E1 244: Homework - 5

Assigned on: 24 Mar 2011

1 Topics

- Introduction, probability review
- Baye's Hypothesis Testing
- Minimax

2 Problems

1. Show that, if X is a non-negative random variable, $\mathbf{E}(X) = \int_0^\infty Pr(X > t)\, dt = \int_0^\infty (1 - F(t))\, dt$
2. Show that $\mathbf{E}(X) = \arg\min_\mu \mathbf{E}(X - \mu)^2$.
3. **Baye's.** Consider the hypothesis testing problem

$$\begin{aligned} H_0 &: Y = X - s \\ H_1 &: Y = X + s \end{aligned}$$

where $s > 0$ is a fixed real number and X is random with probability density function

$$p_X(x) = \frac{1}{\pi(1 + x^2)}, \qquad x \text{ real}$$

Find the Baye's rule and the minimum Baye's risk for testing H_0 versus H_1 with uniform costs and equal priors.

4. **Minimax.** You are asked to decide whether a coin is fair (HT) or double-headed (HH). A single flip of the coin is performed and you are told the outcome. The cost of making a wrong decision is Rs. 1000, and zero otherwise. What strategy minimizes the maximum cost? (Hint. May have to randomize.)