E1 244: Homework - 7

Assigned: 06 Apr 2011

1 Topics

- NP testing

2 Problems

1. **Poor Chapter II 2(b)** Given the parametric model

$$f(y|\theta_1, \theta_2) = \begin{cases} \theta_1 y + \theta_2, & 0 \le y \le 1 \\ 0 & \text{otherwise} \end{cases}$$

consider the following binary hypotheses

$$\mathcal{H}_0 : (\theta_1, \theta_2) = \left(\frac{2}{3}, \frac{2}{3}\right) \quad \text{vs.} \quad \mathcal{H}_1 : (\theta_1, \theta_2) = (0, 1).$$

Find the NP detector for an α-level test (i.e., $P_F = \alpha$) and plot the ROC.

2. **Poor Chapter II 10** Suppose a single observation Y is generated from

$$Y = \theta\lambda + N$$

Where $N \sim \mathcal{U}(-1, 2)$ and $\lambda \in (0, 2)$ a known constant. The two hypotheses are given by

$$\mathcal{H}_0 : \theta = 0 \quad \text{vs.} \quad \mathcal{H}_1 : \theta = 1.$$

   (a) Find the NP detection rule for an α-level test.

   (b) Find the probability of detection P_D as a function of α and λ. Plot the ROC.

3. **Gaussian vs. Uniform** Consider the following simple binary hypotheses:

$$\mathcal{H}_0 : Y \sim \mathcal{N}(0, \sigma^2) \quad \text{vs.} \quad \mathcal{H}_1 : Y \sim \mathcal{U}(-\sqrt{3}\sigma, \sqrt{3}\sigma).$$

Both distributions have mean zero and variance σ^2.

(a) First, think intuitively about how you would detect $\mathcal{H}_0$ versus $\mathcal{H}_1$. For example, what would $\mathcal{Z}_0$ and $\mathcal{Z}_1$ look like?

(b) Derive the Neyman-Pearson detector for a given $\alpha \in (0, 1)$. Verify your intuition.

(c) Plot the ROC curve for $\sigma = 1$. How does it change as σ^2 increases?

4. Rework the above problem replacing $\mathcal{H}_0$ with $\mathcal{H}_1$.

5. **Gaussian vs. Exponential** Consider

$$\mathcal{H}_0 : Y \sim \begin{cases} \frac{2}{\sqrt{2\pi}\sigma} e^{-\frac{y^2}{2\sigma^2}}, & y \geq 0 \\ 0 & \text{otherwise} \end{cases} \quad \text{vs.} \quad \mathcal{H}_1 : Y \sim \frac{1}{\sigma} e^{-\frac{x}{\sigma}},$$

both having the same variance but different means μ_0 and μ_1 respectively.

(a) Consider first the simple threshold detector

$$\delta(y) = \begin{cases} 1, & y > \frac{\mu_0 + \mu_1}{2} \\ 0 & \text{otherwise} \end{cases}$$

Find the P_F and P_D for this detector.

(b) Derive the NP detector for a given $P_F = \alpha$. Is the above detector the NP detector for some value of α?

(c) Plot the ROC curves of the two detectors.