Construction of Binary Sensing Matrices Using Extremal Set Theory

R. Ramu Naidu and Chandra R. Murthy

***Abstract*—The construction of binary sensing matrices is one of the active directions in the emerging field of compressed sensing (CS). Due to their sparse structure and competitive performance, they provide multiplier-less and faster dimensionality reduction in applications such as data compression. This letter attempts to relate the notion of extremal set theory to the construction of good CS matrices. In particular, we show that extremal set theory is useful for constructing binary sensing matrices and bounding their maximum column size (i.e., number of columns). We also prove the existence of binary sensing matrices whose column size meets the upper bound. Simulation results show that the constructed matrices outperform Gaussian and Bernoulli random matrices, as well as other deterministic binary and bipolar constructions from the literature.**

***Index Terms*—Binary sensing matrices, compressed sensing (CS), extremal set theory, restricted isometry property (RIP).**

I. Introduction

In recent years, compressed sensing (CS) and sparse representation have become a powerful tool for efficiently compressing and processing data. It has been shown to have tremendous potential for several applications, in areas such as image/signal processing [1]–[5] and numerical computation [6], to name a few. One of the central problems in CS, which is also the focus of this letter, is the construction of *sensing matrices* $\Phi \in \mathbb{R}^{m\times M}$ such that an arbitrary s-sparse vector[1] $u \in \mathbb{R}^M$ can be efficiently reconstructed using the underdetermined linear projection $y = \Phi u$.

In order to state the results in this letter in the right context, we begin by reviewing desirable properties of sensing matrices and the best known bounds on their size. One can find u from its underdetermined linear projection $y = \Phi u$ by solving the following l_0-minimization problem:

$$\min_{v} \|v\|_0 \text{ subject to } \quad \Phi v = y \tag{1}$$

where $\|v\|_0 \triangleq |\{i \mid v_i \neq 0\}|$. The l_0-minimization problem (1) is an NP-hard problem [7]. Chen *et al.* [7] have proposed the following l_1-minimization problem in place of l_0-minimization problem, making it a computationally tractable linear program:

$$\min_{v} \|v\|_1 \text{ subject to } \quad \Phi v = y, \tag{2}$$

where $\|v\|_1$ denotes the l_1-norm of the vector $v \in \mathbb{R}^M$.

One of the important breakthroughs in the CS literature is the characterization of the conditions under which problems (1) and (2) admit the same solution. An $m \times M$ matrix Φ is said to satisfy the restricted isometry property (RIP) [8], [9] of order s with constant δ_s $(0 < \delta_s < 1)$ if for all vectors $u \in \mathbb{R}^M$ with $\|u\|_0 \leq s$, we have

$$(1 - \delta_s)\, \|u\|_2^2 \leq \|\Phi u\|_2^2 \leq (1 + \delta_s)\, \|u\|_2^2 \,. \tag{3}$$

RIP measures the degree to which each submatrix consisting of at most s columns of Φ is close to being an isometry. The following theorem [7], [10] establishes the equivalence between l_0- and l_1-minimization problems through RIP:

Theorem 1: Suppose an $m \times M$ matrix Φ has the $(2s, \delta)$ restricted isometry property for some $\delta < \sqrt{2} - 1$, then (1) and (2) have the same solution if (1) has an s-sparse solution.

A drawback of the RIP-based sparse vector recovery guarantees is that verifying a given sensing matrix satisfies the RIP property is itself NP-hard. As a consequence, most known constructions of matrices satisfying RIP are random constructions. An alternative, easily computable metric for the goodness of a sensing matrix is the mutual coherence.

The mutual coherence $\mu(\Phi)$ of a given matrix Φ is the largest absolute inner product between different normalized columns of Φ, that is,

$$\mu(\Phi) = \max_{1\leq i,j\leq M,\, i\neq j} \frac{|\, \phi_i^T \phi_j|}{\|\phi_i\|_2 \|\phi_j\|_2} \tag{4}$$

where ϕ_i is the ith column of Φ. The following proposition [11] relates the RIP constant δ_s and μ.

Proposition 1: Suppose that $\phi_1, \ldots, \phi_M$ are the unit norm columns of the matrix Φ with coherence μ. Then, Φ satisfies RIP of order s with constant $\delta_s = (s-1)\mu$, when $\delta_s < 1$.

As a consequence of the above, one approach in designing good sensing matrices is to construct matrices with low mutual coherence. In the recent years, the construction of binary CS matrices (i.e., containing 0 and 1 as elements) has attracted significant attention. Binary CS matrices are attractive because they can be implemented without the use of multipliers, thereby leading to faster dimensionality reduction. The construction of

Manuscript received July 21, 2016; revised December 7, 2016; accepted December 7, 2016. Date of publication January 28, 2017; date of current version January 24, 2017. The work of R. Ramu Naidu was supported by the Postdoctoral Fellowship (Ref No. 2/40(63)/2015/R&D-II/4270) from National Board for Higher Mathematics, Government of India. The associate editor coordinating the review of this manuscript and approving it for publication was Dr. Michael A. Lexa. (*Corresponding author: Chandra R. Murthy*.)

The authors are with the Department of Electrical Communication Engineering, Indian Institute of Science, Bangalore 560012, India (e-mail: ramhcu10@gmail.com; cmurthy@ece.iisc.ernet.in).

Color versions of one or more of the figures in this letter are available online at http://ieeexplore.ieee.org.

Digital Object Identifier 10.1109/LSP.2016.2638426

[1]A vector $u \in \mathbb{R}^M$ is said to be s-sparse, if it has at most s nonzero coordinates.

binary sensing matrices is an area of active research, and there are several known constructions [12]–[19] using ideas from algebra, graph theory, and coding theory. All of these constructions possess an (r,k)-structure.

Definition 1: A matrix Φ is said to have an (r,k)-structure, if every column of Φ contains k ones and the inner product between any two columns is at most r, that is, the mutual coherence of Φ is at most $\frac{r}{k}$.

The first constructions of binary sensing matrices were developed by DeVore [14]. The constructed matrices are of size $p^2 \times p^{l+1}$ with coherence $\frac{l}{p}$, where p is any prime power and $1 < l < p$. This construction has an (l,p)-structure. Li *et al.* [16] generalized the work in [14] using algebraic curves over finite fields. The size of the matrices constructed is $|\mathcal{P}|q \times q^{\mathcal{L}(G)}$, where q is any prime power and $\mathcal{P}$ is the set of all rational points on an algebraic curve $\mathcal{X}$ over a finite field $\mathbb{F}_q$ and G is a divisor of $\mathcal{X}$ such that $\deg(G) < |\mathcal{P}|$. This construction has a $(\deg(G), |\mathcal{P}|)$-structure. Using Euler squares, Naidu *et al.* [12] constructed binary sensing matrices of size $nk \times n^2$ and coherence $\frac{1}{k}$, where $n = p_1^{r_1} p_2^{r_2} \ldots p_l^{r_l}$ and $k = \min\{p_1^{r_1}, p_2^{r_2}, \ldots, p_l^{r_l}\} - 1$ with p_i prime and r_i any positive integer for $1 \le i \le l$. This construction has a $(1,k)$-structure. Li and Ge [17] constructed binary sensing matrices using finite geometry. These matrices possess $(1,q)$-, $(1,q+1)$-, and $(2,q+1)$-structure for a prime power q. All the existing constructions have an (r,k)-structure for a particular family of numbers. In the context with the previous discussion, the novel contributions of this work may be summarized as follows:

1) We relate extremal set theory to the problem of constructing matrices with low-mutual coherence. This connection allows us to utilize tools from extremal set theory to construct good binary sensing matrices for CS applications.
2) We derive an upper bound on the column size M of a binary sensing matrix $\Phi_{m\times M}$ in terms of r, k, and m, which possess $(r-1,k)$-structure.
3) We show that, for every pair of integers (r,k) with $r < k$, there exists a binary CS matrix, which possesses an (r,k)-structure, with optimal column size.

This letter is organized as follows. In Section II, we present the basics of extremal set theory and show how they are useful for constructing binary sensing matrices and bounding its maximum column size. Further, we establish the existence of sensing matrices that meet the upper bound on the column size. In Section III, we present the simulation results. In Section IV, we present our concluding remarks.

II. Extremal Set Theory

Let r, k, and m be positive integers such that $r < k < m$ and X an m-element set, that is, $X = \{1,2,\ldots,m\}$. Define $2^X = \{H, H \subseteq X\}$ and $[X]^k = \{H \subseteq X, |H| = k\}$. Any subset $\mathcal{F}$ of $[X]^k$ is called a k-uniform family.

Definition 2: Any subset $\mathcal{F}_d(r,k,m)$ of $[X]^k$ is called r-dense if any r-element subset of X is contained in at least one member of $\mathcal{F}_d$.

Definition 3: Any subset $\mathcal{F}_s(r,k,m)$ of $[X]^k$ is called r-sparse if any r-element subset of X is contained in at most one member of $\mathcal{F}_s$, that is, $|F_i \cap F_j| \le r-1, \forall F_i, F_j \in \mathcal{F}_s$.

Definition 4: Any subset $\mathcal{F}_S(r,k,m)$ of $[X]^k$ is called a Steiner system if every r-element subset of X belongs to exactly one member of $\mathcal{F}_S$.

Note 1: Let m, k, r, and λ be positive integers such that $2 \le r < k < m$. We say that a set $\mathcal{S}$ of k-element subsets of an m-element set X is an r–(m,k,λ) packing design, if, any r-element subset E of X is contained in at most λ members of $\mathcal{S}$. In particular, if $\lambda = 1$, then $\mathcal{S}$ is r-sparse. We say that the set $\mathcal{S}$ is a covering design or a covering, if any r-element subset of X is contained in at least one member of $\mathcal{S}$. That is, covering designs are nothing but r-dense sets.

A necessary *divisibility condition* [20] for the existence of Steiner systems is as follows:

$$\binom{k-i}{r-i} \text{ divides } \binom{m-i}{r-i} \; \forall\, 0 \le i \le r-1. \tag{5}$$

Clearly, the Steiner system $\mathcal{F}_S(r,k,m)$ is a subset of the r-sparse set $\mathcal{F}_s(r,k,m)$.

A. Extremal Set Theory for Binary Sensing Matrices

In this section, we discuss how the r-sparse sets may be related to the construction of binary CS matrices. Further, we determine the maximum possible column size of $(r-1,k)$-structure binary CS matrix and establish the existence of matrices possessing optimal column size, for every pair of integers (r,k) with $r < k$. The following proposition relates the r-sparse sets and binary sensing matrices that possess $(r-1,k)$-structure.

Proposition 2: There is a one-to-one correspondence between the set of all r-sparse k-uniform families and binary sensing matrices that possess $(r-1,k)$-structure.

Proof: Let $\mathcal{F}$ be an r-sparse family of a set $X = \{1,2,\ldots,m\}$. Define the matrix $\Phi = \{\phi_{ij}\}$ to be the $m \times |\mathcal{F}|$ binary incidence matrix of $\mathcal{F}$, that is, the columns of Φ correspond to the sets of $\mathcal{F}$. The characteristic vector on each jth set F_j, gives the jth column in Φ, that is

$$\phi_{ij} = \begin{cases} 1 & \text{if } i \in F_j \\ 0 & \text{otherwise.} \end{cases} \tag{6}$$

Since $\mathcal{F}$ is an r-sparse family, it follows that the cardinality of overlap between any two columns of Φ is at most $r-1$ and the number of ones in each column is k. Hence, the matrix Φ has $(r-1,k)$-structure and the coherence is at most $\frac{r-1}{k}$.

Suppose $\Phi_{m\times M}$ is a binary sensing matrix with $(r-1,k)$-structure. Define $X = \{1,2,\ldots,m\}$, $F_i = supp(\phi_i)$, where ϕ_i is the ith column of Φ and $supp(\phi_i)$ is the support vector of ϕ_i. Let $\mathcal{F} = \{F_1, F_2, \ldots, F_M\}$, clearly $\mathcal{F} \subseteq [X]^k$.

Suppose $\mathcal{F}$ is not an r-sparse family, which implies that there exists some $T \in [X]^r$ and $F_i, F_j \in \mathcal{F}$ such that $T \subseteq F_i \cap F_j$. Therefore, $|F_i \cap F_j| \ge r$. This is contradiction, as F_i and F_j are support vectors of ϕ_i and ϕ_j, respectively, and the cardinality of overlap between any two columns of Φ is at most $r-1$. Hence, $\mathcal{F}$ is an r-sparse family. ■

Therefore, using $r-$sparse sets, one can construct binary sensing matrices with coherence at most $\frac{r-1}{k}$.

Proposition 3: If $\mathcal{F}$ is an r-sparse $k-$uniform family with cardinality M on an m-element set X, then the incidence matrix $\Phi_{m\times M}$ of $\mathcal{F}$ has coherence at most $\frac{r-1}{k}$ and $\Phi = \frac{1}{\sqrt{k}}\Phi$ satisfies RIP with $\delta_s = (s-1)(\frac{r-1}{k})$, for any $s < \frac{k}{r-1}+1$.

Proof: Follows from propositions 1 and 2. ■

B. Construction of r-Sparse Sets

In this section, we give a construction of 2-sparse k uniform family on a set $X = \{0, 1, 2, \ldots, kp-1\}$ by using polynomials over finite field $\mathbb{F}_p = \{f_1 = 0, f_2 \ldots, f_p\}$, where p is any prime power and $2 \le k \le p-1$. For simplicity of notation, let us denote $f_i = i$ to form an order among the elements of $\mathbb{F}_p$. Let us denote the set of all polynomials of degree at most one over $\mathbb{F}_p$ as $\Gamma(x) = \{f_i x + f_j : f_i, f_j \in \mathbb{F}_p\}$. Note that $|\Gamma(x)| = p^2$. If we consider each polynomial $P(x) \in \Gamma(x)$ as a mapping from $\mathbb{F}_p$ to $\mathbb{F}_p$ then $P(x) \in \mathbb{F}_p, \forall x \in \mathbb{F}_p$. Form a $k-$tuple $S_k = \{0, 1, \ldots, k-1\} \subset \mathbb{F}_p$, for $2 \le k \le p-1$. Define $P(S_k) = \{P(0), P(1), \ldots, P(k-1)\}$ for all $P(x) \in \Gamma(x)$. Since the degree of the polynomial $P(x) - T(x)$ is at most one for all $P(x), T(x) \in \Gamma(x)$, the set $\mathcal{F}_\Gamma = \{\{P(0), P(1)+p, \ldots, P(k-1)+(k-1)p\} : \forall P(x) \in \Gamma(x)\}$ forms a 2-sparse k-uniform family on the set $X = \{0, 1, 2, \ldots, kp-1\}$. Since there are p^2 polynomials, it follows that the cardinality of $\mathcal{F}_\Gamma$ is p^2. The above construction procedure may be summarized as follows:

Proposition 4: If $\mathbb{F}_p$ is a finite field of order p and $\Gamma(x)$ is the set of all polynomials of degree at most one over $\mathbb{F}_p$, then, the set $\mathcal{F}_\Gamma$ constructed above is a 2-sparse k-uniform family $\mathcal{F}_s(2, k, kp)$ on the set X with cardinality p^2.

From Propositions 2 and 4, the incidence matrix of $\mathcal{F}_\Gamma$ will have a $(2, k)$-structure and size $kp \times p^2$. Further, from Proposition 2, it can be observed that any binary sensing matrix Φ of size $m \times M$ with $(r-1, k)$-structure will fall in the set of all r-sparse k-uniform families on a set X of cardinality m. For example:

1) The binary construction in [14] has an (r, p)-structure with size $p^2 \times p^{r+1}$. This construction falls in the $(r+1)$-sparse p-uniform family of the set X with cardinality p^2, for every pair (r, p) with $r < p$ and p being a prime power. That is, it is an $(r+1)$-sparse set on a set $X = \{1, 2, \ldots, p^2\}$.
2) The construction in [12] has a $(1, k)$-structure with size $nk \times n^2$. This construction falls in the 2-sparse k-uniform family of the set X with cardinality nk, where $n = p_1^{r_1} p_2^{r_2} \ldots p_l^{r_l}$ and $k = \min\{p_1^{r_1}, p_2^{r_2}, \ldots, p_l^{r_l}\} - 1$ with p_i a prime for $1 \le i \le l$ and r_i a positive integer. That is, it is a 2-sparse set on a set $X = \{1, 2, \ldots, nk\}$.
3) The construction in [17] falls in the r-sparse families of $\mathcal{F}_s(2, q, q^3)$, $\mathcal{F}_s(2, q+1, q^3+1)$, and $\mathcal{F}_s(3, q+1, q^2+1)$.
4) The Steiner system $\mathcal{F}_S(r, k, m)$ falls in the r-sparse family of $\mathcal{F}_s(r, k, m)$.

Remark 1: The r-sparse family is the superclass of all binary constructions that have an $(r-1, k)$-structure.

C. Upper Bound on the Column Size

So far, we have discussed how one can construct binary sensing matrices with small coherence using r-sparse sets. We also proved that there is a one-to-one correspondence between the set of all r-sparse k-uniform families and binary sensing matrices. Now, using results from extremal set theory on r-sparse sets, we bound the maximum possible column size of these matrices. Katona and Nemetz [21] proved the following result:

Proposition 5: If $\mathcal{F}_s \subseteq [X]^k$ and $\mathcal{F}_s$ is an r-sparse family, then

$$|\mathcal{F}_s| \le \frac{\binom{m}{r}}{\binom{k}{r}}. \tag{7}$$

Therefore, the column size of a binary sensing matrix that possesses $(r-1, k)$-structure is at most $\frac{\binom{m}{r}}{\binom{k}{r}}$, where m is the row size, k is the number of ones each column contains, and $r-1$ is the inner product between any two columns.

Let $n(m, k, r)$ denote the maximum possible cardinality of $\mathcal{F}_s$ in Definition 3. The following theorem from [22] establishes the existence of binary sensing matrices with optimal column size.

Theorem 2: $\lim_{m\to\infty} n(m, k, r)\frac{\binom{k}{r}}{\binom{m}{r}} = 1$, for every pair (r, k) with $r < k$.

The proof of the above theorem is based on constructing an r-sparse family $\mathcal{F}_s(r, k, m)$ with size M using probabilistic methods, such that $M\left(\frac{\binom{k}{r}}{\binom{m}{r}}\right) \in (1-\epsilon, 1+\epsilon)$ for every fixed r and k with $r < k$ and m is sufficiently large, with ϵ being a small positive number.

Remark 2: For sufficiently large m, by using the Rodl construction [22], one can generate (r, k)-structure binary sensing matrices with asymptotically optimal column size for any r and k with $r < k$.

A drawback of the Rodl construction is that the row size m does not have an explicit value. However, the recent breakthrough construction on Steiner systems by Keevash [20] gives the explicit value of m for every r and k. He proved the following theorem.

Theorem 3: For fixed r and k there exists an $m_0(r, k)$ such that if $m > m_0(r, k)$ satisfies the divisibility condition (5), then a Stenier system $\mathcal{F}_S(r, k, m)$ exists.

Therefore, using the construction in [20] and Proposition 2, we can deduce the following theorem.

Theorem 4: For every pair of integers (r, k) with $r < k$ there exists a binary sensing matrix Φ of size $m \times M$ having (r, k)-structure with optimal column size.

In the above theorem, row size m is a positive integer greater than $m_0(r, k)$ that satisfies the divisibility condition (5) and the column size is $M = \frac{\binom{m}{r}}{\binom{k}{r}}$.

Thus, we have established the existence of matrices possessing optimal column size, for every pair of integers (r, k) with $r < k$. In the next section, we illustrate the performance of the proposed binary sensing matrix constructions via simulations.

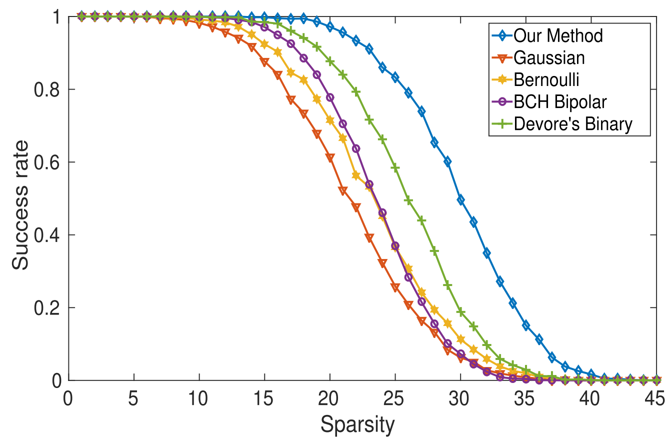

Fig. 1. Comparison of the reconstruction performances of the matrices constructed from r-sparse sets, Gaussian and Bernoulli random matrices, bipolar construction from BCH codes [13], and DeVore's binary construction [14].

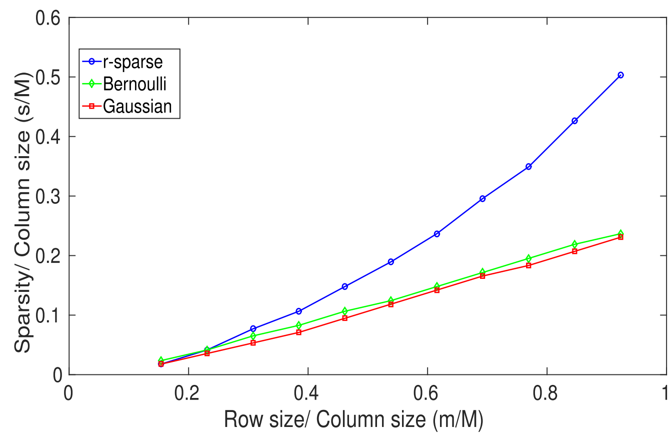

Fig. 2. Comparison of the matrices constructed from r-sparse sets with Gaussian and Bernoulli random matrices through phase transition.

III. Simulation Results

In this section, we compare the sparse vector recovery performance of matrices formed by r-sparse sets, the standard Gaussian (with entries drawn from $\mathcal{N}(0, \frac{1}{m})$), Bernoulli (with entries $\phi_{ij} = \pm\frac{1}{\sqrt{m}}$, each with probability $\frac{1}{2}$) random matrices, deterministic bipolar matrices constructed using BCH codes [13], and deterministic binary matrices [14], via numerical simulations. The binary matrix of size 66×121 is constructed as the incidence matrix of 2-sparse set $\mathcal{F}_s(2, 6, 66)$ using the construction in Section II-B with $p = 11$ and $k = 6$. To compare with DeVore's binary construction [14] and bipolar construction using BCH codes [13], we generate matrices of size of 64×512 and 63×512, respectively, and consider the first 121 columns. It is easy to see that the coherence of the matrix cannot increase by removing columns.

For each sparsity level s, 2000 s-sparse vectors x with nonzero indices chosen uniformly at random and the entries drawn from $\mathcal{N}(0, 1)$ are considered. The measurements $y = \Phi x$ are constructed using the sensing matrices being compared. The vector y and matrix Φ are input to the orthogonal matching pursuit (OMP) algorithm [23], and its output is denoted by $\tilde{x}$. For purposes of comparison, the signal-to-noise ratio (SNR) of x is defined as

$$\mathrm{SNR}(x) = 10\log_{10}\left(\frac{\|x\|_2}{\|x - \tilde{x}\|_2}\right) \text{ dB}.$$

The recovery is considered successful if $\mathrm{SNR}(x) \geq 100$ dB. The success probability is plotted against the sparsity level in Fig. 1. From the plot, we observe that the binary sensing matrices constructed from r-sparse sets outperform the other constructions considered.

Figure 2 shows the phase transition curves for r-sparse, Gaussian and Bernoulli random matrices for different values of $\delta = \frac{m}{M}$ that were given by the matrices of size $m \times M$, where $m = 26, 39, 52, 65, 78, 91, 104, 117, 130$ and $M = 169$. The phase transition diagrams depict the largest k (with fixed m and M) for faithfully recovering k-sparse vectors via l_1-minimization. In Fig. 2, successful reconstruction is not possible in the region above the curve while the recovery is successful in the region below the curve. As mentioned earlier, recovery is considered to be successful if the SNR is greater than 100 dB. Given a set of points $\delta = \frac{m}{M}$, we generate the phase transition curve by finding the largest sparsity k such that the probability of successful recovery is at least 90%. From the plot, we observe that the matrices constructed from the r-sparse sets outperform Gaussian and Bernoulli random matrices.

Next, we demonstrate the relative performances of the Gaussian and Bernoulli random measurement matrices and the matrix obtained using r-sparse sets in the context of image reconstruction from lower dimensional patches. The image is divided into smaller patches $\{I_l | \, l = 1, 2, ..., N\}$ of equal sizes. For each patch, compressed measurements I_l'' are obtained by multiplying vectorized versions of I_l with the product of the sensing matrix and the wavelet transform matrix.

For image recovery, I_l'' and the corresponding binary, Bernoulli, or Gaussian matrix is input to the OMP algorithm, and the resulting higher dimensional sparse vector I_l' is inverse wavelet transformed to reconstruct the image.

We reconstruct the images from lower dimensional patches with respect to the matrices constructed from the r-sparse set $\mathcal{F}_s(2, 8, 32)$, Bernoulli and Gaussian matrices of size 32×64. The associated reconstruction accuracy in terms of SNR are 15.91, 14.55, and 13.28 dB, respectively. Thus, the matrices constructed from r-sparse sets outperform the Gaussian matrix by about 2.63 dB and the Bernoulli matrix by about 1.36 dB.

IV. Concluding Remarks

In this letter, we showed the utility of extremal set theory for constructing binary sensing matrices with optimal column size. We also presented bounds on the column size as a function of the row size of the matrix, number of ones each column contains and the cardinality of overlap between any two columns. In addition, we showed that there exists a binary sensing matrix with optimal column size for every pair (r, k) with $r < k$. The resulting binary sensing matrix was shown to outperform random Bernoulli and Gaussian sensing matrices of the same size via numerical experiments. These binary constructions are useful because of the computational advantage they offer during the measurement and the reconstruction.

REFERENCES

[1] J. Bowley and L. Rebollo-Neira, "Sparsity and something else: An approach to encrypted image folding," *IEEE Signal Process. Lett.*, vol. 18, no. 3, pp. 189–192, Mar. 2011.

[2] R. Jagannath and K. V. S. Hari, "Block sparse estimator for grid matching in single snapshot DoA estimation," *IEEE Signal Process. Lett.*, vol. 20, no. 11, pp. 1038–1041, Nov. 2013.

[3] R. Prasad, C. R. Murthy, and B. D. Rao, "Joint channel estimation and data detection in MIMO-OFDM systems: A sparse bayesian learning approach," *IEEE J. Sel. Topics Signal Process.*, vol. 63, no. 20, pp. 5369–5382, Oct. 2015.

[4] Y. Rivenson and A. Stern, "Compressed imaging with a separable sensing operator," *IEEE Signal Process. Lett.*, vol. 16, no. 6, pp. 449–452, Jun. 2009.

[5] M. Srinivas, R. Ramu Naidu, C. S. Sastry, and C. Krishna Mohan, "Content based medical image retrieval using dictionary learning," *Neurocomputing*, vol. 168, pp. 880–895, 2015.

[6] A. M. Bruckstein, D. L. Donoho, and M. Elad, "From sparse solutions of systems of equations to sparse modeling of signals and images," *SIAM Rev.*, vol. 51, no. 1, pp. 34–81, 2009.

[7] E. J. Candès and T. Tao, "Decoding by linear programming," *IEEE Trans. Inf. Theory*, vol. 51, no. 12, pp. 4203–4215, Dec. 2005.

[8] E. J. Candès, J. K. Romberg, and T. Tao, "Stable signal recovery from incomplete and inaccurate measurements," *Commun. Pure Appl. Math.*, vol. 59, no. 8, pp. 1207–1223, Aug. 2006.

[9] E. J. Candès, "The restricted isometry property and its implications for compressed sensing," *Comptes Rendus Mathematique*, vol. 346, no. 9, pp. 589–592, 2008.

[10] E. J. Candès, J. Romberg, and T. Tao, "Robust uncertainty principles: Exact signal reconstruction from highly incomplete frequency information," *IEEE Trans. Inf. Theory*, vol. 52, no. 2, pp. 489–509, Feb. 2006.

[11] J. Bourgain, S. Dilworth, K. Ford, S. Konyagin, and D. Kutzarova, "Explicit constructions of RIP matrices and related problems," *Duke Math. J.*, vol. 159, no. 1, pp. 145–185, Jul. 2011.

[12] R. R. Naidu, P. Jampana, and C. S. Sastry, "Deterministic compressed sensing matrices: Construction via Euler squares and applications," *IEEE J. Sel. Topics Signal Process.*, vol. 64, no. 14, pp. 3566–3575, Jul. 2016.

[13] A. Amini and F. Marvasti, "Deterministic construction of binary, bipolar, and ternary compressed sensing matrices," *IEEE Trans. Inf. Theory*, vol. 57, no. 4, pp. 2360–2370, Apr. 2011.

[14] R. A. DeVore, "Deterministic constructions of compressed sensing matrices," *J. Complexity*, vol. 23, no. 4, pp. 918–925, 2007.

[15] P. Indyk, "Explicit constructions for compressed sensing of sparse signals," in *Proc. 19th Annu. ACM-SIAM Symp. Discrete Algorithms*, 2008, pp. 30–33.

[16] S. Li, F. Gao, G. Ge, and S. Zhang, "Deterministic construction of compressed sensing matrices via algebraic curves," *IEEE Trans. Inf. Theory*, vol. 58, no. 8, pp. 5035–5041, Aug. 2012.

[17] S. Li and G. Ge, "Deterministic construction of sparse sensing matrices via finite geometry," *IEEE J. Sel. Topics Signal Process.*, vol. 62, no. 11, pp. 2850–2859, Jun. 2014.

[18] P. Sasmal, R. R. Naidu, C. S. Sastry, and P. Jampana, "Composition of binary compressed sensing matrices," *IEEE Signal Process. Lett.*, vol. 23, no. 8, pp. 1096–1100, Aug. 2016.

[19] J. Zhang, G. Han, and Y. Fang, "Deterministic construction of compressed sensing matrices from protograph LDPC codes," *IEEE Signal Process. Lett.*, vol. 22, no. 11, pp. 1960–1964, Nov. 2015.

[20] P. Keevash, "The existence of designs," arXiv e-prints, Jan. 2014.

[21] G. Katona and T. Nemetz, "On a graph-problem of Turan," *Mat. Lapok*, vol. 15, pp. 228–238, 1964.

[22] V. Rodl, "On a packing and covering problem," *Eur. J. Combinatorics*, vol. 6, no. 1, pp. 69–78, 1985.

[23] J. A. Tropp and A. C. Gilbert, "Signal recovery from random measurements via orthogonal matching pursuit," *IEEE Trans. Inf. Theory*, vol. 53, no. 12, pp. 4655–4666, Dec. 2007.