_2)}$) denotes the difference of the in-phase (quadrature-) components of $a^{(i_1)}$ and $a^{(i_2)}$ and $\mathcal{D}_I$ ($\mathcal{D}_Q$) denotes the set of all possible such differences. Let

$$d_{I,\theta}^{(i_1,i_2)}, \quad d_{Q,\theta}^{(i_1,i_2)}, \quad \mathcal{D}_{I,\theta} \quad \text{and} \quad \mathcal{D}_{Q,\theta}$$

denote the corresponding quantities for the rotated signal set $\mathcal{A}e^{\mathbf{j}\theta}$. We have the following relation among the set of differences of the in- and quadrature phase components of the rotated and the original signal set:

$$\begin{bmatrix} d_{I,\theta}^{(i_1,i_2)} \\ d_{Q,\theta}^{(i_1,i_2)} \end{bmatrix} = \begin{bmatrix} cos(\theta) & -sin(\theta) \\ sin(\theta) & cos(\theta) \end{bmatrix} \begin{bmatrix} d_I^{(i_1,i_2)} \\ d_Q^{(i_1,i_2)} \end{bmatrix}. \tag{29}$$

We need to choose θ such that

$$\mathcal{D}_I \cap \mathcal{D}_{I,\theta} = \Phi \text{ and } \mathcal{D}_Q \cap \mathcal{D}_{Q,\theta} = \Phi$$

where Φ denotes the empty set. Since $\mathcal{D}_I$ and $\mathcal{D}_Q$ are finite sets, only for finite number of values of θ these can have non-empty intersection with $\mathcal{D}_{I,\theta}$ and $\mathcal{D}_{Q,\theta}$. Since θ can take continuous values we can always choose an angle such that $\mathcal{D}_I \cap \mathcal{D}_{I,\theta} = \Phi$ and $\mathcal{D}_Q \cap \mathcal{D}_{Q,\theta} = \Phi$ as well as the CPD of $\mathcal{A}e^{\mathbf{j}\theta}$ is not zero. For all such choices the STBC from the CID S_8 gives full diversity order. In the following example we illustrate one such choice for 4-QAM signal set.

Example 4.1 *Consider the regular* 4*-QAM signal set (square) consisting of the following four points* $\{(\pm d, \pm d)\}$*. The CPD of this* 4*-QAM is zero. So, we rotate this by an angle* $\phi = 15$ *degrees clockwise to get the signal set* $\mathcal{A}$ *with CPD not equal to zero. We have*

$$\mathcal{A} = \{A = (\sqrt{2}d\ cos(30), \sqrt{2}d\ sin(30)), B = (-\sqrt{2}d\ sin(30), \sqrt{2}d\ cos(30)), C = (-\sqrt{2}d\ cos(30), -\sqrt{2}d\ sin(30)), D = (\sqrt{2}d\ sin(30), -\sqrt{2}d\ cos(30))\}$$

with the CPD equal to d^2. *We have*

$(x,y)=$	(A,B)	(A,C)	(A,D)	(B,C)	(B,D)	(C,D)
$d_I^{(x,y)}$	$\frac{d}{\sqrt{2}}(\sqrt{3}+1)$	$\frac{d}{\sqrt{2}}(2\sqrt{3})$	$\frac{d}{\sqrt{2}}(\sqrt{3}-1)$	$\frac{d}{\sqrt{2}}(\sqrt{3}-1)$	$\frac{d}{\sqrt{2}}(-2)$	$\frac{d}{\sqrt{2}}(-\sqrt{3}-1)$
$d_Q^{(x,y)}$	$\frac{d}{\sqrt{2}}(1-\sqrt{3})$	$\frac{d}{\sqrt{2}}2$	$\frac{d}{\sqrt{2}}(1+\sqrt{3})$	$\frac{d}{\sqrt{2}}(\sqrt{3}+1)$	$\frac{d}{\sqrt{2}}(2\sqrt{3})$	$\frac{d}{\sqrt{2}}(\sqrt{3}-1)$

and

$$\mathcal{D}_I = \left\{\pm\tfrac{d}{\sqrt{2}}(\sqrt{3}+1), \quad \pm\tfrac{d}{\sqrt{2}}(2\sqrt{3}), \quad \pm\tfrac{d}{\sqrt{2}}(\sqrt{3}-1), \quad \pm\tfrac{d}{\sqrt{2}}2\right\} = \mathcal{D}_Q.$$

We consider the rotated signal $\mathcal{A}e^{\mathbf{j}\theta}$ *with* $\theta = -15$ *degrees. With this choice we have*

$$\mathcal{A}e^{-\mathbf{j}15} = \{A' = (\sqrt{2}d\ cos(15), \sqrt{2}d\ sin(15)), B' = (-\sqrt{2}d\ sin(15), \sqrt{2}d\ cos(15)), C' = (-\sqrt{2}d\ cos(15), -\sqrt{2}d\ sin(15)), D' = (\sqrt{2}d\ sin(15), -\sqrt{2}d\ cos(15))\}$$

with the CPD equal to $\sqrt{3}d^2$. *We have*

$(x,y)=$	(A',B')	(A',C')	(A',D')
$d_I^{(x,y)}$	$\sqrt{2+\sqrt{3}}+\sqrt{2-\sqrt{3}}$	$2\sqrt{2+\sqrt{3}}$	$\sqrt{2+\sqrt{3}}-\sqrt{2-\sqrt{3}}$
$d_Q^{(x,y)}$	$\sqrt{2-\sqrt{3}}-\sqrt{2+\sqrt{3}}$	$2\sqrt{2-\sqrt{3}}$	$\sqrt{2-\sqrt{3}}+\sqrt{2+\sqrt{3}}$

$(x,y)=$	(B',C')	(B',D')	(C',D')
$d_I^{(x,y)}$	$\sqrt{2+\sqrt{3}}-\sqrt{2-\sqrt{3}}$	$-2\sqrt{2-\sqrt{3}}$	$-\sqrt{2+\sqrt{3}}-\sqrt{2-\sqrt{3}}$
$d_Q^{(x,y)}$	$\sqrt{2+\sqrt{3}}+\sqrt{2-\sqrt{3}}$	$2\sqrt{2+\sqrt{3}}$	$\sqrt{2+\sqrt{3}}-\sqrt{2-\sqrt{3}}$

with each entry multiplied by $\frac{d}{\sqrt{2}}$ *and*

$$\mathcal{D}_{I,-15} = \{\pm\tfrac{d}{\sqrt{2}}(\sqrt{2+\sqrt{3}}+\sqrt{2-\sqrt{3}}), \pm\tfrac{d}{\sqrt{2}}(2\sqrt{2-\sqrt{3}}), \pm\tfrac{d}{\sqrt{2}}(\sqrt{2+\sqrt{3}}-\sqrt{2-\sqrt{3}}), \pm\tfrac{d}{\sqrt{2}}2(\sqrt{2-\sqrt{3}})\} = \mathcal{D}_{Q,-15}.$$

Since $\mathcal{D}_I \cap \mathcal{D}_{I,-15} = \Phi$ *the angle of rotation* $\theta = -15$ *degrees will give full diversity.*

Assuming that full-rank is achieved, the resulting coding gain is given by the minimum of the square root of the matrix in (27) over all possible pairs of distinct codeword matrices [4]. From the discussion just before the Definition 4.1 it follows that this coding gain is upper bounded by

$$min\{CPD(\mathcal{A}), CPD(\mathcal{A}e^{\mathbf{j}\theta})\}.$$

5 Simulation Results

Figure 2 shows simulation results for data rate of 1.5 bits/sec/Hz over a quasi-static Rayleigh fading channel. The fading is assumed to be constant over a fade length of 120 symbol durations.

We compare our scheme for 8 transmitters with rate 3/4 QODs and rate 1/2 real OD at same energy per bit (E_b/N_0). For the proposed scheme we choose signals from 3-PSK rotated by angles $15°$ for the first two variable and $-15°$ for the last two. This results in a rate of 1.58 bits/sec/Hz. For QODs we select symbols from 4-QAM and for OD from 8-QAM. Two curves for QODs are shown: one with the symbols rotated to achieve full diversity and other where the symbols are not rotated. Note that the scheme of this correspondence out-performs QODs at low ρ (SNR), but is inferior at $\rho > 5.5dB$. This is due to different coding gains of these schemes. The coding gain of the proposed scheme is 0.09 while that of quasi-orthogonal design (full diversity) is 0.27 and that of orthogonal designs is 0.24. It is well known that the rank and determinant criteria apply at high SNR's and at low SNR's and/or when the number of antennas is large the performance is determined by the trace criterion [18]. This explains the behavior of the curves shown in the figure.

6 Discussion

We have presented a STBC scheme for eight transmit antennas that achieves full-diversity of 8 and symbol-rate 1 based on a non-orthogonal design. That a rate-one design is not possible with complex orthogonal design for any number of antennas more than 2 has been shown in [16, 17]. Our construction is based on a coordinate interleaved design, first proposed for 4 transmit antennas in [11]. The conditions for the resulting STBC to have full diversity of 8 turns out to be the union of the conditions given in [11] and [15].

An apparent drawback is a large peak-to-average power ratio transmission due to zeros in the transmission matrix. This can be removed by "power uniformization" methods similar to those discussed in [9]. An interesting direction for further research is to investigate the usefulness of the several generalized Walsh-Hadamard transform matrices discussed in [19, 20, 21] for issues related to space-time coding.

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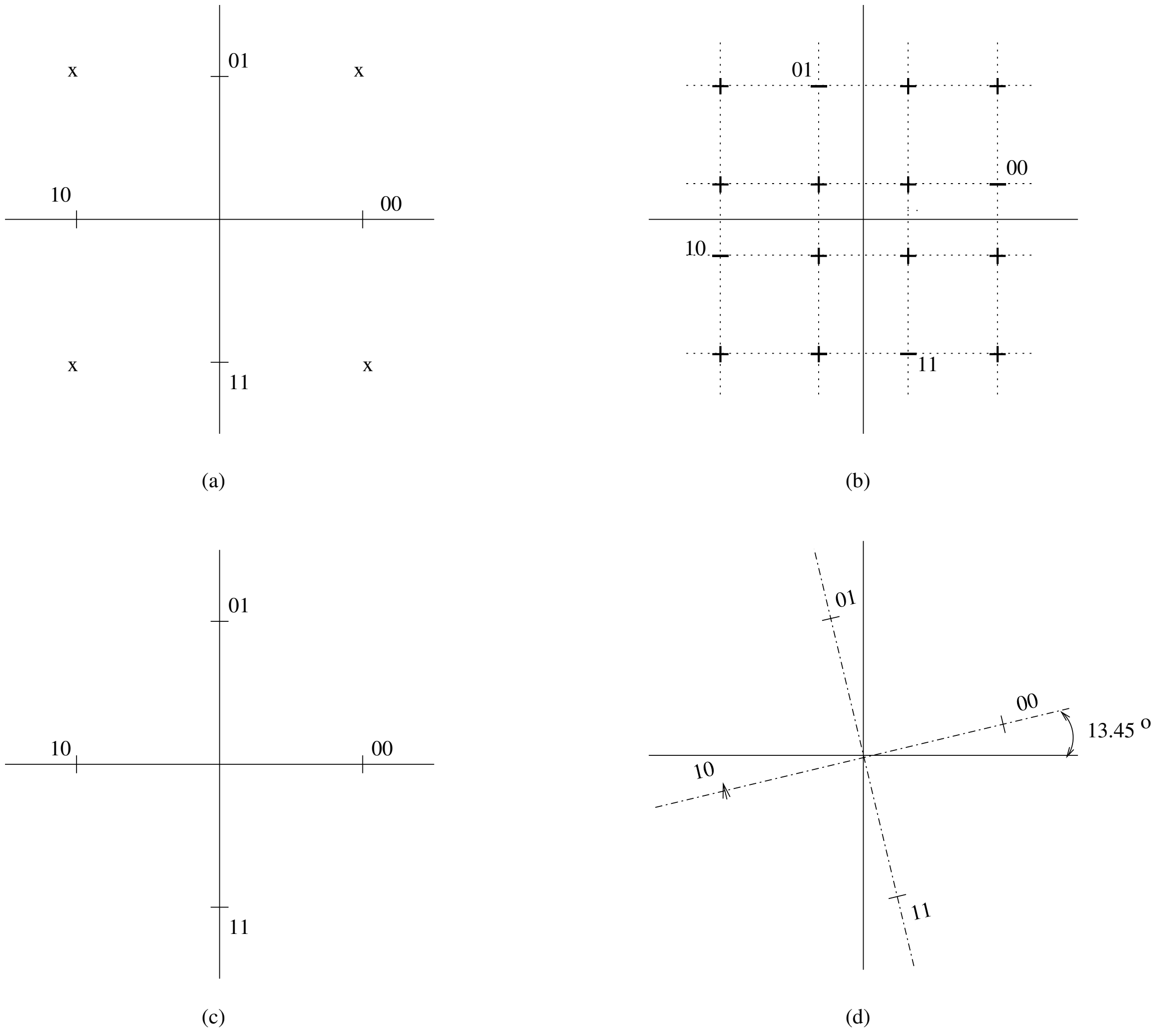

Figure 1: Expanded signal sets $\tilde{\mathcal{A}}$ for $\mathcal{A} = \{1, -1, \mathbf{j}, -\mathbf{j}\}$ and a rotated version of it.

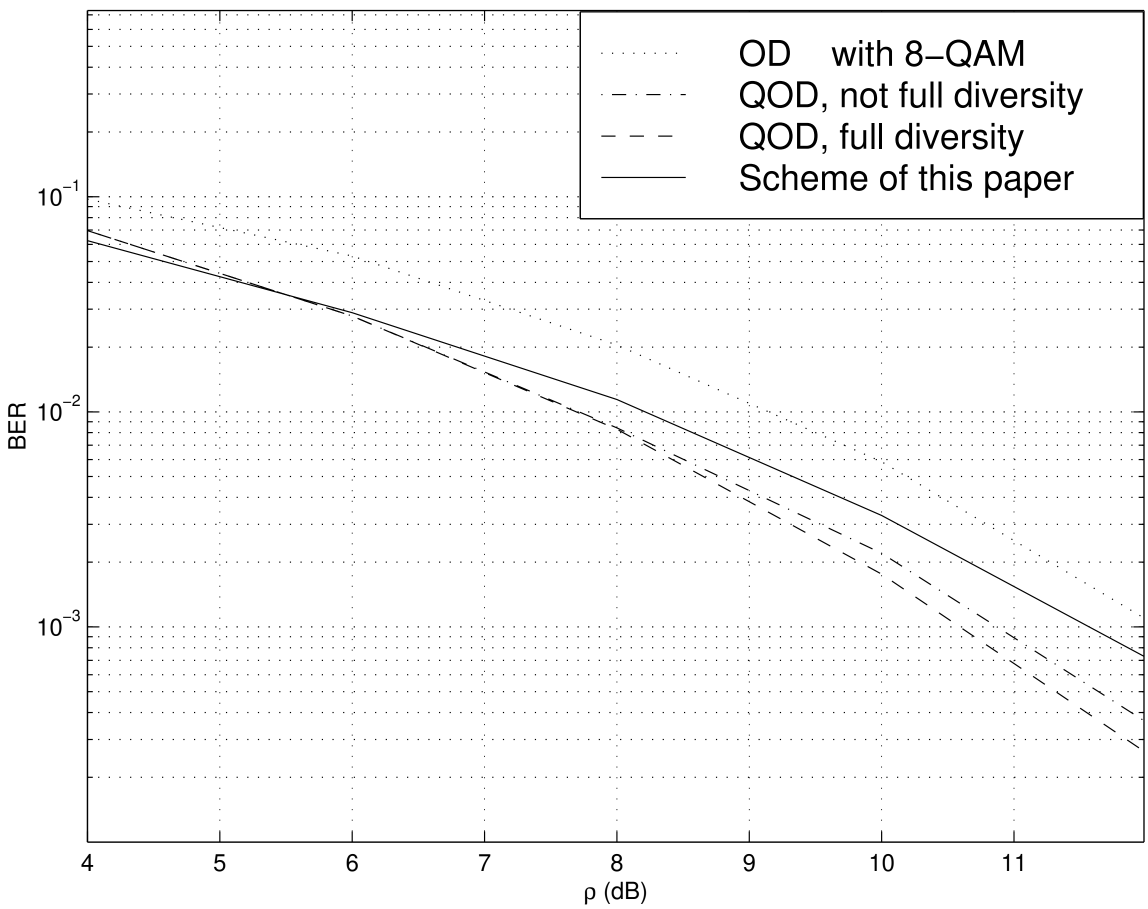

Figure 2: The BER performance of STBCs from OD, QODs and the design of this paper at 1.5 bits/sec/Hz in quasi-static Rayleigh fading channel).