Radar CFAR Thresholding in Clutter and Multiple Target Situations

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Radar detection procedures involve the comparison of the received signal amplitude to a threshold. In order to obtain a constant false-alarm rate (CFAR), an adaptive threshold must be applied reflecting the local clutter situation. The cell averaging approach, for example, is an adaptive procedure.

A CFAR *method* is discussed using as the CFAR threshold one single value selected from the so-called ordered statistic (this method is fundamentally different from a rank statistic). This procedure has some advantages over cell averaging CFAR, especially in cases where more than one target is present within the reference window on which estimation of the local clutter situation is based, or where this reference window is crossing clutter edges.

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I. INTRODUCTION AND FORMULATION OF THE PROBLEM

The task of primary radars used in air or vessel traffic control is to detect all objects within the area of observation and to estimate their positional coordinates. Generally speaking, target detection would be an easy task if the echoing objects were located in front of an otherwise clear or empty background. In such a case the echo signal can simply be compared with a fixed threshold, and targets are detected whenever the signal exceeds this threshold. In real radar application, however, the target practically always appears before a background filled (mostly in a complicated manner) with point, area, or extended clutter. Frequently the location of this background clutter is additionally subject to variations in time and position. This fact calls for adaptive signal processing techniques operating with a variable detection threshold to be determined in accordance to the local clutter situation. In order to obtain the needed local clutter information, a certain environment defined by a window around the radar test cell must be analyzed.

Usually the background reflectors, undesired as they are from the standpoint of detection and tracking, are denoted by the term "clutter," and in the design of the signal processing circuits the assumption is made that this clutter is uniformly distributed over the entire environment. Signal processing is designed so that, whenever possible, target reports are received from useful targets only, rather than from background reflectors.

In practice, however, clutter phenomena may be caused by a number of different sources. Improvements in target detection and clutter suppression over the present state of the art can be effected only by removing the simplifying assumptions step by step and introducing a more differentiating way of argumentation. Ultimately it may become necessary to identify clutter regions of differing clutter type and to describe their properties such as type, size and borders, power, and spectral features rather than trying to suppress and ignore them at an early stage of signal processing. Thus for discriminating targets from clutter, it might be useful to build up a complete "image" of the clutter situation encountered in the overall observation space.

These ideas reflect a trend presently observed in radar signal processing philosophy, a trend to regard the problem of target detection and clutter suppression more and more as a problem of image processing and image analysis [1]. The procedure outlined in the following is one step in this direction. The assumption of a uniform clutter situation within the reference window is no longer maintained. Instead, provisions are made to handle transitions in clutter characteristics, clutter areas of small extensions, and interfering target echos occurring within the reference window of the radar test cell. The idea is to modify the common CFAR techniques by replacing the usual clutter power estimation based on arithmetic averaging by a new procedure which has proven useful for similar tasks in general image processing applications.

In existing CFAR systems target decision is commonly performed using the sliding window technique. The data available in the reference window enter into an algorithm for the calculation of the decision threshold. These procedures are nearly the same in all CFAR systems and are illustrated by Fig. 1 which shows a certain reference window and the signal processing structure for target detection as a flowchart. The first step is to measure the mean clutter power level Z. The second step is to multiply this estimation Z by a scaling factor T depending first on the estimation method applied and secondly on the false alarm rate required. The resulting product TZ is directly used as the threshold value.

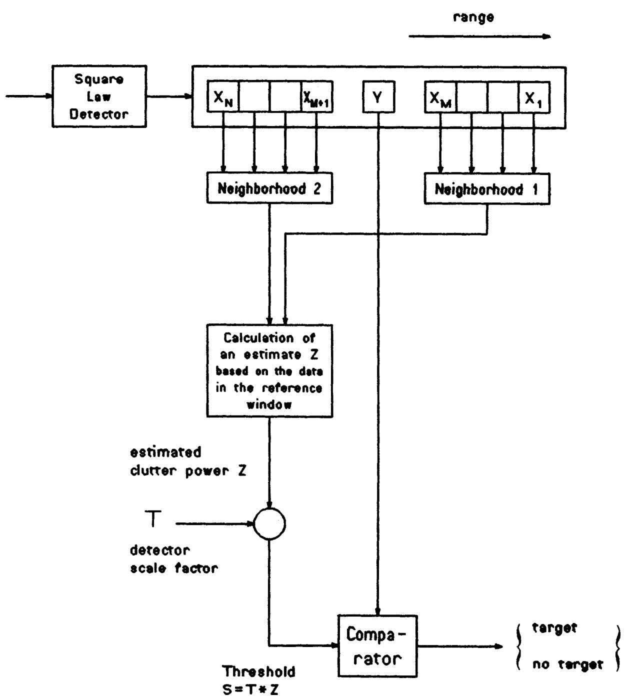

Fig. 1. General CFAR processor. The central operation of the CFAR system is to estimate the clutter power level Z.

Whereas common CFAR systems apply estimation techniques which are mainly based on arithmetic averaging, in this paper a different estimation procedure is proposed which derives the clutter power estimation from the so-called ordered statistic. The rest of the signal processing procedure remains almost unchanged.

In order to demonstrate the advantages of the proposed method, it is necessary to recall the CFAR methods presently used to handle nonstationary clutter situations. These are the cell averaging CFAR (CA CFAR) and the cell averaging CFAR with greatest of (GO) selection (CAGO CFAR) (see [2] and [3]). Both processing methods can be described in terms of a split neighborhood. From each of the two neighborhood areas the arithmetic mean of the amplitude contained therein is obtained. The two clutter power estimators are then combined into one single value either by further averaging or by maximum selection. The main difference between CA CFAR and CAGO CFAR is that the former is implicitly based on the assumption of a clutter situation uniform in the entire neighborhood area whereas the latter makes allowance for clutter edges occurring within the reference area. The differences in processing are shown schematically in Fig. 2.

In situations with clutter edges, this organization makes the transient behavior of the CAGO CFAR superior to that of the CA CFAR, although it is known [3] that its performance is only slightly inferior in the case of stationary clutter where it exhibits a loss in sensitivity of only 0.3 dB in its signal-to-noise ratio (SNR).

For the discussion of advantages and disadvantages of CA and CAGO CFAR, two different background situations have been considered: uniform (stationary) and nonuniform (clutter edges) clutter within the reference window. These are two selected idealized examples of the multitude of different situations which may occur in practice, and they are not sufficient for a comprehensive assessment of adaptive procedures in radar signal processing. In this context the attention of the reader is drawn to the behavior of both CFAR methods in multiple (dense) target situations [4–6].

Such a dense target situation as illustrated in Fig. 3 occurs whenever two targets come close in range and azi-

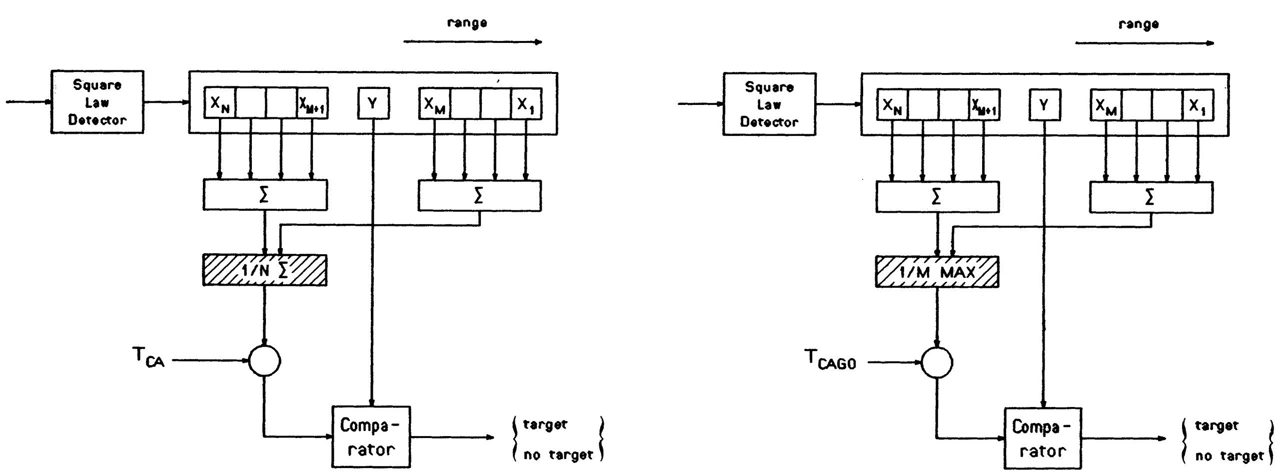

Fig. 2. Comparison of CA and CAGO CFAR procedures.

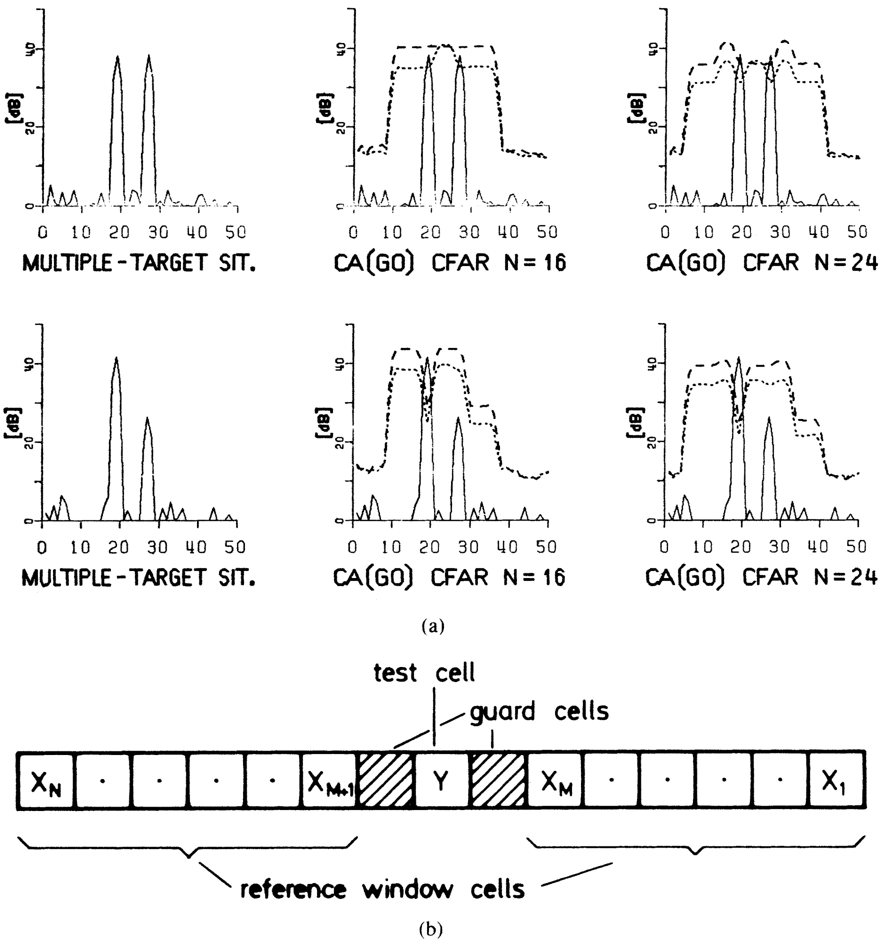

Fig. 3. (a) Double target situation with equal and differing target amplitudes, scaling factor T adjusted to $P_{fa} = 10^{-6}$. Threshold response for CA CFAR is shown by dotted lines and for CAGO CFAR by dashed lines. (b) Definition of the reference window for CA and CAGO CFAR used in the simulation program with two guard cells directly adjacent to the test cell.

muth even if they may be clearly separated in elevation. The echoes of both targets are within one reference window. During clutter power estimation the signal power encountered in the neighborhood of the cell under test is, due to the oversimplified model assumptions and the absence of any further signal analysis, simply interpreted as clutter power.

Accordingly, the decision threshold is drastically raised (see dotted lines in Fig. 3), which often leaves one of the two targets or even both undetected even in the case of high signal amplitudes. The CAGO CFAR procedure with $N = 16$ and two targets of equal strength is, independent of the signal amplitude, unable to detect any of the two targets.

For simulating the CA and CAGO CFAR procedures a reference window was used as shown in Fig. 3(a). The two cells (guard cells) directly adjacent to the cell under test have been completely ignored. This is usually also done in the single target case because signal energy can spill over into the adjacent range cells and may affect the CA and CAGO CFAR clutter power estimation.

By modifying the parameters N and T the problem of resolving closely neighbored targets can to a certain degree be reduced but not really be solved. In the case of different amplitudes the target with the smaller amplitude is frequently "masked" by the other one as shown in the bottom row of Fig. 3. Weiss [4] and Rickard and Dillard [6] suggested eliminating the maximum amplitude(s) from the reference window, which hopefully should come from the interfering target. The idea is helpful in a multiple target situation, but has undesired effects in ordinary clutter situations.

Signal situations as described by Fig. 3 motivate deliberations aiming at new CFAR methods which should retain the advantages of the methods already known but avoid their drawbacks. In this paper a new class of CFAR detectors is discussed which are based on the following simple ideas:

(1) Conventional CFAR procedures suffer from the fact that they are specifically tailored to the assumption of a uniform statistic in the reference window. Based on this assumption, they derive the clutter power estimate using the unbiased and most efficient arithmetic mean estimator.

(2) Improved CFAR procedures should be robust with respect to interfering signal and amplitudes generated by target returns or by transition areas of different clutter sources. Also in such situations CFAR methods should remain able to provide reliable clutter power estimations.

(3) The desired insensitivity to violation of the above statistical assumptions can be obtained by using quantiles instead of statistical moments as clutter power estimators.

Clutter power corresponds to a defined parameter of the clutter probability density function (pdf). This parameter must be estimated based on samples collected within the reference window. The first result of this data acquisition is a histogram which may be considered as an estimation of the clutter pdf. The conventional CFAR procedures generate the clutter power parameter by taking the first moment of the histogram. The proposed CFAR procedures take quantiles.

In practice the proposed method is performed by rank-ordering the values encountered in the neighborhood area according to their magnitude and by selecting a certain predefined value from the ordered sequence. This can be the median, the minimum, the maximum, or any other value. In the following, such signal processing methods are denoted as methods with an *ordered statistic (OS CFAR)*.

These methods, although being rank-order methods, must not be confused with the so-called rank-order tests (in the field of distribution-free and nonparametric tests) [7, 8], where only the information on the rank number encountered in the individual resolution cells is taken into account in the decision function, instead of the full amplitude information. The adaptive techniques proposed here do not belong to the distribution-free procedures since they actually make use of the respective pdf.

The model situations on which the discussion of CFAR procedures is based will be introduced in Section II and a detailed motivation for new CFAR techniques will be presented in Section III. The advantages of these new ordered statistic CFAR techniques over the conventional CA and CAGO CFAR will then be demonstrated in Section IV. The conventional CFAR techniques are designed for simple situations and fail for difficult ones. Remarkably the OS CFAR designed for difficult situations retains its superiority even for the uniform clutter situation for which the CA CFAR was specifically designed.

II. MODEL DESCRIPTION

The statistical model with the uniform clutter background originally had been exclusively used for the development of CFAR procedures has lost its predominance since it is a rather true approximation only for one of the numerous different clutter situations occurring in practice. With the discrimination between different clutter situations each representing a certain realistic problem, a more deterministic point of view is introduced into the discussion of clutter models.

Real radar environment cannot be described by a single model, yet consideration of a larger number of different situations might be confusing. For these reasons, in the following, three different signal situations are selected: uniform clutter, clutter edges, and double target. Each situation is represented by a distinct signal model. The different CFAR procedures are investigated and compared on the background of these three signal situations.

Model 1: Uniform Clutter

This model describes the classical signal situation with stationary noise in the reference window. In this model two signal situations are of interest:

(1) one target in the test cell in front of an otherwise uniform background
(2) uniform noise situation throughout the reference window.

In both cases the noise neighborhood has a uniform statistic, i.e., the random variables $X_1, \ldots, X_N$ in the reference window are assumed to be statistically independent and identically distributed. The random variable Y of the cell under test is denoted by Y_1 in the case of a target and by Y_0 in the case of noise, with Y_0 being statistically independent of the neighborhood and subject to the same distribution function as the random variables $X_1, \ldots, X_N$.

Model 2: Clutter Edges

This model serves to describe the situation which is encountered in the transition area between background regions of very different characteristics. A typical example of such a situation is the periphery of precipitation areas. The model consists of two areas, clutter and background noise. With respect to clutter two signal situations are discussed:

(1) clutter amplitudes statistically independent, Rayleigh distributed
(2) clutter represented by a constant amplitude response.

Both cases are illustrated by Fig. 4. They represent extreme cases, the truth lying in between, since weather

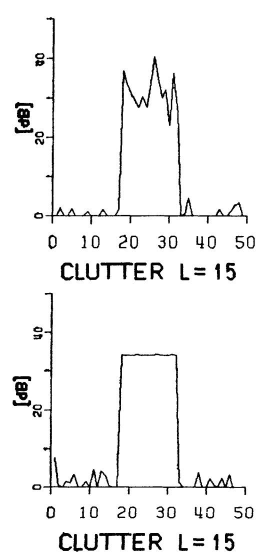

Fig. 4. Signal situations for model 2 with a clutter area extended over L range cells, here $L = 15$.

clutter will be always subject to statistical fluctuations with partially correlated amplitudes. Nevertheless the simplified assumptions permit useful conclusions on the general behavior of CFAR systems.

The statistical assumptions for model 1 are no longer valid in this case because, for instance, the random variables $X_1, \ldots, X_N, Y_0$ are no longer identically distributed. Applying the usual methods for clutter power estimation, too low a value will be calculated for the decision threshold in the transition area between clear background and clutter resulting in clutter detections at the clutter edges.

Model 3: Double Target

Model 3 describes a situation occurring occasionally in radar signal processing when two useful targets are closely spaced in range. With many CFAR procedures, such a signal situation results in undesired effects such as masking of closely spaced targets [4–6].

The assumption is made that the two useful targets are located in front of an otherwise uniform clutter background. The amplitude response of the targets is corrupted by additive background clutter but otherwise deterministic (see Fig. 3). This point of view differs from the statistical model used in [4] for the same multiple target situation.

Here a deterministic target model is preferred in order not to mix up the effects of mutual masking and of fluctuation. Masking is a signal processing problem which concerns the single scan. Fluctuating targets as generated by the Swerling I or Swerling III models more or less eliminate the masking problems since, due to the statistical fluctuation, one or the other of the two targets occasionally gets dominating amplitude allowing it to be detected.

The uniform clutter model 1 serves as the basis for defining the performance measures P_d, the probability of detection, and P_{fa}, the probability of false alarm, generally used in radar literature and for calculating the decision threshold necessary to achieve a certain required P_{fa} for a given signal-to-noise ratio (SNR). For any given P_{fa} and a certain variety of CFAR methods to be considered, a family of curves as shown in Fig. 5 can be obtained.

The figure displays the dependency of the probability of detection P_d on the SNR. The threshold value to be applied is not indicated, but there exists a fixed relationship between P_{fa} and the decision threshold value for any given clutter pdf.

Typically the curves of Fig. 5, representing different CFAR procedures, differ mainly in translations along the SNR axis. There are only slight changes in the overall appearance of the functions P_d(SNR), at least as long as a fixed fluctuation model is used. Fig. 5 is based on a nonfluctuating target model.

For comparing different CFAR procedures the translations along the SNR axis expressed in SNR units are usually denoted by the term "additional detectability loss" or "CFAR loss" [2]. Since the functional relationship P_d

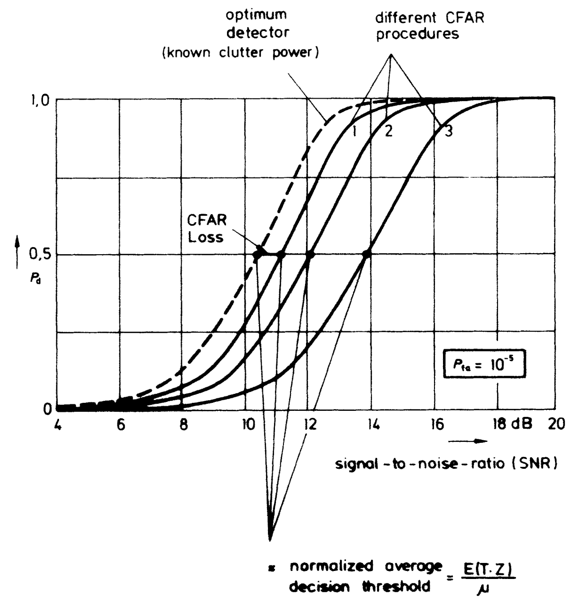

Fig. 5. Explanation of the performance measure "average decision threshold" ADT by means of the usual P_d SNR diagrams. The SNR of the minimum detectable signal ($P_d = 0.5$) is approximately the same as the ADT = $E(TZ)/\mu$ of each CFAR system. The random variable Z is the result of the clutter level estimation (see Fig. 1), T is the scaling factor controlling P_{fa}, μ is the actual mean clutter power level, and E stands for the expectation.

(SNR) remains mainly unchanged, the CFAR loss can be determined by measuring the translation at a given arbitrary P_d value. Here the reference value $P_d = 0.5$ will be used.

In the case of low false alarm rates ($P_{fa} = 10^{-6}$), the SNR required for the adjustment of $P_d = 0.5$ is obtained with sufficient accuracy by calculating the average decision threshold (ADT) or mean threshold normalized with the average noise power μ. A formal definition of the ADT meausre is

$$\text{ADT} = E(TZ)/\mu \tag{1}$$

where the random variable Z is the result of the estimation method used in the CFAR system, T is the scaling factor for threshold adjustment adapted to the estimation method and the required P_{fa}, and μ is the mean clutter power level. E stands for the expectation.

Deviating from the methods usually described in the radar literature, we use for the comparison of various CFAR procedures the normalized average decision threshold ADT (1). This provides the advantage that the difference existing between various CFAR systems (with reference to the background situation given in model 1) are then expressed by a single-valued measure.

These differences between two CFAR systems (indicated by the indices 1 and 2) in a homogeneous clutter situation (model 1) can be expressed by the ratio of the two ADT's measured in dB

$$\Delta\ [\text{dB}] = 10 \log \frac{E(T_1 Z_1)}{E(T_2 Z_2)}. \tag{2}$$

This measure reflects the separation between two correspondent P_d curves valid for a homogeneous clutter situation as given by model 1; see Fig. 5.

III. ANALYSIS OF CA AND CAGO CFAR AND MOTIVATION FOR NEW CFAR METHODS

In this section the response of CA and CAGO CFAR procedures to the three model situations is discussed. The results are described in an easily understandable manner.

With all CFAR systems considered here the decision is realized by simple thresholding

$$e(Y) = \begin{cases} \text{target}, & \text{if } Y \geq S \\ \text{no target}, & \text{if } Y < S \end{cases} \tag{3}$$

where the task of the CFAR system essentially is to provide the threshold value S needed. Different CFAR systems are distinguished by the way this threshold value is obtained. In calculating the threshold, allowance must be made for two aspects, one being the average clutter power in the reference window and the other being the P_{fa} specified. Accordingly, the threshold S is always calculated as the product

$$S = TZ \tag{4}$$

where Z is the estimate for the average clutter power μ and T is a scaling factor used to adjust the P_{fa}.

Different CFAR procedures are characterized by the method used for estimating the average clutter power μ. In the following the index of Z and T indicates what estimation method is being used, e.g., Z_{CA} for cell averaging CFAR. The scaling factor T is a function of the method used to compute Z and also a function of the P_{fa}. For each estimation method discussed, the value of T is given for a P_{fa} of 10^{-6}.

Although the value of T for a given estimation method and P_{fa} depends on the model being considered, in the following discussion it is computed exclusively in the model 1 situation of uniform clutter even though the CFAR procedure may be tailored to another model. Considerations of this kind constitute additional constraints on threshold optimization which may result in a loss of performance in model 1 situations (CFAR loss), but which are advantageous in other more complicated signal situations such as those in models 2 or 3.

In this paper it is assumed that the random variables $Y_0, X_1, \ldots, X_N$ of clutter (model 1) follow an exponential distribution with the pdf

$$p_{Y_0}(x) = p_{X_i}(x) = \begin{cases} 1/\mu \, e^{-x/\mu}, & x \geq 0 \\ 0, & \text{otherwise} \end{cases} \tag{5a}$$

and the distribution function (df)

$$P_{Y_0}(x) = P_{X_i}(x) = \begin{cases} 1 - e^{-x/\mu}, & x \geq 0 \\ 0, & \text{otherwise} \end{cases} \tag{5b}$$

where $i = 1, \ldots, N$. The argument of the functions pdf $p(x)$ and df $P(x)$ is uniformly denoted by the symbol x. For distinction the distribution functions belonging to different random variables are indexed according to their respective random variables.

The assumption of an exponential distribution is justified for the square-law detector in the case of complex normally distributed noise in the video range. Other pdf's may also be used, however. In this case separate analyses (in particular calculation of the factor T) are necessary [9].

The statistical behavior of the random variables $X_1, \ldots, X_N$ is fully known except for the parameter μ (5a) describing the (unknown) average clutter power. Based on empirical values collected from the respective reference window, therefore, an estimate of this parameter μ must be computed.

For this purpose the CA CFAR uses the estimate

$$Z_{CA} = (1/N) \sum_{i=1}^{N} X_i. \tag{6}$$

In order to cope with local clutter phenomena, like clutter edges of weather clutter, Moore et al. [3] proposed a different estimation method. The CAGO CFAR applies the maximum of two arithmetic means gained from two different reference windows, one on the leading and one on the lagging side of the cell under test (see Fig. 2).

$$Z_{CAGO} = (1/M) \max\left(\sum_{i=1}^{M} X_i, \sum_{i=M+1}^{N} X_i\right). \tag{7}$$

The average decision threshold ADT on which the comparison of different CFAR systems is based in this paper is given for the CA CFAR by

$$\text{ADT}_{CA} = E(T_{CA} Z_{CA})/\mu = T_{CA} E(Z_{CA})/\mu = T_{CA} \tag{8a}$$

and for the CAGO CFAR by

$$\text{ADT}_{CAGO} = E(T_{CAGO} Z_{CAGO})/\mu = T_{CAGO} E(Z_{CAGO})/\mu. \tag{8b}$$

In Table I the scaling factor T and the average decision threshold ADT are listed for some selected values of N for both the CA and the CAGO CFAR. Whereas the scaling factor T turns out to be characteristically smaller for the CAGO CFAR, the higher estimate $E(Z_{CAGO})$ compensates this effect and leads to an almost unchanged ADT measure for the CAGO CFAR compared with the CA CFAR, at least for the higher values of N.

Hansen et al. [2] have investigated CAGO CFAR and have found that it has but minor losses (less than 0.3 dB SNR) over CA CFAR for the model 1 case. This result corresponds with minor differences to the average decision threshold recorded in Table I and further justifies using the average decision threshold ADT for comparing different CFAR procedures. The CFAR losses calculated in [2] for a P_d of 0.5 are directly comparable to differences in ADT as used in this paper.

Except for these minor differences of little practical relevance, the two CFAR procedures CA and CAGO

TABLE I
Scaling Factor T and average decision threshold $ADT = E(TZ)/\mu$ given in (1) for CA and CAGO CFAR and $P_{fa} = 10^{-6}$ (Square Law Detector)

	$N=16$		$N=32$		$N=64$	
	T	ADT	T	ADT	T	ADT
CA	21.94	21.94	17.28	17.28	15.42	15.42
CAGO	19.36	23.16	15.72	17.92	14.35	15.78
$\Delta[dB]$	—	0.24	—	0.16	—	0.10

CFAR can be considered equivalent for model 1 situations.

This statement, however, becomes uncertain if the assumptions of model 1 are violated. In model 2 situations with clutter edges crossing the reference window, for example, the CAGO CFAR should prove superior since this is exactly the situation for which it was designed.

These effects are demonstrated in Fig. 6 for three different simulated signal situations. There is a clutter area of limited length L with independent clutter amplitudes following a Rayleigh distribution. The clutter area length L is varied in three steps as well as the length N of the reference window in two steps. For a fixed probability of false alarms $P_{fa} = 10^{-6}$ (referring to model 1) the thresholds gained from the two CFAR procedures are indicated by dotted lines for the CA CFAR and by dashed lines for the CAGO CFAR.

It can be seen from Fig. 6 that interdependencies exist between the clutter area length L and the length N of the reference window. For N larger than L, the clutter power tends to be underestimated for both CFAR procedures, which results in an increased clutter detection rate. Depending on the reference window length N the area of raised detection threshold can be considerably extended over the actual value L which must lead to a decreased probability of detection in the neighborhood of clutter edges.

The response of the CAGO CFAR to the jump in clutter power is distinctly steeper than that of the CA CFAR as is to be expected. The difference vanishes with increasing distance from the clutter edge (see Fig. 6 bottom row). Particularly for smaller clutter areas (L small) both methods will produce undesired clutter detections.

These considerations are confirmed by Fig. 7 which displays the threshold curves for CA and CAGO CFAR in a clutter environment with fully correlated amplitudes (deterministic model) and two different values of the clutter area length L. The interpretation here is eased by the lack of random variations in clutter power. Problems with undesired clutter detections arise in the center of clutter areas which are small with respect to the reference window size. The reference window in such a case is only partially covered with clutter. This leads to underestimations of clutter power for both CFAR procedures. Due to symmetry, the splitting of the reference window of the CAGO CFAR does not help in this situation.

In model 3 double target situations are illustrated by Fig. 3, the CA and CAGO CFAR procedures exhibit considerable deficiencies since any amplitude observed within the reference area simply is taken as clutter and no discrimination is possible between actual clutter returns and echoes of neighboring targets. The result is that one target may suppress the other or both targets may even remain undetected.

The drawbacks of CA and CAGO CFAR discussed in the foregoing sections motivate the design of other methods than those based on an ordered statistic.

What Is an Ordered Statistic?

The amplitude values taken from the reference window (see Fig. 1) are first rank-ordered according to increasing magnitude. The sequence thus achieved is

$$X_{(1)} \le X_{(2)} \le \cdots \le X_{(N)}. \tag{9}$$

The indices in parentheses indicate the rank-order number. $X_{(1)}$ denotes the minimum and $X_{(N)}$ the maximum value. The sequence given in (9) is called an ordered statistic.

The central idea of an ordered statistic CFAR procedure is to select one certain value $X_{(k)}$, $k \in \{1,2, \ldots, N\}$ from the sequence in (9) and to use it as an estimate Z for the average clutter power as observed in the reference window

$$Z = X_{(k)}. \tag{10}$$

In this connection reference should be made to a rather similarly motivated way of proceeding in digital picture processing [10] where so-called rank-order operators are used for image filtering. In order to explain the relationships, the terms used in image processing are translated to the problem at hand.

The radar data available after it has been processed can be thought of as representing a two-dimensional raster image which could be entered into a conventional image processing system. The fact that radar images commonly are sampled along polar coordinates instead of the Cartesian coordinates of conventional image processing does not affect the following considerations. The sliding window of the radar CFAR system corresponds to the local operator as used in image processing. The calculation of a threshold value individually for every test cell is identical to the generation of a threshold image with the dimensions and the resolution of the input image. In image processing the local operator is called a rank-order operator if it outputs a preselected value from the ordered statistic.

Rank-order operators are used in image processing whenever a smoothing filter effect is desired which at the same time should preserve steps in brightness at the borderlines between larger areas of different power (median filter), or whenever defined image structures are to be deliberately intensified or suppressed (erosion by minimum filter, dilatation by maximum filter, opening by a sequence of erosion and dilatation, closing by the reverse order of these two operators).

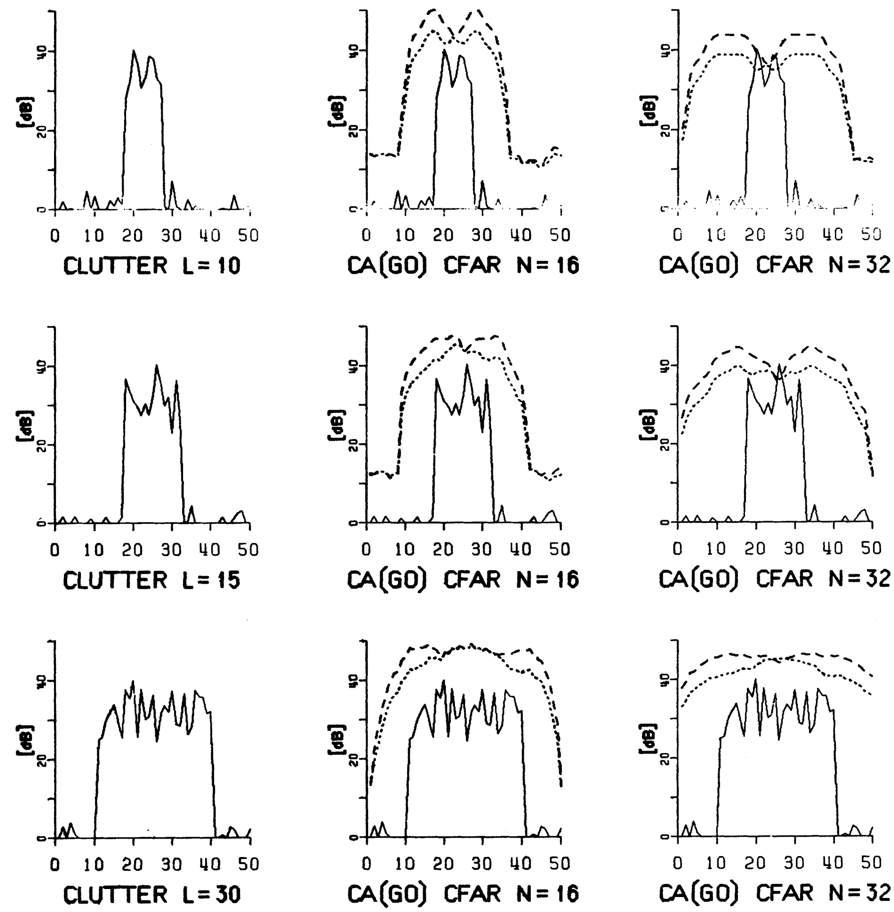

Fig. 6. Some examples for the CA and CAGO CFAR behavior in model 2 clutter situations with uniformly distributed statistically independent clutter amplitudes and different clutter area lengths L. Dotted lines show the threshold response for CA CFAR and dashed lines for CAGO CFAR.

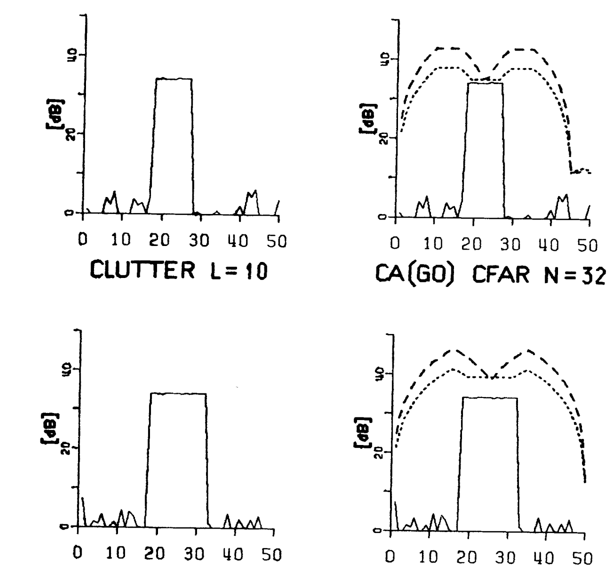

Fig. 7. Behavior of CA and CAGO CFAR in model 2 clutter situations with fully correlated clutter amplitudes and different clutter area lengths L, dotted and dashed lines as in Fig. 6.

In contrast to image processing where, in general, a strictly statistical model generation is not adequate, the situation in radar signal detection is such that rank-order operators can be analytically investigated based on statistical (and partly deterministic) models which are known for their property of coming close to reality.

IV. ANALYSIS OF OS CFAR

To start with, the model 1 uniform clutter situation is considered. In this case the random variables $X_1, \ldots, X_N$ are statistically independent and identically distributed. From these assumptions for any given P_{fa} the decision threshold S as well as the scaling factor T can be derived; see (4).

P_{fa} indicates the probability that a noise random variable Y_0 (a single value from the noise process with the pdf of (5)) is interpreted as target echo during the threshold decision (3). Formally, this probability is given by

$$P_{fa} = P\,[Y_0 \geq TZ]. \tag{11}$$

In order to calculate P_{fa} according to (11) both the pdfs of Y_0 and of Z must be known.

Since $Z = X_{(k)}$ is an ordered statistic value, its pdf can be determined according to [11]. The derivations are given in the Appendix. For the pdf of the kth value of the ordered statistic for exponentially distributed random variables $X_1, \ldots, X_N$ we have

$$P_{X_{(k)}}(x) = p_k(x)$$

$$= k/\mu \binom{N}{k} (e^{-x/\mu})^{N-k+1} (1 - e^{-x/\mu})^{k-1}. \qquad (12)$$

Using (12) the P_{fa} can be calculated for a fixed factor T

$$P_{fa} = P\,[Y_0 \geq TX_{(k)}]$$

$$= \int_0^{\infty} P[Y_0 \geq Tx]\, p_k(x)\, dx$$

$$= \int_0^{\infty} e^{-Tx/\mu}\, k/\mu \binom{N}{k} (e - x^{/\mu})^{N+1-k}$$

$$(1 - e^{-x/\mu})^{k-1}\, dx$$

$$= k \binom{N}{k} \int_0^{\infty} e^{-(T+N+1-k)y} (1 - e^{-y})^{k-1}\, dy. \qquad (13)$$

From (13) a first important conclusion can be drawn, namely, that the scaling factor T controlling the false alarm probability P_{fa} does not depend on the average clutter power μ of the exponentially distributed parent population. Thus these methods may actually be considered as CFAR methods. In the following paragraphs they are denoted by the term OS CFAR.

The use of the ordered statistic in the context of CFAR processing does not define a single CFAR method but rather a series of several different CFAR methods. For any given random variable $X_{(k)}$ a distinct CFAR procedure is established. For practical application, however, only a few of the N possible values k are of interest.

The false alarm probability can be derived from (13) and is given by

$$P_{fa} = k \binom{N}{k} \frac{(k-1)!\,(T+N-k)!}{(T+N)!}. \qquad (14)$$

The scaling factors T required for $P_{\text{fa}} = 10^{-6}$ are listed for some combinations of the parameters N and k in Table II.

At this point in the analysis the probability of detection would normally have to be considered. In this paper we use instead the single-valued measure ADT as defined in (1) and (2).

For the exponentially distributed random variables $X_1, \ldots, X_N$, the mean values $E(X_{(k)})$ of the random variables $X_{(k)}$ are given by

$$E(X_{(k)}) = \mu \sum_{j=1}^{k} 1/(N-k+j). \qquad (15)$$

According to (10) $X_{(k)}$ is used as the clutter power estimate Z. Therefore ADT $= E(ZT)/\mu$ is calculated from the mean value of Z given in (15), multiplied by the scaling factor T, given in (14). Fig. 8 displays the three relevant variables, expectation $E(X_{(k)})$ of the rank-ordered random variables, scaling factor T, and ADT as functions of the rank-order index k. The parameters used for this representation are the reference window length $N = 24$ and the false alarm probability $P_{\text{fa}} = 10^{-6}$.

The function ADT(k) exhibits a broad minimum, the absolute minimum lying at $k = 20$ (approximately $7N/8$)

TABLE II

Scaling Factor T for OS CFAR for $P_{\text{fa}} = 10^{-6}$ (Square Law Detector)

k	$N=8$	$N=16$	$N=24$	$N=32$
1	7 999 992.0	15 999 984.0	23 999 976.0	31 999 968.0
2	7 475.8	15 476.4	23 471.2	31 464.5
3	688.2	1 482.8	2 275.5	3 067.9
4	196.0	442.7	688.1	933.3
5	86.4	206.7	326.0	444.9
6	46.7	120.4	192.8	265.0
7	27.8	79.4	129.5	179.2
8	16.8	56.6	94.1	131.3
9		42.4	72.1	101.4
10		32.9	57.3	81.4
11		26.1	46.8	67.2
12		20.9	39.1	56.7
13		16.9	33.1	48.5
14		13.7	28.3	42.2
15		10.9	24.5	37.0
16		8.3	21.3	32.7
17			18.6	29.2
18			16.3	26.1
19			14.3	23.5
20			12.5	21.2
21			10.8	19.2
22			9.3	17.4
23			7.9	15.8
24			6.3	14.4
25				13.1
26				11.9
27				10.7
28				9.6
29				8.6
30				7.6
31				6.6
32				5.4

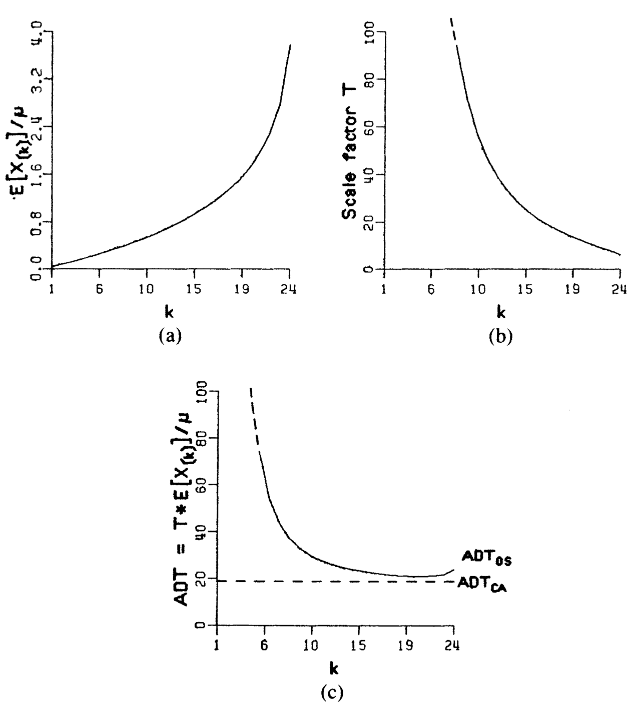

Fig. 8. (a) Mean value $E(X_{(k)})/\mu$ of the random variable $X_{(k)}$ for $N = 24$. (b) Scaling factor T for the OS CFAR and $P_{\text{fa}} = 10^{-6}$, $N = 24$. (c) Average decision threshold ADT given in (1) for OS CFAR with $N = 24$ and $P_{\text{fa}} = 10^{-6}$. For reference the ADT for CA CFAR with the same reference window size $N = 24$ is shown by the dashed line.

for $N = 24$. It approaches the reference value of ADT = 18.67 (dashed line) valid for the CA CFAR procedure with identical window length $N = 24$ and the same fixed $P_{fa} = 10^{-6}$. The average decision thresholds ADT of some OS CFAR procedures with the same parameter combinations of k and N as in Table II are listed in Table III.

TABLE III
Average Decision Threshold ADT = $E(TZ)/\mu$ Given in (1) for OS CFAR Systems

k	$N=8$	$N=16$	$N=24$	$N=32$
1	999 998.9	999 998.9	999 998.9	999 998.9
2	2 002.4	1 999.0	1 998.5	1 998.3
3	299.0	297.4	297.2	297.1
4	124.3	122.8	122.6	122.6
5	76.4	74.6	74.4	74.3
6	56.9	54.4	54.2	54.1
7	47.7	43.8	43.5	43.5
8	45.6	37.5	37.2	37.1
9		33.4	33.0	32.9
10		30.6	30.1	29.9
11		28.6	27.9	27.8
12		27.2	26.3	26.1
13		26.2	25.0	24.8
14		25.7	24.0	23.8
15		25.9	23.2	22.9
16		28.0	22.5	22.2
17			22.0	21.6
18			21.6	21.1
19			21.3	20.7
20			21.1	20.3
21			21.1	20.0
22			21.2	19.7
23			21.8	19.5
24			23.9	19.3
25				19.2
26				19.1
27				19.0
28				19.0
29				19.2
30				19.5
31				20.1
32				22.1
ADT_{CA} from (8a)	36.99	21.94	18.67	17.28

Note: The average decision thresholds of CA CFAR are indicated in the bottom line for the same parameters N for reference; see Table I, $P_{fa} = 10^{-6}$.

The OS and CA CFAR procedures are compared using (2), thus yielding

$$\Delta[\mathrm{dB}] = 10 \log \frac{E(T_{OS} Z_{OS})}{E(T_{CA} Z_{CA})}. \tag{16}$$

Expressed in dB, the quotient of (16) represents the difference of the respective ADT values if they are in the same way measured in dB. It directly corresponds to the CFAR loss to be suffered in a model 1 situation if the CA CFAR procedure is replaced by an OS CFAR one.

These differences between CA CFAR and OS CFAR are illustrated by the data given in Table IV. Here for

TABLE IV
Comparison of CA CFAR and OS CFAR with $k = 3N/4$ for Various Reference Window Lengths N and Fixed $P_{fa} = 10^{-6}$ Showing the Normalized Average Decision Thresholds ADT for the Different CFAR Procedures

	$N=16$	$N=24$	$N=32$				
CA	21.94	18.67	17.28	($N=16$)	21.94	($N=16$)	21.94
OS	27.2	21.6	19.3	($N=24$)	21.6	($N=32$)	19.3
Δ(dB)	0.93	0.63	0.48		−0.07		−0.56

Note: In the bottom line the ADT ratios (relative CFAR loss) are given in dB.

specifying the OS CFAR procedure the parameter $k = 3N/4$ is chosen. This choice is motivated by the fact that it results in only a negligible additional loss in ADT due to the broad minimum of the function ADT versus rank order index k (see Fig. 8(c)), and that this value compared with the value $k = 7N/8$ corresponding to minimum ADT leads to less expansion of clutter areas. A value of k about $3N/4$ is well suited for practical application.

The response of OS CFAR when applied to a model 2 situation basically differs from that of CA and CAGO CFAR. Compare Fig. 6 with Fig. 9 which shows the reaction of the OS CFAR system to the same clutter edge situation with uncorrelated Rayleigh-distributed clutter. The parameters specifying the OS CFAR procedure are $N = 24$ and $k = 17$. The reference window for OS CFAR, Fig. 9(a), differs slightly from that used for the CA and CAGO CFAR simulations, Fig. 3(a). The directly adjacent cells to the test cell are not omitted since that would provide no advantage for the OS CFAR procedure.

Fig. 9 shows an almost ideal reaction of a CFAR system to unexpected clutter areas. The threshold (dotted line) immediately follows the clutter amplitudes with sufficient distance to avoid undesired clutter detections. There remains a kind of safety distance between clutter area and background. The width of the safety zone depends on the relation k/N. With k greater $N/2$ clutter areas are expanded, with k less than $N/2$ they are shrunk. These effects are called dilatation and erosion in image processing. With $k = N/2$ the so-called median filtering is performed.

These relations are illustrated by Fig. 10. It is obvious that for CFAR applications a value of k greater than $N/2$ should be used in order to avoid clutter detections at clutter edges.

For Figs. 6 and 9 the clutter was assumed uncorrelated. In contrast, in Fig. 7 the response of CA and CAGO CFAR was demonstrated for a fully correlated clutter situation. The results of OS CFAR processing applied to the same situation are shown in Fig. 11. Again an excellent CFAR-behavior can be observed.

From the discussion of Figs. 6 and 7 it was stated in Section III that the performance of CA and CAGO CFAR depends on the relation of the clutter area extension L to the reference window size N. Smaller values of L/N lead to underestimations of clutter power and thus to an increased clutter detection rate. It should be noted that

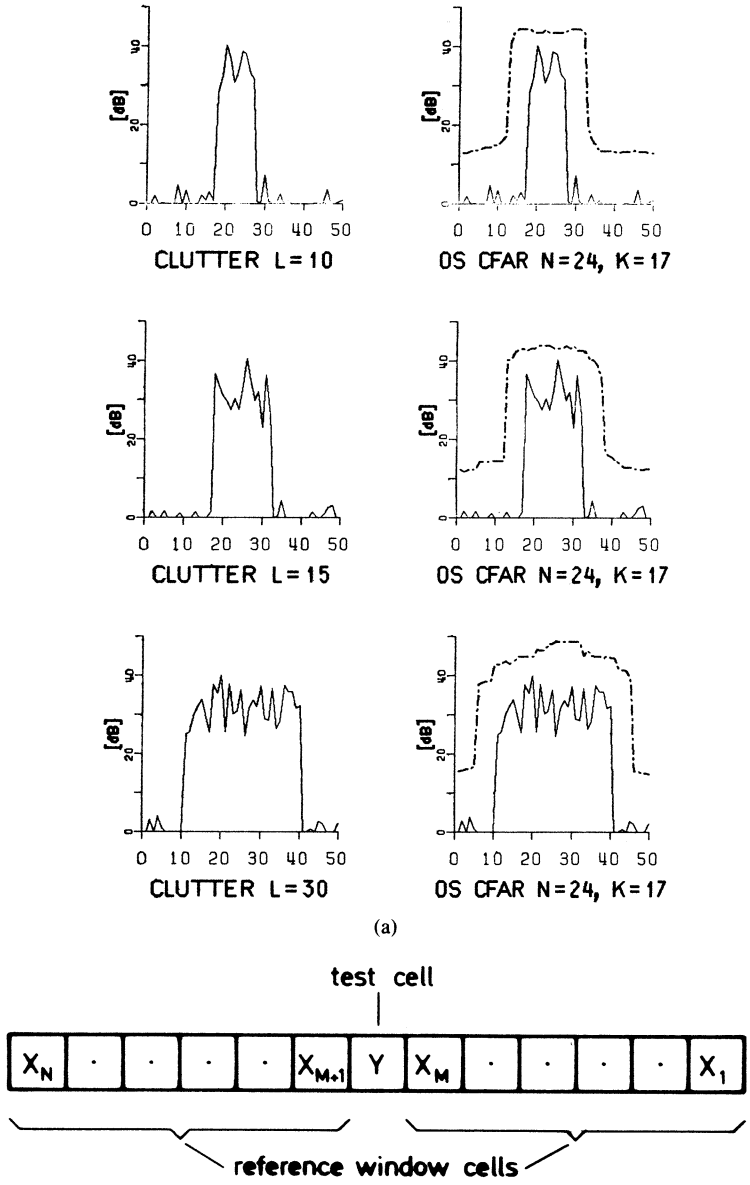

Fig. 9. (a) Behavior of OS CFAR ($N = 24$, $k = 17$) in model 2 clutter situations with statistically independent clutter amplitudes. (b) Definition of the reference window for OS CFAR used in the simulation program.

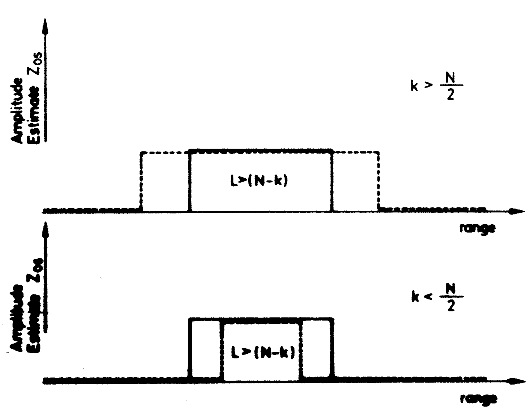

Fig. 10. Principal impact of the rank-order parameter k on the clutter power estimation in OS CFAR. The clutter power is shown in solid lines and the clutter power estimation Z in dotted lines. For $k > N/2$ the clutter area appears extended (dilatation), for $k < N/2$ it appears shrunk (erosion).

these problems to a large extent are removed when an OS CFAR system is applied. This becomes obvious by comparison of Figs. 7 and 11. Independently of the clutter length L clutter detections are sufficiently suppressed even if a reference window size N distinctly larger than L is used.

These arguments lead to the conclusion that the parameter N (reference window size) of a CFAR procedure must be treated differently for OS CFAR and the conventional CA and CAGO CFAR systems.

In both cases the reference window size N impacts the average clutter power estimate. As long as the model 1 assumption of uniformity (stationarity) is valid any increase in N will increase the accuracy of the clutter power estimate and thus improve the CFAR performance.

With the conventional CA and CAGO CFAR systems, however, the reference window size N additionally im-

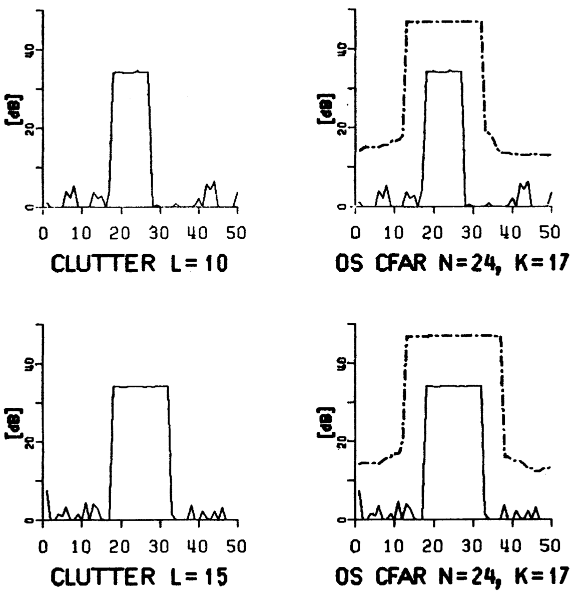

Fig. 11. Behavior of OS CFAR ($N=24$, $k=17$) in model 2 clutter situations with fully correlated clutter amplitudes (deterministic model).

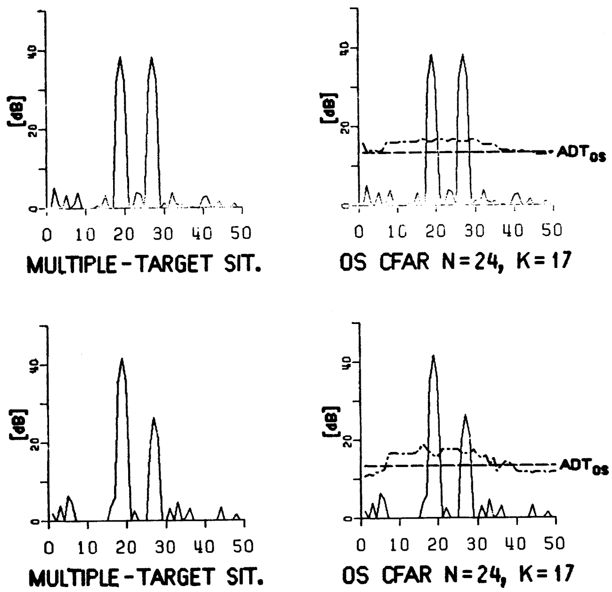

Fig. 12. Behavior of OS CFAR ($N=17$) in the same model 3 double target situation as in Fig. 3.

pacts the adaptability of the clutter power estimate Z_{CA} to changing clutter situations. The greater N the more slowly the estimate Z_{CA} is able to follow changes in clutter power in transition areas. This corresponds to a spatial lowpassing effect. The tradeoff between clutter power estimation (improving with increasing N), and adaptability to inhomogeneous clutter situations (improving with decreasing N), calls for a compromise resulting in a certain medium value of N not too large with respect to the smallest clutter areas still to be suppressed.

With OS CFAR, however, the influence of the reference window size N on the system adaptability is distinctly diminished. Therefore the reference window size N can be largely defined according to other aspects. It becomes particularly possible to operate with larger values of N than adequate with conventional CFAR processing. This allows OS CFAR system with larger N to be compared with conventional CA and CAGO CFAR systems with smaller N and justifies the statement that OS CFAR is superior to CA and CAGO CFAR also in model 1 uniform clutter situations; see Table IV.

One of the most important arguments for necessary improvements of conventional CFAR techniques was the masking, or range resolution, problem exemplified by the model 3 double target situation. The discussion of OS CFAR processing can be completed by analyzing the behavior of OS CFAR confronted with this situation. It is obvious that minor inhomogeneities occurring within the reference window do not modify, or only modify to a small extent, the clutter power estimation of an OS CFAR. In this context "minor" means anything that affects less than $(N-k)$ resolution cells.

The results of processing the simulated double target input data of Fig. 3 with the OS CFAR procedure is shown in Fig. 12. For specifying the OS CFAR procedure the parameters $N=24$, $k=17$ are used. The detection threshold (dotted line) is almost imperceptibly raised over the ADT_{OS}-level in background noise (dashed line).

Therefore, the probability of detection P_d for both targets is comparable to that of the targets in white noise; see Fig. 5. In this case only negligible differences in P_d are observed in signal situations with one or with several targets. Weak targets are not masked by stronger targets. Multiple target situations thus lead to almost negligible losses in OS CFAR processing compared with conventional CFAR processing.

V. DISCUSSION OF RESULTS

The comparisons of the different CFAR procedures in different model situations have clearly demonstrated the superiority of OS CFAR processing over conventional CA and CAGO CFAR processing. In the following some additional considerations referring to practical application of OS CFAR procedures are given.

For CA and CAGO CFAR reference window sizes of about $N = 16 \ldots 24$ are commonly used. For OS CFAR window sizes of about $N = 24 \ldots 32$ and more are applicable as discussed in the foregoing section. The rank-order parameter k of the OS CFAR procedure should be greater than $N/2$ which results in selecting a value greater than the median from the ordered sequence and has the effect of expanding the clutter areas. On the other hand, the difference $N-k$ should not be less than double the target length in order to avoid two targets from being mutually blanked.

This leaves a certain range for defining the parameter k within which the clutter level estimation performance of the OS CFAR procedure is only slightly changed; see the broad minimum in Fig. 8(c). The rank-order parameter k can therefore be deliberately determined within this range according to geometric considerations on reference window size N, minimum suppressible clutter size L, and maximum target width. For $N = 32$, e.g., k should be chosen between $N/2$ and $3N/4$.

In practical CA CFAR application, guard cells are used for separating the cell under test from the reference area in order to prevent target returns from falsifying the clutter level estimation; see Fig. 3(a). In OS CFAR processing these guard cells become unnecessary since a small number of target amplitudes occurring within the reference area have almost no influence on the clutter level estimation by quantiles. This is clearly confirmed by the fact that even a second target within the reference area does barely change the clutter level measurement; see Fig. 12. Therefore a reference window without guard cells can be used with OS CFAR processing; see Fig. 9(a).

In contrast to the foregoing theoretical treatments of CFAR processing which were based on the use of the squared magnitude (square law detector) of the coherent receiver output signals in the video domain, in practice the absolute value (linear detector) is most frequently used. The random variables $X_1, \ldots, X_N$ of model 1 are then no longer exponentially distributed (5a), but obey a Rayleigh distribution.

As far as the OS CFAR procedure is concerned, only the scaling factor T is affected. The scaling factors T_{lin} for the linear detector can easily be derived from the scaling factors T_q valid for the square law detector and given in Table II. The conversion rule can be derived as follows [12]:

$$\begin{aligned} P[Y_q \geq T_q Z_q] &= P[\sqrt{Y_q} \geq \sqrt{T_q Z_q}] \\ &= P[Y_{\text{lin}} \geq T_{\text{lin}} Z_{\text{lin}}] \\ T_{\text{lin}} &= \sqrt{T_q}. \end{aligned} \tag{17}$$

In accordance with (17) it needs no more than taking the square root of the scaling factors T_q listed in Table II in order to obtain the scaling factors T_{lin} for the linear detector. This simple conversion, however, does not apply for CA or CAGO CFAR.

APPENDIX

Let $X_1, \ldots, X_N$ be a sequence of statistically independent, identically distributed random variables. The pdf of the random variables is denoted by $p(x)$, and their distribution function by $P(x)$. The pdf of the kth value of the ordered statistic is given in [11].

$$\begin{aligned} P_{X_{(k)}}(x) = p_k(x) &= k \binom{N}{k} (1 - P_X(x))^{N-k} \\ &\qquad (P_X(x))^{k-1} \, p_X(x). \end{aligned} \tag{A1}$$

For the minimum $X_{(1)}$,

$$p_1(x) = N(1 - P_X(x))^{N-1} \, p_X(x) \tag{A2}$$

applies, and for the maximum $X_{(N)}$,

$$p_N(x) = N(P_X(x))^{N-1} \, p_X(x) \tag{A3}$$

applies.

In addition to the minimum and the maximum, the median of the ordered sequence is frequently of particular significance in actual practice. For an odd number $N = 2M - 1$, the middle value $X_{(M)}$ of the ordered sequence is designated "empirical" median. The pdf can be calculated directly from (A1).

$$p_M(x) = M \binom{N}{M} (1 - P_X(x))^{M-1} (P_X(x))^{M-1} \, p_X(x). \tag{A4}$$

For an exponentially distributed parent population (5a), the pdf of the kth value of the ordered statistic is given in (12).

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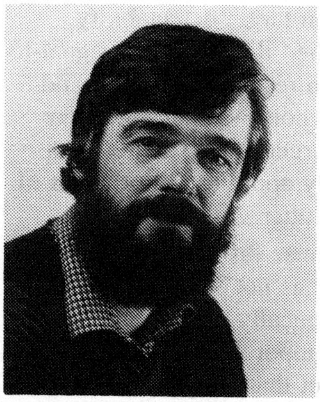

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