MMSE Optimal Space-Time Block Codes from Crossed Product Algebras

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Abstract

Design of Space-Time Block codes (STBCs) for Maximum Likelihood (ML) reception has been predominantly the main focus of researchers. However the ML decoding complexity of STBCs become prohibitive large as the number of transmit and receive antennas increase. To solve this problem, orthogonal designs, single symbol ML decodable and double symbol ML decodable STBCs have been proposed in the literature. But the rate of these STBCs fall exponentially with increase in the number of transmit antennas. Hence it is natural to resort to suboptimal reception techniques like linear Minimum Mean Squared Error (MMSE) receiver. Barbarossa et al and Liu et al have independently derived necessary and sufficient conditions for a full rate linear STBC to be MMSE optimal, i.e achieve least Symbol Error Rate (SER). Motivated by this problem, we construct a new class of full rate MMSE optimal STBCs for any number of transmit antennas using crossed product algebras. It is also shown that codes from cyclic division algebras which are special cases of crossed product algebras are also MMSE optimal. Hence these STBCs achieve least SER when MMSE reception is employed and are fully diverse when ML reception is employed.

1. Introduction & Background

It is well known that employing multiple transmit and receive antennas is an effective means to combat fading in a wireless channel. Also MIMO(Multiple input Multiple Output) systems offer significant capacity gains compared to single input single output systems. Coding for such situations is called space-time coding. Space-Time Block Codes(STBCs) were formally introduced in [1], wherein a design criteria for achieving full diversity (equal to product of number of transmit and receive antennas) with a ML(Maximum Likelihood) receiver was derived. But the ML decoding complexity of STBCs become prohibitively large for large number of transmit and receive antennas. The sphere decoder helps to some extent in reducing the complexity but is still far away from practicality for large number of transmit antennas. In [2, 3, 4], orthogonal designs, single and double symbol ML decodable STBCs have been proposed to solve this problem. But unfortunately, the rate of such codes fall exponentially with increase in the number of transmit antennas which leads to inefficient usage of the capacity gains offered by MIMO systems. This led to the study of suboptimal reception strategies such as linear MMSE(Minimum Mean Square Error) and linear ZF(Zero Forcing) receivers [5]-[9],[14]. It is then natural to address the question of how to design STBCs which are optimal for a linear MMSE receiver. This problem was addressed in [5, 6, 7, 8, 9].

Definition 1 *A $N \times N$ linear STBC S in variables $x_1, \ldots, x_k$ given by*

$$S = \sum_{i=1}^{k} x_i A_i \tag{1}$$

is called a unitary trace-orthogonal STBC if the set of matrices $A_i, i = 1, \ldots k$ satisfy the following conditions

$$A_i A_i^H = \frac{N}{k} I_N \tag{2}$$

$$Tr(A_i^H A_j) = 0, \forall\, i \neq j \tag{3}$$

If $k = N^2$ we refer to it as full rate transmission.

It was shown in [5, 6, 7, 8, 9] that if full rate transmission is considered, unitary trace-orthogonality is the necessary and sufficient condition for a linear STBC to achieve minimum bit error rate when the variables take values from a QPSK(Quadrature Phase Shift Keying) constellation. Further it was shown that full rate unitary trace orthogonal STBC achieve minimum mean squared error when other 2-dimensional constellations are used. Also it was shown that at high SNR, the predominant metric that decides probability of symbol error is optimized only by unitary trace orthogonal STBCs. Hence we refer to full rate unitary trace orthogonal STBCs as MMSE optimal STBCs. Few constructions of such codes are given in [5, 6, 7, 8, 11]. However these constructions were based on matrix manipulations and lacked an algebraic theory behind them.

The contributions of this paper are as follows.

- Construct a new class of MMSE optimal STBCs for arbitrary number of transmit antennas using crossed product algebras.

- Since the code constructions are algebraic, the description of the code is elegant and it also simplifies the study of their properties.
- Few of the existing code constructions [5, 6, 8, 11] are shown to be special cases of the constructions in this paper.
- By restricting to cyclic division algebras, we obtain STBCs which are simultaneously MMSE optimal as well as fully diverse for ML reception.

The rest of the paper is organized as follows. In Section 2, we briefly describe our main algebraic tool, i.e crossed product algebras and also explicitly construct STBCs from crossed product algebras. In Section 3, we provide sufficient conditions as to when STBCs from crossed product algebras are MMSE optimal. Then we focus on a proper subclass of crossed product algebras called cyclic algebras and show that they are MMSE optimal. Few illustrative examples of code constructions are provided for better explanation. Discussions on future work comprise Section 4.

Notation:
For a matrix A, the matrix A^H denotes the conjugate transpose of A. $End_K(A)$ denotes the set of all K-linear maps from A to A. The symbols j and ω_n denote square root of -1 and nth root of unity respectively.

2. STBCs from Crossed Product Algebras

In this section, we briefly introduce crossed product algebras and obtain STBCs using matrix representations of crossed product algebras. We refer the readers to [12] for a detailed explanation of crossed product algebras.

Let F be a field. Then, an associative algebra F-algebra A is called a central simple algebra if the center of A is F and A is a simple algebra, i.e., A does not have nontrivial two-sided ideals. Simple examples of central simple algebras are division algebras and matrix algebras over fields. It is well known that the dimension $[A : F]$ of A over its center is always a perfect square, say n^2 [12, 15]. The square root of $[A : F]$ is called the degree of A. Let K be a strictly maximal subfield of A, i.e., $K \subset A$ and K is not contained in any other subfield of A and the centralizer of K in A is K itself. It is well known that $[K : F] = n$, the degree of the algebra. In addition, let the extension K/F be a Galois extension and let $G = \{\sigma_0 = 1, \sigma_1, \sigma_2, \dots \sigma_{n-1}\}$ be the Galois group of K/F. Let ϕ be a map from $G \times G$ to $K\backslash\{0\}$ called the cocycle which satisfies the cocycle condition ($\phi(\sigma, \tau\gamma)\phi(\tau, \gamma) = \phi(\sigma\tau, \gamma)\gamma(\phi(\sigma, \tau))$ for all $\sigma, \tau, \gamma \in G$). Then, the algebra A is called a Crossed Product Algebra if

$$A = \bigoplus_{\sigma_i \in G} u_{\sigma_i} K$$

where, equality and addition are component-wise and where u_σ are symbols such that i) $\sigma(k) = u_\sigma^{-1} k u_\sigma$ and ii) $u_\sigma u_\tau = u_{\sigma\tau}\phi(\sigma, \tau)$ for all $k \in K, \sigma, \tau \in G$. It is clear that A can be seen as a right K-space of dimension n over K. Also multiplication between two elements of A, say $a = \sum_{i=0}^{n-1} u_{\sigma_i} k_{\sigma_i}$ and $a' = \sum_{j=0}^{n-1} u_{\sigma_j} k'_{\sigma_j}$ is given by

$$\left(\sum_{i=0}^{n-1} u_{\sigma_i} k_{\sigma_i}\right)\left(\sum_{j=0}^{n-1} u_{\sigma_j} k'_{\sigma_j}\right) = \sum_{l=0}^{n-1} u_{\sigma_l} k''_{\sigma_l}$$

where, $k''_{\sigma_l} = \sum_{\sigma_i\sigma_j=\sigma_l} \phi(\sigma_i, \sigma_j)\sigma_j(k_{\sigma_i})k'_{\sigma_j}$ We will denote the crossed product algebra A by (K, G, ϕ). The field K can be seen as an n-dimensional F-vector space. Let $B = \{t_0, t_1, \dots t_{n-1}\}$ be a basis of K over F. Then, the left regular representation [12] of A in $End_K(A)$ is given by the map $L : A \mapsto End_K(A)$ which is defined as follows.

$$L(a) = \lambda_a \text{ where, } \lambda_a(u) = au, \forall u \in A$$

The matrix representation M_a of the linear transformation λ_a with respect to the basis $\{u_{\sigma_i} : \sigma_i \in G\}$ is given by (4) shown at the top of the next page where, $f_{\sigma_j}^{(i)} \in F, \forall\ 0 \le i, j \le n-1$, $\mu_{i,j} = \sigma_i\sigma_j^{-1}$, $\beta_i^{(j)} = \phi(\sigma_i\sigma_j^{-1}, \sigma_j)$ and P is a scaling factor to normalize the average total power of a codeword to n^2. Thus we have obtained a full rate linear STBC M_a in variables $f_{\sigma_j}^{(}i), 0 \le i, j \le n-1$ from the crossed product algebra A. M_a can expressed in a linear dispersion form [**?**] as follows.

$$M_a = \sum_{j=0}^{n-1}\sum_{i=0}^{n-1} f_{\sigma_j}^{(i)} W_{i,j}$$

where, the matrices $W_{i,j}$ are called the 'weight matrices' of M_a. Then, we have

$$W_{i,j} = \frac{1}{\sqrt{P}} P_j Q_i \tag{5}$$

where,

$$Q_i = \begin{bmatrix} t_i & 0 & \cdots & 0 \\ 0 & \sigma_1(t_i) & \ddots & \vdots \\ \vdots & \ddots & \ddots & 0 \\ 0 & \cdots & 0 & \sigma_{n-1}(t_i) \end{bmatrix} \tag{6}$$

and the matrix P_j can be described as follows. Let us index the rows and columns of P_j with the elements of G. Then the (σ_k, σ_l)th entry of P_j is equal to $\phi(\sigma_j, \sigma_l)$ if $\sigma_j\sigma_l = \sigma_k$ and 0 otherwise.

The matrices P_j and Q_i are nothing but the images of u_{σ_j} and t_i respectively under the map L. Note that every column of P_j has exactly one non-zero entry and any two columns of P_j have their non-zero entries in completely disjoint set of rows.

$$M_a = \frac{1}{\sqrt{P}} \begin{bmatrix} \sum_{i=0}^{n-1} f^{(i)}_{\sigma_0} t_i & \beta_0^{(1)} \sum_{i=0}^{n-1} f^{(i)}_{\mu_{0,1}} \sigma_1(t_i) & \beta_0^{(2)} \sum_{i=0}^{n-1} f^{(i)}_{\mu_{0,2}} \sigma_2(t_i) & \cdots & \beta_0^{(n-1)} \sum_{i=0}^{n-1} f^{(i)}_{\mu_{0,n-1}} \sigma_{n-1}(t_i) \\ \sum_{i=0}^{n-1} f^{(i)}_{\sigma_1} t_i & \beta_1^{(1)} \sum_{i=0}^{n-1} f^{(i)}_{\mu_{1,1}} \sigma_1(t_i) & \beta_1^{(2)} \sum_{i=0}^{n-1} f^{(i)}_{\mu_{1,2}} \sigma_2(t_i) & \cdots & \beta_1^{(n-1)} \sum_{i=0}^{n-1} f^{(i)}_{\mu_{1,n-1}} \sigma_{n-1}(t_i) \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ \sum_{i=0}^{n-1} f^{(i)}_{\sigma_{n-1}} t_i & \beta_{n-1}^{(1)} \sum_{i=0}^{n-1} f^{(i)}_{\mu_{0,1}} \sigma_1(t_i) & \beta_{n-1}^{(2)} \sum_{i=0}^{n-1} f^{(i)}_{\mu_{0,2}} \sigma_2(t_i) & \cdots & \beta_{n-1}^{(n-1)} \sum_{i=0}^{n-1} f^{(i)}_{\mu_{0,n-1}} \sigma_{n-1}(t_i) \end{bmatrix} \tag{4}$$

3. MMSE optimal STBCs

In this section, we provide sufficient conditions as to when STBCs from crossed product algebras are MMSE optimal. Then, we focus on a proper subclass of crossed product algebras called cyclic algebras and obtain a class of STBCs meeting the required conditions for MMSE optimality. It turns out that the codes in [5, 6, 11] are special cases of our construction. Few illustrative code construction examples are also provided. Finally, we discuss the decoding procedure for the codes in this paper and also highlight its simplicity as compared to ML decoding.

Theorem 1 *The STBC M_a constructed as in* (4) *using the crossed product algebra $A = (K, G, \phi)$ is an MMSE optimal code if*

$$|\sigma_j(t_i)| = |t_i| = |\phi(\sigma_i, \sigma_j)| = 1, \forall\, 0 \leq i, j \leq n-1 \tag{7}$$

$$\sum_{i=0}^{n-1} \sigma_j(t_i)(\sigma_{j'})(t_i))^* = 0, \text{ if } j \neq j'. \tag{8}$$

Proof: We need to show that the weight matrices of M_a satisfy (2) and (3). (7) implies that the matrices P_j and Q_i are scaled unitary matrices. The scaling factor P here equals N. Therefore $W_{i,j} W_{i,j}^H = \frac{I_n}{n}$ which implies (2) is satisfied.

It can be shown [7] that condition (3) is equivalent to the condition that the matrix Φ which is shown in (9) at the top of the next page satisfies $\Phi\Phi^H = nI_n^2$. The (k, l)th element of $\Phi\Phi^H$ is given by

$$\sum_{a=0}^{n-1} \phi(\sigma_i \sigma_j^{-1}, \sigma_j) \sigma_j(t_a) \left(\phi\left(\sigma_{i'} \sigma_{j'}^{-1}, \sigma_{j'}\right) \sigma_{j'}(t_a) \right)^*$$

which simplifies to

$$\phi(\sigma_i \sigma_j^{-1}, \sigma_j) \phi\left(\sigma_{i'} \sigma_{j'}^{-1}, \sigma_{j'}\right) \sum_{a=0}^{n-1} \sigma_j(t_a)(\sigma_{j'}(t_a))^*$$

which is equal to zero from the statement of the theorem. If $k = l$, then we have

$$(\Phi\Phi^H)_{k,k} = \sum_{a=0}^{n-1} |\sigma_j(t_a)|^2 = n$$

Thus, $\Phi\Phi^H = nI_n^2$ which in turn implies (3) is satisfied.

Theorem 1 gives conditions on the basis of a Galois extension and on the cocycle which result in MMSE optimal STBCs.

3.1. STBCs from Cyclic Algebras

In this subsection, we study a proper subclass of crossed product algebras called cyclic algebras and give an explicit construction of MMSE optimal codes satisfying the conditions of Theorem 1.

An F-central simple algebra is called a cyclic algebra, if A has a strictly maximal subfield K which is a cyclic extension of the center F. Clearly, a cyclic algebra is a crossed product algebra. Let σ be a generator of the Galois group G. If $u_{\sigma^i}, i = 0, 1, \ldots, n-1$ is a basis for the algebra A over K, then we have

$$\begin{aligned} u_{\sigma^i} &= u_\sigma^i \\ \phi(\sigma^i, \sigma^j) &= \left\{ \begin{array}{ll} 1, & \text{if } i + j < n \\ \delta, & \text{if } i + j \geq n \end{array} \right\} \end{aligned}$$

where, $u_\sigma^n = \delta$. Since the cocycle can now be described by just one element δ and similarly G can be described by σ, we denote the crossed product algebra (K, G, ϕ) with (K, σ, δ). Thus, with $z = u_\sigma$, we have

$$A = (K, \sigma, \delta) = \bigoplus_{i=0}^{n-1} z^i K$$

where, $z^n = \delta$ and $kz = z\sigma(k), \forall k \in K$.

Note that if the smallest positive integer t such that δ^t is the norm of some element in $K \backslash \{0\}$ is n, then the cyclic algebra $A = (K, \sigma, \delta)$ is a cyclic division algebra [13].

Construction 1 *Let K/F be a cyclic extension of degree n with $K = F(t_n = t^{1/n})$, $t, \omega_n \in F$, $|t| = 1$. Here ω_n denotes the nth root of unity and $\sigma : t_n \mapsto \omega_n t_n$ is the generator of the Galois group. Let δ be a transcendental element over K. Then the STBC arising from the cyclic division algebra $(K(\delta)/F(\delta), \sigma, \delta)$ is MMSE optimal. MMSE optimality follows because of the following identities and Theorem 1.*

$$\begin{array}{c} |t| = |\delta| = |\sigma^i(t_n)| = 1, \; i = 0, 1, \ldots, n-1 \\ \sum_{i=0}^{n-1} (t_n)^i (\sigma^k(t_n^i))^* = 0, \text{ if } k \neq 0 \end{array} \tag{10}$$

The MMSE optimal STBC M_a is given by

$$M_a = \sum_{j=0}^{n-1} \sum_{i=0}^{n-1} f_j^{(i)} W_{i,j}, \quad f_j^{(i)} \in F \tag{11}$$

$$\Phi = \left[\begin{array}{cccccc} vec(W_{0,0}) & vec(W_{1,0}) & \ldots & vec(W_{n-1,0}) & vec(W_{0,n-1}) & \ldots & vec(W_{n-1,n-1}) \end{array} \right] \quad (9)$$

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where, the weight matrices $W_{i,j} = t_n^i P^j Q^i$. The matrices P and Q are as shown below.

$$\begin{aligned} P &= \begin{pmatrix} 0 & \ldots & \ldots & 0 & \delta \\ 1 & 0 & \ldots & 0 & 0 \\ 0 & 1 & \ddots & \vdots & \vdots \\ \vdots & \ddots & \ddots & 0 & \vdots \\ 0 & \ldots & 0 & 1 & 0 \end{pmatrix} \\ Q &= \begin{pmatrix} 1 & 0 & \ldots & 0 & 0 \\ 0 & \omega_n & \ddots & 0 & \vdots \\ \vdots & \ddots & \omega_n^2 & \ddots & \vdots \\ \vdots & 0 & \ddots & \ddots & 0 \\ 0 & \ldots & \ldots & 0 & \omega_n^{n-1} \end{pmatrix} \end{aligned} \quad (12)$$

Choosing a number δ which is transcendental over K is always possible for any given n by virtue of the Lindemann-Weierstrass theorem [13, 16]. We would like to emphasize here that the codes in [5, 6, 8, 11] can be obtained as a special case of our construction by simply choosing $\delta = 1$. If $\delta = 1$ then the algebra A will be a cyclic algebra and is not guaranteed to be a division algebra.

Some of the salient features of the codes in this paper are listed below.

1. Full rate
2. Information lossless [12, 13]. The capacity of the equivalent MIMO channel is same as that of the MIMO channel.
3. Fully diverse when a ML receiver is employed since they arise from matrix representation of division algebras [13].
4. MMSE optimal

Example 1 *This example illustrates our construction procedure for $n = 2$. Let $F = \mathbb{Q}(j, t)$, where t is transcendental over $\mathbb{Q}(j)$. Then $K = F(t_2 = \sqrt{t})$ is a cyclic extension of F of degree 2. The generator of the Galois group is given by $\sigma : t_2 \mapsto -t_2$. Let δ be any transcendental over K. Then $(K(\delta)/F(\delta), \sigma, \delta)$ is a cyclic division algebra. For example, we can choose $t = e^j$ and $\delta = e^{j\sqrt{5}}$. Then, we have*

$$M_a = \frac{1}{\sqrt{2}} \begin{bmatrix} f_0^{(0)} + f_0^{(1)} t_2 & \delta(f_1^{(0)} - f_1^{(1)} t_2) \\ f_1^{(0)} + f_1^{(1)} t_2 & f_0^{(0)} - f_0^{(1)} t_2 \end{bmatrix} \quad (13)$$

Example 2 *This is an example of a MMSE optimal code which is not obtained from a cyclic division algebra. Let $n = 4$ and $F = \mathbb{Q}(j, x, y)$ where x and y are two transcendental numbers independent over $\mathbb{Q}(j)$. We choose these transcendental numbers to lie on the unit circle (this is possible because of the Lindemann-Weierstrass theorem [13, 16]). Then $K = F(\sqrt{x}, \sqrt{y})$ is a Galois extension of F with the Galois group $G = \langle \sigma_x, \sigma_y \rangle$, where $\sigma_x : \sqrt{x} \mapsto -\sqrt{x}$ and $\sigma_y : \sqrt{y} \mapsto -\sqrt{y}$. The cocyle ϕ is defined as follows.*

$$\begin{gathered} \phi(\sigma_x, \sigma_x) = \phi(\sigma_x\sigma_y, \sigma_x) = \delta_1 \\ \phi(\sigma_y, \sigma_y) = \phi(\sigma_x\sigma_y, \sigma_y) = \delta_2 \\ \phi(\sigma_x, \sigma_y) = 1 \text{ and } \phi(\sigma_x\sigma_y, \sigma_x\sigma_y) = \delta_1\delta_2 \end{gathered}$$

Then, the algebra

$$\begin{aligned} (K(\delta_1, \delta_2), G, \phi) \quad = \quad & K(\delta_1, \delta_2) \oplus u_{\sigma_x} K(\delta_2, \delta_2) \\ & \oplus u_{\sigma_y} K(\delta_1, \delta_2) \oplus u_{\sigma_x} u_{\sigma_y} K(\delta_1, \delta_2) \end{aligned}$$

is a crossed product algebra where, δ_1, δ_2 are independent transcendental numbers over K. We choose to pick δ_1 and δ_2 to also lie on the unit circle. The matrix representation of this crossed product algebra will give rise to an MMSE optimal STBC since the conditions of Theorem 1 are met. The codewords of this STBC have the form

$$\frac{1}{\sqrt{P}} \begin{bmatrix} k_{0,0} & \delta_2\sigma_y(k_{0,1}) & \delta_1\sigma_x(k_{1,0}) & \delta_1\delta_2\sigma_x\sigma_y(k_{1,1}) \\ k_{0,1} & \sigma_y(k_{0,0}) & \delta_1\sigma_x(k_{1,1}) & \delta_1\sigma_x\sigma_y(k_{1,0}) \\ k_{1,0} & \delta_2\sigma_y(k_{1,1}) & \sigma_x(k_{0,0}) & \delta_2\sigma_x\sigma_y(k_{0,1}) \\ k_{1,1} & \sigma_y(k_{1,0}) & \sigma_x(k_{0,1}) & \sigma_x\sigma_y(k_{0,0}) \end{bmatrix} \quad (14)$$

where, $k_{i,j} = f_{i,j}^{(0)} + f_{i,j}^{(1)} \sqrt{(}x) + f_{i,j}^{(2)} \sqrt{(}y) + f_{i,j}^{(3)} \sqrt{xy}$ and $f_{i,j}^{(l)} \in \mathbb{Q}(j) \subset F$.

3.2. Decoding procedure

In this subsection, we explain the decoding procedure for the codes in this paper and highlight its receiver simplicity. Let the encoded matrix $X = \sum_{j=0}^{n-1} \sum_{i=0}^{n-1} f_j^{(i)} W_{i,j}$. Let the number of receive antennas be m. We assume that $m \geq n$ in the sequel since otherwise there will be an error floor [10] when linear MMSE reception is employed. The received matrix Y can be expressed as

$$Y = HX + N \quad (15)$$

where, H is the channel matrix of size $m \times n$ and N is the $m \times n$ matrix representing the additive noise at the receiver whose entries are i.i.d $\mathcal{CN}(0, 1)$. Then, the linear MMSE receiver can be implemented in its simplest form as a symbol-by-symbol decoder [10], as described below. Let

$$\widehat{f_j^{(i)}} = tr(W_{i,j}^H J^H Y). \quad (16)$$

with $J = (H^H H + \frac{1}{\rho} I_n)^{-1} H^H$ where ρ is the Signal to Noise ratio (SNR). Computation of $\widehat{f_j^{(i)}}$ is then followed by hard decision ,i.e., it is decoded to the nearest point (in the sense of Euclidean distance) in the constellation. Note that the decoding complexity is linear in the size of the signal set as compared to exponential in the case of ML reception.

4. Discussion

An algebraic construction of MMSE optimal STBCs has been given for any number of antennas. Few constructions from tensor products of division algebras and brauer division algebras were omitted due to lack of space. On similar lines, it will be interesting to study design of optimal STBCs for linear ZF receivers. Some initial work in this direction has been reported in [14] wherein it has been shown that Toeplitz STBCs achieve full transmit diversity even with a linear ZF receiver.

5. Acknowledgements

This work was supported through grants to B.S. Rajan; partly by the IISc-DRDO program on Advanced Research in Mathematical Engineering, and partly by the Council of Scientific & Industrial Research (CSIR, India) Research Grant (22(0365)/04/EMR-II). The authors gratefully acknowledge Dr.T.Kiran and Dr.V.Shashidhar of Beceem Communications for useful discussions on the subject of this matter. The authors thank Jing Liu and Professor Sergio Barbarossa for providing us with preprints of their recent works.

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