New Modulation Waveforms for Delay and Time-scale Spread Wideband Channels

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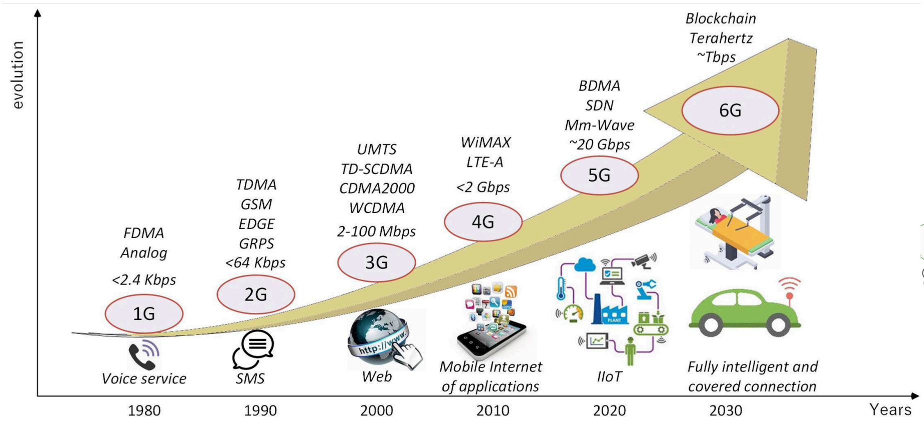
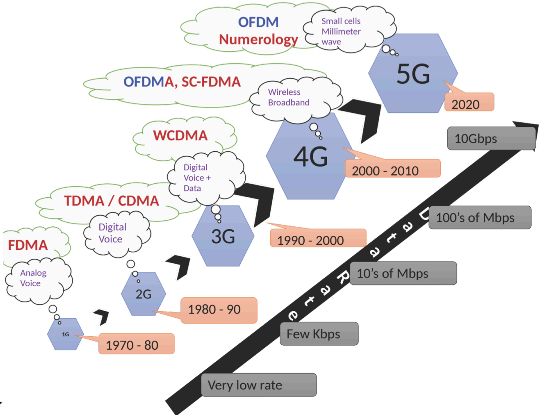
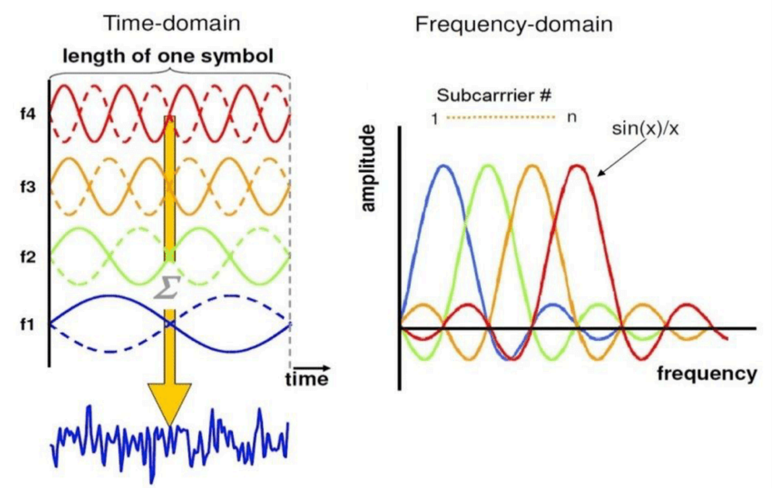
Waveform Evolution in Terrestrial RF Communication

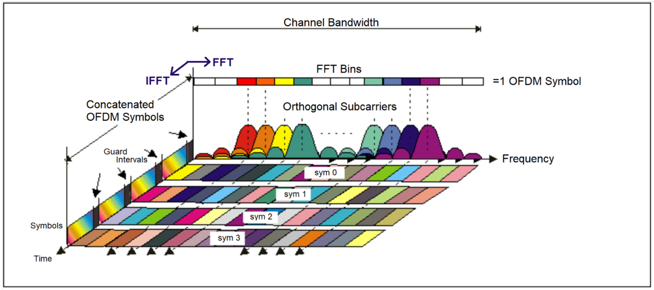




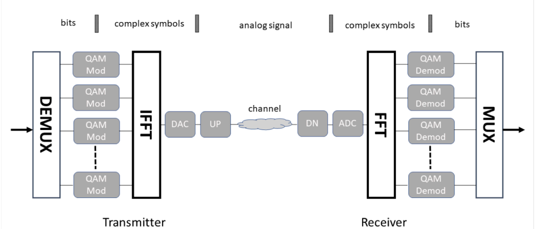
- High mobility communications in rural areas
  - Low frequencies preferred for long range
  - Large bandwidth desirable for high data rates
- UnderWater Acoustic (UWA), Ultra WideBand (UWB) channels need new waveforms

OFDM: A Widely Employed Waveform



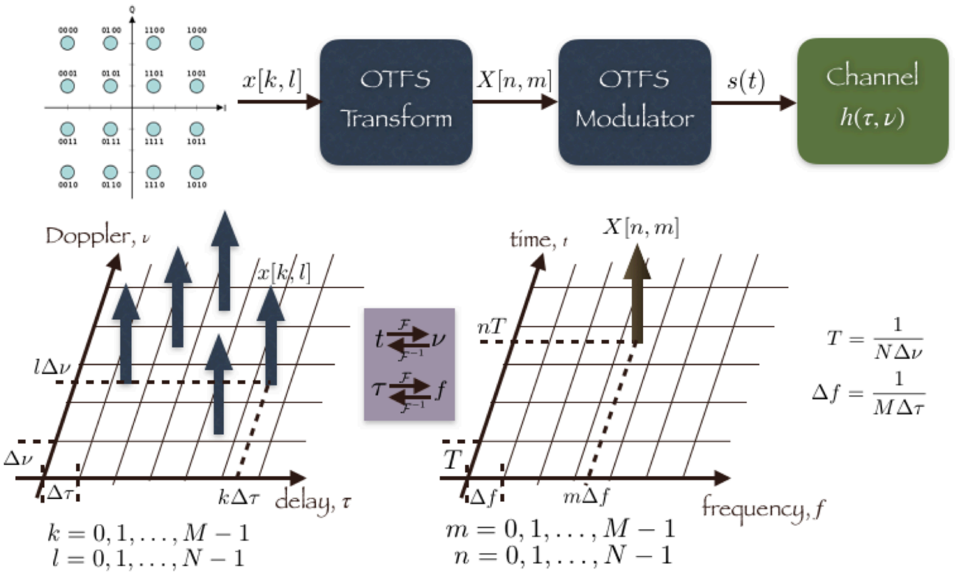


Frequency-Time Representative of an OFDM signal



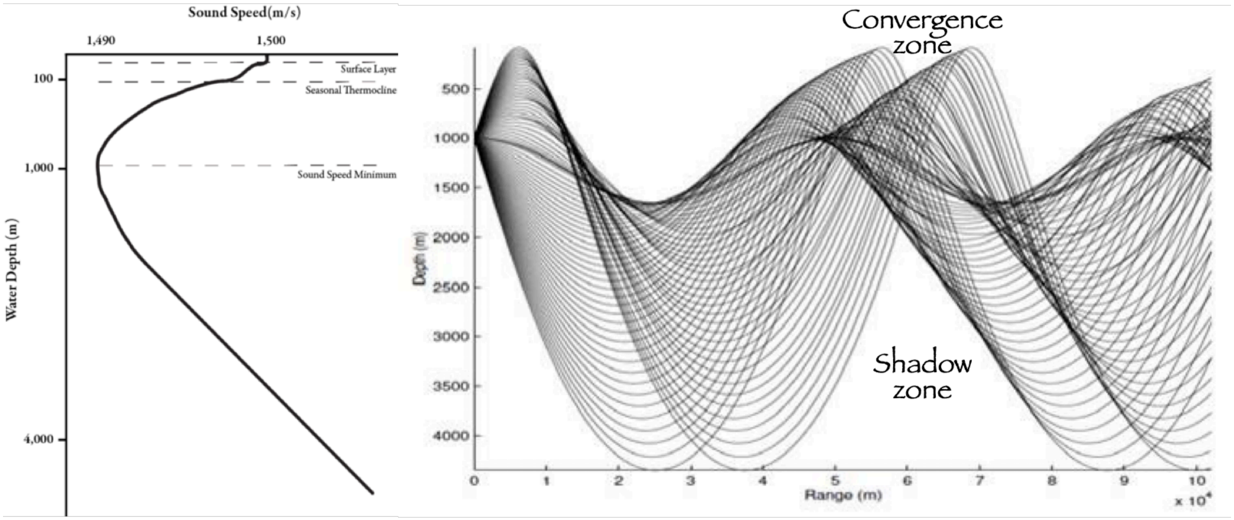
- Designed for delay-spread channels
  - dominant waveform in 4G, 5G
- Known to be sensitive to
  - high vehicle speeds
  - temporal fluctuations (in the channel)

OTFS: A Recent Waveform



- Designed for narrowband, underspread channels
  - Narrowband: Effect of Doppler $\approx$ frequency shift
  - Underspread: (Doppler spread) $\times$ (Delay spread) $\ll 1$
- Useful in high-mobility scenarios

The UWA Channel



- Troubles of both **deep space** and **terrestrial** radio channels
  - large propagation delay (as in deep space radio channels)
  - fading and frequency dependent path loss (as in terrestrial radio channels)
- Large delay and Doppler spread $\implies$ frequency and time selective
- Large fractional bandwidth: $\frac{B}{f_c}$ $\implies$ UWA is a wideband channel

Doppler in UWA versus RF Communication

- Doppler causes time-scaling: $s(t) \to \sqrt{\alpha} s(\alpha t)$, where $\alpha \approx 1 \pm \frac{2|v|}{c}$
- If $s(t) \rightleftharpoons S(f)$ are Fourier transform pairs, then $\sqrt{\alpha} s(\alpha t) \rightleftharpoons \frac{1}{\sqrt{\alpha}} S(\frac{f}{\alpha})$

	UWA	**RF**
Communication Band, f_L–f_H	10 kHz – 20 kHz	400 MHz – 420 MHz
Fractional bandwidth, $\frac{B}{f_c}$	0.6667	0.0488
Wave speed, c (m/s)	1500	3×10^8
Relative speed, v (m/s)	1.5	278
Doppler shift, δf_L (Hz)	20	741.3
Doppler shift, δf_H (Hz)	40	778.4
No. of subcarriers, N_{FFT}	1024	1024
Subcarrier spacing, Δf (Hz)	9.8	19500
Fractional Doppler shift, $\frac{\delta f_H}{\Delta f}$	4.1	0.04

- Delay-scale spread channel:

$$r_s(t) = \iint h(\tau,\alpha)\sqrt{\alpha}s\left(\alpha(t-\tau)\right)d\tau d\alpha$$

time-scale $\alpha = \frac{c-v}{c+v}$ (v: radial velocity of the scatterer, c: wave sound)

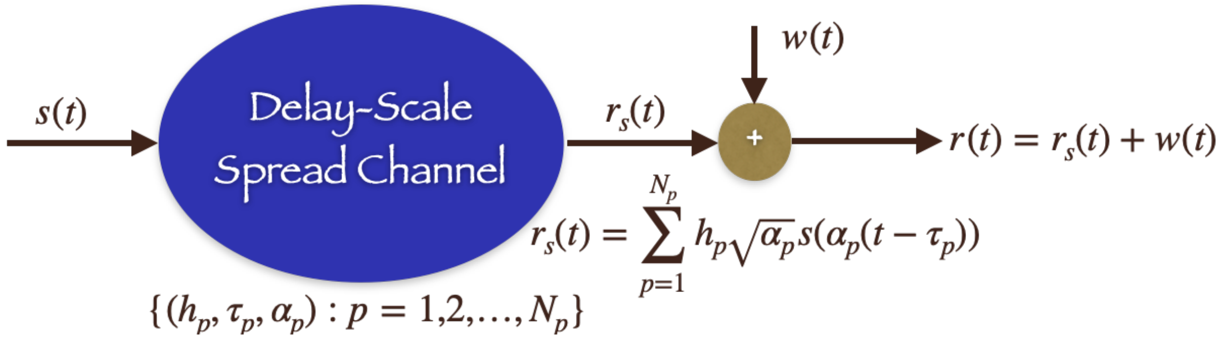
- Narrowband channel: if $B/f_c \ll 1$ **and** $v \ll \frac{c}{2BT}$ are satisfied, Doppler can be approximated by a frequency shift, $\nu \approx (\alpha - 1)\, f_c$,

$$r_s(t) = \iint h(\tau,\nu)s\left(t-\tau\right)e^{j2\pi\nu(t-\tau)}d\tau d\nu$$

- In a wideband channel: **either** $B/f_c \ll 1$ **or** $v \ll \frac{c}{2BT}$ is **violated**
- **Most RF channels are narrowband, UWA & UWB channels are wideband**

Outline

Transmitter & Channel Model



$$s(t) = \sum_{n=0}^{N-1} s_{BB}[n] \, \mathrm{sinc}\left(B(t - nT_s)\right) e^{j2\pi f_c t} \quad \text{(Transmitted waveform)}$$

$$r_s(t) = \sum_{p=1}^{N_p} h_p \sqrt{\alpha_p} s(\alpha_p(t - \tau_p)) \quad \text{(Propagation channel)}$$

$$= \sum_{p=1}^{N_p} h_p \sqrt{\alpha_p} \sum_{n=0}^{N-1} s_{BB}[n] \, \mathrm{sinc}\left(B(\alpha_p \overline{t - \tau_p} - nT_s)\right) e^{j2\pi f_c \alpha_p \overline{t - \tau_p}}$$

$$r(t) = r_s(t) + w(t) \quad \text{(Received signal)}$$

Receiver

- Received signal in the baseband: $r_{\mathrm{BB,s}}(t) = r_s(t)e^{-j2\pi f_c t}$
- The samples of the received signal in the baseband, $r_{\mathrm{BB,s}}[m'] = r_{\mathrm{BB,s}}(m'/F_s)$, can be expressed as

$$r_{\mathrm{BB,s}}[m'] = \sum_{m=0}^{M-1} s_{\mathrm{BB}}[m] \sum_{p=1}^{P} h_p \sqrt{\alpha_p} e^{-j2\pi f_c \alpha_p \tau_p} \times e^{j2\pi f_c(\alpha_p - 1)m'/F_s} \mathrm{sinc}\left(B(\alpha_p m' - m)/F_s - \alpha_p \tau_p B\right),$$

for $m' = 0, 1, \ldots, M' - 1$, where $M' = \lfloor F_s T + F_s \tau_{\max} \rfloor$ is the number of signal samples at the delay-scale channel output

Receiver Measurement Model

- Received signal vector including the additive receiver noise:

$$\mathbf{r} = H\mathbf{s} + \mathbf{w},$$

where $\mathbf{w} \in \mathbb{C}^{M'}$ is the vector receiver noise samples $w[m]$,

$$\begin{aligned} \mathbf{s} &= \left[s_{\text{BB}}[0], s_{\text{BB}}[1], \ldots, s_{\text{BB}}[M-1]\right]^T \in \mathbb{C}^{M}, \\ \mathbf{r} &= \left[r_{\text{BB}}[0], r_{\text{BB}}[1], \ldots, r_{\text{BB}}[M'-1]\right]^T \in \mathbb{C}^{M'}, \end{aligned}$$

$r_{m'} = r_{\text{BB}}[m'] = r_{\text{BB,s}}[m'] + w[m']$, and $H \in \mathbb{C}^{M' \times M}$ is the delay-scale channel matrix whose $(m', m)^{\text{th}}$ entry is given by

$$H_{m',m} = \sum_{p=1}^{P} h_p \sqrt{\alpha_p} e^{-j2\pi f_c \alpha_p \tau_p} e^{j2\pi f_c (\alpha_p - 1) m'/F_s} \times \text{sinc}\left(B(\alpha_p m' - m)/F_s - \alpha_p \tau_p B\right)$$

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1. VBMC Communications
   - Delay-Scale Channel Model
   - Core Idea: Variable Subcarrier Bandwidth
   - Framework for Waveform Evaluation
   - Numerical Results
2. ODSS Communications
   - Mellin Transform & ODSS Transmission Scheme
   - Numerical Results

Core Idea: Variable Subcarrier Bandwidth

- Signal arriving along pth path: $r_s^{(p)}(t) = h_p\sqrt{\alpha_p}s(\alpha_p(t-\tau_p))$
- pth path signal spectrum: $R_s^{(p)}(f) = h_p\frac{1}{\sqrt{\alpha_p}}S\left(\frac{f}{\alpha_p}\right)e^{-j2\pi f\tau_p}$
- A component at f_0 in $s(t)$ appears at $\alpha_p f_0$ in $r_s^{(p)}(t)$
- Spectrum expansion: $R_s^{(p)}(\alpha_p f_0) = h_p\frac{1}{\sqrt{\alpha_p}}S(f_0)\,e^{-j2\pi\alpha_p f_0\tau_p}$

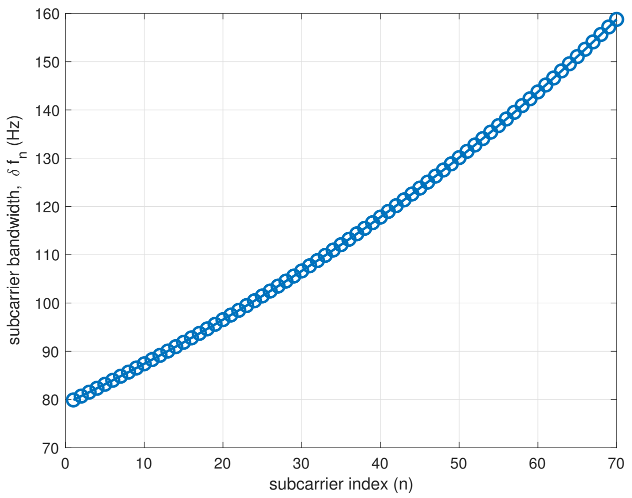


Figure 1: $f_L = 10$ kHz, $f_H = 20$ kHz, $\alpha_{\max} = 1.001$, $T = 20$ ms, $\beta = 2$

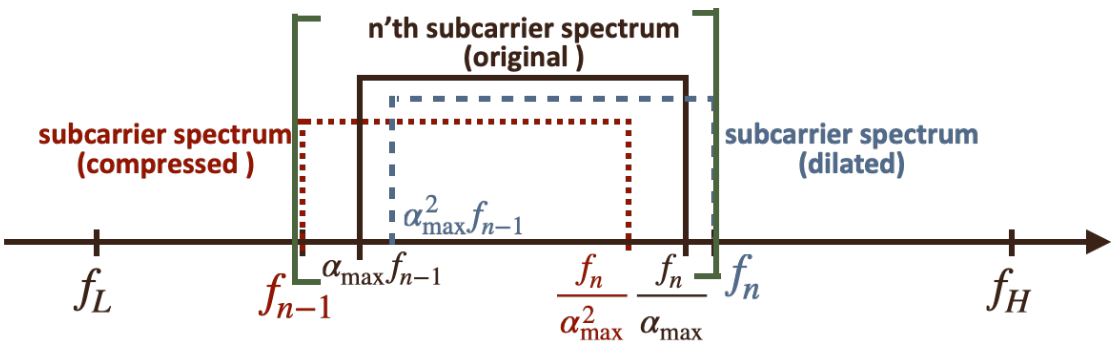
VBMC Waveform: Parameters[1]

- **Communication band**: $f_L - f_H$, **Symbol bandwidth**: $\Delta f = \frac{\beta}{T}$, where T: symbol duration, β: pulse shaping factor
- N frequency cells of varying widths: $f_0 = f_L, f_1, f_2, \ldots, f_N$

$$N = \left\lfloor \frac{\log\left(f_H/f_L\right)}{\log\left(1 + \Delta f/f_L\right)} \right\rfloor$$

$$f_{n+1} = f_n\left(1 + \Delta f/f_L\right) = f_L\left(1 + \Delta f/f_L\right)^n, n = 0, 1, \ldots, N-1$$

- Subcarrier bandwidth varies with n: $\delta f_n = \alpha_{\max}^{-1} f_n - \alpha_{\max} f_{n-1}$



[1] Arunkumar K. P. and Chandra R. Murthy, "Variable Bandwidth Multicarrier Communications: A New Waveform for the Delay-Scale Channel", *SPAWC*, Oulu, Finland, July 2022

VBMC Waveform: Construction

START

INPUT f_L, f_H, T, β

- Communication band: $[f_L, f_H]$
- Symbol duration: T
- Pulse shaping factor: β
- n: subcarrier index
- Subcarrier bandwidth $\delta f_n = \alpha_{\max}^{-1} f_n - \alpha_{\max} f_{n-1}$

COMPUTE SYMBOL BANDWIDTH

$$\Delta f = \frac{\beta}{T}$$

→ **COMPUTE NUMBER OF SUBCARRIERS**

$$N = \left\lfloor \frac{\log(f_H/f_L)}{\log(1+\Delta f/f_L)} \right\rfloor$$

→ **COMPUTE SUBCARRIER BAND EDGE FREQUENCIES**

$$f_{n+1} = f_n(1+\Delta f/f_L) = f_L(1+\Delta f/f_L)^n, \quad n = 0, 1, \ldots, N-1$$

→ **COMPUTE SUBCARRIER PULSE DURATION**

$$T_n = \left(\frac{\alpha_{max}^{-1} f_1 - \alpha_{max} f_0}{\alpha_{max}^{-1} f_n - \alpha_{max} f_{n-1}} \right) T$$

→ **COMPUTE SUBCARRIER CHIRP RATE**

$$k_n = \frac{\alpha_{max}^{-1} f_n - \alpha_{max} f_{n-1}}{T_n}$$

→ **COMPUTE SUBCARRIER CHIRP CENTRE FREQUENCY**

$$f_{c,n} = \frac{\alpha_{max} f_{n-1} + \alpha_{max}^{-1} f_n}{2}$$

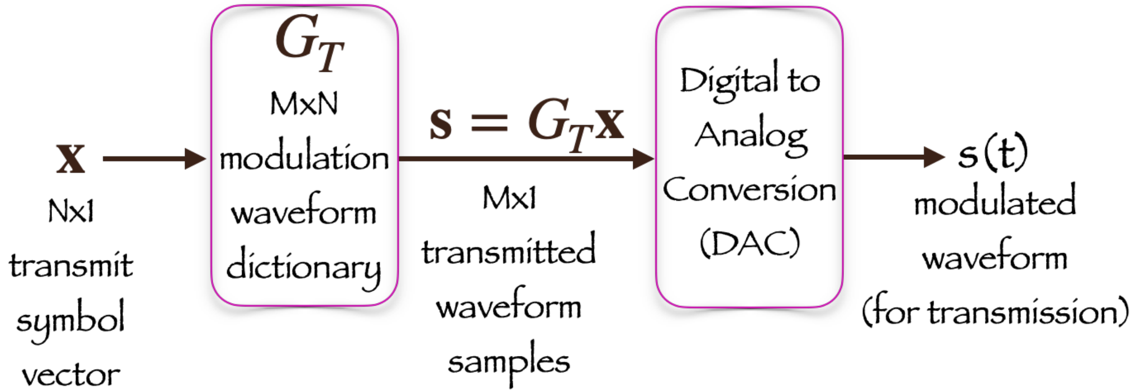
→ **COMPUTE VBMC SUBCARRIER WAVEFORMS**

$$c_n(t) = \frac{1}{\sqrt{T_n}} e^{j2\pi\left(f_{c,n} t + \frac{1}{2} k_n t^2\right)}, \quad -T_n/2 \le t \le T_n/2$$

→ **END**

Outline

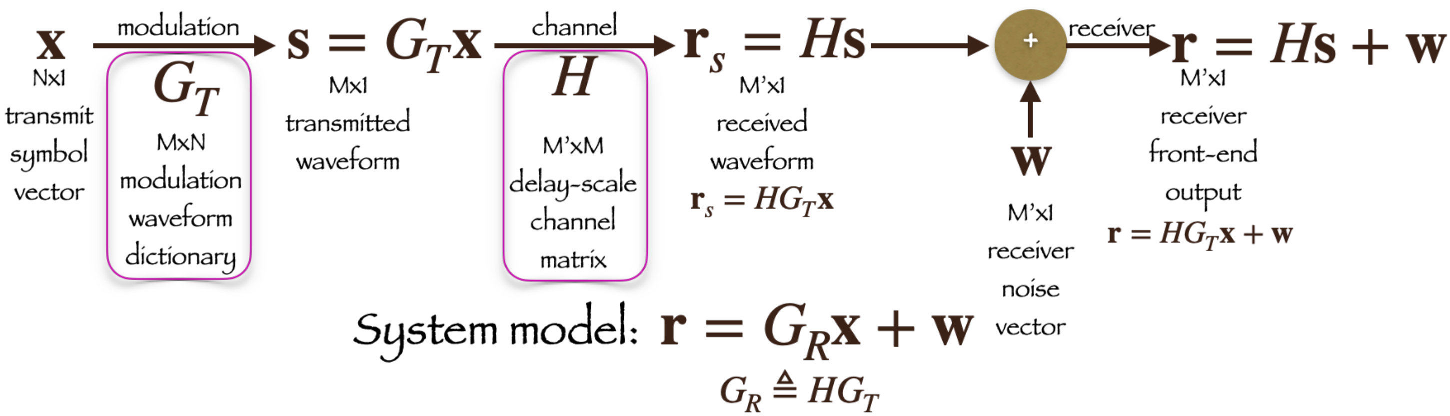
VBMC Waveform: Digital Modulation

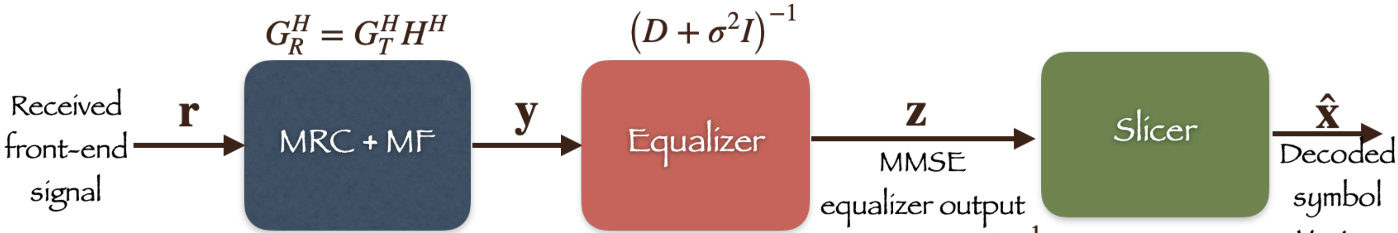


nth column of $\mathbf{G}_T$ contains M_n samples of the nth VBMC subcarrier:

$$G_T(m,n) = \begin{cases} c_n\left(\frac{m-m_n}{F_s} - \frac{T_n}{2}\right), & m_n \le m \le m_n + M_n - 1 \\ 0, & \text{otherwise} \end{cases}$$

System Model: A Common Framework





$$\mathbf{r} = G_R\mathbf{x} + \mathbf{w}$$

$$\mathbf{w} \sim \mathcal{CN}\left(0, \sigma^2 I\right)$$

$$\mathbf{y} = G_R^H \mathbf{r}$$

$$D \triangleq G_R^H G_R$$

$$\mathbf{v} \triangleq G_R^H \mathbf{w}$$

$$\mathbf{y} = G_R^H G_R \mathbf{x} + G_R^H \mathbf{w}$$

$$\mathbf{y} = D\mathbf{x} + \mathbf{v}$$

$$\mathbf{z} = \left(D + \sigma^2 I\right)^{-1} \mathbf{y}$$

$$\mathbf{u} \triangleq \left(D + \sigma^2 I\right)^{-1} \mathbf{v}$$

$$\mathbf{z} = \left(D + \sigma^2 I\right)^{-1} D\mathbf{x} + \mathbf{u}$$

- Full Complexity Equalizer (FC-EQ): uses entire $D = G_R^H G_R$
- One-tap Equalizer (1-tap EQ): uses diagonal approximation of D

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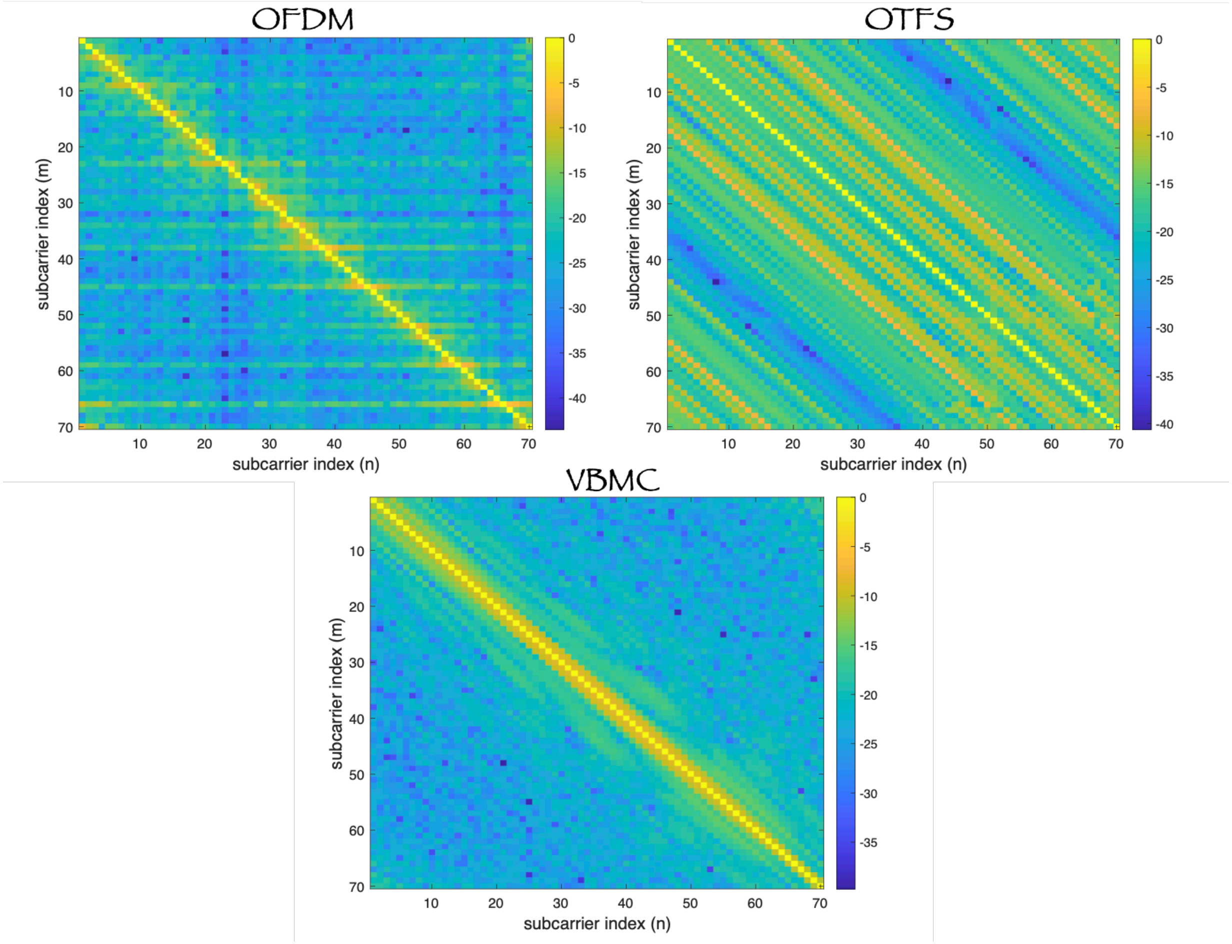
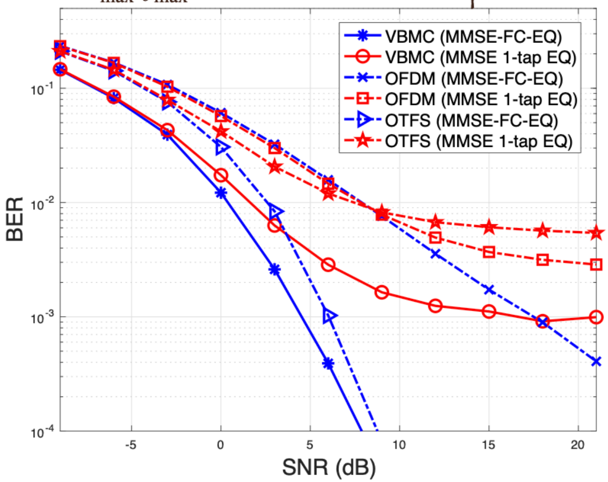
Composite Channel Matrices



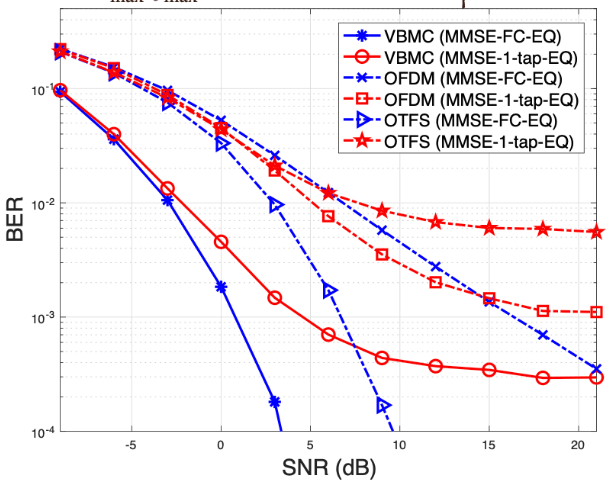
BER Performance: Underspread and Overspread channels

$\tau_{\max}\delta f_{\max} = 0.2 < 0.25 \Rightarrow$ underspread



$\tau_{\max} = 10$ ms, $\alpha_{\max} = 1.001$

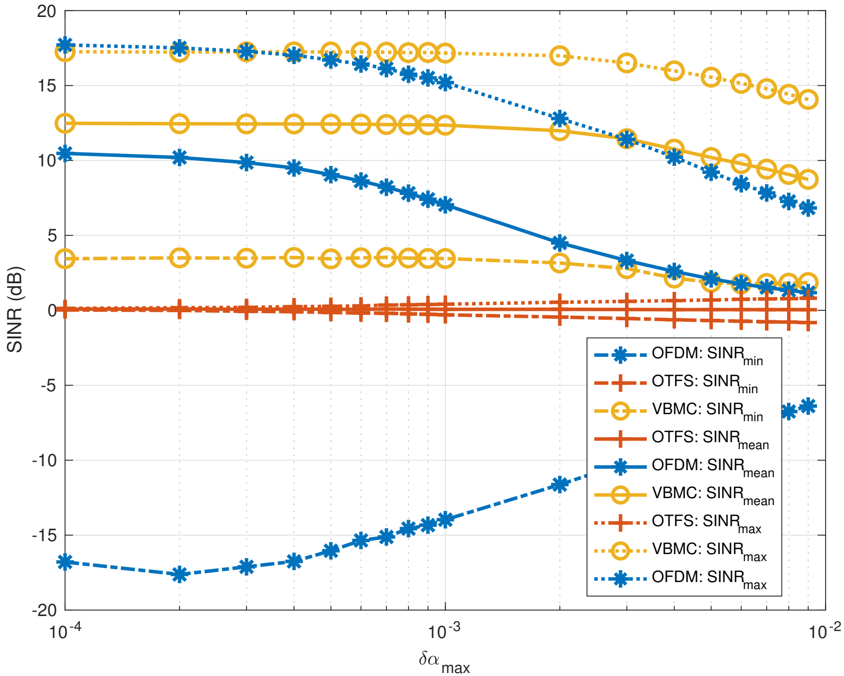
$\tau_{\max}\delta f_{\max} = 0.8 > 0.25 \Rightarrow$ overspread



$\tau_{\max} = 20$ ms, $\alpha_{\max} = 1.002$

SINR Statistics

$$\mathrm{SINR}_n = \frac{|D_{n,n}|^2}{\sum_{n' \neq n} |D_{n,n'}|^2 + \sigma^2_{v,n}}$$



Jain's Fairness Index

$$\mathcal{J}_D = \frac{|\sum_n D_{n,n}|^2}{N \sum_n |D_{n,n}|^2}$$

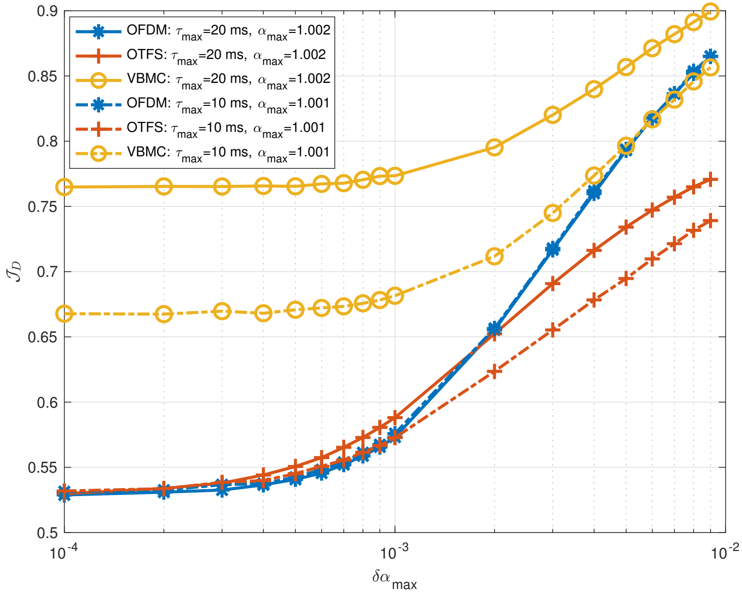
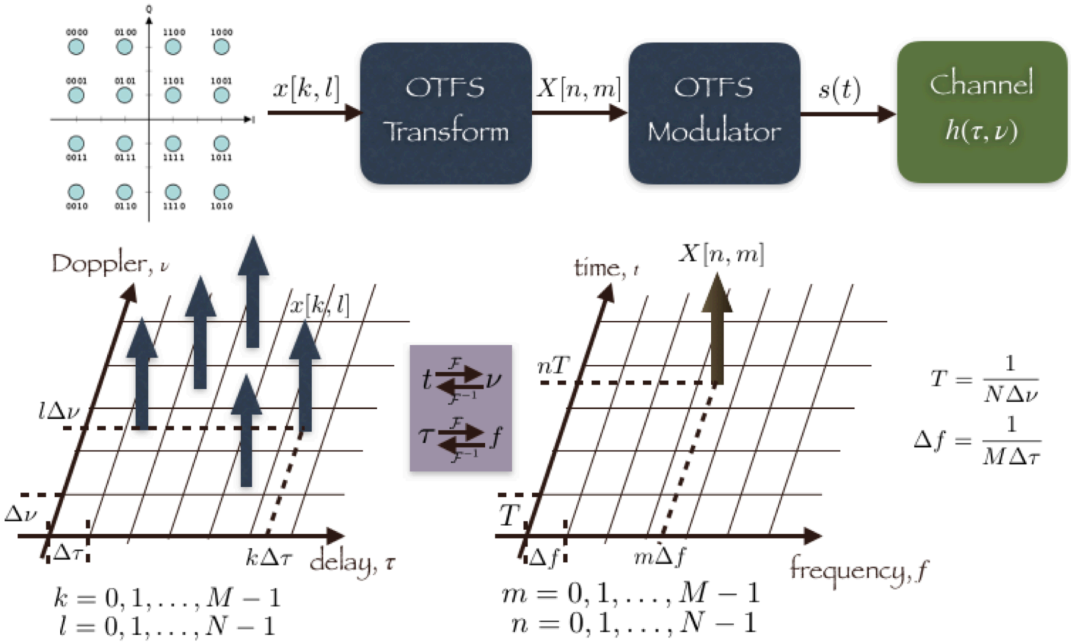


Figure 3: Jain's fairness index of the diagonal entries of OFDM, OTFS and VBMC composite channel matrix, D.

OTFS: Waveform for Narrowband Delay-Doppler Channels



- Designed for narrowband, underspread channels
- Useful in high-mobility scenarios
- Need new waveforms for wideband, overspread channels (UWA, UWB)

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Key Ingredient: Mellin Transform[2]

- Mellin transform is matched to scale changes (geometrically samples the scale parameter):

$$\mathcal{M}_x(\beta) \triangleq \int_0^{\infty} \frac{1}{\sqrt{\alpha}} x(\alpha) e^{j2\pi\beta \log(\alpha)} d\alpha$$

and the inverse Mellin transform is given by:

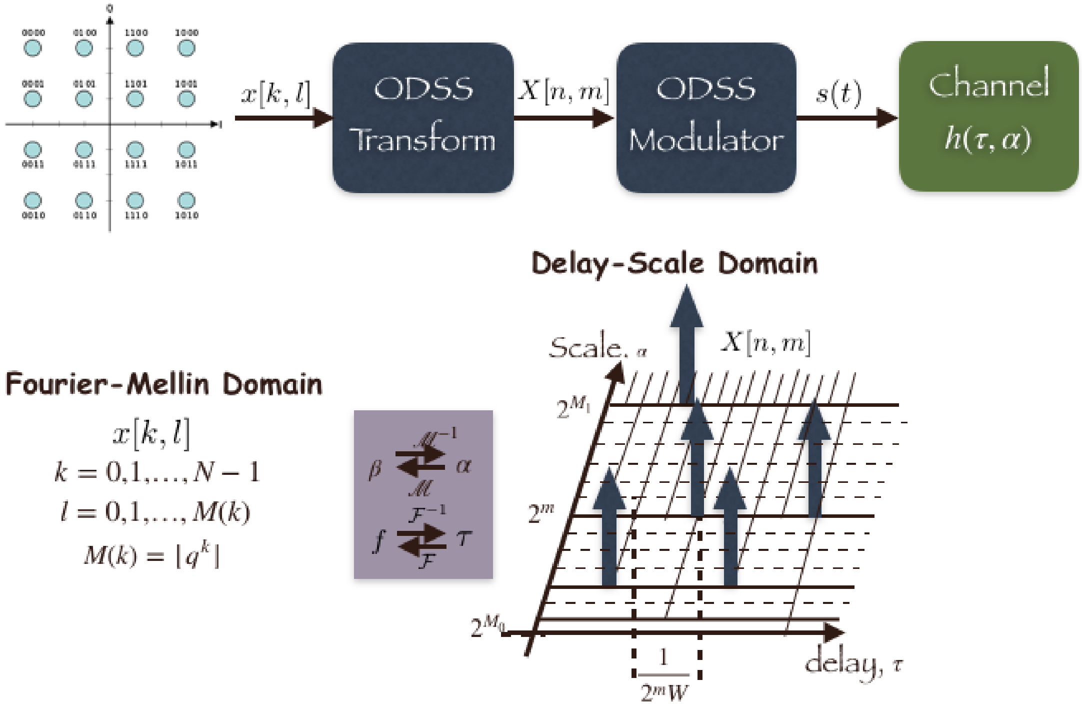
$$x(\alpha) \triangleq \frac{1}{\sqrt{\alpha}} \int_{-\infty}^{\infty} \mathcal{M}_x(\beta) e^{-j2\pi\beta \log(\alpha)} d\beta, \alpha > 0.$$

- Mellin transform is invariant to time-scale changes (up to a phase shift): for $a > 0$, $x(\alpha) \xrightarrow{\mathcal{M}} \mathcal{M}_x(\beta) \implies \sqrt{a}x(a\alpha) \xrightarrow{\mathcal{M}} e^{-j2\pi\beta \log a}\mathcal{M}_x(\beta)$

- *Multiplicative* convolution between two scale-domain signals corresponds to the product of their Mellin transforms in the Mellin domain:

$$(x_1 \vee x_2)(\alpha) = \int_0^{\infty} \sqrt{\alpha} x_1(\alpha') x_2\left(\frac{\alpha}{\alpha'}\right) \frac{d\alpha'}{\alpha'} \xrightarrow{\mathcal{M}} \mathcal{M}_{x_1}(\beta)\mathcal{M}_{x_2}(\beta)$$

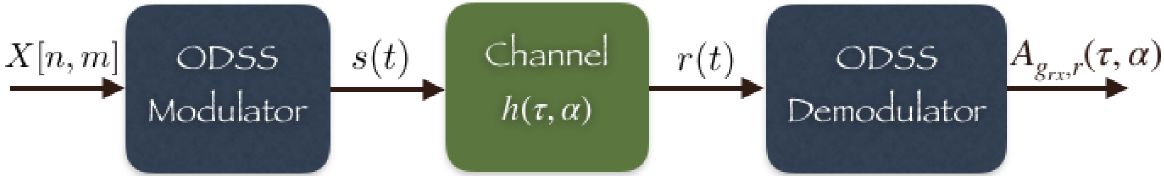
[2] J. Bertrand, P. Bertrand, and J.-P. Ovarlez, *The Mellin Transform*, 2nd ed. Boca Raton: CRC Press LLC, 2000



ODSS Transform (discrete IMFT):

$$X[n,m] = \frac{q^{-n/2}}{N} \sum_{k=0}^{N-1} \frac{\sum_{l=0}^{M(k)-1} x[k,l] e^{j2\pi\left(\frac{ml}{M(k)} - \frac{nk}{N}\right)}}{M(k)}$$

ODSS for Wideband Delay-Scale Channel



$$s(t) = \sum_{n=0}^{N-1} \sum_{m=0}^{M(n)-1} X[n,m] q^{m/2} g_{tx}\left(q^m\left(t - \frac{n}{\alpha_0^m W}\right)\right) \quad \text{(Modulator)}$$

$$\begin{aligned} r(t) &= \int\int h(\tau,\alpha)\sqrt{\alpha}\, s\left(\alpha(t-\tau)\right) d\tau d\alpha \quad \text{(Propagation channel)} \\ A_{g_{rx},r}(\tau,\alpha) &= \int \sqrt{\alpha}\, g_{rx}^*\left(\alpha(t-\tau)\right) r(t) dt \quad \text{(Demodulator)} \\ &= \sum_n \sum_m X[n,m] H_{n,m}(\tau,\alpha) \end{aligned}$$

$$H_{n,m}(\tau,\alpha) = \iint h(\tau'',\alpha'') A_{g_{rx},g_{tx}}\left(\alpha'' q^n\left(\tau - \frac{m}{\alpha'' q^n W} - \tau''\right), \frac{\alpha}{\alpha'' q^n}\right) d\tau'' d\alpha''$$

ODSS: Choice of q and W

- ICI is avoided if we choose –
  1. $q \geq \alpha_{\max}^2$, and
  2. $W = \min\left(W_{m'>m}, W_{m'<m}\right)$, where,

$$W_{m'>m} \triangleq \frac{1}{(1+\alpha_{\max})\,\tau_{\max}}$$

$$W_{m'<m} \triangleq \frac{1}{\left(1+\alpha_{\max}^{2N-3}\right)\tau_{\max}}$$

- Under the following assumptions:
  1. $h(\tau, \alpha)$ has a finite support: is non-zero only for $-\tau_{\max} \leq \tau \leq \tau_{\max}$ and $\frac{1}{\alpha_{\max}} \leq \alpha \leq \alpha_{\max}$, where $\alpha_{\max} \geq 1$, and
  2. *robust bi-orthogonality* holds between the transmit and receive pulses: the cross-ambiguity function vanishes in the neighborhood of all lattice points $\left(\frac{m}{q^n W}, q^n\right)$ *except* $(0, 1)$ corresponding to $m = 0$ and $n = 0$. That is, $A_{g_{\text{rx}}, g_{\text{tx}}}(\tau, \alpha) = 0$ for $\tau \in \left(\frac{m}{q^n W} - \tau_{\max}, \frac{m}{q^n W} + \tau_{\max}\right)$ and $\alpha \in (q^n/\alpha_{\max}, q^n \alpha_{\max})$ except when $m = 0$ and $n = 0$.

ODSS: Input-Output Relation

- ODSS scheme results in an ISI/ICI free symbol measurements (in the delay-scale domain), and time-independent scalar (complex) channel gains:

$$\hat{Y}[n,m] = H_{n,m}[n,m]X[n,m] + W[n,m]$$

- Matrix-vector form

$$\hat{\mathbf{Y}} = \mathbf{D}\mathbf{X} + \mathbf{W}$$

where $\hat{\mathbf{Y}} \in \mathbb{C}^{M_{\text{tot}} \times 1}$: obtained by stacking $\hat{Y}[n,m]$, $\mathbf{D} \in \mathbb{C}^{M_{\text{tot}} \times M_{\text{tot}}}$: diagonal matrix formed by stacking $H_{n,m}[n,m]$ along diagonal, $\mathbf{X} \in \mathbb{C}^{M_{\text{tot}} \times 1}$: data symbol vector obtained by stacking $X[n,m]$, $\mathbf{W} \in \mathbb{C}^{M_{\text{tot}} \times 1}$: additive noise

ODSS: Data Decoding

- Data decoding proceeds after an MMSE equalizer on $\hat{\mathbf{Y}}$:

$$\hat{\mathbf{Z}} = \mathbf{D}^H \left(\mathbf{D}\mathbf{D}^H + \sigma_W^2 I \right)^{-1} \hat{\mathbf{Y}},$$

  where σ_W^2 is the noise variance in the delay-scale domain

- The data symbol vector is then obtained as follows:

$$\hat{\mathbf{x}} = \mathcal{S}\left(\mathcal{T}_{\text{iMF}}^{-1} \hat{\mathbf{Z}} \right),$$

  where the operator $\mathcal{S}(.)$ slices each entry in the input vector to the nearest symbol in the transmitted constellation

ODSS Subcarriers

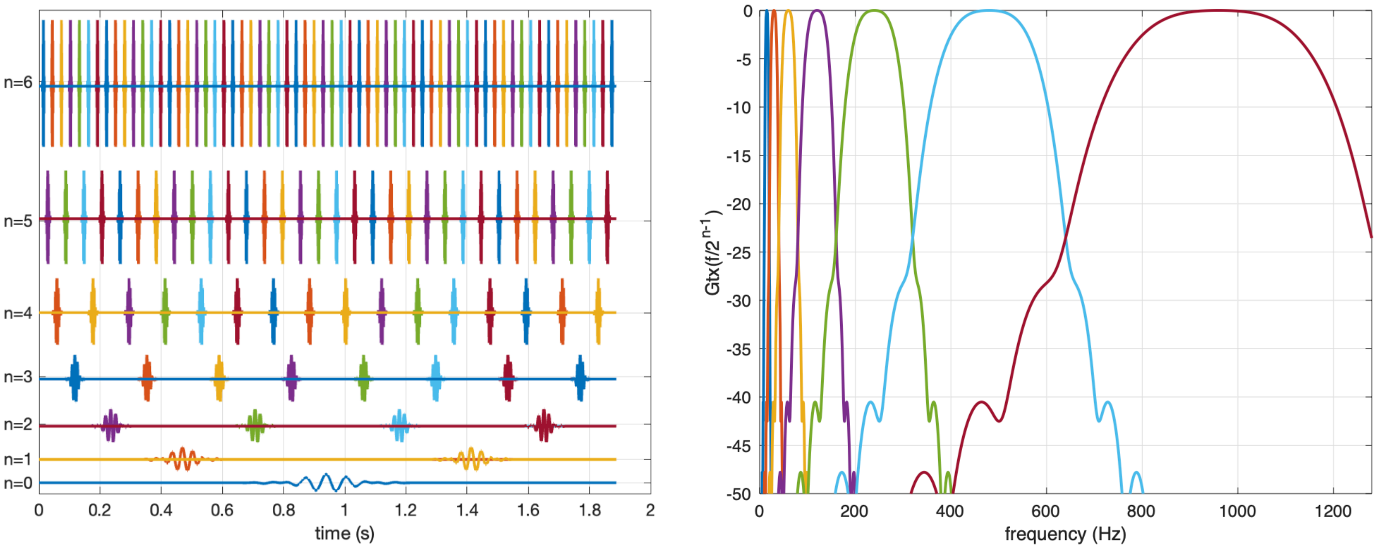
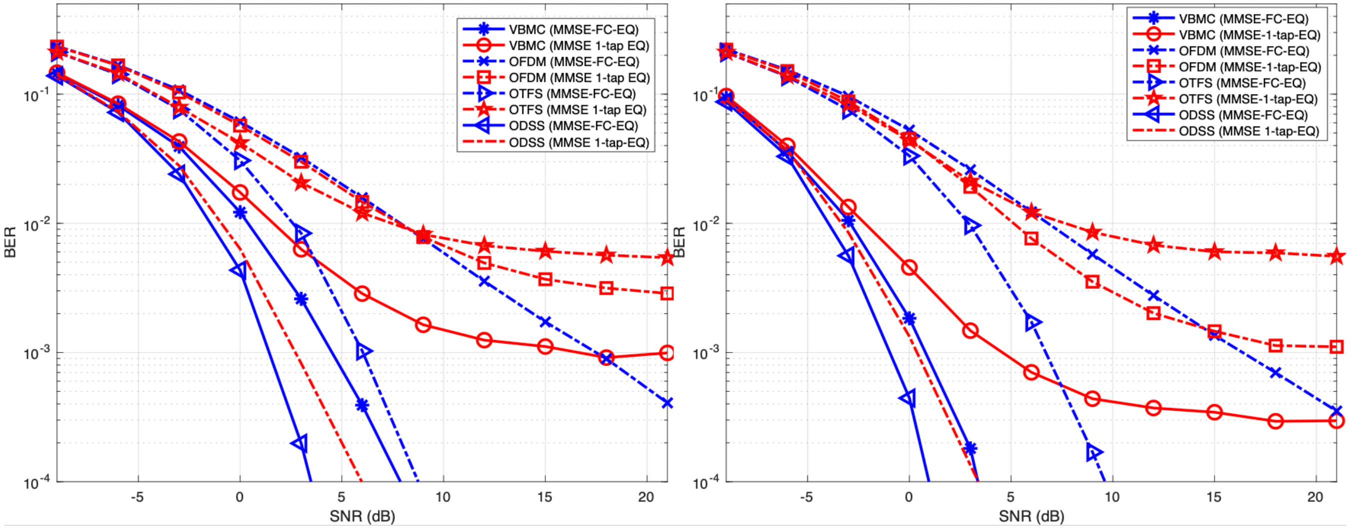


Figure 4: **Panel 1**: Subcarrier waveforms, for $n = 0, 1, \ldots, 6$, on an example dyadic ($q = 2$) tiling. A total of $N_7 = 127$ subcarriers are tiled in the symbol duration. **Panel 2**: ODSS subcarrier spectra (for $n = 0, 1, \ldots, 6$).

Outline



$\tau_{\max} = 10$ ms, $\alpha_{\max} = 1.001$ $\quad$ $\tau_{\max} = 20$ ms, $\alpha_{\max} = 1.002$

Our Publications

1. Modulation scheme for wideband time-varying channels
   - Arunkumar K.P. & C.R. Murthy, "Orthogonal Delay Scale Space Modulation: A New Technique for Wideband Time-Varying Channels", *IEEE TSP*, June 2022. (IEEE ComSoc's best reading list in the area of OTFS and Delay Doppler Signal Processing)
2. Variable bandwidth multicarrier communications
   - Arunkumar K.P. & C.R. Murthy, "Variable Bandwidth Multicarrier Communications: A New Waveform for the Delay-Scale Channel", *SPAWC*, Oulu, Finland, July 2022. (student best paper award)

Matlab codes available: `https://ece.iisc.ac.in/~cmurthy/ODSS_VBMC_Performance_Simulation.zip`

Summary

- New chirp based waveforms (VBMC, ODSS) for ultra-wideband com.
- New waveforms performs well under 1-tap equalizer (due to low ICI, high SINR & Jain Index)
- In the over-spread scenario, the relative performance is even better

Thank You!

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