On the Performance of Distributed Antenna Array Systems with Quasi-Orthogonal Pilots

Anubhab Chowdhury, Pradip Sasmal, Chandra R. Murthy, and Ribhu Chopra

***Abstract*—In this paper, we address the problem of channel estimation in a single cell massive multiple-input multiple-output (mMIMO) system with distributed antenna arrays (DAA) under the availability of limited pilots. Specifically, we propose to use pilots that are not only orthogonal within a cluster but also minimally correlated across clusters. We show that pilot sets that form mutually unbiased orthonormal bases (MUOB) minimize both inter- and intra-cluster pilot contamination. This also allows us to optimally choose the number of pilots needed to serve a given number of user equipments (UEs) within a DAA-mMIMO system. Following this, we develop an access point (AP) centered UE clustering algorithm to optimally cluster UEs for MUOB pilot allocation. Our experiments reveal that in a DAA-mMIMO system, MUOB not only offers better fairness compared to the traditional orthogonal pilot reuse scheme in terms of channel estimation error and per-UE throughput, but also delivers a higher sum system throughput.**

***Index Terms*—Distributed antenna array, channel estimation, orthonormal bases, pilot contamination, massive MIMO**

I. Introduction

DISTRIBUTED antenna array (DAA) massive multiple input multiple output (mMIMO) systems refer to the architecture where multiple antenna access points (APs) distributed at arbitrary locations within a cell serve a number of user equipments (UEs) [1]. In contrast to co-located mMIMO, DAA systems can potentially achieve better coverage and higher macro-diversity even under LoS channels [2]. Cell-free massive MIMO is a special case of DAA-mMIMO [3].

Typically, DAA systems operate in the time division duplex (TDD) mode in order to exploit channel reciprocity and thereby avoid the large training overhead required for estimating the downlink channels at UEs [1]. However, even with TDD mode of operation, the large number of UEs whose channel state information (CSI) is required at the access points (APs) results in either an inordinately high pilot overhead, or in severe pilot contamination. Hence, the development of pilot allocation strategies to minimize pilot contamination is an important research problem in the context of DAA-mMIMO. In [4], a structured pilot assignment scheme aiming to maximize the distance between pilot-sharing UEs is discussed. The authors in [5] refine this via an iterative graph coloring based AP selection algorithm.

Another way of looking at the problem of pilot allocation in DAA-mMIMO systems is to allocate pilots from a set of non-orthogonal yet distinct sequences (in the sequel, we will refer to these as quasi-orthonormal sequences) to minimize the effect of pilot contamination. It has been shown that mutually unbiased orthonormal bases (MUOB) [6] can be used to generate pilot sequences satisfying this property. Furthermore, Zadoff-Chu (ZC) sequences with different roots have been found to be efficient implementations of MUOB and have been employed in 3GPP LTE and 5G NR [7], [8]. It has been shown that that in cellular mMIMO systems, the use of MUOB pilots can deliver *uniform quality of service irrespective of the underlying pilot assignment strategy [9]*.

Note that, in cellular mMIMO systems, UEs are naturally clustered based on the base station (BS) serving them. However, no such clusters exist in the DAA case, since all the APs can potentially serve all the UEs. This makes the problem of CSI acquisition more challenging in DAA-mMIMO systems compared to their cellular counterparts, and necessitates clustering of the UEs before pilot allocation. The current state of the art [1]–[3] in DAA-mMIMO systems considers the use of orthogonal pilot reuse (OPR) among different UE clusters. One issue with this approach is that a large amount of pilot contamination can potentially be incurred if adjacent cluster-edge UEs from two physically proximal clusters share same pilot sequence. This, in turn, substantially degrades the quality of the channel estimates for that UE at all nearby APs. However, this problem can be circumvented via suitably designing non-orthogonal pilots. To the best of our knowledge, the problem of channel estimation with non-orthogonal pilots (and, in particular, mutually unbiased orthogonal bases pilots) has not yet been explored in the context of a DAA-mMIMO system. Therefore, our goal in this paper is to analyze the performance of DAA-mMIMO systems with quasi-orthogonal pilots, and to develop a strategy for pilot allocation to the UEs that can minimize the effects of pilot contamination.

Our main contributions are:

1) We first derive a lower bound on the mean squared pilot contamination power in a DAA-mMIMO system with non-orthogonal pilots under arbitrary pilot assignment. We show that pilots drawn from an MUOB codebook achieve this lower bound. (See Theorem 1.)
2) We develop a low complexity AP-centric UE clustering algorithm for pilot allocation to the UEs (See Algo-

A. Chowdhury and C. R. Murthy are with the Indian Institute of Science, Bangalore 560012, India. (e-mails:{anubhabc, cmurthy}@iisc.ac.in.)

P. Sasmal was with the Indian Institute of Science, Bangalore 560012, India. He is now with the Indian Institute of Technology Hyderabad, Hyderabad 502329, India. (e-mail: pradipsasmal@iisc.ac.in)

R. Chopra is with the Indian Institute of Technology Guwahati, Guwahati 781039, India (e-mail: ribhufec@iitg.ac.in.)

This work was financially supported by research grants from Intel India and the Young Faculty Research Fellowship from the Ministry of Electronics and Information Technology, Govt. of India.

rithm 1.). The algorithm aims to minimize the pilot contamination across the UEs for a given pilot length.
3) We derive the achievable uplink and downlink rates for this system. (See Theorem 2.) Note that these expressions are developed for arbitrarily correlated pilots and are applicable for both MUOB and OPR.
4) Via numerical simulations, we validate our derived results and prescribe parameter values that optimize the achievable rates in the system under study. We also benchmark the performance of MUOB against the OPR based channel estimation technique, which has previously been used in [1], [3]. We observe that MUOB pilots with a pilot length 13 achieve a Jain's fairness index value of above 0.999 for a 50 UE DAA-mMIMO system, which is comparable to the case with no pilot contamination, i.e., pilot length 50 (see Fig. 3). Also, with optimized pilot length, both cluster wise MUOB and unclustered MUOB uniformly outperform adaptive OPR as well as unclustered OPR in terms of the achievable rates (see Fig. 4).

The key takeaway of this work is that MUOB pilots can minimize the effects of pilot contamination in a DAA-mMIMO system for a given pilot length. Also, we can arbitrarily allot pilots to UEs within each cluster, and do not require computationally expensive pilot allocation algorithms. Furthermore, optimizing the pilot length significantly improves the throughput achievable with MUOB pilots due to the inverse scaling of the correlation between non-orthogonal pilots. Such properties make MUOB-codebooks an attractive choice as training signals in distributed systems such as DAA-mMIMO.

The rest of the paper is organized as follows. In Sec. II, we present the system model and evaluate the channel estimates under any quasi-orthonormal pilot codebook. Next, in Sec. III, we formulate the pilot sequence design problem, and prove that pilot codebooks designed via MUOB sequences attain the lower bound on inter-pilot correlation. Following this, in Sec. IV, we present our clustering algorithms for pilot allocation. Section V derives the achievable rates under the proposed pilot allocation schemes. We present our experimental results and benchmark our proposed solution against the existing OPR technique in Sec. VI, and conclude the paper in Sec. VII.

Notation: Matrices and vectors are represented using boldface upper and lower case alphabet, respectively. Sets are denoted by calligraphic letters; $|\cdot|$ denotes the cardinality of a set or the magnitude of a scalar depending on the context; and $\setminus$ and $\cup$ denote the set difference and set union operations, respectively. $\langle\cdot,\cdot\rangle$ denotes the inner-product of two vectors. Other notations are described in Tables I and II.

TABLE I
SYMBOLS

Notation	Description
$\texttt{Cont}_{mk}$	Pilot contamination in the kth UE's channel estimated at the mth AP
$\mathcal{U}$	Set of all UE indices
$\mathcal{O}_k$	Set of UE indices whose pilots are orthogonal to the kth UE's pilot
$\mathcal{U}_m$	Indices of the UEs clustered with mth AP
$\tilde{\mathcal{U}}_m$	Indices of the UEs whose data is processed by the mth AP
$\mathcal{A}_k$	Indices of the APs that jointly process the kth UE's data
$\mathbf{\Phi}$	The set of all pilot sequences with the kth sequence (column) $\boldsymbol{\varphi}_k \in \mathbb{C}^{\tau_p}$ being allocated to the kth UE

TABLE II
ACRONYMS

Abbreviation	Description
MUOB	Mutually unbiased orthonormal bases
OPR	Orthogonal pilot reuse
DAA-mMIMO	Distributed antenna array massive MIMO
TDD	Time division duplexing
LLSF	Largest large-scale fading

II. System Model

We consider a TDD DAA-mMIMO system consisting of M APs equipped with N antennas each, jointly serving K single antenna UEs. The channel vector between the mth AP and kth UE is modeled as $\mathbf{h}_{mk} = \sqrt{\beta_{mk}}\mathbf{f}_{mk} \in \mathbb{C}^N$, where the pathloss component β_{mk} is assumed to be constant for several coherence blocks, and the fast fading channel, $\mathbf{f}_{mk} \sim \mathcal{CN}(\mathbf{0}, \mathbf{I}_N)$, is estimated at the start of each coherence interval. Let $\mathcal{U} = \{1, 2, \ldots, K\}$ be the index set of all the UEs, and let their corresponding set of pilot sequences be $\mathbf{\Phi} \triangleq \{\boldsymbol{\varphi}_1, \boldsymbol{\varphi}_2, \ldots, \boldsymbol{\varphi}_K\}$, with the pilot sequence $\boldsymbol{\varphi}_k \in \mathbb{C}^{\tau_p}$ allocated to the kth UE, such that $\langle\boldsymbol{\varphi}_k, \boldsymbol{\varphi}_k\rangle = 1$ [1]. Without loss of generality, we group the K pilot sequences (correspondingly, UEs) into L clusters, with each cluster containing at most τ_p sequences, such that any pair of pilots within a cluster are mutually orthogonal. Thus, $\tau_p L \geq K$. Note that $L = 1$ if all the pilot sequences are orthogonal (this requires $\tau_p \geq K$), while $L = K$ if no pair of the pilot sequences is orthogonal. In the sequel, for simplicity, we assume that the pilot sequences can be grouped into L clusters, each containing τ_p mutually orthogonal pilots such that $\tau_p L = K$.

Let the kth UE transmit the pilot signal with an energy $\mathcal{E}_{p,k}$. Also, let the index set of UEs whose pilots are orthogonal to the pilot transmitted by the kth UE be denoted as $\mathcal{O}_k$. That is, $\mathcal{O}_k \triangleq \{k' : \langle\boldsymbol{\varphi}_k, \boldsymbol{\varphi}_{k'}\rangle = 0, k' \in \mathcal{U}\}$.

All the APs use the received pilot symbols to obtain minimum mean square error (MMSE) estimates of the channel vectors to the corresponding UEs. Let $\hat{\mathbf{h}}_{mk}$ be the estimate of $\mathbf{h}_{mk}$, such that $\mathbf{h}_{mk} = \hat{\mathbf{h}}_{mk} + \tilde{\mathbf{h}}_{mk}$, with $\tilde{\mathbf{h}}_{mk} \sim \mathcal{CN}(\mathbf{0}, (\beta_{mk} - \sigma_{mk}^2)\mathbf{I}_N)$ being the channel estimation error orthogonal to $\hat{\mathbf{h}}_{mk}$, where

$$\sigma_{mk}^2 \triangleq \frac{\mathcal{E}_{p,k}\beta_{mk}^2\tau_p}{N_0 + \mathcal{E}_{p,k}\beta_{mk}\tau_p + \texttt{Cont}_{mk}}.$$

Here, $\texttt{Cont}_{mk}$ represents the amount of pilot contamination in the kth UE's channel estimate, and is given as

$$\texttt{Cont}_{mk} = \sum_{j\in\mathcal{U}\setminus\{\mathcal{O}_k\cup k\}} \tau_p\mathcal{E}_{p,j}\beta_{mj}|\langle\boldsymbol{\varphi}_j, \boldsymbol{\varphi}_k\rangle|^2. \tag{1}$$

Next, we formulate the pilot design problem as one of min-max optimization based on the contamination derived in (1).

III. Minimizing Pilot Contamination

The contribution of pilot contamination to the channel estimation error is minimized when the inner product term $|\langle\boldsymbol{\varphi}_j, \boldsymbol{\varphi}_k\rangle|^2$ is uniformly zero. However, this is not possible in a system with $\tau_p < K$. Hence, we seek to minimize the maximum inter-pilot correlation to minimize (1), that is,

$$\mathrm{P}: \min_{\mathbf{\Phi}} \max_{\substack{k \in \mathcal{U}, \\ j \in \mathcal{U} \setminus \{\mathcal{O}_k \cup k\}}} |\langle \boldsymbol{\varphi}_j, \boldsymbol{\varphi}_k \rangle|^2. \tag{2}$$

We have the following theorem.

Theorem 1. *For a given pilot length τ_p satisfying $\sqrt{K} \leq \tau_p < K$ and for $\tau_p L = K$, the optimal value of* P *is $\frac{1}{\tau_p}$, and is attained when distinct MUOB-pilot codebooks are allocated across clusters and the chosen pilot length τ_p is either a prime number or a power of a prime number.*

Proof. See Appendix A. ■

Also, from [6], τ_p distinct orthogonal pilot codebooks can be constructed and allotted to τ_p^2 UEs using MUOB-codebooks. In practice, we can choose the smallest prime or prime-powered $\dot{\tau}_p$, for a given number of UEs, such that $\tau_p \geq \dot{\tau}_p$ and $\dot{\tau}_p^2 \geq K$. This way, we can generate sufficiently many pilot sequences of length $\dot{\tau}_p$ to allot to all K UEs.

We note that for prime values of τ_p, ZC sequences allow for a fast implementation of MUOBs [8]. Since sequences generated by circular shifts of a ZC sequence with a given root are orthogonal to each other, and since a ZC sequence of length τ_p has $\tau_p - 1$ roots, we can generate $\tau_p^2 - \tau_p$ MUOB pilots using ZC sequences. These $\tau_p^2 - \tau_p$ sequences coupled with the columns of the $\tau_p \times \tau_p$ identity matrix form τ_p^2 distinct pilot sequences of length τ_p.

Using the correlation structure of MUOB pilots, we can write the overall pilot contamination at the kth UE as

$$\mathtt{Cont}_{mk} = \sum_{j \in \mathcal{U} \setminus \{\mathcal{O}_k \cup k\}} \mathcal{E}_{p,j} \beta_{mj}. \tag{3}$$

For comparison, the overall pilot contamination in a system that uses OPR based pilot assignment is

$$\mathtt{Cont}_{mk} = \tau_p \sum_{j \text{ s.t.} \langle \boldsymbol{\varphi}_k, \boldsymbol{\varphi}_j \rangle = 1} \mathcal{E}_{p,j} \beta_{mj}. \tag{4}$$

Note that the contamination power in (4) scales with the number of clusters (also τ_p) in this case, as opposed to (3). The latter is due to the fact that the mutual correlation of non-orthogonal MUOB pilots scales inversely as $\sqrt{\tau_p}$.

Having developed a technique for the optimal design of pilot codebooks, in the next section, we present an AP-centric UE-clustering algorithm for pilot assignment to the UEs.

IV. AP-Centric Pilot Assignment

We note that the natural UE-grouping by associating each UE to its nearest base station (BS) of a cellular mMIMO system is not appropriate in a DAA system, as multiple APs cooperatively process each UE's signal. Consequently, AP-centric clustering is necessary to minimize the pilot contamination among geographically close UEs. Thus, we now discuss our proposed AP-centric UE clustering strategy, with each AP forming non-overlapping clusters with at most τ_p UEs.[1] Hence, given M APs we set $L = M$. Our proposed strategy is summarized in Algorithm 1, with d_{mk} being the distance between the mth AP and the kth UE. We declare an AP as available if the associated cluster size is less than τ_p, and as overloaded if the cluster size exceeds τ_p. In each iteration of this algorithm, we associate unclustered UEs with the nearest available APs and ensure that none of the APs is overloaded. The outputs of this algorithm are index sets $\mathcal{U}_m, m \in \{1 \ldots M\}$, containing UE indices clustered with the corresponding AP.

[1]The association/clustering is used only for assigning pilot sequences to the UEs. In a cell-free system, the APs collaboratively serve all UEs.

Algorithm 1: AP-centric UE clustering

Input: τ_p, M, K, $d_{mk}, \forall m$ & $\forall k$
Initialization: $\bar{\mathcal{A}}_m = \{1, 2, ..., M\}$, $\bar{\mathcal{U}} = \{1, 2, ..., K\}$, $\mathcal{U}_i = \emptyset$, $\forall i \in \bar{\mathcal{A}}_m$.
Check: $\tau_p \geq \max\{M - 1, K/M\}$.
while ($|\bar{\mathcal{U}}| \neq 0$) **do**
    for $i \in \text{length}(\bar{\mathcal{U}})$ **do**
        Find: $m' = \min_{m \in \bar{\mathcal{A}}_m} d_{mi}$
        Update: $\mathcal{U}_{m'} = \mathcal{U}_{m'} \cup \{i\}$, $\bar{\mathcal{U}} = \bar{\mathcal{U}} \setminus \{i\}$
    end
    /* Manage the overloaded APs */
    for $j \in \bar{\mathcal{A}}_m$ **do**
        if $|\mathcal{U}_j| > \tau_p$ **then**
            Retain only the τ_p nearest UEs in $\mathcal{U}_j$
            Move the dropped UEs back to $\bar{\mathcal{U}}$
            Update the available APs: $\bar{\mathcal{A}}_m = \bar{\mathcal{A}}_m \setminus \{j\}$
        end
    end
end
Output: $\mathcal{U}_m$: UEs associated with mth AP, $\forall m$.

Remark 1. MUOB pilot codebooks are generated via ZC-sequences with τ_p being a prime number satisfying $\tau_p \geq \max\{M-1, K/M\}$. We cluster UEs into M groups, each containing at most τ_p UEs. Now, since $\tau_p \geq K/M$, the UEs within a cluster can be assigned orthonormal pilots. This avoids intra-cluster pilot contamination. Also, since $\tau_p \geq M - 1$, we can assign a distinct block of pilots to each cluster.

Following this, the UEs within each cluster are assigned pilot sequences that are randomly chosen from a unique block of MUOB-pilots, without replacement. For OPR, a set of orthonormal pilots are assigned to UEs within a cluster and repeated across the clusters [1], [3], [9]. We then employ the largest large-scale fading (LLSF) based AP selection [10] to find the set of the UE indices whose data will be processed by the mth AP, denoted by $\tilde{\mathcal{U}}_m$. This is a superset of the mth AP's pilot cluster. The cardinality of the set $\tilde{\mathcal{U}}_m$ is controlled by a threshold parameter denoted by $\delta \in [0, 1]$. Setting $\delta = 1$ leads to $\tilde{\mathcal{U}}_m = \mathcal{U}, \forall m$. Following this, using $\tilde{\mathcal{U}}_m, \forall m$, we can easily find $\mathcal{A}_k$, the set of AP indices associated with the kth UE. As an example, we demonstrate the clusters formed for one given UE distribution, in Fig. 1, such that each distinctly colored cluster is assigned pilots from one distinct MUOB-pilot codebook. We observe that the UE of interest (as indicated) is jointly served by two APs.

Remark 2. The clusters formed according to Algorithm 1

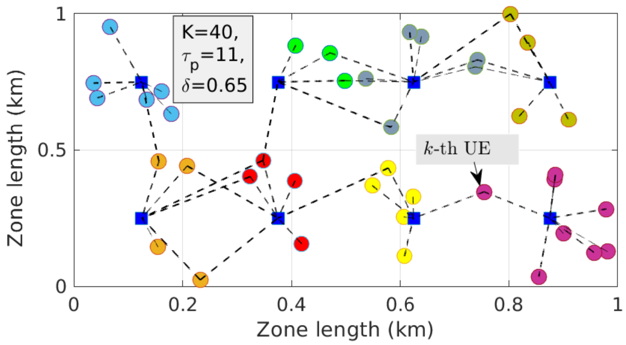

Fig. 1. A realization of the system model. The blue squares denote the AP positions and solid circles denote the UEs.

are used to allocate MUOB pilots across clusters. Due to the constant correlation (i.e., $1/\sqrt{\tau_p}$) among the inter-cluster UEs, the contamination strength is independent of how we assign pilots from each MUOB codebook to the UEs within a given cluster. Therefore, the UEs within each cluster are randomly assigned pilot sequences drawn from one block of MUOB-pilots without replacement. Then, there is no pilot contamination from users within a cluster, and a fixed contamination from users in other clusters regardless of how the pilot sequences are assigned to the users within each cluster. This a key advantage of MUOB, namely, that we completely obviate the need to solve a pilot assignment problem based on inter-cluster UE distances.

Now, in DAA mMIMO systems, OPR may lead to poor performance because each UE is served by multiple APs. Due to this, using $\tau_p \ll K$ may result in multiple UEs being served by the same AP using the same pilot sequence, leading to severe pilot contamination and loss of performance. Therefore, for fair comparison with OPR based pilot allocation, we allow the pilot length with OPR to exceed τ_p, and propose a technique for generating pilot sequences in Algorithm 2, which we call adaptive OPR. The algorithm ensures that if any UE is being jointly served by more than one AP, then the assigned pilot of that particular UE is orthogonal to all the other UEs being served by the corresponding APs. We note that the threshold parameter δ acts as a trade off between the amount of pilot contamination and the pilot length (τ_p'). In the case of adaptive OPR, the parameter δ also controls the pilot length τ_p', unlike MUOB, where τ_p is independent of δ.

Remark 3. The worst case complexity of our proposed clustering Algorithm 1 is $\mathcal{O}(K^2M)$. However, the clustering only needs to be performed in the time scale over which the large scale fading coefficients change. Further, Algorithm 1 is used to allocate MUOB pilots because of the constant correlation property as discussed in Remark 2. For adaptive OPR, we need Algorithm 2 to mitigate inter-cluster pilot contamination, which has worst case order complexity of $\mathcal{O}(M)$. Therefore, with a very low complexity, the clustering algorithm can procure the benefits of offered by MUOB codebooks.

Algorithm 2: Adaptive OPR

Initialization: $\dot{\mathbf{\Phi}}_j = \emptyset, \forall j = 1, 2, \ldots, M$
Find $\tau_p' = \max\{|\tilde{\mathcal{U}}_1|, |\tilde{\mathcal{U}}_2|, \ldots, |\tilde{\mathcal{U}}_M|\}$
Define $\mathcal{I} = \{i_1, \ldots, i_M\}$ s.t. $|\tilde{\mathcal{U}}_{i_1}| \geq |\tilde{\mathcal{U}}_{i_2}| \geq \ldots |\tilde{\mathcal{U}}_{i_M}|$
if $\tau_p' > \tau_p$ **then**
    Generate: New pilot codebook: $\bar{\mathbf{\Phi}} \in \mathbb{C}^{\tau_p' \times \tau_p'}$
else
    Set: $\bar{\mathbf{\Phi}} = \mathbf{\Phi} \in \mathbb{C}^{\tau_p \times \tau_p}$ (initial codebook)
end
for $j = 1 : \text{length}(\mathcal{I})$ **do**
    Find the UEs in $\tilde{\mathcal{U}}_{i_j}$, if any, to which pilots have been assigned in previous iteration(s)
    Store those pilots in $\dot{\mathbf{\Phi}}_j$
    Randomly assign pilots to the remaing UEs in $\tilde{\mathcal{U}}_{i_j}$ from $\bar{\mathbf{\Phi}} \backslash \dot{\mathbf{\Phi}}_j$ without replacement
end

V. Performance Analysis

In this section, we analyze the throughput of the proposed system. Our analysis is applicable to any choice of pilot codebooks, including pilots from MUOB codebooks and pilots allocated using OPR. Let the kth UE's transmitted symbol be $s_{u,k}$ ($\mathbb{E}[|s_{u,k}|^2] = 1$) with energy $\mathcal{E}_{u,k}$. The uplink signal transmitted by the kth UE is processed by the APs whose indices are included in the index set $\mathcal{A}_k$. Each AP processes these uplink signals via maximal ratio combining. Therefore, the processed kth stream of the received signal at the CPU becomes $\sum_{m \in \mathcal{A}_k} \sum_{i \in \mathcal{U}} \sqrt{\mathcal{E}_{u,i}} \hat{\mathbf{h}}_{mk}^H \mathbf{h}_{mi} s_{u,i} + \sum_{m \in \mathcal{A}_k} \hat{\mathbf{h}}_{mk}^H \mathbf{w}_m$, with $\mathbf{w}_m \sim \mathcal{CN}(\mathbf{0}, N_0 \mathbf{I}_N)$.

Similarly, let the downlink symbol intended for the kth UE be denoted by $s_{d,k}$, with $\mathbb{E}[|s_{d,k}|^2] = 1$. In the downlink, the mth AP serves the UEs whose indices are contained in the index set $\tilde{\mathcal{U}}_m$. For simplicity, we assume equal power distribution among the APs, and let ρ_d be the maximum normalized (as a multiple of the noise variance N_0) power transmitted by each AP [10]. Assuming reciprocity based matched filter precoding in the downlink, the signal transmitted by the mth AP can be expressed as $\mathbf{r}_{d,m} = \sum_{i \in \tilde{\mathcal{U}}_m} \sqrt{\rho_d \zeta_{mi}} \hat{\mathbf{h}}_{mi}^* s_{d,i}$, where the power control coefficients, $\zeta_{mk}, \forall k \in \tilde{\mathcal{U}}_m$, are designed such that $\mathbb{E}\left[\|\mathbf{r}_{d,m}\|^2\right] \leq \rho_d$. Also since, $\hat{\mathbf{h}}_{mi}^* \in \mathcal{CN}(\mathbf{0}, \sigma_{mi}^2 \mathbf{I}_N)$,

$$\sum_{i \in \tilde{\mathcal{U}}_m} \zeta_{mi} \mathbb{E}\left[\|\hat{\mathbf{h}}_{mi}^*\|^2\right] \leq 1 \implies \sum_{i \in \tilde{\mathcal{U}}_m} \zeta_{mi} \sigma_{mi}^2 \leq \frac{1}{N}. \quad (5)$$

Optimally solving (5) is beyond the scope of this paper, however, considering each AP to transmit at the maximum allowable power, we can set $\zeta_{mk} = 1/(N \sum_{i \in \tilde{\mathcal{U}}_m} \sigma_{mi}^2), \forall k \in \tilde{\mathcal{U}}_m$. We consider that a fraction λ ($\lambda \in [0, 1]$) of data transmission duration, i.e., $(\tau - \tau_p)$, is allotted for uplink.

Theorem 2. *The achievable rate of the kth UE can be expressed as*

$$R_k = \left(1 - \frac{\tau_p}{\tau}\right) \left[\lambda \log_2(1 + \gamma_k^u) + (1 - \lambda) \log_2(1 + \gamma_k^d)\right],$$

where

$$\gamma_k^u = \frac{N\mathcal{E}_{u,k}(\sum_{m\in\mathcal{A}_k}\sigma_{mk}^2)^2}{N\ \mathtt{CohI}_k^u + \mathtt{NCohI}_k^u + N_0\sum_{m\in\mathcal{A}_k}\sigma_{mk}^2}, \quad (6a)$$

$$\gamma_k^d = \frac{N^2\rho_d(\sum_{m\in\mathcal{A}_k}\sqrt{\zeta_{mk}}\sigma_{mk}^2)^2}{N^2\ \mathtt{CohI}_k^d + N\ \mathtt{NCohI}_k^d + 1}, \quad (6b)$$

with $\mathtt{CohI}_k^u \triangleq \sum_{i\in\mathcal{U}\setminus\{k\}}\mathcal{E}_{u,i}|\boldsymbol{\varphi}_k^H\boldsymbol{\varphi}_i|^2(\sum_{m\in\mathcal{A}_k}\sigma_{mk}^2\sqrt{\frac{\mathcal{E}_{p,i}}{\mathcal{E}_{p,k}}\frac{\beta_{mi}}{\beta_{mk}}})^2$, $\mathtt{NCohI}_k^u \triangleq \sum_{i\in\mathcal{U}}\mathcal{E}_{u,i}\sum_{m\in\mathcal{A}_k}\sigma_{mk}^2\beta_{mi}$, $\mathtt{CohI}_k^d \triangleq \sum_{i\in\mathcal{U}\setminus\{k\}}\rho_d|\boldsymbol{\varphi}_i^H\boldsymbol{\varphi}_k|^2(\sum_{m\in\mathcal{A}_i}\sigma_{mi}^2\sqrt{\zeta_{mi}}\sqrt{\frac{\mathcal{E}_{p,k}}{\mathcal{E}_{p,i}}\frac{\beta_{mk}}{\beta_{mi}}})^2$, *and* $\mathtt{NCohI}_k^d \triangleq \rho_d\sum_{i\in\mathcal{U}}\sum_{m\in\mathcal{A}_i}\sigma_{mi}^2\zeta_{mi}\beta_{mk}$.

Proof. The proof is available in the supplementary material available at https://ece.iisc.ac.in/∼cmurthy/Papers/supp_DAA_MUOB.pdf. ■

We observe that the choice of pilot sequences controls the coherent interference power in the uplink and the downlink, i.e., $\mathtt{CohI}_k^u$ and $\mathtt{CohI}_k^d$, that in turn determines the achievable rates. We have earlier shown (See Theorem 1) that for any given pilot length, MUOB pilot code-books minimize the coherent interference regardless the underlying pilot assignment strategy, hence maximizing the achievable rate.

VI. Numerical Results

We use the setup in Fig. 1 with $M = 8$ APs, each equipped with $N = 32$ antennas. The UEs are deployed uniformly at random over a square area of size 1 km^2 and we consider 10^5 realizations of the channels. The pathloss exponent and the reference distance with respect to each AP are taken as 3.76 and 10 m, respectively [1]. We assume a coherence block to consist of 200 channel uses, corresponding to a coherence time of 1 ms [1], [5]. The pilot and data SNRs are taken as 10 dB, with λ being 0.5. We compare the proposed MUOB pilot codebook based channel estimation with the established OPR technique as presented in [1]. We also compare the performance of MUOB with adaptive OPR where the pilot contamination between inter-cluster UEs are mitigated as described in Sec. IV. We now state the three schemes of pilot allocation and data processing employed in our experiments:

1) *Cluster Wise MUOB [$\tau_p = x, \delta = y$]:* We form the clusters using Algorithm 1 setting $\tau_p = x$, and then assign pilots from a distinct MUOB codebook at each cluster. Then, we apply LLSF-based AP section with $\delta = y$ for joint data processing.
2) *Cluster wise OPR [$\tau_p = x, \delta = y$]*: We form the clusters using Algorithm 1 with $\tau_p = x$, and reuse a single set of orthogonal pilots across clusters. After that, we use LLSF-based AP section method with $\delta = y$ to find the APs that jointly process the data of each UE.
3) *Adaptive OPR [$\delta = y$]:* We first form the clusters using Algorithm 1. Next, we apply LLSF-based AP selection with $\delta = y$ to find the sets $\widetilde{\mathcal{U}}_m, \forall m$. Then we assign pilots using Algorithm 2 which will result in a pilot length $\tau_p' \in [x, |\mathcal{U}|]$ depending on δ.

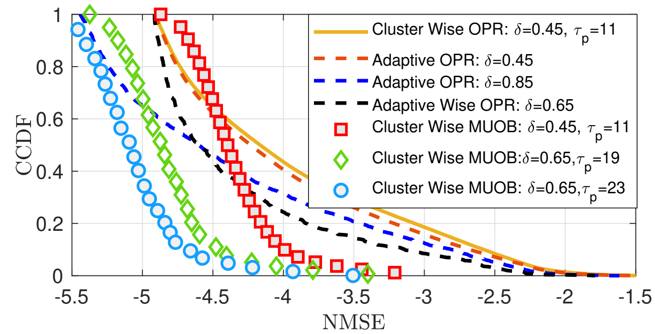

Fig. 2. CCDF of NMSE of the estimated channels with $K = 40$.

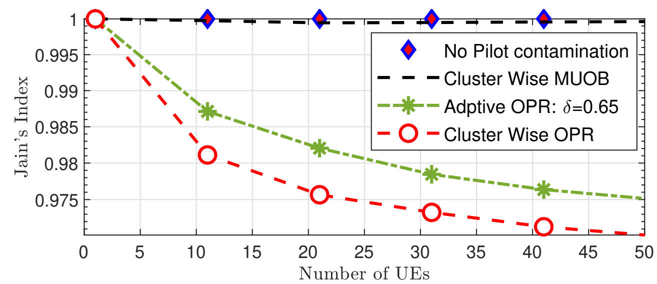

Fig. 3. CDF of Jain's Index and fairness variation with UE load.

We first evaluate the effectiveness of MUOB pilots for channel estimation against cluster wise OPR and adaptive OPR. We do this by plotting the complementary cumulative distribution functions (CCDFs) of the NMSE of estimated channels in Fig. 2. For each of the three schemes, we measure the NMSE of a particular UE at the APs that are involved in joint processing, and average the error variances over the number of associated APs. Thus, the x-axis of Fig. 2 is $\frac{1}{|\mathcal{A}_k|}\sum_{m\in\mathcal{A}_k}(1-\sigma_{mk}^2/\beta_{mk}), \forall k \in \mathcal{U}$. We can observe that MUOB pilots render channel estimates with considerably lower error variance as compared to cluster wise OPR. As adaptive OPR reduces contamination by increasing the pilot length, for certain UEs it can achieve better channel estimates. However, the probability that the NMSE is greater than -3.5 dB with MUOB is almost zero, whereas, the NMSE under adaptive OPR exceeds -3.5dB at least in 20% of the cases even with $\delta = 0.85$. Also, even though the pilot length is increased in adaptive OPR, MUOB significantly outperforms adaptive OPR. Therefore, a simple clustering-based algorithm can attain better channel estimates with MUOB pilots.

Next, in Fig. 3, we compare the fairness offered by these pilot allocation schemes via Jain's utility index [9], which is defined as $J(\sigma) = (\sum_{k=1}^K\sigma_{mk}^2/\beta_{mk})^2 \ /\ K\sum_{k=1}^K(\sigma_{mk}^2/\beta_{mk})^2$. For $\delta = 1$, Algorithm 2 generates orthogonal pilot codebooks (i.e. $\tau_p' = K$) for all the UEs, which results in a Jain's index of unity for all UEs. We observe that MUOB pilots can achieve nearly the same fairness as a system with no pilot contamination. Furthermore, even as the number of UEs increases, MUOB-pilots retain the overall fairness.

We observe that the pilot length represents an important trade-off between the amount of pilot contamination and the usable frame duration. We observe in Theorem 2 that $\mathtt{CohI}_k^u$ and $\mathtt{CohI}_k^d$ are dependent on $|\langle\boldsymbol{\varphi}_k, \boldsymbol{\varphi}_i\rangle|^2$ which scales as

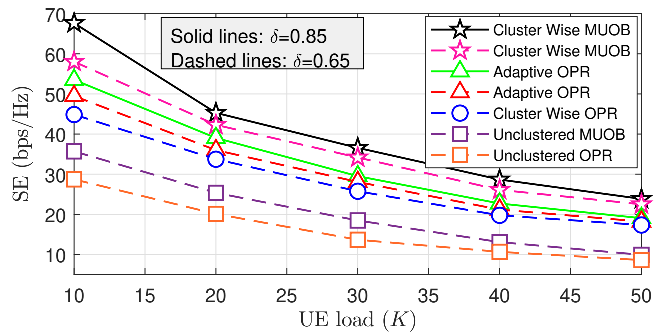

Fig. 4. Pilot length optimized SE vs. UE load (K).

$1/\tau_p$ under MUOB. However, if τ_p becomes comparable with coherence interval (τ), the pre-log factor $(1-\tau_p/\tau)$ degrades the SE. Thus, the right choice of the pilot length with MUOB is important for obtaining optimal performance. Although optimally solving for τ_p in Theorem 2 is beyond the scope of this work, we numerically solve the following problem:

$$\max_{\tau_p} \sum_{k\in\mathcal{U}} R_k, \text{ subject to}$$

$$\tau_p \in \begin{cases} [x,\tau], \text{Cluster wise OPR \& Cluser wise MUOB} \\ [\tau_p',\tau], \text{Adaptive OPR,} \end{cases}$$

where x and τ_p' are as defined at the beginning of this section. We plot the average optimized SE against the UE load in Fig. 4. We observe that MUOB pilot codebooks with optimized pilot lengths substantially improve the average throughput compared to adaptive OPR. Also, with increasing δ, more APs contribute to joint data processing which improves the per user rate. Furthermore, at optimal pilot length, the coherent interference is also minimized under MUOB-pilot books unlike any OPR technique, where both the interference and the effective channel gain increase linearly with the pilot length.

VII. Conclusions

The benefits of DAA-massive MIMO rely heavily on the locally available CSI-quality at the APs. We tackled the CSI-acquisition problem via quasi-orthonormal pilots and user clustering. We showed that among the set of quasi orthogonal pilot sequences, pilots from MUOB codebooks minimize the coherent interference in DAA-mMIMO systems. We then argued that MUOB pilot matrices can be generated using ZC-sequences, which is compatible with the 5G-NR standard. We also provided an AP-centric clustering algorithm assigning pilots from MUOB codebooks. Our numerical results revealed that MUOB-based pilots can achieve better system fairness as well throughput compared to OPR. The performance of MUOB can be further improved via suitable power control strategies, which can be a potential future direction of the current work.

Appendix A
Proof of Theorem 1.

Proof. First, note that since the pilot length is τ_p and $L\tau_p = K$, we can always generate a pilot codebook $\mathbf{\Phi}_p \triangleq [\boldsymbol{\varphi}_k]_{k=1:\tau_p L} \in \mathbb{C}^{\tau_p\times K}$, such that pilots within each cluster are mutually orthogonal. Then, $\mathbf{\Phi}_p\mathbf{\Phi}_p^H = L\mathbf{I}_{\tau_p}$ follows from the fact that the intra-cluster pilots are orthonormal and there are L clusters. Hence, all nonzero singular values of $\mathbf{\Phi}_p$ are equal to $\sqrt{L}$, and consequently, $\|\mathbf{\Phi}_p^H\mathbf{x}\|^2 = L\|\mathbf{x}\|^2$, $\forall \mathbf{x} \in \mathbb{C}^{\tau_p}$. Therefore,

$$\|\mathbf{\Phi}_p^H\mathbf{\Phi}_p\|_{\text{F}}^2 = \sum_{1\le i\le \tau_p L} \|\mathbf{\Phi}_p^H\boldsymbol{\varphi}_i\|_2^2 = L\sum_{1\le i\le \tau_p L}\|\boldsymbol{\varphi}_i\|_2^2 = L^2\tau_p,$$

where the last equality follows from the fact that $\|\boldsymbol{\varphi}_i\|_2^2 = 1, \forall i$. Recall that $\mathcal{U}\setminus\{\mathcal{O}_k\cup k\}$ indicates the set of all UEs that do not share the same cluster as kth UE, then,

$$\sum_{k\in\mathcal{U}}\sum_{j\in\mathcal{U}\setminus\{\mathcal{O}_k\cup k\}} |\langle\boldsymbol{\varphi}_j,\boldsymbol{\varphi}_k\rangle|^2 = \|\mathbf{\Phi}_p^H\mathbf{\Phi}_p\|_{\text{F}}^2 - \sum_{k\in\mathcal{U}}\sum_{j\in\mathcal{O}_k\cup\{k\}}|\langle\boldsymbol{\varphi}_j,\boldsymbol{\varphi}_k\rangle|^2 = \tau_p L(L-1). \quad (7)$$

Since the summation in (7) contains $\tau_p^2L(L-1)$ non-negative entries, it is easy to infer that

$$\max_{\substack{k\in\mathcal{U},\\ j\in\mathcal{U}\setminus\{\mathcal{O}_k\cup k\}}} |\langle\boldsymbol{\varphi}_j,\boldsymbol{\varphi}_k\rangle|^2 \ge \frac{\tau_p L(L-1)}{\tau_p^2 L(L-1)} = \frac{1}{\tau_p}. \quad (8)$$

Hence, $|\langle\boldsymbol{\varphi}_j,\boldsymbol{\varphi}_k\rangle|^2$ is lower bounded by $\frac{1}{\tau_p}$.

We now need to design pilots codebooks that satisfy $|\langle\boldsymbol{\varphi}_j,\boldsymbol{\varphi}_k\rangle|^2 = \frac{1}{\tau_p}$, $\forall k$, $\forall j \notin \mathcal{O}_k\cup k$. Invoking the definition of MUOB[2] [6], when τ_p is a prime or a power of a prime, MUOB codebooks exist provided $\sqrt{K} \le \tau_p < K$. Then, (8) is satisfied with equality when $\boldsymbol{\varphi}_k$ and $\boldsymbol{\varphi}_j$ are chosen from two distinct MUOB-codebooks. ■

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[2] A set of orthonormal bases $\{\varPhi_j\}_{j=1,\dots,L}$ of $\mathbb{C}^{\tau_p}$ are said to be *mutually unbiased* if $|\langle\boldsymbol{\varphi}_1,\boldsymbol{\varphi}_2\rangle|^2 = 1/\tau_p$ for any $\boldsymbol{\varphi}_1 \in \varPhi_l$ and $\boldsymbol{\varphi}_2 \in \varPhi_m$, $l \ne m$.