

Distributed Power Control for Multi-hop Energy Harvesting Links with Retransmission

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Abstract—In this work, we consider an energy harvesting (EH) node that periodically takes a measurement and conveys it to a destination over multiple EH relays operating in the decode-and-forward fashion, using the *automatic repeat request (ARQ)* protocol. Packets that are not delivered to the destination before the next measurement is taken are *dropped*. We seek to design an online retransmission-index based power control policy (RIP) for each node which minimizes the packet drop probability (PDP). To this end, we first derive an expression for the PDP in terms of the RIPs at the nodes. Next, when the energy cost for decoding a packet is negligible, we obtain *closed form* expressions for the optimal RIPs. We also extend the results to the case where the peak transmit power is constrained. When the energy cost of decoding is non-negligible, we present a *geometric programming* based iterative algorithm to obtain near-optimal RIPs. In both the scenarios, in order to obtain insight on the impact of channel coherence time, we design the RIPs for both slow and fast fading channels. Through Monte Carlo simulations, we show that the proposed policies significantly outperform state-of-the-art solutions.

Index Terms—Energy harvesting, ARQ, Multi-hop links, packet drop probability, geometric programming, battery size, receiver.

I. INTRODUCTION

In many prototypical Internet-of-Things (IoT) applications, measurements are taken by remote sensor nodes, and are conveyed to a central fusion center over multiple short-range hops [2], [3]. Often, these sensors and relays are powered by energy harvested from the environment, to ensure long lifetime of the network and reduce maintenance overheads such as battery replacement [4], [5]. In this scenario, conveying measurements reliably and in an up-to-date fashion while operating within the power constraints imposed by the random and sporadic nature of energy availability at the nodes is an important challenge.

Retransmission schemes such as the automatic repeat request (ARQ), along with power control, is a popular approach for ensuring reliable packet delivery. They are part of various low power communications standards such as IEEE 802.15.4 and the bluetooth low energy specification [6], [7]. Further, in EH networks, retransmission protocols extenuate the impact of both small scale fading and the randomness of energy availability [8]. In this paper, we consider an ARQ based retransmission protocol for communication between

EH nodes (EHNs) of a multi-hop link, and propose near-optimal power control policies that minimize the packet drop probability (PDP). When the energy cost of decoding a packet at a receiver is non-negligible, the power control policies are coupled across the nodes.¹ This makes the design of optimal power policies significantly more challenging than designing policies for point-to-point EH links [9].

We consider a monitoring system, where a sensor node takes periodic measurements which are to be delivered to a destination over a multi-hop link formed between EHNs. Each relay node in the multi-hop link operates in a decode-and-forward manner. In addition, the packet transmission over each hop follows the ARQ protocol to deliver a given packet to the next node, with a predetermined number of slots allocated to it. For each attempt of the packet, the receiving node sends an acknowledgment (ACK) or negative ACK (NACK) to indicate the success or failure of the previous attempt, respectively. Once the transmitter receives an ACK for the current packet, it goes to sleep till it is time for it to receive the next packet from the preceding node. A node in the sleep mode does not consume energy, but continues to harvest energy from the environment. Packets that are not delivered to the destination before the next measurement is taken are *dropped*. Thus, a packet is dropped if and only if any node fails to deliver it to the next node within its allocated number of slots. A packet failure may happen either due to the energy outage at the transmitter or receiver, or due to channel fading/noise at the receiver. For ARQ-based links with periodic transmission of fixed-size measurement packets, the PDP is used as a metric for reliability [8], [10], [11]. For this system, the PDP is defined as the fraction of packets that are dropped.

We present the design of optimal online power control policies in two extreme scenarios of channel variation: slow fading and fast fading. In case of slow fading, the channel remains constant for all the attempts of a given packet made by an EHN, while in contrast, for fast fading, it changes in an independent, identically distributed (i.i.d.) fashion in every slot. Clearly, the fast fading channel offers higher diversity. Our results show that, due to this, the structure of the optimal policy in the two scenarios is completely different. In the following paragraphs, we briefly discuss related work, before summarizing the main contributions of this paper.

The design and analysis of power management policies for multi-hop EH links has been studied with various objectives such as the long-term rate [12], energy efficiency [13],

¹For example, if a node uses higher packet transmit power, it implies that the next node consumes less energy for packet reception, since fewer retransmissions will be required on average.

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transmission reliability [14], distortion [15], fairness [16], utility [17], [18], throughput [19], and sensing rate [20]. The design of power management policies for retransmission based point-to-point EH links is considered in [8]–[11], [21], [22]. However, the design of retransmission based *multi-hop* EH links has not been considered in the literature.

An important issue in the design of the power control policies is the availability of the *state-of-charge* (SoC) information of the battery. In practice, the estimation of SoC could be energy-expensive [23], [24] as well as inaccurate. In [9], we presented a method to systematically design PDP-optimal SoC-independent policies for retransmission-based point-to-point EH links, where both the transmitter and receiver are EHNs. We note that, due to the coupling among the policies of different nodes, policies that are optimal for point-to-point EH links could be highly suboptimal for multi-hop EH links. Hence, the design procedure presented in [9] does not extend to multi-hop EH links. This motivates us to consider the design of near-optimal, SoC-independent, online power control policies for multi-hop EH links. Further, our policies require minimal coordination among the nodes, and can be implemented in a distributed fashion across the nodes without using any additional control overhead. Our main contributions are as follows:

- We derive approximate closed-form expressions for the PDP of ARQ-based multi-hop EH links equipped with finite sized batteries for both slow and fast fading channels. We illustrate the accuracy of the analysis over a wide range of system parameters via simulations.
- Using the closed-form expressions, we derive an analytically tractable lower bound on the PDP. We show that the *gap* between the lower bound and the PDP of our system decays exponentially fast with the battery size at each node.
- When the energy required for receiving and decoding a packet is negligible compared to that required for transmitting a packet, we present *closed-form* expressions for the *optimal* SoC-independent retransmission-index based policy (RIP) for transmit power control, in both slow and fast fading scenarios. Furthermore, when there is a peak transmit power constraint at the transmitter, we provide a provably convergent algorithm to determine the optimal transmit power control policy.
- Finally, when the energy required for receiving and decoding a packet is non-negligible, the problem becomes a mixed-integer nonlinear program. Using tools from geometric programming (GP), we obtain near-optimal RIPs in the general case.

Our results not only reveal the inter-dependence among the transmit power levels of the RIPs, but also on the system parameters, e.g., harvesting rate, number of retransmissions, etc. For example, when the channel is slow fading and the energy required to receive and decode a packet is negligible, we show that the optimal transmit power is geometrically increasing. In contrast, for fast fading links, the transmit power levels are exponentially increasing.

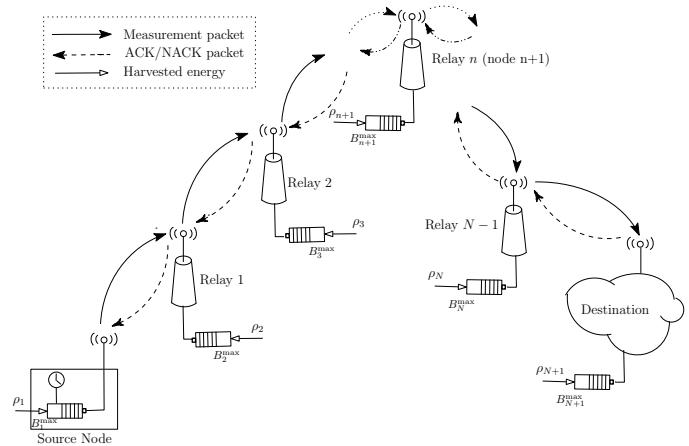


Fig. 1: System model. Each node transmits and receives in its assigned sub-frame.

II. SYSTEM MODEL

We consider an N -hop link formed by $N+1$ EHNs as shown in Fig. 1. The first EHN (source) takes a measurement at the beginning of every *frame* of duration T_f . The measurement packet needs to be delivered to the last node (destination), before the end of the frame. If a packet does not reach the destination by the end of the frame, it is *dropped*. Each packet is relayed to the destination using $N - 1$ half-duplex relays which operate in a decode and forward manner.

A. Transmission Protocol on Each Hop

The transmission of a packet between two successive EHNs follows the ARQ protocol where each packet attempt by the transmitter is followed by an acknowledgment (ACK) or negative ACK (NACK) signal from the receiver, indicating the success or failure of the attempt, respectively. The ACK/NACK messages are assumed to be received without any error and delay [8]–[11], [21], [22]. This is a reasonable assumption because compared to a measurement packet, the ACK/NACK messages are smaller in the size and can be transmitted with significant protection to keep the error rate negligibly small. If the transmitter receives an ACK, then it does not retransmit the packet and goes to sleep and harvests the energy until it is time to receive the next packet. On the other hand, reception of a NACK results in retransmission of the packet, provided both the transmitter and receiver have sufficient energy to make the next attempt.

We consider a time-slotted system, and let T_s denote the duration of a slot, which is the total time required to make an attempt and receive the ACK/NACK from the receiver. Hence, a frame contains $K = \lfloor \frac{T_f}{T_s} \rfloor$ slots. Out of these K slots, the n^{th} hop is allocated K_n slots, such that $\sum_{n=1}^N K_n = K$. Thus, the n^{th} node remains awake for at most $K_{n-1} + K_n$ slots in a frame, and receives in a slot s if $s \in \left\{ \sum_{p=1}^{n-2} K_p + 1, \dots, \sum_{p=1}^{n-1} K_p \right\}$ and trans-

mits if $s \in \left\{ \sum_{p=1}^{n-1} K_p + 1, \dots, \sum_{p=1}^n K_p \right\}$. The duration $\left\{ \sum_{p=1}^{n-1} K_p + 1, \dots, \sum_{p=1}^n K_p \right\}$ is called the n^{th} sub-frame, and is of duration K_n slots. A packet received in $(n-1)^{\text{th}}$ sub-frame needs to be delivered to the $(n+1)^{\text{th}}$ node within the n^{th} sub-frame, otherwise it is dropped. This type of fixed slot allocation can be pre-programmed during the network deployment phase and is more energy efficient compared to dynamic slot allocation which requires a node to remain awake in anticipation of a transmission by the previous node. However, the PDP analysis presented in the following sections can be extended to the scenario when the slot allocation among the nodes is not fixed, and a node starts forwarding a packet immediately after receiving it. Then, the complementary geometric program based design approach presented in this paper can be directly used to obtain near-optimal online policies for multi-hop links with dynamic slot allocation.

B. Energy Harvesting Model

The energy harvesting process at the nodes is modeled as a temporally i.i.d. Bernoulli process, independent across nodes [5], [8]–[11], [22], [25]. That is, in a slot, node n harvests energy E with probability ρ_n , and does not harvest energy with probability $1 - \rho_n$, for $1 \leq n \leq N + 1$. Without loss of generality, we normalize $E = 1$ throughout the paper. The Bernoulli model is motivated by switch-based and vibration based harvesting mechanisms [11], [26]. The simplicity of Bernoulli model facilitates the exposition of the key ideas presented in the paper, while still capturing the sporadic and random nature of the energy availability at the EHNs. However, the Markov chain based framework presented in the sequel directly extends to more general models, e.g., the stationary Markov model [5] and generalized Markov model [27] as well as to account for spatial correlation in the harvesting process [8], [9]. The extension of the presented results to these cases is explicitly detailed in [28].

C. Power Management Policy

The transmit power policy of node n is an RIP denoted by $\mathcal{P}^n \triangleq \{P_1^n = \frac{E_1^n}{T_s}, P_2^n = \frac{E_2^n}{T_s}, \dots, P_{K_n}^n = \frac{E_{K_n}^n}{T_s}\}$, for $1 \leq n \leq N$. The RIP \mathcal{P}^n is an *attempt* based prescription, i.e., the n^{th} node uses E_ℓ^n amount of energy to make its ℓ^{th} attempt,² $1 \leq \ell \leq K_n$. In addition, due to the restriction imposed by the RF-front end, $E_\ell^n \leq E_{\max}$, where E_{\max} is the maximum allowed transmission energy per slot. At the receiver, since the size as well as the modulation and coding scheme remain fixed for each packet, we assume that a node consumes R units of energy to receive and decode a packet, including the energy required to transmit the ACK/NACK message [8], [22], [29].

The energy consumption at each node is governed by the *energy neutrality constraint* (ENC), which requires that, at any time instant, the difference between the total amount of energy harvested and consumed by a node up to that point in time must be non-negative. The Markovian evolution of the battery

at each node ensures that the operation of the node satisfies the ENC, and is given as follows

$$B_{s+1}^n = \min \left(\left(B_s^n + \mathbb{1}_{\{\mathcal{H}_s^n\}} - E_\ell^n \mathbb{1}_{\{\mathcal{E}_{t,s}^n\}} - R \mathbb{1}_{\{\mathcal{E}_{r,s}^n\}} \right)^+, B_n^{\max} \right), \quad (1)$$

for $1 \leq n \leq N + 1$. In the above, $B_n^{\max} < \infty$ denotes the size of the battery at the n^{th} node and $(x)^+ \triangleq \max(0, x)$. Also, $\mathbb{1}_{\{\mathcal{E}\}}$ denotes an indicator function which takes the value one when event \mathcal{E} occurs, and takes the value zero otherwise; and $\mathcal{E}_{t,s}^n$ and $\mathcal{E}_{r,s}^n$ denote the events that node n is acting as a transmitter and receiver, respectively, in the s^{th} slot. The event that node n harvests energy in the s^{th} slot is denoted by \mathcal{H}_s^n . We let U_s^n denote the *local transmission index* of the n^{th} node in the s^{th} slot, $s \geq \sum_{p=1}^{n-1} K_p + 1$. It is defined as

$$U_s^n \triangleq \begin{cases} -1 & \text{ACK received,} \\ \ell & \ell - 1 \text{ NACKs received, } \ell \in \{1, \dots, K_n\}. \end{cases} \quad (2)$$

For $s \leq \sum_{p=1}^{n-1} K_p$, $U_s^n = 0$, i.e., the U_s^n is zero until the start of n^{th} sub-frame, and at the start of n^{th} sub-frame the local transmission index is set to one. It is incremented by one each time a NACK is received, and set to -1 if an ACK is received. Thus, the n^{th} node makes the ℓ^{th} attempt in the s^{th} slot if and only if all the following conditions are satisfied:

- 1) The n^{th} node has received the packet successfully, i.e., the local transmission index of all the previous $n - 1$ nodes is equal to -1 .
- 2) The s^{th} slot is a slot in the n^{th} sub-frame.
- 3) $U_s^n = \ell$, i.e., the previous $\ell - 1$ attempts made by the n^{th} node has failed.
- 4) Both the n^{th} and $(n + 1)^{\text{th}}$ nodes have sufficient energy in the battery to transmit and receive the packet, respectively. That is, $E_\ell^n \leq B_s^n$ and $R \leq B_s^{n+1}$.

Based on the above, we can define $\mathcal{E}_{t,s}^n$ and $\mathcal{E}_{r,s}^n$ in (1) as $\mathcal{E}_{t,s}^n \triangleq \{B_s^n \geq E_\ell^n, B_s^{n+1} \geq R, (U_s^i = -1)_{i=1}^{n-1}, U_s^n \neq -1, \sum_{p=1}^{n-1} K_p + 1 \leq s \leq \sum_{p=1}^n K_p\}$, and $\mathcal{E}_{r,s}^n \triangleq \{B_s^{n-1} \geq E_\ell^{n-1}, B_s^n \geq R, (U_s^i = -1)_{i=1}^{n-2}, U_s^{n-1} \neq -1, \sum_{p=1}^{n-2} K_p + 1 \leq s \leq \sum_{p=1}^{n-1} K_p\}$. Note that, $\mathcal{E}_{t,s}^n$ and $\mathcal{E}_{r,s}^{n+1}$ are the same events. The dynamics of our system is pictorially illustrated in Fig. 2.

Note that, to ensure that an attempt is made only if $E_\ell^n \leq B_s^n$ and $R \leq B_s^{n+1}$, the transmitter and receiver need to have one bit information about the SoC of the other node. This can be obtained using a coordinated sleep-wake protocol between the transmitter and receiver [8], [30]. Furthermore, to implement the power control prescribed by the RIPs, a node only needs to know its own battery state B_s^n , the RIP \mathcal{P}^n , and the local transmission index U_s^n . Hence, the proposed RIPs can be implemented in a *distributed* fashion without any additional control overhead.

D. Channel Model

The wireless channel between two consecutive nodes is modeled as a *block fading* channel [8], [11], [22] with two different scenarios for the block duration. In the first scenario, named as *slow fading*, the channel remains constant for the duration of a sub-frame and changes in an i.i.d. fashion at

²The subscript is used for either time or node index, depending on the context. However, when both time index and node index appear together, they are indicated in the subscript and superscript, respectively.

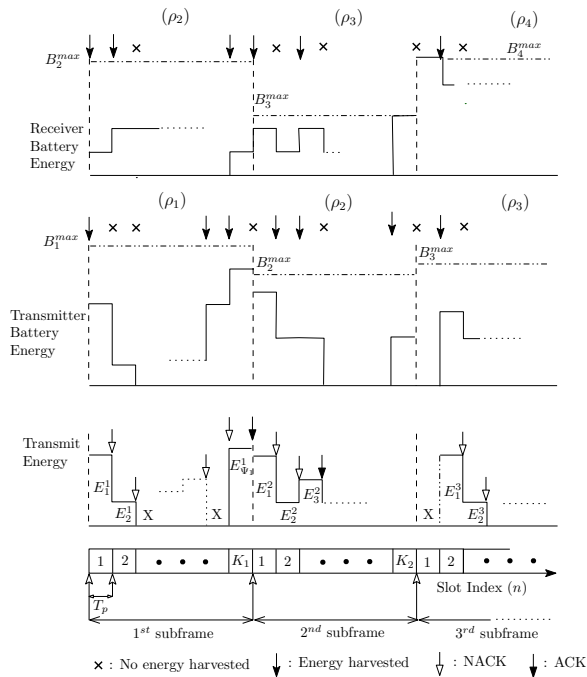


Fig. 2: Evolution of the batteries at the transmitter and receiver during the transmission of a packet. In the first sub-frame, the source node is the transmitter and the 2nd node is the receiver. More generally, in the n^{th} sub-frame, the n^{th} node transmits to the $(n+1)^{\text{th}}$ node. In the illustrated scenario, the first node delivers the packet in the K_1^{th} slot, while the second node receives an ACK in the 3rd slot. Note that, after receiving the ACK signal, the 2nd node does not make further attempts and harvests energy for the rest of the frame. Also, after receiving the packet successfully, the 3rd node starts its transmission only at the start of 3rd sub-frame. A packet is dropped if any node in the multi-hop link fails to deliver the packet to next node.

the start of next sub-frame. In the second scenario, called as *fast fading*, the channel stays constant for a slot duration and changes in an i.i.d. fashion at the beginning of a new slot. The transmitting node does not have access to channel state information, but it can possibly infer about the channel using the received ACK/NACK messages. In both slow and fast fading cases, the channel is assumed to be Rayleigh distributed, with the complex baseband channel distributed as $\mathcal{CN}(0, \sigma_c^2)$. The probability that the ℓ^{th} attempt of the n^{th} node is in *outage* is given as [31]–[33]

$$P_e(E_\ell^n, \gamma) = \exp\left(-\frac{E_\ell^n \gamma}{N_0}\right), \quad (3)$$

where γ and N_0 denote the instantaneous channel gain and power spectral density of the AWGN at the receiver, respectively.

The goal in this work is to design the set of RIPs $\{\mathcal{P}^n\}_{n=1}^N$ such that the PDP is minimized. To do so, we need to characterize the dependence of the PDP on the system parameters and transmit power levels of the RIPs. In the next section, we derive the approximate closed-form expressions for the PDP which are accurate over a wide range of system parameters. Using the closed-form expressions, we formulate our main optimization problem.

III. PACKET DROP PROBABILITY

The system described in the previous section can be modeled as a discrete time Markov chain (DTMC). The state of the DTMC in slot s is represented by the tuple $(\mathbf{B}_s, \mathbf{U}_s, s)$, where $\mathbf{B}_s \triangleq (B_s^1, B_s^2, \dots, B_s^{N+1})$ and $\mathbf{U}_s \triangleq (U_s^1, U_s^2, \dots, U_s^N)$ are the vectors denoting the battery state and local transmission index of all the nodes. For a *slow* fading channel, the state transition probability matrix (TPM) is denoted by $\mathbf{G}(\gamma)$ and its $(a, b)^{\text{th}}$ entry denotes the probability of transitioning from state $\mathbf{a} \triangleq (\mathbf{B}_a, \mathbf{U}_a, s)$ to $\mathbf{b} \triangleq (\mathbf{B}_b, \mathbf{U}_b, s+1)$, i.e.,

$$G_{a,b}(\gamma) \triangleq \Pr\left[(\mathbf{B}_{s+1} = \mathbf{B}_b, \mathbf{U}_{s+1} = \mathbf{U}_b, s+1) \mid (\mathbf{B}_s = \mathbf{B}_a, \mathbf{U}_s = \mathbf{U}_a, s), \gamma\right], \quad (4)$$

The transition probabilities are determined by the RIPs, $\{\mathcal{P}^n\}_{n=1}^N$, and the channel and EH statistics. For a *fast* fading channel, entries of the TPM, \mathbf{G} , are written similarly. The expressions for the transition probabilities are provided in Appendix A.

Using the above DTMC, for a given set of RIPs $\mathcal{P} \triangleq \{\mathcal{P}^n\}_{n=1}^N$, the PDP can be written as

$$P_D = \sum_{\mathbf{B}} \pi(\mathbf{B}) \mathbb{E}_\gamma \{P_D(K | \mathbf{B}, \mathbf{U} = \mathbf{0}, \gamma)\}, \quad (5)$$

where $\pi(\mathbf{B})$ denotes the stationary probability that, at the start of the frame, the battery states of the nodes in the system is \mathbf{B} , and $P_D(K | \mathbf{B}, \mathbf{U} = \mathbf{0}, \gamma)$, termed as the *conditional PDP*, denotes the probability that the packet is dropped after K slots, given that at the start of the frame the battery state is \mathbf{B} and the channel encountered by the packet is γ . For a slow fading channel $\gamma \triangleq (\gamma_1, \gamma_2, \dots, \gamma_N)$, where γ_n denotes the channel in the n^{th} subframe, while for a fast fading channel $\gamma \triangleq (\gamma_1, \gamma_2, \dots, \gamma_K)$, where γ_s denotes the channel in s^{th} slot. Conditioning on $\mathbf{U} = \mathbf{0}$ signifies that the local transmission index at all the nodes is reset to zero at the start of the frame. Thus, to compute the PDP using (5), we need to find the stationary distribution of the DTMC, π , and the average conditional PDP, $\mathbb{E}_\gamma \{P_D(K | \mathbf{B}, \mathbf{U} = \mathbf{0}, \gamma)\}$, where $\mathbb{E}_\gamma(\cdot)$ denotes the expectation over the channel state γ .

Now, since the DTMC is irreducible and the number of states is finite, the DTMC is positive recurrent. This ensures the existence of the stationary distribution π . The stationary distribution over the battery states at the start of the frame is given as [8]

$$\pi = (\mathbb{E}[\mathbf{G}'(\gamma)] - \mathbf{I} + \mathbf{A})^{-1} \mathbf{1}, \quad (6)$$

where $\mathbf{1}$ is a $\prod_{n=1}^{N+1} (B_n^{\max} + 1)$ dimensional all ones vector, \mathbf{A} is an all ones matrix, \mathbf{I} is the identity matrix. $\mathbf{G}'(\gamma)$ is the K -step TPM with entries $\Pr[\mathbf{B}_{(M+1)K} = \mathbf{B}_2 | \mathbf{B}_{MK} = \mathbf{B}_1, \gamma]$, where M is the frame index. The entries of $\mathbf{G}'(\gamma)$ are computed using $\mathbf{G}^K(\gamma)$, by marginalizing out the local transmission index vector \mathbf{U} . The details are presented in [28].

A. Average Conditional PDP

For a given channel state, γ , the conditional PDP is determined by the number of transmit and receive attempts supported by each node which, in turn, is determined by the

battery states of the nodes at the start of the frame, \mathbf{B} , and the amount of energy harvested by the nodes during the frame. To elaborate, the total energy that can be used by a node is limited by the energy available in the battery at the start of the frame and the energy harvested by it from the start of the frame till the last slot of the sub-frame in which it is scheduled to be active. Let $M_n \triangleq (m_{r,n}, m_{t,n})$ denote the *harvesting pattern* of the n^{th} node, where, for Bernoulli energy harvesting model, $0 \leq m_{r,n} \leq \sum_{i=1}^{n-1} K_i$ and $0 \leq m_{t,n} \leq K_n$ denote the total amount of energy harvested by the n^{th} node in the first $n-1$ sub-frames and during the n^{th} sub-frame, respectively. Note that, $m_{r,1} = 0$ and $m_{t,1} \leq K_1$. Similarly, $m_{r,N+1} \leq K$ and $m_{t,N+1} = 0$. The following Lemma expresses the average conditional PDP in terms of the probability that the packet is dropped when the initial battery state vector and harvesting pattern vector are \mathbf{B} and $\mathbf{M} = (M_1, \dots, M_{N+1})$, respectively, when the channel encountered by the packet is γ , denoted by $p_D(\mathbf{B}, \mathbf{M}, \gamma)$. The result directly follows from the spatial independence of the harvesting processes across the nodes, and hence its proof is omitted.

Lemma 1. *The average conditional PDP can be written as*

$$\mathbb{E}_\gamma \{P_D(K|\mathbf{B}, \mathbf{U} = \mathbf{0}, \gamma)\} = \sum_{\mathbf{M}=(M_1, \dots, M_{N+1})} q(\mathbf{M}) \mathbb{E}_\gamma \{p_D(\mathbf{B}, \mathbf{M}, \gamma)\}, \quad (7)$$

where $q(\mathbf{M})$ denotes the probability of a harvesting pattern \mathbf{M} , and is given by

$$q(\mathbf{M}) = \prod_{n=1}^{N+1} \binom{\sum_{i=1}^{n-1} K_i}{m_{r,n}} \binom{K_n}{m_{t,n}} \rho_n^{m_{r,n} + m_{t,n}} \times (1 - \rho_n)^{\sum_{i=1}^{K_n} K_i - m_{r,n} - m_{t,n}}. \quad (8)$$

Next, to compute the conditional PDP using Lemma 1, we need to find $p_D(\mathbf{B}, \mathbf{M}, \gamma)$, which can be expressed in terms of the outage probability of the individual hops as follows.

Lemma 2. *Let the battery state at the start of the frame and the harvesting pattern be \mathbf{B} and \mathbf{M} , respectively. When the channel encountered by the packet is γ , the probability that the packet is dropped, $p_D(\mathbf{B}, \mathbf{M}, \gamma)$ can be expressed as*

$$p_D(\mathbf{B}, \mathbf{M}, \gamma) = \sum_{n=1}^N p_{D,n} \prod_{i=1}^{n-1} (1 - p_{D,i}), \quad (9)$$

where $p_{D,n}$ denotes the probability that the packet is dropped at the n^{th} hop.

Proof: The proof follows from the fact that the packet drop event can be written as the union of N mutually exclusive events, where the n^{th} event is that the packet is dropped at the n^{th} hop, for all $1 \leq n \leq N$. The probability that the packet is dropped in the n^{th} hop is written using the independence of channel states across the sub-frames. ■

Observe that, $p_{D,n}$ is a function of $B^n, B^{n+1}, m_{t,n}, m_{r,n+1}$, and the channel between the n^{th} and $(n+1)^{\text{th}}$ node. Here, B^n and B^{n+1} denote the n^{th} and $(n+1)^{\text{th}}$ components of \mathbf{B} . Furthermore, $p_{D,n}$ depends only on the channel state and the number of feasible attempts supported in the n^{th} subframe,

Ψ_n , which, in turn, depends on the initial battery states and the harvesting patterns of the n^{th} and $(n+1)^{\text{th}}$ node, respectively. Note that, the probability $p_{D,n}$ does not depend on the exact slot indices in which the attempts are made. Based on this, $p_{D,n}$ can be written as

$$p_{D,n} = \prod_{\ell=1}^{\Psi_n} P_e \left(\frac{E_\ell^n \gamma_n}{N_0} \right). \quad (10)$$

To compute $p_{D,n}$ using (10), we need to determine Ψ_n . A method to compute Ψ_n is provided in Appendix B.

Thus, using Lemmas 1, 2 and the procedure to compute the number of feasible attempts in Appendix B, $\mathbb{E}_\gamma \{p_D(\mathbf{B}, \mathbf{M}, \gamma)\}$ in (7) can be written as

$$\mathbb{E}_\gamma \{p_D(\mathbf{B}, \mathbf{M}, \gamma)\} = \sum_{n=1}^N \mathbb{E}_\gamma \{p_{D,n}\} \prod_{i=1}^{n-1} (1 - \mathbb{E}_\gamma \{p_{D,i}\}), \quad (11)$$

In the above equation, computing $\mathbb{E}_\gamma \{p_{D,n}\}$ depends on whether the channel is slow or fast fading. In the slow fading case, from (3) and (10), and since the channel state γ is exponentially distributed and constant through the subframe, we get

$$\mathbb{E}_\gamma \{p_{D,n}\} = \frac{1}{1 + \sum_{\ell=1}^{\Psi_n} \frac{E_\ell^n}{N_0}}. \quad (12)$$

Similarly, in the fast fading case, we have

$$\mathbb{E}_\gamma \{p_{D,n}\} = \frac{1}{\prod_{\ell=1}^{\Psi_n} \left(1 + \frac{E_\ell^n}{N_0}\right)}. \quad (13)$$

This completes the derivation of the PDP expressions in both the cases. In the next section, we use the PDP expressions derived in this section to formulate the PDP optimization problem.

IV. PACKET DROP PROBABILITY MINIMIZATION

In this section, we formulate an optimization problem for obtaining the RIPs that minimize the PDP. Using (5), we can express the optimization problem as:

$$\min_{\{\mathcal{P}^n\}_{n=1}^N} \sum_{\mathbf{B}} \pi(\mathbf{B}) \mathbb{E}_\gamma \{P_D(K|\mathbf{B}, \mathbf{U} = \mathbf{0}, \gamma)\}, \quad (14a)$$

$$0 \leq E_\ell^n \leq E_{\max} \text{ for all } 1 \leq \ell \leq K_n \text{ and } 1 \leq n \leq N. \quad (14b)$$

In (14), π is obtained using (6), in which the TPM \mathbf{G}' , in turn, is determined by the energy neutrality constraint (ENC). This implicit dependence on the ENC makes the above problem hard to solve. Furthermore, due to the large state space of the problem, it is challenging to find a numerical solution using dynamic programming techniques [10]. Hence, in the following, we reformulate the above optimization problem by finding tight bounds on the objective function. The following Lemma provides an upper bound and a lower bound on the PDP, and is a generalization of a result in [9] for point-to-point links. The proof is similar to the proof of Lemma 1 in [9], and hence is omitted.

Lemma 3. *Let $\mathcal{P} \triangleq \{\mathcal{P}^n\}_{n=1}^N$ be a set of RIPs satisfying $E_\ell^n \leq E_{\max}$ for all $1 \leq \ell \leq K_n$ and $1 \leq n \leq N$. Let $\mathcal{I}_A \triangleq \{\mathbf{B} | 0 \leq B^n \leq B_n^{\max}, B^n - \sum_{\ell=1}^{K_n} E_\ell^n - K_{n-1}R \geq$*

0, for all $1 \leq n \leq N + 1$ } and $\mathcal{I}_A^c \triangleq \mathcal{I} \setminus \mathcal{I}_A$, where $\mathcal{I} \triangleq \{\mathbf{B} | 0 \leq B_n \leq B_n^{\max} \text{ for all } 1 \leq n \leq N + 1\}$ is the set of all battery state tuples. Then, for a multi-hop EH link operating using RIP \mathcal{P} ,

$$\begin{aligned} P_{D_\infty}^* &\leq \min_{\{\mathcal{P}^n\}_{n=1}^N} \sum_{\mathbf{B}} \pi(\mathbf{B}) \mathbb{E}_\gamma \{P_D(K|\mathbf{B}, \mathbf{U} = \mathbf{0}, \gamma)\} \\ &\leq P_{D_\infty}^* + \sum_{\mathbf{B} \in \mathcal{I}_A^c} \pi(\mathbf{B}) \Big|_{\mathcal{P}^*}, \end{aligned} \quad (15)$$

where $P_{D_\infty}^* \triangleq \min_{\{\mathcal{P}^n\}_{n=1}^N} \mathbb{E}_\gamma \{P_D(K|\mathbf{B}, \mathbf{U} = \mathbf{0}, \gamma)\}$ and $\mathcal{P}^* \triangleq \arg \min_{\{\mathcal{P}^n\}_{n=1}^N} \mathbb{E}_\gamma \{P_D(K|\mathbf{B}, \mathbf{U} = \mathbf{0}, \gamma)\}$ for any \mathbf{B} such that $\mathbf{B} \in \mathcal{I}_A$.

In the above, the lower bound $P_{D_\infty}^*$ is the minimum PDP obtainable, when, at the start of the frame, each node has sufficient energy in its battery to support all the possible transmit and receive attempts, regardless of its harvesting pattern. This set of ‘‘good’’ initial battery state tuples is denoted by \mathcal{I}_A . On the other hand, the difference between the two bounds, $\sum_{\mathbf{B} \in \mathcal{I}_A^c} \pi(\mathbf{B}) \Big|_{\mathcal{P}^*}$, is the sum of the stationary probabilities of the battery state vectors that do not necessarily support all transmission and reception attempts. Its value depends on the policy \mathcal{P}^* as well as the size of the batteries at the nodes.

Remark 1. The lower bound $P_{D_\infty}^*$ can be interpreted as the minimum PDP achievable for a multi-hop link whose nodes are equipped with infinite sized batteries. This is because, as noted in [16], with infinite battery, it is necessary and sufficient to operate under an average power constraint³ (APC) to minimize the PDP while satisfying the ENC.

Intuitively, a policy designed to operate under the APC induces a (small) positive drift on the battery states at the nodes, causing the states to drift away from the set \mathcal{I}_A^c , and thereby reducing the gap between the upper and lower bounds. In addition, this ensures that an event where the node does not have sufficient energy to transmit or receive a packet occurs with very low probability which, in turn, also reduces the loss in utility due to energy outages. Hence, policies designed under the APC are likely to be near-optimal for links with finite battery nodes also. In effect, when the average energy harvesting rate exceeds the energy consumption rate, energy outages become low probability events. Hence, in the sequel, the APC is also referred to as the energy unconstrained regime (EUR). In the following subsection, we prove that for a multi-hop EH link with each node satisfying the APC, the difference between the upper and lower bound decays exponentially with the size of the battery at the nodes. This, in turn, allows us to replace the objective function in (14) by the lower bound and the ENC by the APC, to obtain near-optimal policies.

A. Tightness of the Bounds

In this subsection, we show that the difference between the upper and lower bound in (15) can be expressed as the sum

³When operating under an average power constraint, a node consumes ϵ less power than it harvests, on average, where $\epsilon > 0$ can be arbitrarily small. Due to this, the battery states drift to infinity over time, and energy outages do not occur.

of $N + 1$ terms, and each term decays exponentially with the size of the battery at a node, provided the multi-hop EH link is operating in the EUR.

Theorem 1. *If each node of a multi-hop EH link operates using a policy \mathcal{P} with finite power levels and satisfying the EUR constraint, then $\sum_{\mathbf{B} \in \mathcal{I}_A^c} \pi(\mathbf{B}) = \sum_{n=1}^{N+1} \Theta(e^{r_n^* B_n^{\max}})$ where r_n^* is a negative root of the asymptotic log moment generating function (MGF) of the battery drift process $(Y_s^n \triangleq B_s^n + \mathbb{1}_{\{\mathcal{H}_s^n\}} - L(B_s^n, B_s^{n+1}, \{U_s^i\}_{i=1}^n) - R(B_s^{n-1}, B_s^n, \{U_s^i\}_{i=1}^{n-1}))$ of the n^{th} node. Here, $L(\cdot)$ and $R(\cdot)$ denote the energy consumed for transmission and reception, respectively. The asymptotic log MGF is defined as $\Lambda(r_n) = \lim_{T \rightarrow \infty} \frac{1}{T} \log \mathbb{E} \left[\exp \left(r_n \sum_{s=1}^T Y_s^n \right) \right]$.*

Proof: The proof follows using arguments similar to the proof in [9, Theorem 1]. ■

In the above, $r_n^* \approx -\frac{2\delta_n}{\sigma_e^2} + o(\delta_n)$ [34], where δ_n is the battery drift (difference between the average energy harvested and average energy consumed) at node n and σ_e^2 is the asymptotic variance of the harvesting process. For further details, see [34, Lemma 3].

We note that Theorem 1 generalizes the result in [9, Theorem 1], to multi-hop case. Theorem 1 implies that, for a multi-hop EH link operating in EUR, the probability that the battery state of a node at the start of the frame cannot support all the receive and transmit attempts that could occur during the frame can be made arbitrarily small by choosing a sufficiently large battery at each node. Thus, for a multi-hop EH link with *large but finite* sized battery at each node, the difference between the upper and lower bounds in (15) is negligible, when operating in the EUR. Hence, in the large battery regime, we can replace the objective in (14) by the lower bound obtained in Lemma 3. Moreover, the stringent energy neutrality requirement can be replaced by the relaxed EUR constraint. This leads to the following *reformulated optimization problem*:

$$\min_{\{\mathcal{P}^n\}_{n=1}^N} \mathbb{E}_\gamma \{P_D(\mathbf{B}, \mathbf{M}, \gamma)\}, \quad (16a)$$

$$\text{subject to } T_n + R_n \leq K\rho_n, \text{ for all } 1 \leq n \leq N + 1, \quad (16b)$$

$$0 \leq E_\ell^n \leq E_{\max} \text{ for all } 1 \leq \ell \leq K_n \text{ and } 1 \leq n \leq N, \quad (16c)$$

where

$$T_n \triangleq \Pr [n] \left(\sum_{\ell=1}^{K_n} E_\ell^n \mathbb{E}_\gamma \left\{ \prod_{i=1}^{\ell-1} P_e(E_i^n, \gamma) \right\} \right), \text{ and} \quad (17)$$

$$R_n \triangleq \Pr [n-1] \left(\sum_{\ell=1}^{K_{n-1}} R \mathbb{1}_{\{E_\ell^{(n-1)} \neq 0\}} \mathbb{E}_\gamma \left\{ \prod_{i=1}^{\ell-1} P_e(E_i^{(n-1)}, \gamma) \right\} \right) \quad (18)$$

denote the average energy consumed by the n^{th} node for transmission and reception, respectively. The average energy consumed for transmitting the packet, T_n , is written by accounting for the following events:

- 1) The packet reaches the n^{th} node, the probability of which is denoted by $\Pr [n]$.

- 2) The ℓ^{th} attempt is made only if all the previous $\ell - 1$ attempts have failed. This happens with probability $\mathbb{E}_\gamma \left\{ \prod_{i=1}^{\ell-1} P_e(E_i^n, \gamma) \right\}$.

The average energy consumed by n^{th} node in receiving a packet, R_n , is written similarly. Note that, in the expression for R_n , $\mathbb{1}_{\{E_\ell^{(n-1)} \neq 0\}}$ is an indicator function which captures the fact that the receiving node spends R units of energy only if the transmitter attempts the packet at nonzero power. Also, the average energy consumed for reception and transmission at the source and destination node, i.e., R_1 and T_{N+1} , respectively, are defined to be equal to zero.

In the above,

$$\Pr[n] \triangleq \prod_{m=1}^{n-1} \mathbb{E}_\gamma \left(1 - \prod_{i=1}^{K_m} P_e(E_i^m, \gamma) \right) \quad (19)$$

denotes the probability that a given packet reaches the n^{th} node, and is written as the product of the probabilities of $n - 1$ independent events that the packet is delivered successfully at the previous $n - 1$ hops. Further, at m^{th} hop, the probability of successful delivery is written using the fact that a packet is dropped if all K_m transmission attempts fail.

The constraint (16b) requires that the average energy consumed by each node in both transmission and reception, i.e., $T_n + R_n$, must be less than the average energy harvested by it, $K\rho_n$. Note that, the average power consumed by the n^{th} node in receiving a packet, R_n , depends on \mathcal{P}^{n-1} , the transmit policy of $(n - 1)^{\text{th}}$ node. In turn, R_n determines T_n , the average amount of energy left at the n^{th} node to transmit the packet. Hence, \mathcal{P}^n depends on \mathcal{P}^{n-1} , and so on. This coupling between the policies of all the nodes necessitates the joint design of policies and renders the design problem in (16) challenging. However, for multi-hop links where the distance between consecutive nodes is large, the transmit energy dominates the power consumption of a node. In such a scenario, one can neglect the energy consumed in the reception, and the R_n term in constraint (16b) can be dropped. This relaxes the coupling between the policies of the nodes and admits a closed-form optimal solution. Therefore, before presenting the solution for the general problem in (16) with non-negligible R_n , in the following section, we present the solution for the special case when the energy cost of receiving a packet is negligible.

V. SPECIAL CASE: NEGLIGIBLE RECEPTION COST

When the energy cost for receiving a packet is negligible, by dropping the R_n term in constraint (16b), the optimization problem in (16) can be written as

$$\max_{\{\mathcal{P}^n\}_{n=1}^N} \Pr[N + 1], \text{ subject to} \quad (20a)$$

$$\Pr[n] \left(\sum_{\ell=1}^{K_n} E_\ell^n \mathbb{E}_\gamma \left\{ \prod_{i=1}^{\ell-1} P_e(E_i^n, \gamma) \right\} \right) \leq K\rho_n, \forall 1 \leq n \leq N, \quad (20b)$$

$$0 \leq E_\ell^n \leq E_{\max} \text{ for all } 1 \leq \ell \leq K_n \text{ and } 1 \leq n \leq N. \quad (20c)$$

In (20), the objective is written in terms of the probability of reaching the destination node, $\Pr[N + 1]$, which is given by

(19) and is a product of N terms with disjoint optimization variables. Hence, the problem splits as N subproblems, one corresponding to each hop. Given $\Pr[n]$, the optimal power control for the n^{th} hop can be derived by solving the subproblem corresponding to the n^{th} hop. In the following, we show that the optimization problem in (16) admits a closed-form solution for both fast and slow fading cases, which can be obtained by sequentially solving the optimization subproblems in (20). We discuss the slow fading case next, and relegate the fast fading case to Sec. VI.

Consider the optimization problem for the n^{th} hop and with a slow fading channel:

$$\min_{\mathbf{E}^n = \{E_1^n, \dots, E_{K_n}^n\}} \mathbb{E}_\gamma \left\{ \prod_{\ell=1}^{K_n} P_e(E_\ell^n, \gamma) \right\}, \quad (21a)$$

$$\text{subject to } \Pr[n] \left(\sum_{\ell=1}^{K_n} E_\ell^n \mathbb{E}_\gamma \left\{ \prod_{i=1}^{\ell-1} P_e(E_i^n, \gamma) \right\} \right) \leq K\rho_n, \quad (21b)$$

$$0 \leq E_\ell^n \leq E_{\max} \text{ for all } 1 \leq \ell \leq K_n. \quad (21c)$$

Note that, in the above $\Pr[n]$ is function of power control policies of the previous $n - 1$ nodes. Hence, it does not depend on the optimization variables of the problem (21) and can be treated as a constant for solving the problem (21). In the following, we first solve (21) without the peak power constraint and then we adapt the solution to satisfy (21c).

A. Optimal Policy without the Peak Power Constraint

Using (3), for slow fading channels, the optimization problem (21) with only the EUR constraint can be written as

$$\min_{\mathbf{E}^n = \{E_1^n, \dots, E_{K_n}^n\}} \left(1 + \sum_{\ell=1}^{K_n} \frac{E_\ell^n}{N_0} \right)^{-1}, \quad (22a)$$

$$\text{subject to: } \sum_{\ell=1}^{K_n} E_\ell^n \left(1 + \sum_{i=1}^{\ell-1} \frac{E_i^n}{N_0} \right)^{-1} \leq \frac{K\rho_n}{\Pr[n]}, \quad (22b)$$

and $E_\ell^n \geq 0$ for $1 \leq \ell \leq K_n$. Due to the monotonic relationship between the transmit power level and the objective, the optimal policy satisfies the constraint in (22b) with equality. Hence, in what follows, we consider (22) with the equality constraint. Also, the objective function can be equivalently simplified to maximize $E_{\text{sum}} = \sum_{\ell=1}^{K_n} E_\ell^n$. Note that the above optimization problem is nonconvex, as the constraint set defined by (22b) is nonconvex.

The following result provides a closed-form expression for the optimal policy. It has been proved in [35] in the context of point-to-point links and for slow fading channels; the same proof is applicable here also.

Theorem 2. *The unique optimal solution to (22) is given by*

$$E_k^{n*} = \frac{\rho_n K}{K_n \Pr[n]} \left(1 + \frac{\rho_n K}{K_n N_0 \Pr[n]} \right)^{k-1}, \quad k = 1, 2, \dots, K_n. \quad (23)$$

The above result shows that the transmit power levels in the optimal policy increase monotonically and geometrically with the transmission index. Note that, the optimal policy ensures that the average power consumed in an attempt equals the average harvested energy that is available per active slot, $\frac{\rho_n K}{K_n \text{Pr}[n]}$. Next, we adapt the solution obtained in Theorem 2 to the case where the peak transmit power is constrained.

B. Optimal Power Control Policy with Peak Power Constraint

In the rest of this section, we drop the superscript n (the node index) to simplify the notation. Let $\mathbf{E}^* \triangleq \{E_1^*, \dots, E_{K_n}^*\}$ be the RIP obtained from Theorem 2, and let E_i^p denote the i^{th} component of a feasible power vector of the original problem (21) under the peak power constraint. Consider forcing the solution \mathbf{E}^* to satisfy the peak power constraint by setting $E_i^p = E_{\max}$ for all $i \in \mathcal{I}_p \triangleq \{i : E_i^* > E_{\max}, 1 \leq i \leq K_n\}$, and E_j^p for all $j \notin \mathcal{I}_p$ obtained by solving a reduced dimensional optimization problem with the energy levels for indices $j \notin \mathcal{I}_p$ determined using Theorem 2. We recursively apply this procedure until a vector feasible to the original problem is obtained. We have the following Lemma, which is an immediate consequence of Lemma 2 in [36].

Lemma 4. Let $\mathcal{I}_p \triangleq \{i : E_i^* > E_{\max}, 1 \leq i \leq K_n\}$, and let E_i^{p*} denote the optimal solution to (21). If $\mathcal{I}_p = \phi$ then $E_i^{p*} = E_i^*$ for all $1 \leq i \leq K_n$, else $E_i^{p*} = E_{\max}$ for all $i \in \mathcal{I}_p$.

The above Lemma shows that limiting the components of the closed-form solution (23) to take a value at most E_{\max} yields the corresponding components of the optimal solution to (21). Now, the solution in (23) is nondecreasing in the attempt index. Hence, we can set the transmit power for the last K' attempts to E_{\max} , where K' is cardinality of \mathcal{I}_p . This results in the average energy consumption being strictly less than the energy harvested, leaving room for further optimizing the first $K_n - K'$ power levels. Let $t \triangleq \sum_{\ell=1}^{K_n - K'} E_\ell$ denote the sum of the first $K_n - K'$ power levels. Considering t to be an auxiliary optimization variable, and ignoring the constant terms in the objective, we obtain the following reduced dimensional version of (22):

$$\max_{\{E_1, \dots, E_{K_n - K'}, t\}} t, \quad \text{subject to} \quad (24a)$$

$$\sum_{\ell=1}^{K_n - K'} E_\ell \frac{1}{1 + \sum_{i=1}^{\ell-1} \frac{E_i}{N_0}} \leq \frac{K \rho_n}{\text{Pr}[n]} - \sum_{i=1}^{K'} \frac{E_{\max}}{1 + \frac{t}{N_0} + \frac{(i-1)E_{\max}}{N_0}}, \quad (24b)$$

with $t = \sum_{\ell=1}^{K_n - K'} E_\ell$. From Theorem 2, the optimal solution to the problem (24) is given as

$$E_k^* = \frac{K}{K_n - K'} \left(\frac{\rho_n}{\text{Pr}[n]} - \frac{F(t^*)}{K} \right) \times \left(1 + \frac{K}{(K_n - K')N_0} \left(\frac{\rho_n}{\text{Pr}[n]} - \frac{F(t^*)}{K} \right) \right)^{k-1}, \quad (25)$$

for $1 \leq k \leq K_n - K'$. In the above, $F(t)$ is given by

$$F(t) \triangleq \sum_{i=1}^{K'} \frac{E_{\max}}{1 + \frac{t}{N_0} + \frac{(i-1)E_{\max}}{N_0}}. \quad (26)$$

Since $\sum_{\ell=1}^{K_n - K'} E_\ell = t^*$, we compute t^* as the solution to the fixed point equation

$$N_0 \left[1 + \frac{K}{(K_n - K')N_0} \left(\frac{\rho_n}{\text{Pr}[n]} - \frac{F(t^*)}{K} \right) \right]^{K_n - K'} - 1 = t^*. \quad (27)$$

It can be shown that a fixed point exists for the above equation [28]. In case there are multiple fixed points, we pick the largest one among them, since the goal is to maximize the objective function. Thus, we obtain the optimal solution to (21) in closed form.

Using the optimal power vectors of the individual hops, we can now obtain the optimal solution in the multi-hop EH case. We set $\text{Pr}[1] = 1$. For $n = 1, 2, \dots, N$, we compute \mathbf{E}^n , the power vector of the n^{th} hop, using the procedure described above. From \mathbf{E}^n , compute $\text{Pr}[n+1]$ using (19), which, in turn, is used for computing \mathbf{E}^{n+1} . The output is the set of optimal RIPs $\{\mathbf{E}^1, \dots, \mathbf{E}^N\}$. This completes the description of the solution to the optimization problem with negligible reception cost in the slow fading case. We next turn to the case where the channel is fast fading.

VI. NEGLIGIBLE RECEPTION COST: FAST FADING CHANNEL

In this section, we first find the optimal RIP with only the EUR constraint and then adapt it to obtain the optimal RIP under both EUR and peak power constraints. For a point-to-point link with fast fading channel, the optimization problem with only the EUR constraint can be written using (21) and (13) as

$$\min_{\mathbf{E} = \{E_1^n, \dots, E_{K_n}^n\}} \frac{1}{\prod_{\ell=1}^{K_n} (1 + E_\ell^n)}, \quad (28a)$$

$$\text{subject to} \quad \sum_{\ell=1}^{K_n} E_\ell^n \frac{1}{\prod_{i=1}^{\ell-1} (1 + E_i^n)} \leq \frac{K \rho_n}{\text{Pr}[n]}. \quad (28b)$$

The following Theorem provides a recursive relationship between the power levels for successive attempts in the optimal solution $\mathbf{E}^* = \{E_1^{n*}, \dots, E_{K_n}^{n*}\}$ to (28).

Theorem 3. For all $1 \leq \ell \leq K_n$, the optimal solution to (28) satisfies

$$E_{\ell+1}^{n*} = \frac{E_\ell^{n*} (E_\ell^{n*} + 2)}{2}. \quad (29)$$

Proof: See Appendix C. ■

Based on the above Theorem, all the power levels can be expressed in terms of E_1^n , and the objective function in (28a) is a monotonically decreasing function of E_1^n . Hence, the optimal E_1^n is simply the largest value that satisfies the constraint (28b). This is easy to find using the bisection method, as the left hand side of (28b) is monotonically increasing in E_1^n .

Also, we see that the optimal RIP in the fast fading case increases exponentially in the attempt index. This is in contrast to the slow fading case in Theorem 2, where the power levels increased geometrically in the attempt index.

Finally, it is straightforward to extend the solution in Theorem 3 to handle peak power constraints, using the procedure described in Sec. V-B, since Lemma 4 is valid for both slow and fast fading channels. The only difference from the slow fading case is that, at each iteration, we need to solve a fixed-point equation obtained by expressing all other power levels in terms of E_1^n to find its optimal value. Since the details are identical, we skip them.

This completes our discussion of the optimal RIPs when the reception cost is negligible. In the next section, we present the solution for the general problem in (16).

VII. GENERAL CASE: NONZERO RECEPTION COST

In this section, we present the solution in the scenario when the energy cost of packet reception is non-negligible. The nonzero reception cost leads to a coupling of the policies across the nodes and makes the problem challenging. In the following, we transform the optimization problem in (16) to a complementary geometric program (CGP), and then solve it iteratively through a series of geometric program (GP) approximations. In the next subsection, we present the solution for the slow fading case. The solution in the fast fading case is similar, and can be found in [1].

A. Multi-hop Links with Slow Fading Channel

In the slow fading case, the optimization problem in (16) can be rewritten as

$$\min_{\{\mathcal{P}^n\}_{n=1}^N} 1 - \Pr[N + 1], \quad (30a)$$

$$\text{subject to: } \Pr[n - 1] \left(\sum_{\ell=1}^{K_{n-1}} \frac{\mathbb{1}_{\{E_{\ell}^{n-1} > 0\}} R}{1 + \sum_{i=1}^{\ell-1} E_i^{n-1}} \right) + \Pr[n] \left(\sum_{\ell=1}^{K_n} \frac{E_{\ell}^n}{1 + \sum_{i=1}^{\ell-1} E_i^n} \right) \leq K \rho_n \text{ for all } n, \quad (30b)$$

$$0 \leq E_{\ell}^n \leq E_{\max} \text{ for all } 1 \leq \ell \leq K_n \text{ and } 1 \leq n \leq N + 1,$$

where $\Pr[n] = \prod_{m=1}^{n-1} \left(1 - \frac{1}{1 + \sum_{\ell=1}^{K_m} E_{\ell}^m} \right)$. The objective in the above problem is written using the fact that the PDP can also be expressed in terms of the probability that the packet reaches the destination, $\Pr[N + 1]$. The constraint in (30b) captures the fact that, in each frame, the average energy consumed by n^{th} node, for both transmission and reception, must be less than or equal to the average energy harvested by it. Both the objective and constraint in (30) are non-convex functions. In addition, due to the indicator function involved in the constraint (30b), the feasibility set of the above optimization problem depends on whether or not a particular element of the power control policy is zero. Hence, the optimization problem (30) is a nonconvex mixed integer nonlinear program (NMINLP), which is strongly NP hard to solve in general [37].

Depending on whether the indicator variable in constraint (30b) takes the value zero or one, the problem described in (30) is essentially a set of 2^K subproblems. However, it is interesting to note that the solution of the above optimization problem depends only on the number of nonzero power attempts by each node, i.e., it does not depend on the precise indices of the nonzero attempts at each node. In particular, two policies with equal number of nonzero attempts per node and with equal transmit power levels for nonzero attempts will result in the same PDP values. Hence, the computational complexity of the above problem can be reduced from 2^K to $\prod_{n=1}^N K_n$. Thus, the optimal solution to (30) can be obtained by solving $\prod_{n=1}^N K_n$ subproblems, with each subproblem corresponding to a combination of number of nonzero attempts across the hops. In the following, we focus on solving one subproblem, for a given pattern of nonzero transmit power levels. We solve it by transforming it into a CGP [38].

Without loss of generality, we present the solution for the case when all attempts are made at nonzero power levels. Using the substitution $L_{\ell}^n = 1 + \sum_{i=1}^{\ell} E_i^n$, the above problem can be reformulated as a CGP, as follows

$$\max_{\{\mathcal{P}^n\}_{n=1}^N} V, \quad (31a)$$

$$\text{subject to: } V \leq \prod_{n=1}^N \left(1 - \frac{1}{L_{K_n}^n} \right) \quad (31b)$$

$$\prod_{m=1}^{n-2} \left(1 - \frac{1}{L_{K_m}^m} \right) \left(\sum_{\ell=1}^{K_{n-1}} \frac{R}{L_{\ell-1}^{n-1}} \right) + \prod_{m=1}^{n-1} \left(1 - \frac{1}{L_{K_m}^m} \right) \left(\sum_{\ell=1}^{K_n} \frac{L_{\ell}^n - L_{\ell-1}^n}{L_{\ell-1}^n} \right) \leq K \rho_n \text{ for all } n, \quad (31c)$$

$$L_{\ell-1}^n (L_{\ell}^n)^{-1} \leq 1, \quad \frac{L_{\ell}^n}{L_{\ell-1}^n + E_{\max}} \leq 1 \text{ for all } 1 \leq \ell \leq K_n \text{ and } 1 \leq n \leq N + 1. \quad (31d)$$

Note that (31b) and (31c) can be expressed as a ratio of posynomials. Hence, (31) is a CGP, which is also NP-hard to solve [38]. Therefore, we consider a monomial approximation for the denominator posynomial of the constraints. This results in a GP approximation of (31), which can be solved optimally, since a GP is a convex problem. Thus, we solve the original problem iteratively, by solving a GP approximation to the problem at each iteration.

To construct a monomial approximation for a posynomial $p(\mathbf{x}) = \sum_i m_i(\mathbf{x})$, where $m_i(\mathbf{x})$ are monomials, we proceed as follows. Since the monomials are nonnegative by definition,

$$p(\mathbf{x}) \geq \tilde{p}(\mathbf{x}) \triangleq \prod_i \left(\frac{m_i(\mathbf{x})}{\beta_i} \right)^{\beta_i}, \quad (32)$$

where $\beta_i \triangleq \frac{m_i(\mathbf{x}_0)}{p(\mathbf{x}_0)}$, $0 \leq \beta_i \leq 1$, and \mathbf{x}_0 is any fixed vector in the positive orthant. Then, $\tilde{p}(\mathbf{x}_0) = p(\mathbf{x}_0)$, and $\tilde{p}(\mathbf{x})$ is the best local monomial approximation to $p(\mathbf{x})$ near \mathbf{x}_0 in the sense of the first order Taylor approximation [38].

The recipe to solve the CGP in (31) is summarized in Algorithm 1. This completes our solution to the problem in the case where the energy cost of packet reception is non-negligible. Next, we present simulation results to illustrate the

Algorithm 1 : Solution to the Complementary GP

Initialize: $\mathbf{L}^{(0)} = \{\mathbf{L}_1, \mathbf{L}_2, \dots, \mathbf{L}_N\}$, where $\mathbf{L}^{(0)}$ is a feasible vector for (31). $p \leftarrow 0$, $\epsilon \leftarrow 10^{-6}$.

do

- 1) Evaluate the denominator posynomials $D_a(\mathbf{L})$, $D_b(\mathbf{L})$ and $D_c(\mathbf{L})$ in (31b), (31c) and (31d), respectively, with the given $\mathbf{L}^{(p)}$.
- 2) For each term M_ℓ^q in the denominator posynomials $D_q(\mathbf{Z})$, where $q = a, b$ and c , compute $\beta_\ell^q = \frac{M_\ell^q(\mathbf{L}^{(p)})}{D_q(\mathbf{L}^{(p)})}$.
- 3) Replace the denominator posynomial of (31b), (31c) and (31d) with a monomial using (32), with the weights β_ℓ^q .
- 4) Solve the GP (e.g., using GGPLAB [39]) to obtain $\mathbf{L}^{(p+1)}$; set $p \leftarrow p + 1$.

while $\|\mathbf{L}^{(p+1)} - \mathbf{L}^{(p)}\|_2 > \epsilon$.

Output: The near-optimal RIP and PDP are given by $\mathbf{L}^{(p+1)}$ and V , respectively.

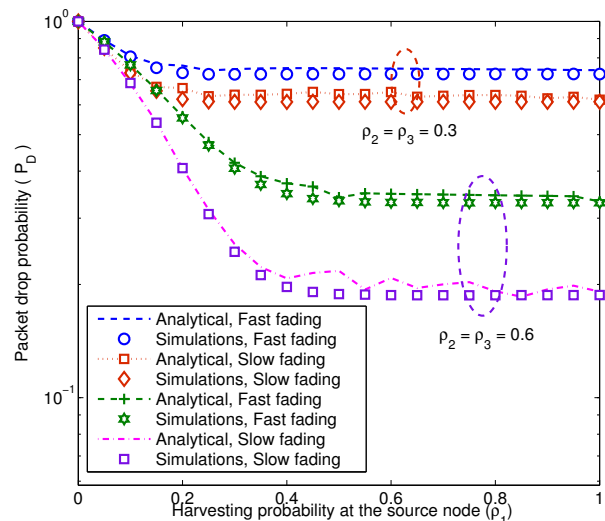


Fig. 3: Accuracy of the closed-form PDP expressions. Parameters used: $K_1 = K_2 = 2$, $R = 1$, and $B^{\max} = 3$ for all the nodes. The RIP is $[1 \ 1]$ at both source and relay nodes. The harvested energy for slow and fast fading cases are $E = 8$ dB and 3 dB, respectively.

performance of the proposed solution.

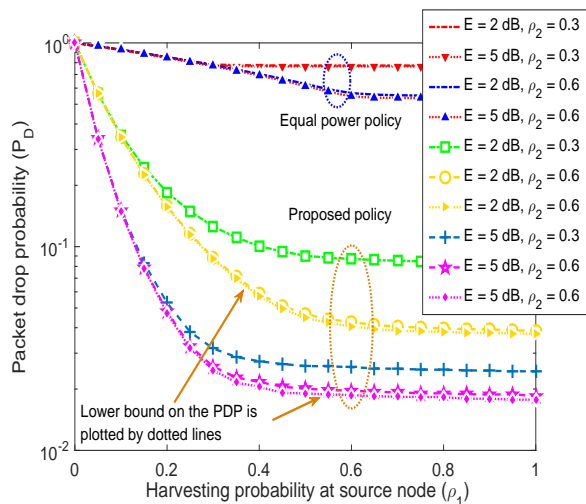
VIII. SIMULATIONS AND DISCUSSION

We consider a two-hop EH link, with a frame and slot duration of 800 ms and 100 ms, respectively. Thus, throughout the simulations, unless stated otherwise, the frame has a total of 8 slots, which are distributed equally between the first and second subframe, i.e., $K_1 = K_2 = 4$. The distance between the transmitter and receiver at both the hops is 500 m, with a reference distance $d_0 = 10$ m and path-loss exponent $\eta = 4$. We consider a typical ZigBee system with the carrier frequency 950 MHz and the system bandwidth 2 MHz [6]. The noise at the receiver corresponds to 300 K. In this system, $E = 0$ dB is equivalent to 25 μJ . The channel is assumed to be i.i.d. Rayleigh faded for both slow and fast fading cases, with the channel remaining constant for the frame and slot duration, respectively. The packets are assumed to be uncoded BPSK modulated with 10 bits [10]. Since, for short-range communications, the value of the energy required for decoding and receiving a packet is of the order of the energy required for transmission [40], we choose $R = 1$. The PDP is measured by averaging the performance over 10^7 packets.

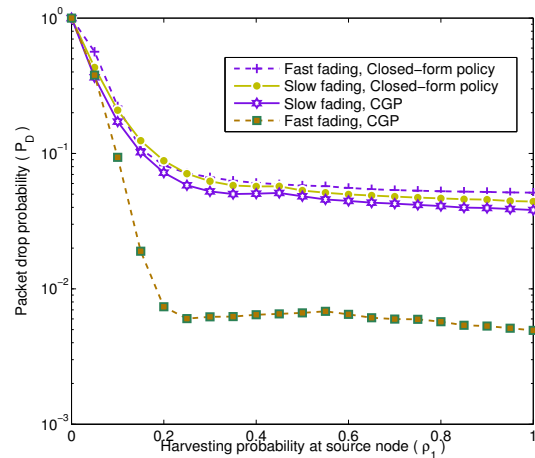
1) *Accuracy of closed-form PDP expressions:* Figure 3 illustrates the accuracy of our closed-form expressions for the PDP of a multi-hop link in both slow and fast fading cases derived in Sec. III. The analytical expressions match closely with the simulation results. We observe that the PDP initially decreases with the harvesting probability at the source node, ρ_1 , and later saturates. The latter regime is the EUR, because, under the RIP $[1, 1]$, the average energy consumed is lower than the average energy harvested for ρ_1 greater than about 0.4. Also, the PDP obtained for $\rho_2 = \rho_3 = 0.6$ is lower than the PDP for $\rho_2 = \rho_3 = 0.3$. This is because, in the latter case, the energy availability at the relay and destination is lower, and therefore fewer attempts are supported at each hop.

2) *Performance under negligible reception cost:* In Fig. 4a, we illustrate the performance of the closed-form RIP derived in Sec. V-B for slow fading links. The performance of the proposed policy for multi-hop links with finite sized battery nodes is close to the lower bound presented in Lemma 3. Also, the proposed RIP offers a ten-fold improvement in the PDP compared to the equal power policy (EPP) which uses $P_{\max} = 10E/T_s$ as the transmit power in every attempt. Note that the EPP is an online policy and operates under the ENC. It is interesting to observe that the PDP of the EPP with ($E = 2$ dB, $\rho_2 = 0.6$) is lower than the PDP of the EPP with ($E = 5$ dB, $\rho_2 = 0.3$), even though the average energy harvested in the two scenarios is the same. On the other hand, for the proposed policy, the PDP with ($E = 2$ dB, $\rho_2 = 0.6$) is higher than the PDP with ($E = 5$ dB, $\rho_2 = 0.3$). This can be explained as follows. The EPP for $E = 2$ dB uses lower power in each attempt than the EPP for $E = 5$ dB. Due to this, the second node runs out of energy less frequently when $E = 2$ dB, ensuring better packet delivery at the destination. For the proposed policy, the range of power values available to the transmitter for designing the RIP is higher when $E = 5$ dB than that with $E = 2$ dB, since $P_{\max} = 10E/T_s$ is set as the maximum allowed transmit power. Therefore, the performance of the optimal policy in the former case is better than the latter.

In Fig. 4b, the performance of the closed-form optimal RIP designed by ignoring the packet reception cost using the approach in Secs. V and VI is compared against the performance of the near-optimal policy for the general case, when $R = 1$. We note that, the PDP of closed-form policy is inferior to the PDP of near-optimal policy for general case. Under the settings considered, when the channel is slow fading, the peak power constraint saturates the transmit power levels of the CGP-optimal solution to P_{\max} for all attempts. Because of this, the performance of the CGP-based



(a) Slow fading case: EPP versus proposed policy.



(b) Comparison of CGP policy with closed-form policy.

Fig. 4: Performance of the policy under negligible reception cost, for a slow fading channel. (a) Our closed-form policy outperforms the equal power policy which attempts the packet with transmit power level P_{\max} . In addition, for the proposed policy, the PDP obtained using a finite sized battery ($B_{\max}^n = 100$ for all n) meets the lower bound presented in Lemma 3. (b) Performance of optimal policy designed by ignoring the energy cost of packet reception, compared to the near-optimal policy for the general case. In Fig. 4b, for all nodes, $B_{\max}^n = 50$ and 200 for the slow and fast fading channels, respectively. The setup considers a multi-hop link with packet reception cost $R = 1$. As before, $P_{\max} = \frac{10E}{T_s}$. The harvested energy in the slow fading case is $E = 5$ dB, while for the fast fading case, $E = 3$ dB.

and the closed-form solution is similar. In the fast fading case, there is a much larger performance improvement over the closed-form solution. This highlights the value of solving the coupled optimization problem when the packet reception cost is nonzero.

3) *Performance under non-negligible reception cost:* Figure 5a illustrates the performance of the proposed iterative GP approximation based solution presented in Sec. VII-A, for slow fading channels. In this case also, the PDP of the proposed policy is close to the lower bound presented in the Lemma 3. The curves corresponding to the lower bound are labeled as Bound. The PDP of our solution is also compared against the EPP that uses power P_{\max} for all attempts. The PDP of the proposed policy is approximately ten-fold better than the PDP obtained by the EPP. Also, we plot the performance of the policy designed by setting the indicator function in (18) to be always one, i.e., by assuming that all the transmit attempts are made at a nonzero power level. We label the corresponding curves as Relaxed. By setting the indicator functions to unity, we only need to solve a single sub-problem instead of $\prod_{n=1}^N K_n$ sub-problems. We see that the performance gap between the two policies is almost negligible. Thus, in a wide range of scenarios, nearly optimal policies can be obtained by solving the relaxed problem. Similar behavior is seen in the fast fading case: in fact, our policy offers over 100 times improvement in PDP compared to the EPP [1].

In Fig. 5b, we illustrate that the PDP at the second hop decreases with increase in the harvesting probability at the source node. This is because, at higher harvesting rate, the source node can attempt the packet transmission at higher transmit power levels. This reduces the average power consumed for packet reception at the 2nd node, which, in turn, allows it to transmit at higher power levels to the destination, while still meeting its own average power constraint. Also, the relatively

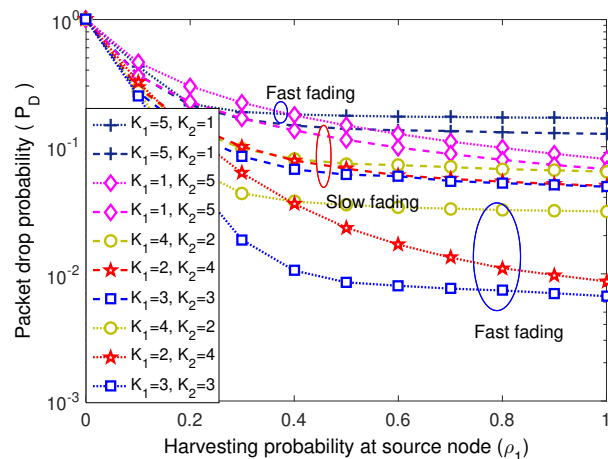


Fig. 6: Impact of slot allocation on the PDP: equal slot allocation performs the best. The harvested energy for slow and fast fading case is $E_s = 5$ dB and $E_s = 3$ dB, respectively, while the size of the battery at each node is $50E_s$ and $200E_s$, respectively. In both the cases: the energy required for decoding at each node is $1E_s$, the harvesting probabilities at relay and destination nodes are $\rho_2 = 0.6$ and $\rho_3 = 0.3$, respectively, and the maximum transmit power allowed is $P_{\max} = \frac{10E}{T_s}$. Further, the channels are assumed to be symmetric at both the hops. $B_{\max}^n = 50$ for all nodes.

smaller improvement in the PDP at higher harvesting rates is because of the peak transmit power constraint, which limits the benefit obtainable by higher harvesting rates.

4) *Effect of slot allocation:* Fig. 6 demonstrates the impact of different slot allocations on the PDP. Here, a total of 6 slots are distributed between two hops and the PDP performance corresponding to each slot allocation is plotted against the harvesting probability at the source node. For both slow and fast fading channels, the PDP is lowest when both the first and second node have equal number of slots to forward the packet

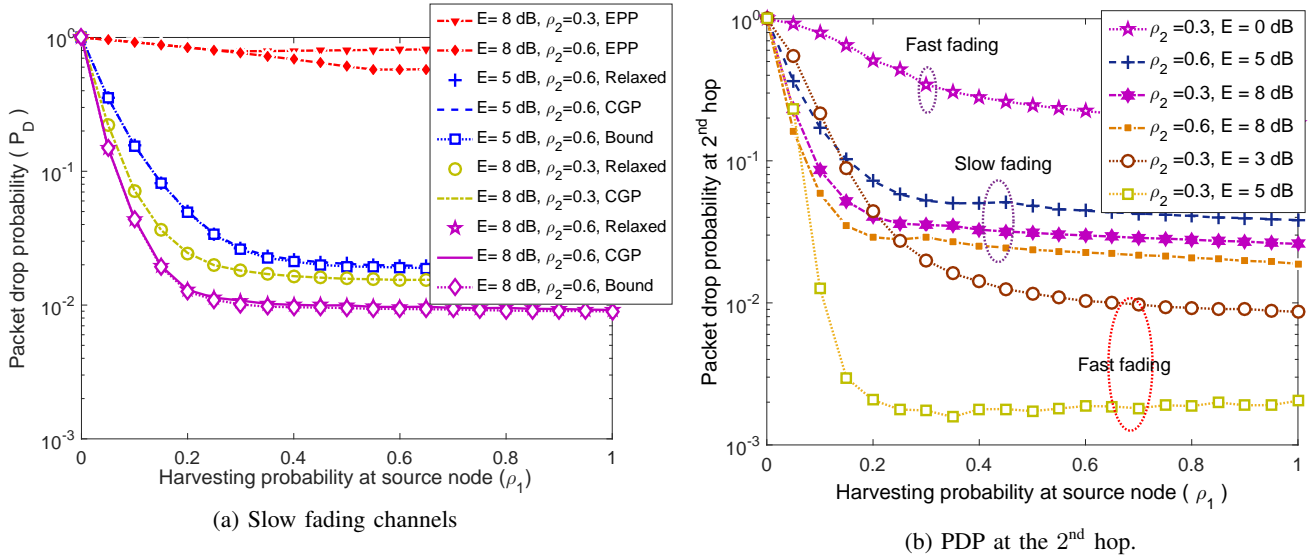


Fig. 5: Performance of the proposed CGP based algorithm for finding near-optimal RIPs, for a slow fading channel: (a) The performance of the CGP based policy meets the lower bound. Also, the policy designed by relaxing the integer constraint performs similar to a CGP based policy. Further, the CGP based policy outperforms the EPP. (b) the PDP at the second hop improves with increase in the harvesting rate at the source node. In this case, $B_n^{\max} = 50$ for all the nodes. The parameters for both the figures are $R = 1$ and $P_{\max} = 10E/T_s$. For both the hops, the EPP is $[P_{\max} P_{\max} P_{\max} P_{\max}]$.

to next node, i.e., $K_1 = K_2 = 3$. This is somewhat surprising, because one might intuitively conjecture that allotting a larger number of slots for the first hop compared to the second hop would result in better performance compared to the vice-versa, because a packet dropped in the first hop will not even utilize the second hop. However, this is countered by two points: first, an equal slot allocation provides roughly equal opportunity for the nodes to harvest energy; and second, the power policy optimizes the PDP within the slots available.

Also, the performance degrades with a more asymmetric distribution. However, between different asymmetric distributions of slots, it is interesting to note that for lower harvesting rates at the source node, it is better to allocate more slots to the first hop, while at higher harvesting rates, it is better to allocate more slots to the second hop. For example, at lower harvesting rates, the PDP for the allocation $K_1 = 5$ and $K_2 = 1$ is lower than that for $K_1 = 1$ and $K_2 = 5$, while at higher harvesting rates, $K_1 = 1$ and $K_2 = 5$ results in lower PDP. This is because of the trade-off between the time-diversity offered by the channel and the harvesting rate.

IX. CONCLUSIONS

In this paper, we designed PDP-optimal RIPs for ARQ-based multi-hop EH links. To this end, we first derived closed-form expressions for the PDP of multi-hop EH links. Using the derived expressions, we setup a RIP optimization problem, which was solved in two different scenarios. First, we considered a scenario when the energy cost for reception is negligible, and derived closed-form expressions for the optimal RIPs. Next, we presented an iterative geometric programming based solution to the RIP optimization problem under non-negligible energy reception cost. Through simulations, we illustrated that our proposed policies significantly outperform equal power policies and achieve a performance close to

the lower bound. In addition, our results provided interesting insights into the trade-offs in the system parameters and highlighted the coupled nature of the problem. Future extensions of this work can consider the design of RIPs for multi-hop links with time-correlated channels, and under different quality of service requirements.

APPENDIX

A. Transition probabilities

For a slow fading channel, the probability of transitioning from state $\mathbf{a} \triangleq (\mathbf{B}_a, \mathbf{U}_a, s)$ to $\mathbf{b} \triangleq (\mathbf{B}_b, \mathbf{U}_b, s + 1)$ is $G_{a,b} \triangleq \Pr((\mathbf{B}_{s+1} = \mathbf{B}_b, \mathbf{U}_{s+1} = \mathbf{U}_b, s + 1) | (\mathbf{B}_s = \mathbf{B}_a, \mathbf{U}_s = \mathbf{U}_a, s), \gamma)$, where $B_a^n, B_b^n \in \{0, 1, \dots, B_n^{\max}\}$ and $U_a^n, U_b^n \in \{-1, 1, \dots, K_n\}$. For $B_a^n \geq L_\ell^n$ and $B_a^{n+1} \geq R$, and \mathbf{U}_a such that $\{U_a^i = -1\}_{i=1}^{n-1}$, $\{U_a^i = 0\}_{i=n+1}^N$ and $U_a^n = \ell$, we can write $G_{a,b}$ as

$$G_{a,b}(\gamma) = \begin{cases} p(\mathbf{I}_s)P_e(L_\ell^n, \gamma), & \mathbf{B}_b = \tilde{\mathbf{B}}, \tilde{\mathbf{U}}_b^{-n} = \tilde{\mathbf{U}}_a^{-n}, \\ & U_b^n = U_a^n + 1, \\ p(\mathbf{I}_s)(1 - P_e(L_\ell^n, \gamma)), & \mathbf{B}_b = \tilde{\mathbf{B}}, \tilde{\mathbf{U}}_b^{-n} = \tilde{\mathbf{U}}_a^{-n}, \\ & U_b^n = -1, \\ 0, & \text{otherwise.} \end{cases} \quad (33)$$

where $p(\mathbf{I}_s) \triangleq \prod_{k=1}^{N+1} \rho_k^{I_k} (1 - \rho_k)^{1-I_k}$ denotes the probability that the nodes harvest energy according to pattern $\mathbf{I}_s \in \{0, 1\}^{N+1}$ in the s^{th} slot, $\tilde{\mathbf{B}} \triangleq (\min\{B_a^k + I_k - \mathbb{1}_{\{k=n\}}L_\ell^n - R\mathbb{1}_{\{k=n+1\}}, B_k^{\max}\})_{k=1}^{N+1}$, and $\tilde{\mathbf{U}}_a^{-n}$ and $\tilde{\mathbf{U}}_b^{-n}$ denote $(N - 1)$ -length vectors obtained by removing the n^{th} component of \mathbf{U}_a and \mathbf{U}_b , respectively. Also, I_k is the k^{th} component of \mathbf{I}_s , and is equal to one if the k^{th} node harvests the energy in the current slot, otherwise it is

equal to zero. Further, $\mathbb{1}_{\{\cdot\}}$ denotes an indicator function which takes the value 1 if its argument is true, and takes the value 0 otherwise. In (33), $P_e(L_\ell^n, \gamma)$ is given by (3) for both slow and fast fading channels. The terms in the above transition probability expression are obtained by considering the events that need to occur for a particular transition to happen. For instance, (33) is written for the case when the transmissions of all the previous nodes were successful, i.e., $\{U_a^i = -1\}_{i=1}^{n-1}$ and the n^{th} node has to make the ℓ^{th} attempt in s^{th} slot. In this case, the transition described in the first case happens if all the nodes harvest energy according to the pattern described by I_s , a decoding failure occurs in the current attempt, and the channel in the n^{th} sub-frame is γ . Note that, during such an event, the battery of a node is incremented by one if it harvests energy in the current slot and its battery is not full. Further, the energy in the battery of the n^{th} and $n+1^{\text{th}}$ nodes are decreased by the amount of energy used to make the ℓ^{th} attempt, i.e., L_ℓ^n and R , respectively. Since only the n^{th} node transmits during the s^{th} slot, the local transmission index of all nodes except the n^{th} node remain unchanged. The transition probabilities for the other cases, e.g., $B_a^n \leq L_\ell$ and $B_a^{n+1} \geq R$, can be written similarly.

In the fast fading case, \mathbf{G} contains the channel-averaged entries, i.e., $P_e(E_\ell^n, \gamma)$ in (33) is replaced by $\mathbb{E}_\gamma(P_e(E_\ell^n, \gamma))$. We omit the details to avoid repetition.

B. Procedure to compute number of feasible attempts Ψ_n

Recall that a packet transmission attempt is made if and only if both the transmitter and receiver have sufficient energy to make the next attempt. This implies, for a given RIP \mathcal{P} , the battery states at the start of the frame B^n and B^{n+1} , the harvesting patterns of n^{th} and $n+1^{\text{th}}$ node, namely, $(m_{r,n}, m_{t,n})$ and $(m_{r,n+1}, m_{t,n+1})$, respectively, determine the number of feasible attempts Ψ_n . To determine Ψ_n , we use the notion of *energy available for transmission and reception*, denoted as $E_{\text{avl},t}^n$ and $E_{\text{avl},r}^n$, respectively, at the n^{th} node, which are random variables determined by the order of energy arrivals and consumption. To obtain closed form expressions, we approximate $E_{\text{avl},r}^n$ and $E_{\text{avl},t}^n$ as $E_{\text{avl},r}^n \approx \min\{B^n + m_{r,n}, B_n^{\text{max}}\}$ and $E_{\text{avl},t}^n \approx \min\{B^n + m_{r,n} + m_{t,n} - \Psi_{n-1}R, B_n^{\text{max}}\}$, respectively. Based on this, Ψ_n can be approximated as $\Psi_n = \min\{\kappa_n, \kappa_{n+1}\}$, where $\kappa_n \triangleq \max\{\kappa : E_{\text{avl},t}^n - \Psi_{n-1}R - \sum_{\ell=1}^{\kappa} E_\ell^n \geq 0\}$ and $\kappa_{n+1} \triangleq \max\{\kappa : E_{\text{avl},r}^{n+1} - \kappa R \geq 0\}$ denote the number of feasible attempts at the transmitting and receiving EHN of the n^{th} hop, respectively. Thus, we have obtained a recursive equation to compute the number of feasible attempts.

C. Proof of Theorem 3

To prove the Theorem, we need the following Lemma, which asserts that the optimal RIP allocates nonzero power to all packet attempts. Its proof is similar to the proof of a corresponding Lemma in the slow fading case in [35].

Lemma 5. *The optimal RIP solution to (28), denoted by $\mathbf{E}^{n*} = \{E_1^{n*}, \dots, E_{K_n}^{n*}\}$, satisfies $E_\ell^{n*} > 0$ for all $1 \leq \ell \leq K_n$.*

Next, we make the substitution $x_k \triangleq \frac{E_k^n}{\prod_{i=1}^{k-1} (1+E_i^n)}$. Thus, $E_1^n = x_1, E_2^n = x_2(1+x_1), E_3^n = x_3(1+x_1)(1+x_2(1+x_1))$, and so on. Hence, we can rewrite (28) as

$$\max_{x_1, \dots, x_{K_n}} f_{K_n-1}(1+x_{K_n}f_{K_n-1}), \text{ subject to } \sum_{\ell=1}^{K_n} x_\ell \leq \frac{K\rho_n}{\text{Pr}[n]}, \quad (34)$$

where $f_\ell \triangleq f_{\ell-1}(1+x_\ell f_{\ell-1})$ for $1 \leq \ell \leq K_n$ and $f_0 \triangleq 1$. We claim that the solution to the transformed problem obeys the following recursive relationship

$$x_{\ell+1} = \frac{x_\ell}{2} \left(\frac{2+x_\ell f_{\ell-1}}{1+x_\ell f_{\ell-1}} \right) = \frac{x_\ell}{2} \left(\frac{f_{\ell-1}+f_\ell}{f_\ell} \right), \quad 1 \leq \ell \leq K_n-1. \quad (35)$$

The proof follows by induction. When $K_n = 2$, using the constraint in (34), the objective function can be written as $f_1 \left(1 + \left(\frac{K\rho_n}{\text{Pr}[n]} - x_1 \right) f_1 \right)$. Let $f'_1 \triangleq df_1/dx_1$. Now, because of Lemma 5, and since the domain of optimization is the positive orthant, $f'_1 = 0$ at the optimal solution of the unconstrained problem [41, Chapter 4]. Hence, we have

$$f'_1 \left(1 + \left(\frac{K\rho_n}{\text{Pr}[n]} - x_1 \right) f_1 \right) + f_1 \left(f'_1 \left(\frac{K\rho_n}{\text{Pr}[n]} - x_1 \right) - f_1 \right) = 0$$

Rearranging the above, we get

$$x_2 = \frac{f_1}{2f'_1} - \frac{1}{2f_1} = \frac{x_1}{2} \left(\frac{2+x_1 f_0}{1+x_1 f_0} \right).$$

This establishes (35) when $K_n = 2$. Suppose (35) holds for $K_n = k$. We proceed to show that it holds for $K_n = k+1$ as well. Towards this end, we derive an alternative induction hypothesis. From the constraint in (34), we substitute $x_k = \frac{K\rho_n}{\text{Pr}[n]} - \sum_{\ell=1}^{k-1} x_\ell$ in the objective function, and differentiate it with respect to x_1 and set equal to zero to obtain

$$x_k = \frac{f_{k-1}}{2f'_{k-1}} \left(1 + \sum_{i=2}^{k-1} \frac{\partial x_i}{\partial x_1} \right) - \frac{1}{2f_{k-1}}, \quad (36)$$

where $f'_{k-1} = f'_{k-2} + 2x_{k-1}f_{k-2}f'_{k-2} + f_{k-2}^2 \frac{\partial x_{k-1}}{\partial x_1}$. Equating the right hand side of the expression for x_k above with the one given by the induction hypothesis in (35) and rearranging, we get

$$f'_{k-1} = f_{k-2}^2 \left(1 + \sum_{i=2}^{k-1} \frac{\partial x_i}{\partial x_1} \right). \quad (37)$$

To complete the proof, we need to show that the solution of $k+1$ -dimensional optimization problem also follows the relation in (35). For the $(k+1)$ -dimensional problem, solving for x_{k+1} from the constraint, substituting into the objective function, and differentiating with respect to x_1 and setting equal to zero, we obtain

$$x_{k+1} = \frac{f_k}{2f'_k} \left(1 + \sum_{i=2}^k \frac{\partial x_i}{\partial x_1} \right) - \frac{1}{2f_k}, \quad (38)$$

where $f'_k = f'_{k-1} + 2x_k f_{k-1} f'_{k-1} + f_{k-1}^2 \frac{\partial x_k}{\partial x_1}$. Now, in the $(k+1)$ -dimensional problem, if we fix x_{k+1} , it reduces to a k -dimensional problem, for which the relation (37) holds. Since it holds at any value of x_{k+1} , it also holds at the optimal

solution to the $(k+1)$ -dimensional problem. In the expression for f'_k , substituting for f'_{k-1} from the new induction hypothesis in (37), we get

$$\begin{aligned} f'_k &= f_{k-2}^2 \left(1 + \sum_{i=2}^{k-1} \frac{\partial x_i}{\partial x_1} \right) (1 + x_{k-1}(f_{k-2} + f_{k-1})) + f_{k-1}^2 \frac{\partial x_k}{\partial x_1} \\ &= f_{k-1}^2 \left(1 + \sum_{i=2}^k \frac{\partial x_i}{\partial x_1} \right). \end{aligned}$$

The first equality above uses (35) and (37); the second equality uses the definition of f_k . Substituting the above expression for f'_k in (38) results in

$$x_{k+1} = \frac{f_k}{2f_{k-1}^2} - \frac{1}{2f_k} = \frac{x_k}{2} \left(\frac{f_{k-1} + f_k}{f_k} \right),$$

which is precisely the induction step for the $(k+1)$ -dimensional problem. This, along with the observation that (35) is equivalent to (29), completes the proof.

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