# Asymptotically Optimal Uncoordinated Power Control Policies for Energy Harvesting Multiple Access Channels with Decoding Costs

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***Abstract*—The objective of this paper is to design a power control policy that maximizes the long-term time-averaged sum throughput of a Gaussian multiple access channel (MAC), where the transmitters as well as the access point (AP) are energy harvesting nodes (EHNs). In addition, the policy is required to facilitate *uncoordinated* operation of the network. That is, in each slot, the transmitting nodes and the AP need to independently take their actions, e.g., the amount of energy to be used for transmission or whether to turn on and receive the data. First, in order to benchmark the performance of any policy, we derive an upper bound on the throughput achievable, by analyzing a centralized genie-aided system where the nodes have infinite capacity batteries and can freely share the available energy among themselves. In addition, the genie-aided system has non-causal knowledge of the energy arrivals at all the nodes. Next, we show that, surprisingly, a simple *time sharing based online policy* which requires no coordination among the transmitters and uses time-dilation at the receiver achieves the upper bound asymptotically in the battery size. We also present a policy that requires an occasional one-bit feedback from the AP about its battery state, and show that it requires a smaller sized battery at the receiver compared to a policy which operates without any feedback from the AP, to achieve the same performance. We use Monte Carlo simulations to validate our theoretical results and illustrate the performance of the proposed policies.**

***Index Terms*—Energy harvesting, uncoordinated multiple-access, power control, decoding cost, receiver.**

## I. Introduction

The next generation wireless networks are envisioned to connect massive number of low power nodes [2], and in particular, nodes that operate using energy harvested from the environment [3]. In many applications, nodes are typically required to send their data to a gateway or an access point (AP), e.g., in low-power wide area networks (LPWAN) [4] or the internet-of-things (IoT). In the absence of coordination among the nodes, multiple nodes may attempt to simultaneously transmit their data to the energy harvesting (EH) AP, which, in turn, may lead to wastage of the energy as well as loss of data due to collisions. In addition, a transmission will fail if the AP does not have sufficient energy to receive the packet. On the other hand, the energy cost of signaling and control to avoid collisions as well as for coordination with the AP can be significant when the number of devices is large and their individual data rate requirements are small. Hence, a key goal in the design of these networks is to achieve optimal performance through uncoordinated operation of the nodes, where a node does not have access to the state and actions of the other nodes as well as the AP, and takes its actions using only locally available information.

In this work, we consider an IoT application where the nodes transmit their delay-insensitive saturated traffic data to an AP over a shared Gaussian multiple access channel (MAC). All the nodes, including the AP, rely solely on energy harvested from the environment for communication. Our goal is to design an online power control policy that maximizes the average sum throughput of the uncoordinated EH MAC with non-negligible decoding cost. For EH communications, the average sum throughput is also a good measure of the energy efficiency of a power control policy [4]–[11]. The optimal policy consists of two parts: (i) a rule to determine when to access the channel; and (ii) a power control policy executed by an individual node upon channel access. The policy needs to be implementable using minimal knowledge at the nodes, e.g., using only the average harvesting rates and the local battery state information. To access the channel, a deterministic time sharing scheme where nodes determine their transmission slots based on the global slot index can avoid collisions among the nodes in an uncoordinated manner. Also, for networks with saturated traffic, time sharing based access schemes are known to outperform their alternatives such as random access and hybrid MAC protocols [12]–[15]. However, the structure of the optimal time-sharing scheme, and the power control policy for an individual node are not obvious. Furthermore, the randomness of the EH process and finite size battery at each node makes the problem challenging.

Note that, the energy availability at the AP for receiving the next packet can be made known to the nodes by simply delaying its one bit acknowledgment (ACK)/negative ACK signal after each transmission attempt by the nodes [16]. However, sending a feedback signal for each transmission attempt is an additional overhead for the energy-starved nodes, and is in fact not used in several present day low power communication standards [2], e.g., in the unconfirmed data

The work in this paper will appear in part in IEEE GLOBECOM 2018 [1].
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This work was financially supported by a research grant from the Aerospace Network Research Consortium and the Indo-French Centre for the Promotion of Advanced Research. The work of C. R. Murthy was also supported by the Young Faculty Research Fellowship from the Ministry of Electronics and Information Technology, Govt. of India.

mode in LoRaWANs [17].

In the next subsection, we discuss the related work and summarize our main contributions.

### A. Related Work and Main Contributions

The capacity region of the EH MAC where the AP is connected to the mains is characterized in [18]. In [19]–[21], the authors design *offline* policies to maximize the *departure region* of an EH Gaussian MAC with [19] and without cooperation [20], [21] among the users, respectively. Offline policies to maximize the sum-rate of a fading MAC were developed in [11]. These policies assume noncausal knowledge of the state of the channel and the EH process, and are designed in a centralized fashion. The issue of uncoordinated multi-access does not arise for offline policies. However, energy harvesting levels and the channel states are rarely predictable far into the future. Hence, offline policies are generally not suitable for practical implementation.

The design of *online* power control policies to maximize the sum throughput of an uncoordinated EH MAC where the AP is connected to the mains is considered in [4]–[10]. The authors in [4] presented randomized suboptimal policies which require partial coordination, achieved with the help of a centralized controller. In [5], the authors proposed a convergent, iterative, and numerical method to obtain the optimal power control policies for individual nodes. However, it may require a large number of iterations to obtain optimal policies, as the convergence rate of the algorithm is not known. In [6]–[8], the authors designed suboptimal fixed-fraction policies which achieve the throughput within a constant gap from the optimality. The work in [5]–[7] does not explicitly deal with the issue of how to achieve the coordination required to implement their proposed policies. In [10], the performance of equal power as well as equal time based transmission schemes is analyzed for time division multiple access (TDMA) based uncoordinated MAC, and the former is shown to outperform the latter when the nodes are equipped with infinite sized batteries. Thus, *optimal* online policies for an EH MAC when the AP is connected to the mains are not yet available in the literature. Moreover, all these studies neglect the energy cost of receiving data, since they assume that the AP is connected to the mains.

The EH MAC with non-negligible decoding cost has been considered in [22] and [23] with an EH AP. The authors designed optimal offline power control policies to maximize the departure region when non-causal information about the energy arrivals is available to a central controller.

In our previous work [24], we presented an online policy for uncoordinated transmission over a point-to-point link between an EH transmitter and EH receiver. Although we *empirically* showed that the policy can achieve optimal throughput in a point-to-point link, theoretical guarantees for the optimality of the policies are still missing in the literature. Implementation of this policy requires an occasional one bit feedback from the receiver about its battery state. Thus, the design of an optimal policy which operates in a fully uncoordinated fashion is not known, even for point-to-point links. Furthermore, the design of online power control policies for the uncoordinated EH MAC with nonzero decoding cost at the EH AP has not been studied in the literature. Our main contributions are as follows:

- We derive an upper bound on the maximum achievable throughput by analyzing a genie-aided centralized system where the nodes can freely share their available energy, are equipped with infinite capacity batteries, and have non-causal knowledge of the energy arrivals at all nodes. (See Sec.III.)
- We analytically show that a *time-division* based schedule where, the transmit duration of each node is in proportion to its harvesting rate, along with an online power control policy can achieve the upper-bound asymptotically in the size of the batteries at the nodes. Also, the policy is computable in closed form, and it does not require any coordination among the nodes, except for an occasional one bit feedback from the AP. (See Sec. IV and V.)
- To dispense with even the occasional one bit feedback, we adapt the above policy to obtain a *fully uncoordinated policy* by restricting the receiver to operate under an average power constraint. The adapted policy also asymptotically achieves the upper bound. However, it requires a strictly larger battery size at the AP compared to the occasional one-bit feedback based policy, to get the same performance. (See Sec. IV and V.)
- Our simulation results confirm the theoretical findings and illustrate the trade-offs between system parameters. For instance, our results quantify the impact of decoding cost on the average sum throughput and show how the energy required for decoding the data received in a slot can be traded-off for the harvesting rate at the AP. (See Sec. VI.)

To the best of our knowledge, for the first time in the literature, we present a closed-form computable, asymptotically optimal policy for the uncoordinated EH MAC with decoding cost. We note that, by setting the decoding cost to zero, the proposed policy reduces to the optimal policy for an EH MAC where the AP is connected to the mains. Moreover, the time-dilation based policies proposed in the paper generalize the time-dilation based policies in [24] to the EH MAC. The policies presented here are easy to implement as they require only the knowledge of the average EH rates of the nodes and the local battery state at a transmitting node. This, in turn, allows the nodes to operate in an uncoordinated fashion, obviating the need for any feedback. Our results remain valid for a network with arbitrary number of nodes and under any general spatio-temporal correlation among the EH processes across the nodes as well as AP. Hence, the policies scale well with the number of nodes and are suitable for large networks.

## II. System Model and Problem Formulation

We consider an EH network where $K$ EH transmitters wish to send their data to an EH AP. Data is transmitted over a time-slotted additive white Gaussian noise (AWGN) multiple access channel. The transmission is successful only if the AP has sufficient energy to receive data. Also, simultaneous transmission by more than one transmitter in the same slot could lead to the data decoding failure, and hence we consider the design of policies that avoid data collisions. Specifically, by assuming that packet collisions lead to loss of all the colliding packets, we obtain a policy whose rate is achievable using single-user detection schemes at the AP. This is reasonable for an EH AP, as the AP may not have sufficient energy and/or computational power to support sophisticated multi-user detection techniques. Further, without loss of generality, each slot is assumed to be of unit length.[1] Also, all the nodes in the network (including the AP) are slot-level synchronized, i.e., they keep track of a global slot index, and each transmission lasts for the entire slot duration [7], [9], [10], [13]. This is consistent with the emerging standards for IoT applications such as DetNet and 6TiSCH [25].

In a slot, the nodes harvest energy according to a general stationary and ergodic discrete-valued harvesting process with joint probability mass function denoted by $f_{\mathcal{E}_1,\mathcal{E}_2,\ldots,\mathcal{E}_K,\mathcal{E}_r}(e_1, e_2, \ldots, e_K, e_r)$, where random variables $\mathcal{E}_k$ and $\mathcal{E}_r$ denote the amount of energy harvested by the $k^{\text{th}}$ transmitter and the AP, respectively, and $e_k$ and $e_r$ denote a realization of $\mathcal{E}_k$ and $\mathcal{E}_r$, respectively. The harvesting rate at the $k^{\text{th}}$ transmitter and AP are denoted by $\mu_k \triangleq E[\mathcal{E}_k]$ and $\mu_r \triangleq E[\mathcal{E}_r]$, respectively. At each node, the harvested energy is stored in a perfectly efficient, finite capacity battery. Moreover, at any node, *only* the harvesting *rates* of the other nodes, $\mu_k$'s, are known; other information such as the instantaneous battery state and transmit power are not available.

The energy used by the AP in the $n^{\text{th}}$ slot is denoted by $p_n^r \in \{0, R\}$. It is binary valued, i.e., if it decides to turn *on* in a slot it consumes $R$ units of energy; and it does not incur any energy cost in the sleep mode[2] [29]–[32] . On the other hand, the power control policy at each transmitter is continuous valued. Let $p_n^k$ denote the amount of energy used by the $k^{\text{th}}$ transmitter in the $n^{\text{th}}$ slot and $\mathcal{P}_n \triangleq \{p_n^k\}_{k=1}^K$ denote the set of power levels used, in the $n^{\text{th}}$ slot, by all the transmitters. The battery at the $k^{\text{th}}$ transmitter evolves as $B_{n+1}^k = \min\{[B_n^k + e_n^k - p_n^k]^+, B_{\max}^k\}$, where $B_{\max}^k$, $B_n^k$ and $e_n^k$ denote the battery capacity, battery level, and the energy harvested at the start of the $n^{\text{th}}$ slot, by the $k^{\text{th}}$ transmitter, respectively, for $1 \le k \le K$, and $[x]^+ \triangleq \max\{0, x\}$. The battery evolution at the AP can be described in a similar manner, with $k$ replaced by $r$.

In addition to the battery, each transmitter is equipped with a finite sized supercapacitor for temporarily storing the energy to be used for transmission [33]–[35]. For example, in a slot when a node is not transmitting, its supercapacitor can be used to withdraw energy from the battery. Here, we use the supercapacitor primarily for the sake of analytical tractability, as it makes the bookkeeping of the energy arrivals and utilization simpler (see footnote 4 for further details). The

[1]Hence, we use the terms energy and power interchangeably.

[2]In practice, the energy consumed for receiving and decoding the data remains roughly independent of the incoming rate and the SNR. For instance, at 315 MHz, the receiver current consumption in CC1101 low power sub 1-GHz RF transceiver [26] increases by only 6% when the input Baud rate varies from 1.2 kBaud to 250 kBaud. A similar trend is also observed for the LTE receivers [27]. This happens because the energy consumption in the RF part of the receiver dominates the energy consumption in the digital part [28], and is nearly independent of the input SNR and the data rate.

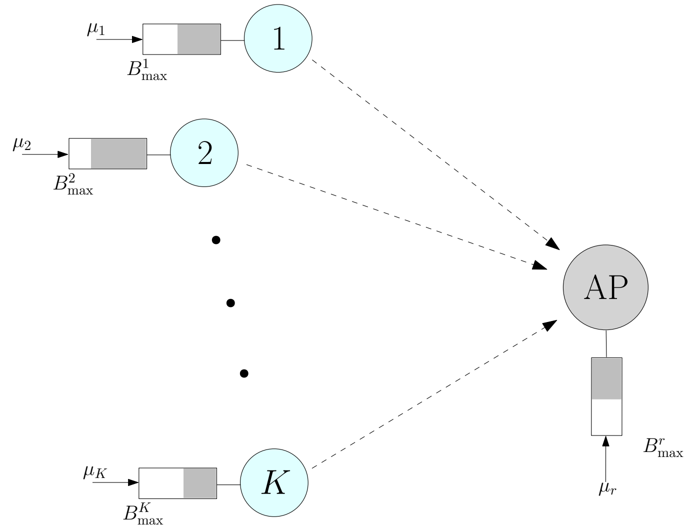



Fig. 1: System model for the uncoordinated multiple access network with an energy harvesting AP. The average harvesting rate at the $k^{\text{th}}$ transmitter and AP are denoted by $\mu_k$ and $\mu_r$, respectively, and the size of the battery at the $k^{\text{th}}$ transmitter and AP are $B_{\max}^k$ and $B_{\max}^r$, respectively. The data is transmitted over a shared Gaussian multiple access channel where the multiple simultaneous transmissions lead to data decoding failure. Note that, the supercapacitor is not shown in the picture.

precise role of supercapacitor in the implementation of the policy and performance analysis will be elaborated upon in the sequel.

Our goal is to find a power control policy, $\{\mathcal{P}_n\}_{n=1}^N$, that maximizes the time-averaged sum throughput. We assume that the rate achieved when a single node (the $k$th node) transmits data at power $p_n^k$ (in the $n$th slot) is well approximated by the capacity expression $\mathcal{R}(p_n^k) \triangleq \frac{1}{2}\log(1 + p_n^k)$ [36], [37]. Without loss of generality, we set the power spectral density of the AWGN at the receiver as unity. Due to the uncoordinated operation, $p_n^k$ is completely determined by its own battery state, $B_n^k$, and the energy harvested by it in the current slot, $e_n^k$, and the slot index $n$. Under this model, the time-averaged sum throughput can be written as

$$\mathcal{T} = \frac{1}{N}\sum_{n=1}^{N} \mathbb{1}_{\{p_n^r \neq 0\}} \left(\sum_{k=1}^{K}\left\{\prod_{\ell=1,\ell\neq k}^{K}\left(1 - \mathbb{1}_{\{p_n^\ell \neq 0\}}\right)\right\}\log(1 + p_n^k)\right), \tag{1}$$

where $\mathbb{1}_{\{p_n^j \neq 0\}}$ is an indicator variable which takes the value one if the energy used by the $j^{\text{th}}$ node in the $n^{\text{th}}$ slot is nonzero, and takes the value zero otherwise, where $j \in \{1, \ldots, K, r\}$. Thus, $\prod_{\ell=1,\ell\neq k}^{K}\left(1 - \mathbb{1}_{\{p_n^\ell = 0\}}\right)$ equals one if and only if all the transmitters, excluding the $k^{\text{th}}$ transmitter, are *off* in the $n^{\text{th}}$ slot, otherwise it equals zero. This captures the fact that multiple simultaneous transmissions result in a packet failure. Similarly, $\mathbb{1}_{\{p_n^r \neq 0\}}$ accounts for the fact that data is received successfully only if the AP is *on* in that slot. The problem of obtaining a decentralized policy that maximizes the time-averaged sum throughput can then be formulated as

$$\max_{\{\mathcal{P}_n, p_n^r, \text{ for } n=1,2,\ldots\}} \liminf_{N\to\infty} \mathcal{T}, \tag{2a}$$

$$\text{s.t.} \quad 0 \le p_n^k \le B_n^k,\ 1 \le k \le K, \tag{2b}$$

$$\text{and} \quad 0 \leq p_n^r \leq B_n^r,\ p_n^r \in \{0, R\}, \text{ for all } n. \tag{2c}$$

In the above, the constraints (2b) and (2c) ensure that, in a slot, the maximum energy used by a node cannot exceed the energy available in its battery, at both the transmitter and receiver. Also, note that the power control policy at the AP is binary valued.

In order to maximize the sum-throughput in (2), an optimal policy needs to ensure not only that a single node is active in a slot, but also that the AP is *on* in that slot. In addition, transmit powers must be chosen optimally at the transmitters using the local information available. The uncoordinated nature of the problem makes it challenging to achieve the optimal throughput. Note that, if the energy arrivals are deterministic and the nodes are slot-level synchronized, the problem is easy to solve. However, when the energy arrivals are random and only causal information about the EH process is available, the structure of the optimal policy is non-trivial. In the next section, we derive an upper bound on the achievable throughput, which serves as a benchmark for the performance of any policy.

## III. Upper Bound on Time-averaged Throughput

Our upper bound on the sum throughput, presented in Lemma 1 below, considers a genie-aided system where the nodes are equipped with infinite sized batteries, can freely share the available energy between them, and have noncausal information about the energy arrivals. We will later present an uncoordinated policy that, surprisingly, very nearly achieves the upper bound, even with finite capacity batteries.

**Lemma 1.** *The long-term time averaged sum throughput of an uncoordinated multi-access channel with energy harvesting nodes satisfies*

*a)* $\liminf_{N\to\infty} \mathcal{T} \leq \frac{1}{2}\log\left(1 + \sum_{k=1}^{K} \mu_k\right)$, *if* $\frac{R}{\mu_r} \leq 1$,

*b)* $\liminf_{N\to\infty} \mathcal{T} \leq \frac{\mu_r}{2R}\log\left(1 + \frac{R}{\mu_r}\sum_{k=1}^{K} \mu_k\right)$, *if* $\frac{R}{\mu_r} > 1$.

*In the above,* $\mu_k$ *denotes the harvesting rate of the* $k^{th}$ *node,* $\mu_r$ *denotes the harvesting rate of the AP, and* $R$ *denotes the energy required per slot for decoding data at the AP.*

*Proof.* See Appendix A. ■

Lemma 1 generalizes the upper bound derived in [24] for point-to-point links ($K = 1$) where both the transmitter and receiver are EHNs. Interestingly, the expression for the upper bound in Case *(a)* is the same as for the sum-capacity of the EH MAC where the AP is connected to the mains [6]. Note that, since the sum-throughput of a MAC depends on the signal-to-noise ratio (SNR) of the cumulative signal received [11], the upper bound presented in the above Lemma is valid for any EH based MAC, regardless of the detection method used at the receiver, and applies to non-orthogonal schemes as well.

In the above Lemma, scenario *(a)* corresponds to the setting where the average harvesting rate at the AP exceeds $R$, the energy consumed by it per slot when it is *on*. Thus, the battery state at the AP has a positive drift even if it remains *on* in all slots, i.e., the AP is energy *unconstrained*. This is equivalent to the case where only the transmitters are EHNs, provided the AP is equipped with a sufficiently large battery. Case *(b)* corresponds to a scenario when the AP is energy-constrained, i.e., the average energy harvested in a slot is less than the energy consumed in one slot. Consequently, the AP can only turn *on* intermittently. To avoid wastage of energy, the transmitters must not send data when the AP is *off*. However, this requires the transmitters to know the battery state at the AP. In the next section, as a first step towards developing optimal policies for the general case, corresponding to the scenario with multiple transmitters, we first solve the problem when $K = 1$, i.e., for point-to-point links with an EH transmitter and receiver.

## IV. Optimal Policies for the $K = 1$ Case

In this section, we first consider Case *(a)* of Lemma 1 and present a policy that achieves the upper bound asymptotically in the battery size at the nodes, without any coordination between the transmitter and receiver. In the sequel, asymptotically optimal implies the optimality as the battery size at the transmitter and the receiver go to infinity. In this section, the transmitter index $k$ in the superscript/subscript is set to 1, since there is only one transmitting node.

### A. Asymptotically Optimal Policy for Energy Unconstrained Receiver, $\frac{R}{\mu_r} < 1$

It is known that, when $\mu_r > R$, the probability that the receiver does not have sufficient energy to turn *on*, $\Pr[B_n^r < R]$, decays exponentially with $B_{\max}^r$ [16]. Consequently, with high probability, the receiver can always remain *on*, making this case equivalent to the scenario where only the transmitter is EH. Hence, when $\mu_r > R$, the optimal policy for the transmitter, denoted by $\mathcal{P}_{\mathrm{u}}^1 \triangleq \{p_n^1, n = 1, 2, \ldots\}$, is the same as the one proposed in [38], which is as follows:

$$p_n^1 = \begin{cases} \mu_1 + \delta_1, & B_n^1 \geq \frac{B_{\max}^1}{2}, \\ \min\{B_n^1, \mu_1 - \delta_1\}, & B_n^1 < \frac{B_{\max}^1}{2}, \end{cases} \tag{3}$$

where $\delta_1 = \beta_1 \sigma_1^2 \frac{\log B_{\max}^1}{B_{\max}^1}$. Here, $\sigma_1^2$ denotes the asymptotic variance of the harvesting process at the transmitter, and $\beta_1 \geq 2$ is a constant. It follows from [38, Theorem 1] that the throughput achieved by this policy converges to the upper bound in Case *(a)* of Lemma 1 at the rate $\Theta\left(\left(\frac{\log B_{\max}^1}{B_{\max}^1}\right)^2\right)$. Thus, for a network with an EH transmitter and an energy unconstrained AP, policy $\mathcal{P}_{\mathrm{u}}^1$, along with the policy under which receiver is always *on*, is *asymptotically optimal*. We note that, the policy is *fully uncoordinated*, i.e., it operates using the knowledge about the local battery state, and does not require the information about the battery state or action taken by the other node.

Next, we consider the Case *(b)* of Lemma 1. Intuitively, in this case, emulating a policy similar to $\mathcal{P}_{\mathrm{u}}^1$, given in (3), at both the nodes can facilitate to achieve a throughput close to upper bound for this case. In particular, the policy $\mathcal{P}_{\mathrm{u}}^1$ can be directly used at the transmitter. On the other hand, the receiver can implement a policy similar to $\mathcal{P}_{\mathrm{u}}^1$ by *deterministically* turning *on* in one out of every $\lfloor N_r \rfloor$ [3] or $\lceil N_r \rceil$ slots, where $N_r \triangleq \frac{R}{\mu_r}$, depending on whether the battery at the receiver is above or below the half-full mark, $\frac{B^r_{\max}}{2}$, respectively. To facilitate coordination, the receiver sends one bit feedback to the transmitter whenever its battery level crosses the halfway mark. This allows the transmitter to determine the index of the next slot in which the receiver will be *on*. However, at the receiver, the drifts in lower and upper halves, $\lceil N_r \rceil - N_r$ and $N_r - \lfloor N_r \rfloor$, respectively, does not decay with the size of its battery. As a consequence, the throughput achieved by this policy can have a one bit gap from the upper bound, as shown in our previous work [24].

Before presenting our fully uncoordinated policy, in the following, we consider a policy similar to $\mathcal{P}^1_u$ in (3), which uses time-dilation (described in the next section) at the receiver to attain a finer control over the drifts and requires one bit feedback about the battery state at the AP. Further, we characterize the performance of this time-dilation based online policy and show that it achieves the upper bound asymptotically with the battery size.

### *B. Asymptotically Optimal Policy for Energy Constrained Receiver, $\frac{R}{\mu_r} > 1$*

The key idea behind time-dilation is to spread the drifts in the lower and upper half of the receiver's battery, $\lceil N_r \rceil - N_r$ and $N_r - \lfloor N_r \rfloor$, respectively, over a large number of slots. This, in turn, results in a smaller per-slot drift. That is, instead of operating in batches of $\lfloor N_r \rfloor$ or $\lceil N_r \rceil$ slots, the time-dilation based policy operates in batches of $N_f \triangleq \lfloor \frac{Rf(B^r_{\max})}{\mu_r} \rfloor$ and $N_c \triangleq \lceil \frac{Rf(B^r_{\max})}{\mu_r} \rceil$ slots, where $f(\cdot) : \mathbb{R}^+ \rightarrow \mathbb{Z}^+$ is a positive integer valued non-decreasing function. A receiver operating with time-dilation turns *on* deterministically in the first $f(B^r_{\max})$ slots out of $N_f$ or $N_c$ consecutive slots, depending on whether its battery is above or below the half-full mark, respectively. In addition, it sends a one bit feedback whenever the battery level crosses the half-full mark. The one bit feedback enables the transmitter to determine the indices of the slots in which the receiver will be *on*. A numerical example to illustrate time dilation is provided in Table I for the $K = 2$ case in Sec. V.

In the slots when the receiver is *off*, the transmitter accumulates the energy prescribed by its own policy, $\mathcal{P}^1_{\mathrm{u}}$, in a supercapacitor and uses the accumulated energy for transmission in the next $f(B^r_{\max})$ slots when the receiver is *on*. The consequence of using a supercapacitor to temporarily store energy is that the energy withdrawn from the battery at the transmitter depends only on its own battery state; specifically, it is independent of the policy at the receiver. The independence between the transmitter and receiver's battery evolution simplifies the performance analysis presented in the ensuing discussion. In the following, we describe the details of the policy for both the transmitter and the receiver.

To mathematically describe the policy at the receiver, we use $N_{\text{off}}$ to denote the index of the last slot in a batch of slots when the receiver is *off*, according to the policy. $N_{\text{off}}$ is initialized to zero at slot index $n = 0$, and is updated as follows. At any slot index $n$ satisfying $n = N_{\text{off}} + N_f \mathbb{1}_{\mathcal{R}^+} + N_c(1 - \mathbb{1}_{\mathcal{R}^+})$ it is updated to the current slot index, i.e., $N_{\text{off}}$ is set to $N_{\text{off}} = n$. Here, $\mathbb{1}_{\mathcal{R}^+}$ denotes an indicator function which takes the value one if $B^r_n \geq \frac{B^r_{\max}}{2}$ and zero otherwise. Note that, due to the one bit feedback, $N_{\text{off}}$ is known at both the transmitter and the receiver. The policy at the receiver, $\mathcal{P}^r_{\text{td}} \triangleq \{p^r_1, p^r_2, \ldots\}$, is given as

$$p^r_n = \begin{cases} R, & R \leq B^r_n, N_{\text{off}} < n \leq N_{\text{off}} + f, \\ 0, & \text{otherwise}. \end{cases} \tag{4}$$

In the ensuing discussion, to keep the notation simple, we do not explicitly write the argument of time-dilation function $f(B^r_{\max})$. We note that as per the above policy, (4), in any slot, the receiver remains *off* if $B^r_n < R$, i.e., if it does not have sufficient energy to receive a packet. This ensures operation under the constraint (2c). Next, we mathematically describe the policy at the transmitter.

Consider a slot $n$ where the transmitter is scheduled to be *off*, i.e., $n > N_{\text{off}} + f$. The energy accumulated in the supercapacitor[4] by the end of $n^{\text{th}}$ slot is given by

$$C_1(n) = \begin{cases} C_1(n-1) + \mu_1 + \delta_1, & \text{if } B^1_n \geq \frac{B^1_{\max}}{2}, \\ C_1(n-1) + \min\{\mu_1 - \delta_1, B^1_n\}, & \text{if } B^1_n < \frac{B^1_{\max}}{2}. \end{cases} \tag{5}$$

In the above, $\delta_1 = \beta_1 \sigma_1^2 \frac{\log B^1_{\max}}{B^1_{\max}}$. For $N_{\text{off}} < n \leq N_{\text{off}} + f$, i.e., when data is transmitted, the energy in the supercapacitor is updated as $C_1(n) = C_1(n-1) - \frac{C_1(N_{\text{off}})}{f}$. That is, the total energy accumulated over the duration when the transmitter was *off* is distributed equally over the next $f$ slots. Note that the size of the supercapacitor required to implement the above policy is at most $(N_c - f)(\mu_1 + \delta_1)$. In the $n^{\text{th}}$ slot, the transmitter's policy $\mathcal{P}^1_{\text{td}} \triangleq \{p^1_1, p^1_2, \ldots\}$ is given by

$$p^1_n = \begin{cases} \frac{C_1(N_{\text{off}})}{f} + \mu_1 + \delta_1, & \text{if } B^1_n \geq \frac{B^1_{\max}}{2} \\ & \text{and } N_{\text{off}} < n \leq N_{\text{off}} + f, \\ \frac{C_1(N_{\text{off}})}{f} + \min\{\mu_1 - \delta_1, B^1_n\}, & \text{if } B^1_n < \frac{B^1_{\max}}{2} \\ & \text{and } N_{\text{off}} < n \leq N_{\text{off}} + f, \\ 0, & \text{otherwise}. \end{cases} \tag{6}$$

In the discussion to follow, let $\mathcal{P}_{\text{td}}$ denote the joint power management policy proposed above, given by (4) and (6). The following Theorem asserts that policy $\mathcal{P}_{\text{td}}$ achieves the upper bound asymptotically with the size of the batteries.

**Theorem 1.** *Let $\mathcal{T}_{td}$ denote the time-averaged throughput achieved by the time-dilation based policy $\mathcal{P}_{td}$, and $\delta_{r,c} \triangleq \lceil \frac{Rf}{\mu_r} \rceil - \frac{Rf}{\mu_r}$ and $\delta_{r,f} \triangleq \frac{Rf}{\mu_r} - \lfloor \frac{Rf}{\mu_r} \rfloor$. If $f$ is a function such that*

[3]This results in a negative drift in the energy level of the battery, since $\lfloor N_r \rfloor \mu_r - R \leq 0$. Note that, in this paper, the difference between the average energy harvesting and consumption rates is termed as the battery *drift*.

[4]In the absence of a supercapacitor, the time-dilation based policy at the transmitter, given by (5) and (6), can be modified by considering a virtual queue, denoted by $C'_e(n)$. When the transmitter is off, the virtual queue $C'_e(n)$ evolves according to (5) with $B^1_n$ replaced by $B'_n \triangleq \max\{0, B^1_n - C_1(n)\}$. The power is prescribed using (6) with $C_1(n)$ and $B^1_n$ replaced by $C'_e(n)$ and $B'_n$, respectively. It can be shown that this modified policy also achieves the upper bound asymptotically in the battery size, even without a supercapacitor. Thus, the results presented here are also applicable when a supercapacitor is not available.

*$\delta_{r,c} \leq \delta_{r,f}$, then the gap between the upper bound and $\mathcal{T}_{td}$, i.e., $\left(\frac{\mu_r}{2R}\right)\log\left(1+\frac{R\mu_1}{\mu_r}\right)-\mathcal{T}_{td}$, decays to zero as $O\left(\frac{\log B^1_{\max}}{B^1_{\max}}\right)+O\left(\frac{P_{d,r}}{f}\right)+O\left(\frac{1}{f^2}\right)$. Here, $P_{d,r} \triangleq \lim_{N\to\infty}\frac{1}{N}\sum_{n=1}^{N}\mathbb{1}_{\{B^r_n=0\}}$ is the battery discharge probability at the receiver, where $\mathbb{1}_{\{B^r_n=0\}}$ equals one if the battery at the receiver is empty in slot n, and equals zero otherwise.*

*Proof.* See Appendix B. ■

Theorem 1 shows that the throughput achieved by the time-dilation based policy converges to the upper bound asymptotically with the battery size at the transmitter $B^1_{\max}$, time-dilation factor $f$, and the ratio $\frac{\mathrm{P}_{\mathrm{d},r}}{f}$. In the following remark, we comment on behavior of the ratio $\frac{\mathrm{P}_{\mathrm{d},r}}{f}$.

*Remark* 1. Lemma 2 in Appendix B shows that the battery discharge probability at the receiver decays to zero exponentially fast with $B^r_{\max}$, i.e., $\mathrm{P}_{\mathrm{d},r} = \Theta\left(\exp\left(\frac{s^r_* B^r_{\max}}{2}\right)\right)$, where $s^r_*$ is the negative root of the asymptotic log moment generating function (MGF) (defined in Lemma 2 in Appendix B) of the drift process $D^r_n \triangleq \mathcal{E}^r_n - p^r_n(B^r_n)$. Using a proof similar to [38, Lemma 3], it can be shown that the negative root of the asymptotic log MGF is $s^r_* = \left[-\frac{2\delta_{r,c}}{\sigma^2_r\lceil\frac{Rf}{\mu_r}\rceil}+o\left(\frac{\delta_{r,c}}{\lceil\frac{Rf}{\mu_r}\rceil}\right)\right]$, where $\sigma^2_r$ is the asymptotic variance of the harvesting process at the receiver. This implies that, roughly speaking, the battery discharge probability, $\mathrm{P}_{\mathrm{d},r} \approx \Theta\left(\exp\left(-\frac{\delta_{r,c}\mu_r B^r_{\max}}{\sigma^2_r Rf}\right)\right)$. For example, if $f = \Theta\left(\frac{B^r_{\max}}{\log B^r_{\max}}\right)$, then both the second and third terms in the expression for the decay rate in Theorem 1 can be made to decrease as the inverse square of $B^r_{\max}$.

Thus, Theorem 1 establishes that the policy $\mathcal{P}_{\mathrm{td}}$ asymptotically achieves the upper bound. Also, the requirement that $\delta_{r,c} \leq \delta_{r,f}$ can be easily ensured by choosing $f$ to be a piecewise constant approximation of an increasing, sublinear function of $B^r_{\max}$. Intuitively, a policy satisfying the condition $\delta_{r,c} \leq \delta_{r,f}$ uses energy more aggressively in the upper half of the battery compared to the lower half of the battery, causing the battery level to decrease faster towards $B^r_{\max}/2$ when the battery is above the half-full mark. This implies that it safeguards against energy wastage (due to the battery getting full) more strongly than a policy with $\delta_{r,c} > \delta_{r,f}$. In fact, if $\delta_{r,c} > \delta_{r,f}$, it can be shown that the convergence of the policy to the upper bound is slower: the last term becomes $O(1/f)$. This is because, when the receiver is highly energy constrained, it is more important not to waste any of the harvested energy, rather than to ensure that no opportunity of reception is missed due to the battery getting empty. Hence, it is better to have a larger (negative) drift in the upper half compared to the (positive) drift in the lower half.

The proof of Theorem 1 also suggests that it is possible to achieve the optimal throughput (albeit at a lower convergence rate) using a simple policy where the receiver deterministically turns *on* in $f$ out of $\lceil\frac{Rf}{\mu_r}\rceil$ slots. This will completely eliminate the overhead for coordination. In the next section, we use this idea to modify $\mathcal{P}_{\mathrm{td}}$, and present a fully uncoordinated policy.

### *C. Fully Uncoordinated Policy for Energy Constrained Receiver, $\frac{R}{\mu_r} > 1$*

Under the fully uncoordinated policy, denoted by $\mathcal{P}_{\mathrm{fu}}$, the receiver operates under average power constraint and turns *on deterministically* in the first $f$ slots of every $N_c = \lceil\frac{Rf}{\mu_r}\rceil$ slots. Thus, the power control policy at the receiver, $\mathcal{P}^r_{\mathrm{fu}} \triangleq \{p^r_n\}_{n=1}^N$, can be mathematically written as $p^r_n = R$ if $N_{\mathrm{off}} < n \leq N_{\mathrm{off}} + f$ and $B^r_n \geq R$, and $p^r_n = 0$ otherwise, where $N_{\mathrm{off}}$ denotes the index of the immediate previous slot when the receiver was *off*. At the start of communication ($n = 0$), $N_{\mathrm{off}}$ is initialized to zero and is updated as $N_{\mathrm{off}} \longleftarrow N_{\mathrm{off}} + N_c$. The transmitter follows the same policy as before, given by (6).

We denote the above policy for the receiver and transmitter, $\mathcal{P}^r_{\mathrm{fu}}$ and $\mathcal{P}^1_{\mathrm{fu}} \triangleq \{p^1_n\}_{n=1}^N$, by $\mathcal{P}_{\mathrm{fu}}$. The following theorem characterizes the performance of $\mathcal{P}_{\mathrm{fu}}$ and shows that it also achieves the upper bound asymptotically with the battery size at the transmitter and the receiver.

**Theorem 2.** *Let $\mathcal{T}_{fu}$ denote the time-averaged throughput achieved by the fully uncoordinated policy $\mathcal{P}_{fu}$. Then, with a sufficiently large battery at the receiver, the gap between the upper bound and $\mathcal{T}_{fu}$, $\left(\frac{\mu_r}{2R}\right)\log\left(1+\frac{R\mu_1}{\mu_r}\right)-\mathcal{T}_{fu}$, goes to zero as $O\left(\frac{\log B^1_{\max}}{B^1_{\max}}\right)+O\left(\frac{P_{d,r}}{f}\right)+O\left(\frac{1}{f}\right)$, where $P_{d,r}$ is the battery discharge probability at the receiver.*

*Proof.* See Appendix D. ■

Theorem 2 shows that removing the feedback results in a slower convergence rate to the upper bound, compared to the policy $\mathcal{P}_{\mathrm{td}}$ given in Theorem 1, i.e., a larger time-dilation factor $f$ is needed to achieve the same gap to the upper bound. As a consequence, in order to maintain the same value of battery discharge probability $\mathrm{P}_{\mathrm{d},r}$ with no feedback, the battery size at the receiver needs to be larger.

Intuitively, for the collision based model considered in the paper, a time-sharing based approach should achieve the maximum throughput. Note that, as also observed in the proof of Lemma 1, time-orthogonal transmissions among the nodes effectively reduces a MAC to a point-to-point link. Thus, the optimal power control policy for the multi-node case (i.e., when $K > 1$) can be potentially derived by simply using a deterministic time-sharing among the nodes, with each node using a power control policy which provides the optimal throughput in a point-to-point link. However, it is not clear whether such a time-sharing based power control scheme can achieve the upper bound provided in Lemma 1. For example, as shown in Fig. 5b, a simple time-sharing scheme which allocates slots equally among all the transmitting nodes (and adapting the policy to account for the fact that a transmitter can only transmit during its allocated slots), does *not* achieve the upper bound. Thus, the structure of the optimal power control for each individual node needs to be derived in conjunction with the optimal time-sharing scheme. An interesting question which needs to be answered is whether the optimal power control policy for the special case ($K = 1$) can be adapted to derive the optimal power control for an individual node in the multi-node case. In the next section, we address this question and show that, surprisingly, a carefully designed time sharing scheme along with a power control policy for the individual node obtained by adapting the time-dilation based policies presented in the previous section asymptotically achieves the upper bound, even without any coordination among the nodes.

## V. Optimal Policies in The General Case ($K > 1$)

In this section, we consider the MAC channel in two cases when the AP is an energy constrained ($\frac{R}{\mu_r} > 1$) and unconstrained ($\frac{R}{\mu_r} \leq 1$) node. For both the cases, we present time sharing based power control policies which asymptotically achieve the upper bound in Lemma 1.

First, recall that, in Case a) of Lemma 1, the AP can remain *on* in all slots, and the probability that the AP does not have sufficient energy to receive data in any slot decays exponentially with the battery size [16]. Thus, if $R \leq \mu_r$ and the AP is equipped with a sufficiently large battery, the transmitting nodes can operate completely oblivious of the battery state at the AP, as the optimal policy for the AP is to always be *on*. We now present an asymptotically optimal policy (in $B^k_{\max}$ for all $1 \leq k \leq K$) for the transmitting EHNs.

### *A. Asymptotically Optimal Power Control for MAC with Energy Unconstrained Receiver, $\frac{R}{\mu_r} < 1$*

To develop a policy for the transmitting nodes, we observe that the upper bound in Case *(a)* in Lemma 1 can be interpreted as the capacity of a point-to-point AWGN link where the transmitter is equipped with an infinite capacity battery and harvests energy at the rate $\sum_{k=1}^{K} \mu_k$, and the receiver is connected to the mains. Since a transmitting node using energy equal to the average harvesting rate achieves the capacity for these links, the upper bound in Lemma 1 can be attained if the nodes are equipped with infinite sized batteries, their transmissions are orthogonal in time, and they use $\sum_{k=1}^{K} \mu_k$ energy for transmission in every slot. This suggests that transmitting for the duration in proportion to the harvesting rate will be optimal in terms of maximizing the long-term time-averaged sum throughput. Such a strategy is asymptotically optimal in the battery size at the transmitting EHNs, as shown below.

For the MAC with finite batteries at the EHNs, we propose that the nodes transmit according to the following *deterministic* time sharing schedule:

1) For each user $k$, compute the ratio $\frac{n_k}{N_m} = \frac{\mu_k}{\sum_{k=1}^{K} \mu_k}$. Here, $N_m$ is the smallest positive integer such that $n_k$ is an integer for all $1 \leq k \leq K$. Note that $N_m = \sum_{k=1}^{K} n_k$.
2) The $k^{\text{th}}$ node transmits in $n_k$ out of $N_m$ slots, in a round-robin manner across the nodes.

When transmitting, each node executes the policy $\mathcal{P}_{\text{u}}$ given in (3) locally, and independent of the other nodes. Further, based on the time sharing scheme proposed above, the transmit power of all inactive nodes is set to zero, i.e., only the active node transmits during its scheduled slots. However, even when the $k^{\text{th}}$ node is inactive, i.e., when it is not transmitting, it still executes the policy $\mathcal{P}^k_{\text{u}}$ and transfers the energy prescribed for the current slot by $\mathcal{P}^k_{\text{u}}$ from the battery to the supercapacitor attached to it. The total energy accumulated in the supercapacitor of the $k^{\text{th}}$ node during the inactive phase is divided equally over the $n_k$ slots for which the node remains active; and this energy is used for transmission. In the following paragraphs, we mathematically analyze the throughput achieved by this multi-node policy, denoted by $\mathcal{P}_{\text{mu}}$.

If the $k^{\text{th}}$ node is *inactive* in the $n^{\text{th}}$ slot, then its transmit power is set to zero, i.e., $p^k_n = 0$, and the energy in its supercapacitor, denoted by $C_k(n)$, evolves according to (5), with $\delta_1$ replaced by $\delta_k$, which is defined for the $k^{\text{th}}$ node in the same way as for the policy $\mathcal{P}_{\text{u}}$. In a slot $n$ such that $\lfloor \frac{n}{N_m} \rfloor N_m + \sum_{\ell=1}^{k-1} n_\ell < n \leq \lfloor \frac{n}{N_m} \rfloor N_m + \sum_{\ell=1}^{k} n_\ell$, i.e., when the $k^{\text{th}}$ node is active, the energy in the supercapacitor of the $k^{\text{th}}$ node is updated as $C_k(n) = C_k(n-1) - \frac{C_k\left(\lfloor \frac{n}{N_m} \rfloor N_m + \sum_{\ell}^{k-1} n_\ell\right)}{n_k}$, and the transmit power is set as

$$p^k_n = \begin{cases} \frac{C_k\left(\lfloor \frac{n}{N_m} \rfloor N_m + \sum_{\ell}^{k-1} n_\ell\right)}{n_k} + \mu_k + \delta_k, & \text{if } B^k_n > \frac{B^k_{\max}}{2}, \\ \frac{C_k\left(\lfloor \frac{n}{N_m} \rfloor N_m + \sum_{\ell}^{k-1} n_\ell\right)}{n_k} + \min(\mu_k - \delta_k, B^k_n), & \text{otherwise.} \end{cases} \tag{7}$$

In the above, $\frac{C_k\left(\lfloor \frac{n}{N_m} \rfloor N_m + \sum_{\ell}^{k-1} n_\ell\right)}{n_k}$ is the accumulated energy used per active slot and $\mu_k + \delta_k$ and $\mu_k - \delta_k$ is the additional amount of energy to be used in the current slot, determined by the policy based on the battery state in the current slot. Recall that under the policy $\mathcal{P}_{\text{mu}}$, the AP is always *on*. The following Theorem asserts that the proposed policy is asymptotically optimal. Its proof is provided in [1].

**Theorem 3.** *Let $\mathcal{T}_{mu}$ denote the time-averaged throughput achieved by the policy $\mathcal{P}_{mu}$. Then, $\mathcal{T}_{mu}$ approaches the upper bound on the throughput in Case a) of Lemma 1 as*

$$\frac{1}{2} \log\left(1 + \sum_{k=1}^{K} \mu_k\right) - \mathcal{T}_{mu} = \sum_{k=1}^{K} O\left(\frac{\sum_{k'=1}^{K} \mu_{k'}}{\mu_k} \frac{\log B^k_{\max}}{B^k_{\max}}\right). \tag{8}$$

From Theorem 3, in a symmetric MAC where the harvesting rates of all the nodes are equal, i.e., $\mu_k = \mu$ for all $1 \leq k \leq K$, the battery size at each node should scale in proportion to the number of nodes in the network. Theorem 3 establishes that the proposed policy is able to asymptotically achieve the upper bound without requiring knowledge of the correlation among the harvesting processes across the nodes or over time. Moreover, the policy does not require any coordination among the nodes.

In the following, we present a variant of the policy $\mathcal{P}_{\text{mu}}$, which, along with time-dilation, achieves the optimal throughput in the scenario corresponding to Case *(b)* in Lemma 1.

### *B. Asymptotically Optimal Power Control for MAC with Energy Constrained Receiver, $\frac{R}{\mu_r} > 1$*

The policy presented in this section uses time-dilation at the receiver in the same manner as in the $K = 1$ case. Depending on whether the battery at the AP is below or above the halfway mark, it turns *on* in the first $f$ out of $N_c = \lceil \frac{Rf}{\mu_r} \rceil$ and $N_f = \lfloor \frac{Rf}{\mu_r} \rfloor$ slots, respectively. That is, the policy at the receiver is given by (4).

At the transmitters, similar to $\mathcal{P}_{\text{mu}}$, the total number of slots when the receiver is *on* is divided among the transmitters

TABLE I: Example of the policy $\mathcal{P}_{\text{mc}}$ for a 2-user EH MAC. The harvesting rates at the nodes are $\mu_1 = 1.5, \mu_2 = 1$ and $\mu_r = 0.8$. The receiver uses a time-dilation factor $f = 5$ and energy required for decoding is $R = 1$. The size of the battery at all the nodes is 10 units, and $\delta_1 = \delta_2 = 0.5$. The AP sends a 1-bit feedback to the nodes in the gray-shaded slot, since its battery level crosses the half-full mark. The x denotes that no energy is harvested in that slot.

| Slot index ($n$) | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| $\mathcal{E}_r$ | x | 1 | x | 1 | 1 | x | x | x | x | x | 3 | 2 | 3 | 1 | x |
| $B^r$ | 3 | 2 | 2 | 1 | 1 | 1 | 1 | 1 | 0 | 0 | 0 | 2 | 3 | 6 | 6 |
| $p^r$ | 1 | 1 | 1 | 1 | 1 | 0 | 0 | 1 | 0 | 0 | 1 | 1 | 0 | 1 | 1 |
| $\mathcal{E}_1$ | 1 | x | 2 | 3 | x | 3 | x | 1 | 2 | 2 | x | x | x | 1 | 1 |
| $B^1$ | 7 | 6 | 4 | 5 | 7 | 5 | 7 | 5 | 5 | 6 | 6 | 4 | 3 | 2 | 2 |
| $C_1$ | 0 | 0 | 0 | 1 | 3 | 4 | 6 | 4 | 2 | 0 | 2 | 3 | 4 | 2.7 | 1.4 |
| $p^1$ | 2 | 2 | 1 | 0 | 0 | 0 | 0 | 3 | 3 | 4 | 0 | 0 | 0 | 2.3 | 2.3 |
| $\mathcal{E}_2$ | 3 | x | 3 | 1 | x | x | x | x | x | x | x | x | 1 | 3 | 2 |
| $B^2$ | 2 | 4.5 | 4 | 6.5 | 6 | 4.5 | 4 | 3.5 | 3 | 2.5 | 2 | 1.5 | 1 | 1.5 | 4 |
| $C_2$ | 0.5 | 1 | 1.5 | 0.8 | 0 | 0.5 | 1 | 1.5 | 2 | 2.5 | 1.3 | 0 | 0.5 | 1 | 1.5 |
| $p^2$ | 0 | 0 | 0 | 2.3 | 2.3 | 0 | 0 | 0 | 0 | 0 | 1.8 | 1.8 | 0 | 0 | 0 |
| $r_n$ | 0.8 | 0.8 | 0.5 | 0.9 | 0.9 | 0 | 0 | 1 | 0 | 0 | 0.7 | 0.7 | 0 | 0.9 | 0.9 |

according to the ratio of their harvesting rates. For ease of presentation, suppose that $f = \ell N_m$, where $\ell \in \mathbb{Z}^+$. The $k^{\text{th}}$ node remains active in the $n^{\text{th}}$ slot if $N_{\text{off}} + \lfloor \frac{n-N_{\text{off}}}{N_m} \rfloor N_m + \sum_{i=1}^{k-1} n_i < n \leq N_{\text{off}} + \lfloor \frac{n-N_{\text{off}}}{N_m} \rfloor N_m + \sum_{i=1}^{k} n_i$, where $N_{\text{off}}$ denotes the index of the immediate previous slot when the AP was off and its value is updated in a similar fashion as described in Section IV-B. A node remains inactive and accumulates the harvested energy when the receiver is *off*, in addition to the slots when the other nodes are transmitting according to a time sharing pattern prescribed by the policy $\mathcal{P}_{\text{mu}}$. That is, all the nodes remain *off* when $N_{\text{off}} + f < n \leq N_{\text{off}} + \lfloor \frac{Rf}{\mu_r} \rfloor$, and the battery at the AP is more than half full; otherwise, they remain *off* when $N_{\text{off}} + f < n \leq N_{\text{off}} + \lceil \frac{Rf}{\mu_r} \rceil$.

In a slot when the $k^{\text{th}}$ node is inactive, the evolution of the energy in its supercapacitor is given by (5). In a slot when the $k^{\text{th}}$ node is active, the energy in its supercapacitor updates as

$$C_k(n) = C_k(n-1) - \frac{C_k(N_{\text{off}})}{\ell n_k} - \frac{C_k\left(N_{\text{off}} + \lfloor \frac{N_{\text{off}}-n}{N_m} \rfloor N_m + \sum_{i=1}^{k-1} n_i\right)}{n_k}.$$

Let $E_{\text{ac}} \triangleq \frac{C_k(N_{\text{off}})}{\ell n_k} + \frac{C_k\left(N_{\text{off}} + \lfloor \frac{N_{\text{off}}-n}{N_m} \rfloor N_m + \sum_{i=1}^{k-1} n_i\right)}{n_k}$ denote the total accumulated energy used per active slot. If the $k^{\text{th}}$ node is active in the $n^{\text{th}}$ slot, the transmit power is given by

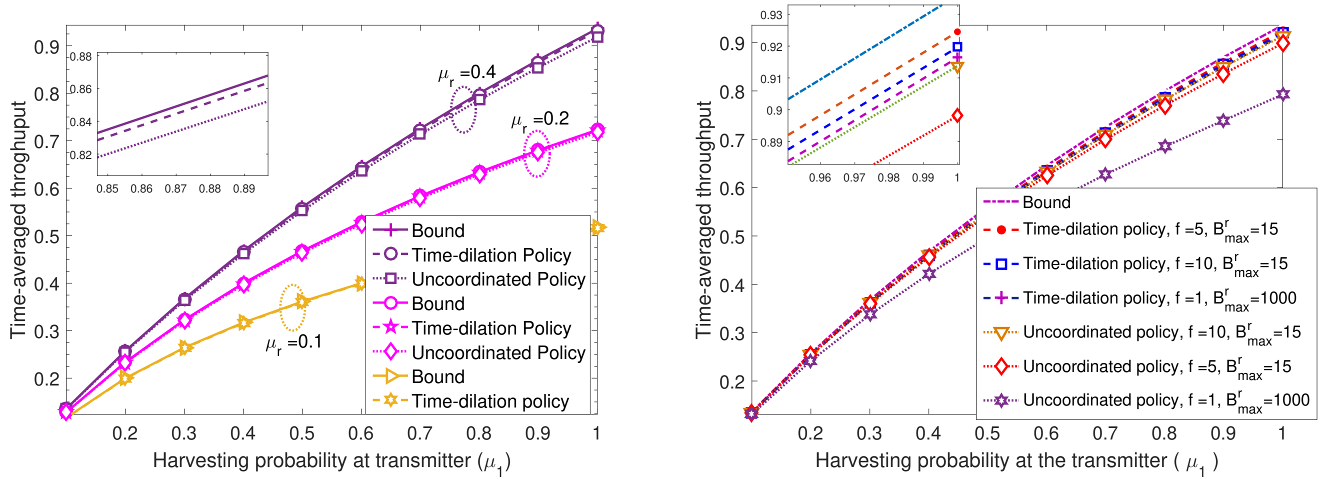
$$p_n^k = \begin{cases} E_{\text{ac}} + \mu_k + \delta_k, & \text{if } B_n^k > \frac{B_{\max}^k}{2}, \\ E_{\text{ac}} + \min\{\mu_k - \delta_k, B_n^k\}, & \text{if } B_n^k \leq \frac{B_{\max}^k}{2}, \end{cases} \quad (9)$$

where $\mu_k + \delta_k$ and $\mu_k - \delta_k$ are the amount of energy to be used in the current slot, determined by the policy based on the battery state. We denote the above policy by $\mathcal{P}_{\text{mc}}$.

In Table I, we illustrate the policy $\mathcal{P}_{\text{mc}}$ for a 2-user MAC with an energy constrained AP. Recall that, $B^k$ and $\mathcal{E}_k$, for $k \in \{1, 2, r\}$, denote the battery state and energy harvested at the $k^{\text{th}}$ node at the start of a slot, respectively. Also, the transmit power level at the two transmitters is denoted by $p^1$ and $p^2$, and the energy accumulated in the supercapacitor of the two transmitters, by the end of a slot, is denoted $C_1$ and $C_2$. Further, $r_n$ denotes the number of bits successfully received by the AP in a slot. The harvesting rate and the energy required for data decoding at the AP are $\mu_r = 0.8$ and $R = 1$, respectively, and the AP uses time-dilation with $f = 5$. Note that, in this scenario, $fN_r = 6.25$. Further, the harvesting rates at the first and second transmitter are $\mu_1 = 1.5$ and $\mu_2 = 1$, respectively. In any slot, the transmit power is determined using (9), where $\delta_1 = \delta_2 = 0.5$. Thus, the AP turns *on* in $f = 5$ out of 6 or 7 slots, depending on whether the battery at the AP is more than half full or not. Among the initial seven slots the AP turns *on* in the first five slots, as at the start of the $7^{\text{th}}$ slot the battery at the AP is less than half full. On the other hand, since at the start of the $14^{\text{th}}$ slot, the battery at the AP is more than half full, the AP remains *on* in the first five out of 6 slots. Note that, in the slots with index 9 and 10, the AP is *off* due to energy unavailability, and no data is received successfully at the AP. Out of the 5 slots where the AP is *on*, the first transmitter uses the initial 3 slots and the last two slots are used by the second transmitter. In addition, both the transmitters remain *off* when the AP is scheduled to be *off*, e.g., during the 6, 7 and 13th slots. Also, in a given slot when a transmitter is *off*, it transfers the energy prescribed by its local policy into its supercapacitor, and uses the accumulated energy in equal amounts over next slots when it is on.

The following Theorem asserts that $\mathcal{P}_{\text{mc}}$ is asymptotically optimal. Its proof can be obtained by combining of the proofs of Theorems 1 and 3 and is therefore omitted.

**Theorem 4.** *Let $\mathcal{T}_{mc}$ denote the time-averaged sum throughput achieved by the proposed policy $\mathcal{P}_{mc}$. Then, the gap between $\mathcal{T}_{mc}$ and the upper bound, $\frac{\mu_r}{2R} \log\left(1 + \frac{R}{\mu_r} \sum_{k=1}^{K} \mu_k\right) - \mathcal{T}_{mc}$, goes*




(a) Performance of the proposed policy, $K = 1$.

(b) Effect of time-dilation factor $f$.

Fig. 2: Performance of the proposed policy for $K = 1$: (a) Evaluation of time-dilation based policy and fully uncoordinated policy: both $\mathcal{P}_{\rm td}$ and $\mathcal{P}_{\rm fu}$ achieve a throughput close to the upper bound. The time-dilation based policy performs better than fully uncoordinated policy. The result corresponds to time-dilation $f = 13$ and $B^1_{\max} = B^r_{\max} = 1000$. (b) Effect of time-dilation factor $f$: compared to the uncoordinated policy $\mathcal{P}_{\rm fu}$, the time-dilation based policy $\mathcal{P}_{\rm td}$ achieves the throughput close to upper bound with a smaller sized battery at the receiver. The fully uncoordinated policy performs better with large time-dilation factor $f$. Simulation parameters are $\mu_r = 0.4$ and $B^1_{\max} = 1000$.

*to zero as* $O\left(\frac{1}{f^2}\right) + O\left(\frac{P_{d,r}}{f}\right) + \sum_{k=1}^{K} O\left(\frac{\sum_{k'=1}^{K}\mu_{k'}}{\mu_k}\frac{\log B^k_{\max}}{B^k_{\max}}\right)$, *when* $\delta_{r,c} \le \delta_{r,f}$.

Finally, we can extend the above to obtain a fully uncoordinated policy by using an approach similar to the one presented in Sec. IV-C at the receiver, and with the transmitter following the policy described above. Its performance guarantees are similar to that presented in Theorem 4, with the first term replaced by $O\left(\frac{1}{f}\right)$.

The above results establish that a simple, time-sharing based closed-form power control policy can achieve the upper bound in Lemma 1. This, in turn, establishes throughput optimality of the time-sharing schemes for the EH MAC. In the next section, we empirically illustrate the performance of our policies.

## VI. Simulation Results

To evaluate the performance of the proposed policy, we consider a $K$-user multi-access channel with a slot duration of 100 ms. The distance between each node and AP is 500 m, with a reference distance $d_0 = 10$ m and path-loss exponent $\eta = 4$. The carrier frequency is 950 MHz. The system bandwidth and the temperature at the receiver are 2 MHz and 300 K, respectively. The harvesting processes at all the nodes are modeled as a spatially as well as temporally independent and identically distributed Bernoulli distribution [16], according to which the $k^{\rm th}$ node harvests $E_s$ units of energy in a slot with probability $\mu_k$. In this system, $E_s = 0$ dB is equivalent to $25\mu$J of energy. The energy consumed for receiving the data in a slot is $R = 0.5$, meaning that the energy for reception is $0.5E_s$. The throughput is evaluated through Monte Carlo simulations of the system over $10^7$ slots.

The results in Fig. 2a show the performance of proposed policies for $K = 1$ with an energy constrained AP. The throughputs achieved by both the time-dilation based policy,

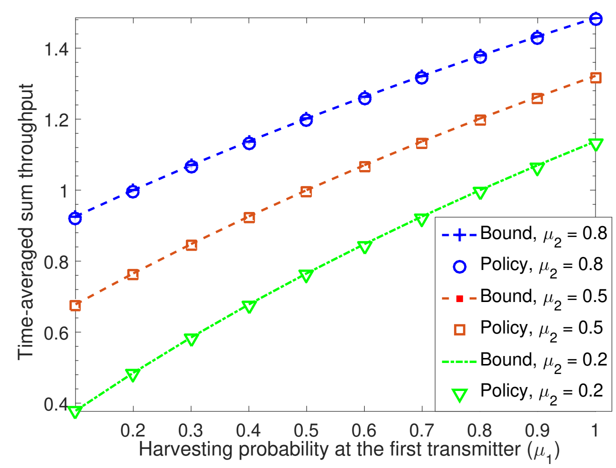
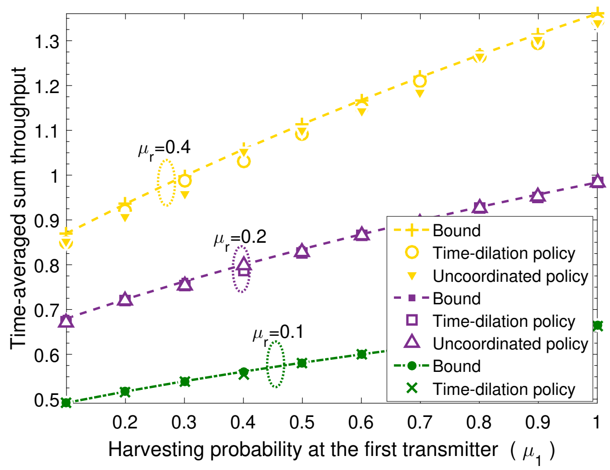



Fig. 3: For a 2-user MAC with an energy *unconstrained* AP, the proposed policy achieves the upper bound. The capacity of the battery at both the nodes as well as at the receiver is $B^1_{\max} = B^2_{\max} = B^r_{\max} = 50$. The size of the supercapacitor is taken to be equal to the size of the battery. The energy harvesting rate at the AP is $\mu_r = 0.75$. The battery size is normalized with respect to the energy harvested in a slot.

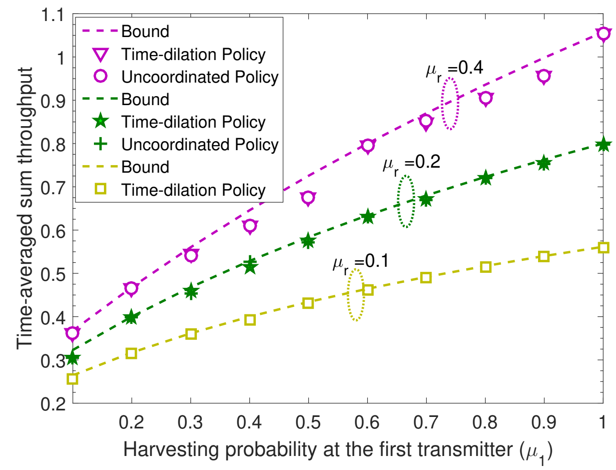
$\mathcal{P}_{\rm td}$, and the fully uncoordinated policy, $\mathcal{P}_{\rm fu}$, are close to upper-bound provided in Lemma 1. However, the throughput achieved by policy $\mathcal{P}_{\rm td}$ is slightly higher than the fully uncoordinated policy $\mathcal{P}_{\rm fu}$.

Fig. 2b illustrates the effect of time-dilation factor $f$ on the performance of both the policies. With no time-dilation, i.e., $f = 1$, both the policies achieve the least throughput among all the scenarios, even if $B^r_{\max} = 1000$, i.e, $1000E_s$ units of energy can be stored.[5] As explained in Sec. IV, this is because

[5]The typical battery size for practical EHNs ranges between 200 mAh-2500 mAh [39]. A 200 mAh capacity battery can deliver 720 J of energy at a nominal voltage of 1 V. Also, using a small solar panel, at 66 % efficiency, NiMH batteries receive 1.3 mJ of energy per 100 ms slot. Thus, with two hours of sunlight, the typical battery size, normalized with respect to $E_s$, equals $5.33 \times 10^5$. Hence, in practice, a battery size of 1000 is very small.




(a) $\mu_2 = 0.8$.




(b) $\mu_2 = 0.2$.

Fig. 4: For a 2-user MAC with an energy *constrained* AP, the proposed policy achieves the upper bound. The capacity of the battery at both the transmitting nodes is $B^1_{\max} = B^2_{\max} = 50$ and at the receiver is $B^r_{\max} = 1000$, normalized with respect to the quantum of energy harvested in a slot. The size of the supercapacitor is taken to be equal to the size of the battery. For both time-dilation based policy and fully uncoordinated policy we use $f = N_m$.

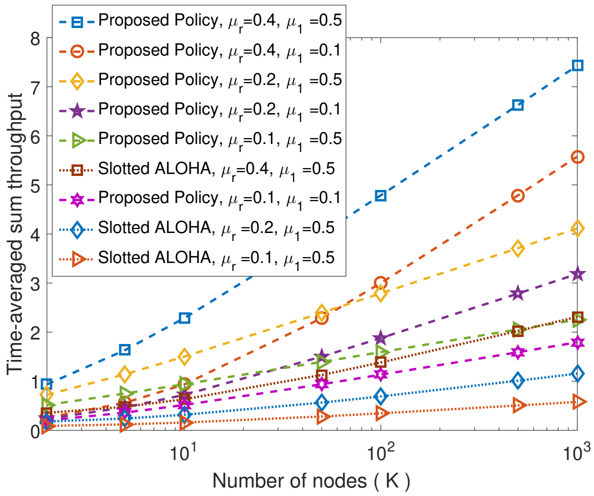
of the binary nature of the power control policy at the receiver, which, in turn, leads to wastage of energy due to poor control over the drift at the receiver.

For the uncoordinated policy, $\mathcal{P}_{\text{fu}}$, the throughput achieved with $f = 10$ is better than that achieved with $f = 5$. This indicates that the choosing a larger $f$ will result in a better throughput, because this facilitates a finer control over the per slot drift at the receiver. However, for a given battery size at the receiver, choosing a very large $f$ will result in a larger battery discharge probability. Hence, increasing $f$ without correspondingly increasing the battery size brings only limited benefits. For example, when $B^r_{\max} = 15$, the time-dilation based policy achieves a better throughput with $f = 5$ in comparison of $f = 10$. This suggests that $f$ must be judiciously chosen based on the battery size at the receiver.

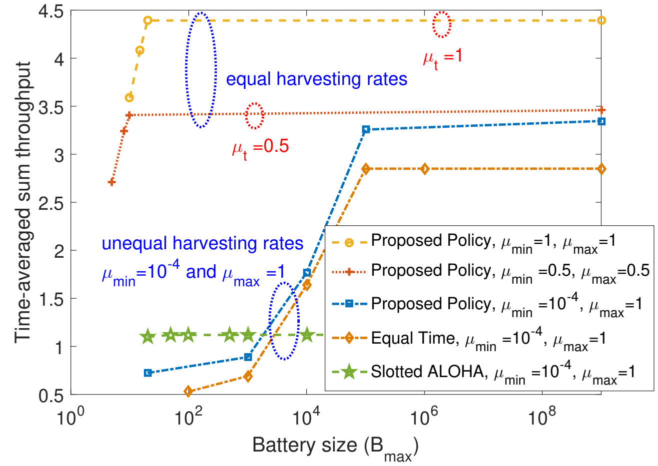
The results in Figs. 3 and 4 illustrate that our policies achieve the upper bound when the AP is energy unconstrained and energy constrained, respectively, for the $K = 2$ user case. For an EH MAC with energy unconstrained AP, the throughput achieved with $B^1_{\max} = B^2_{\max} = B^r_{\max} = 50$ is very close to upper bound. In Fig. 4, for $\mu_r = 0.1$, we note that both time-dilation based policy and the fully uncoordinated policies are the same: the AP turns on deterministically once in every 5 slots. Also, a severely energy constrained AP achieves a much lower throughput than the energy unconstrained AP case, as expected. Also, the results in Figs. 4a and 4b illustrate the time-averaged sum throughout for two different values of the harvesting rate at the second user. It can be observed that for a lower harvesting rate at the transmitters, the sum-throughput obtained is also lower. This is consistent with the lower bound provided in Lemma 1.

In Fig. 5a and 5b, we plot the time-averaged sum throughput achieved by the proposed uncoordinated policy against the number of transmitting nodes and battery size, respectively, and compare the performance with the slotted ALOHA protocol. In slotted ALOHA, the nodes follow a random access policy, i.e., in any slot, the $k^{\text{th}}$ node transmits with probability $\frac{\mu_k}{\sum_{k=1}^{K}\mu_k}$, provided it has nonzero energy in its battery. In Fig. 5a the harvesting rate is the same across the transmitters and is denoted by $\mu_1$ for all $1 \le k \le K$. The $k^{\text{th}}$ node transmits using $\min\{K\mu_k, B^k\}$ units of energy, where $B^k$ denotes the amount of energy available in its battery. We note that the throughput obtained using the proposed policy is significantly higher than the throughput obtained using the slotted-ALOHA based policy. Further, the throughput achieved by our policy increases with the number of transmitting nodes, $K$, however, the rate of increase depends on $\mu_r$, the harvesting rate at the AP. For lower $\mu_r$, the throughput increases at a slower rate. We note that for smaller values of $K$ (e.g., less than 100) the throughput achieved corresponding to $\mu_r = 0.4$ and $\mu_1 = 0.1$ is lower than the throughput obtained for ($\mu_r = 0.2, \mu_1 = 0.5$). In contrast, when the number of nodes in the network is large the throughput achieved for ($\mu_r = 0.4, \mu_1 = 0.1$) is much larger than ($\mu_r = 0.2, \mu_1 = 0.5$). This is because, for a network with smaller number of nodes, the throughput is limited by the energy available for the transmission, while in a network with a large number of nodes, the throughput is limited by the energy availability at the AP.

In order to study the dependence between the time-averaged sum throughput achieved by the proposed policy and the size of the battery, we plot the throughput with $K = 20$ users transmitting to an energy unconstrained AP. In addition, we also plot the performance of a policy obtained using (7), and by allocating the available transmission slots equally among the nodes, i.e., by setting $n_k = \frac{N_m}{K}$ for all $k$. The plot corresponding to this policy is labeled as `Equal Time`. We consider a scenario where the size of the battery at each node is the same. Also, the size of the supercapacitor at each node is equal to the size of the battery at the node. Figure 5b illustrates the time-averaged sum throughput as a function of the size of the battery at the nodes. The size of the battery required to achieve asymptotically optimal performance is roughly equal




(a) MAC with energy unconstrained AP.




(b) MAC with energy constrained AP.

Fig. 5: Comparison of our policy versus the slotted-ALOHA scheme: (a) time-averaged sum throughput of uncoordinated proposed policy and slotted-ALOHA is plotted against number of nodes in a symmetric network where harvesting rates of all the transmitters are equal. Our policy outperforms slotted-ALOHA. The capacity of the battery at all the nodes is $B^k_{\max} = B^r_{\max} = 1000$ for all $1 \le k \le K$. (b) performance of the proposed policy with the battery size: a small sized battery is sufficient to achieve the upper bound for a network with symmetric harvesting rates. On the other hand, for a network with asymmetric harvesting rates, the size of the battery required to achieve the upper bound is large. The number of users is $K = 20$ and the minimum and maximum harvesting rates in the asymmetric scenario are $10^{-4}$ and 1, respectively. The equal-time sharing policy is obtained by setting $n_k = \frac{Nm}{K}$, for all $1 \le k \le K$, in (7). The slotted-ALOHA scheme is outperformed by our policy for the MAC with energy unconstrained AP also.

to the battery size at which the sum throughput saturates. It can be observed that in the scenario where the harvesting rates are symmetric across the nodes, the upper bound can be achieved with a small sized battery, e.g., $B_{\max} = 50$ for $\mu = 0.5$. On the other hand, when the variation in harvesting rates across the nodes is large, the battery size required to achieve a performance close to the upper bound is much larger; it is of the order $10^5$ in the scenario where the ratio of the largest to smallest harvesting rates is of the order $10^4$. This is in line with the remarks following Theorem 3. In contrast, the `Equal Time` policy does not achieve the upper bound, and results in a throughput inferior to the optimal policy. A similar trend is also observed in Figs. 3 and 4, but the performance gap from the optimal is less than that observed in Fig. 5b. This is because, for both the Figs. 3 and 4, the harvesting rates of the nodes are of the same order. Due to this, the performance of the `Equal Time` scheme is also close to the optimal. Further, in the setting with asymmetric harvesting rates also, the throughput achieved by the slotted-ALOHA based policy is inferior to our policy.

## VII. Conclusions

In this paper, we considered the problem of designing power control policies for uncoordinated Gaussian MACs, where the transmitters and the receiver are unaware of the battery state of the other nodes. First, we derived an upper bound on the achievable throughput with the help of a genie-aided system, which has noncausal knowledge of the energy arrivals at all nodes. Next, when the receiver is energy unconstrained, we presented a policy which achieves the upper bound. We then considered the case of an energy constrained receiver, and presented a policy which achieves the upper bound asymptotically through time-dilation and requires occasional one bit feedback. We also presented a fully uncoordinated policy in which the nodes deterministically make their data transmission attempts, and showed that it is also asymptotically optimal. Our results illustrate the tradeoff among system parameters. For example, in the symmetric harvesting rate case, the battery size required to achieve the near-optimal throughput increases in proportion to the number of transmitters. Future work could extend the proposed policies to fading MACs.

## Appendix

### A. *Proof of Lemma 1*

We omit the proof for case *(a)*, as it is provided in [1]. In case *(b)*, i.e., when $\frac{R}{\mu_r} > 1$, the receiver can only turn *on* intermittently. This may result in wastage of the energy as the transmitters do not know the indices of the slots when the receiver will be *on*. To derive an upper bound, we consider a genie-aided centralized system where all the transmitters pool their harvested energy in a common, infinite sized battery, and coordinate their transmissions so that there are no collisions. This facilitates sharing of the harvested energy among the transmitters in a lossless fashion. Also, the entire energy harvested at all the nodes and the AP over the time-horizon of $N$ slots is made available in the first slot itself. This enables the transmitters to perfectly know the number of slots in which the receiver can turn *on*. Since the set of power control policies that are feasible for the uncoordinated MAC is a proper subset of policies that are feasible for the above genie-aided centralized system, the optimal time-averaged sum throughput of the centralized system is an upper bound on the throughput of the uncoordinated MAC.

For large $N$, the receiver can remain *on* in the first $N' = \lfloor \frac{N\mu_r}{R} \rfloor$ out of $N$ slots. Also, since the entire harvested energy is pooled together and made available to the transmitters in

TABLE II: Expressions for $\delta_{s_i}$.

| Cases | $\delta_{s_i}$ where $i = 1, 2, 3$ |
|---|---|
| **Case 2:** Battery at the transmitter in the current slot is less than $\frac{B^1_{\max}}{2}$ and its battery is more than $\frac{B^1_{\max}}{2}$ in $K_c$ out of $N_{oc}$ slots when the receiver is off | $\delta_{s_2} \triangleq -N_r\delta_1 + \frac{\delta_{r,c}(\mu_1-\delta_1)}{f} + \frac{2K_c\delta_1}{f}$ |
| **Case 3:** Battery at the transmitter in the current slot is more than $\frac{B^1_{\max}}{2}$ and its battery is more than $\frac{B^1_{\max}}{2}$ in $K_f$ out of $N_{of}$ slots when the receiver is off | $\delta_{s_3} \triangleq -N_r\delta_1 + 2\delta_1 - \frac{\delta_{r,f}(\mu_1-\delta_1)}{f} + \frac{2K_f\delta_1}{f}$ |
| **Case 4:** Battery at the transmitter in the current slot is less than $\frac{B^1_{\max}}{2}$ and its battery is more than $\frac{B^1_{\max}}{2}$ in $K_f$ out of $N_{of}$ slots when the receiver is off | $\delta_{s_4} \triangleq -N_r\delta_1 - \frac{\delta_{r,f}(\mu_1-\delta_1)}{f} + \frac{2K_f\delta_1}{f}$ |

the first slot itself, by the concavity of the logarithm in the rate expression, it is throughput optimal to equally divide the available energy among the $N'$ slots when the receiver is *on*. Hence, the long-term time-averaged throughput, $\mathcal{T}_g$, of the genie-aided system can be upper bounded as

$$\liminf_{N\to\infty} \mathcal{T}_g \le \liminf_{N\to\infty} \frac{1}{N}\sum_{n=1}^{N'}\sum_{k=1}^{K}\frac{1}{2}\log(1+p_n^k),$$
$$\le \liminf_{N\to\infty} \frac{N'}{N}\frac{1}{2}\log\left(1+\frac{N\sum_{k=1}^{K}\mu_k}{N'}\right).$$

Finally, noting that $\lim_{N\to\infty}\frac{N'}{N} = \frac{\mu_r}{R}$ completes the proof.

### B. Proof of Theorem 1

In order to prove this result, we first compute the total number of bits transmitted by the policy $\mathcal{P}_{\text{td}}$ in batches of $\lfloor \frac{Rf}{\mu_r} \rfloor$ and $\lceil \frac{Rf}{\mu_r} \rceil$ slots. The number of bits transmitted in a given slot depends on the energy used for transmission, which, in turn, depends upon the battery state at the transmitter in the current slot as well as on the energy accumulated in the supercapacitor during the phase when communication was *off*. For convenience, let $N_r \triangleq R/\mu_r$. The number of slots when the receiver is *off* equals either $N_{oc} \triangleq \lceil N_r f \rceil - f$ or $N_{of} \triangleq \lfloor N_r f \rfloor - f$, depending on whether the battery at the receiver was less than or more than half full at the end of a given batch of slots, respectively. The amount of energy accumulated in the supercapacitor depends on the sequence of battery states at the transmitter during these $N_{oc}$ or $N_{of}$ slots. Corresponding to the above, we compute the transmit power in the following cases.

Consider a slot where the receiver is *on* and its battery was less than half full in the immediately preceding slot when the receiver was *off*. Then, the transmitter accumulates energy in the supercapacitor over $N_{oc}$ slots. Out of those $N_{oc}$ slots, let $K_c \in \{0, 1, \cdots, N_{oc}\}$ denote the number of slots in which $\mu_1 + \delta_1$ amount of energy is deposited in the supercapacitor, i.e., the battery at the transmitter is more than half full in $K_c$ out of $N_{oc}$ slots. Then, the total energy accumulated in the supercapacitor by the end of the $N_c$ slots is

$$C_1(N_{\text{off}}) = K_c(\mu_1+\delta_1) + (N_{oc}-K_c)(\mu_1-\delta_1)$$
$$= N_{oc}(\mu_1-\delta_1) + 2K_c\delta_1.$$

The policy $\mathcal{P}_{\text{td}}$ equally distributes the accumulated energy over $f$ slots. Thus, if the battery at the transmitter is more than half full, from (6), the energy used for transmission is

$$p_t = \frac{C_1(N_{\text{off}})}{f} + \mu_1 + \delta_1 = N_r\mu_1 + \delta_{s_1}, \tag{10}$$

where $\delta_{s_1} \triangleq -N_r\delta_1 + 2\delta_1 + \frac{\delta_{r,c}(\mu_1-\delta_1)}{f} + \frac{2K_c\delta_1}{f}$ and $\delta_{r,c} = N_c - fN_r$. We note that here $K_c$ is a random variable whose value depends on the harvesting process at the transmitter. Similar to this, there are three other cases for which the transmit power can be expressed as $N_r\mu_1 + \delta_{s_i}$, which are listed in the Table II. Note that, in Table II, $\delta_{r,f} = fN_r - \lfloor fN_r \rfloor$.

Next, we compute the total number of bits transmitted in batches of $\lceil N_r f \rceil$ and $\lfloor N_r f \rfloor$ slots. Suppose data is transmitted in $f$ out of $\lceil N_r f \rceil$ slots, and the battery at the transmitter is more than half full in $\ell_c$ out of these $f$ slots. Then, the number of bits communicated is

$$\begin{aligned}\mathcal{R}_{\ell_c}(K_c) &= \ell_c\mathcal{R}(N_r\mu_1+\delta_{s_1}) + (f-\ell_c)\mathcal{R}(N_r\mu_1+\delta_{s_2}),\\ &= f\mathcal{R}(N_r\mu_1) + \mathcal{R}^{(1)}(N_r\mu_1)\left[\ell_c\delta_{s_1} + (f-\ell_c)\delta_{s_2}\right]\\ &\quad + \ell_c o(\delta_{s_1}) + (f-\ell_c)o(\delta_{s_2}).\end{aligned} \tag{11}$$

The above follows from the fact that the rate function is analytic. Hence, letting $\mathcal{R}^{(1)}$ denote the first order derivative of $\mathcal{R}$, its Taylor series expansion can be written as $\mathcal{R}(N_r\mu_1+\delta_{s_i}) = \mathcal{R}(N_r\mu_1) + \mathcal{R}^{(1)}(N_r\mu_1)\delta_{s_i} + o(\delta_{s_i})$, where $\delta_{s_i}$, $i = 1, 2, 3, 4$, is as defined above.

Similarly, suppose the communication happens in $f$ out of $\lfloor N_r f \rfloor$ slots and the battery at the transmitter is more than half full in $\ell_f$ out of $f$ slots. Then, the number of bits communicated is

$$\begin{aligned}\mathcal{R}_{\ell_f}(K_f) &= f\mathcal{R}(N_r\mu_1) + \mathcal{R}^{(1)}(N_r\mu_1)\left[\ell_f\delta_{s_3} + (f-\ell_f)\delta_{s_4}\right]\\ &\quad + \ell_f o(\delta_{s_3}) + (f-\ell_f)o(\delta_{s_4}).\end{aligned} \tag{12}$$

In the above, $\ell_c$, $\ell_f$, $K_c$ and $K_f$ are random variables whose values depend on the harvesting process at the transmitter. In the following, we compute the time-averaged throughput by averaging the rates over these random variables. Let $\pi_{\ell_c K_c}$ (or $\pi_{\ell_f K_f}$) denote the fraction of the time when the transmitter's battery is more than half-full in $\ell_c$ and $K_c$ slots (or $\ell_f$

and $K_f$ slots) during the phase when receiver is *on* and *off*, respectively. Then, the throughput achieved by the time-dilation policy, $\mathcal{P}_{\text{td}}$, can be written as

$$\begin{aligned}\mathcal{T}_{\text{td}} &= \sum_{\ell_c=0}^{f}\sum_{K_c=0}^{N_{oc}}\frac{\pi_{\ell_c K_c}\mathcal{R}_{\ell_c}(K_c)}{\lceil fN_r\rceil}+\sum_{\ell_f=0}^{f}\sum_{K_f=0}^{N_{of}}\frac{\pi_{\ell_f K_f}\mathcal{R}_{\ell_f}(K_f)}{\lfloor fN_r\rfloor}\\ &= \frac{1}{\lceil fN_r\rceil}\sum_{\ell_c=0}^{f}\sum_{K_c=0}^{N_{oc}}\pi_{\ell_c K_c}\Big[f\mathcal{R}(N_r\mu_1)+\mathcal{R}^{(1)}(N_r\mu_1)\\ &\qquad\times\left(\ell_c\delta_{s_1}+(f-\ell_c)\delta_{s_2}\right)+o(\delta_{s_1})+o(\delta_{s_2})\Big]\\ &+\frac{1}{\lfloor fN_r\rfloor}\sum_{\ell_f=0}^{f}\sum_{K_f=0}^{N_{of}}\pi_{\ell_f K_f}\Big[f\mathcal{R}(N_r\mu_1)+\mathcal{R}^{(1)}(N_r\mu_1)\\ &\qquad\times\left(\ell_f\delta_{s_3}+(f-\ell_f)\delta_{s_4}\right)+o(\delta_{s_3})+o(\delta_{s_4})\Big].\end{aligned} \tag{13}$$

The last equality in (13) follows from (11) and (12). In the following, to show that $\mathcal{T}_{\text{td}}$ converges to the upper bound asymptotically in the battery size, we analyze the individual terms of (13).

*1) $\mathcal{R}(N_r\mu_1)$ term:* Collecting the $\mathcal{R}(N_r\mu_1)$ terms in (13), we get

$$\begin{aligned}&\sum_{\ell_c=0}^{f}\sum_{K_c=0}^{N_{oc}}\frac{\pi_{\ell_c K_c}\mathcal{R}(N_r\mu_1)f}{\lceil fN_r\rceil}+\sum_{\ell_f=0}^{f}\sum_{K_f=0}^{N_{of}}\frac{\pi_{\ell_f K_f}\mathcal{R}(N_r\mu_1)f}{\lfloor fN_r\rfloor},\\ &=\mathcal{R}(N_r\mu_1)f\left[\frac{\sum_{\ell_c=0}^{f}\pi_{\ell_c}}{(fN_r+\delta_{r,c})}+\frac{\sum_{\ell_f=0}^{f}\pi_{\ell_f}}{(fN_r-\delta_{r,f})}\right],\\ &=\frac{\mathcal{R}(N_r\mu_1)}{N_r\left(1-\frac{\delta_{r,f}}{fN_r}\right)\left(1+\frac{\delta_{r,c}}{fN_r}\right)}\left[1+\frac{\pi_r^+\delta_{r,c}-\pi_r^-\delta_{r,f}}{fN_r}\right].\end{aligned} \tag{14}$$

In the above, $\pi_r^- \triangleq \sum_{\ell_c}^{f}\pi_{\ell_c}$ and $\pi_r^+ \triangleq \sum_{\ell_f=0}^{f}\pi_{\ell_f}$ denote the fraction of time the receiver operates in batches of $\lceil fN_r\rceil$ and $\lfloor fN_r\rfloor$ slots, respectively. We note that $\pi_r^+ = 1-\pi_r^-$. Now, to characterize $\pi_r^+\delta_{r,c}-\pi_r^-\delta_{r,f}$, using the energy conservation principle

$$\begin{aligned}\pi_r^+Rf+(\pi_r^- - \mathrm{P}_{\mathrm{d},r})Rf &= (\pi_r^+ - \mathrm{P}_{\mathrm{o},r})(fN_r-\delta_{r,f})\mu_r\\ &\quad+\pi_r^-(fN_r+\delta_{r,c})\mu_r,\end{aligned}$$

where $\mathrm{P}_{\mathrm{d},r}$ and $\mathrm{P}_{\mathrm{o},r}$ denote the battery discharge and overflow probability at the receiver, respectively. Since $N_r = \frac{R}{\mu_r}$, $N_r f(1-\mathrm{P}_{\mathrm{d},r}) = fN_r + \pi_r^-\delta_{r,c} - \pi_r^+\delta_{r,f} - \mathrm{P}_{\mathrm{o},r} fN_r + \mathrm{P}_{\mathrm{o},r}\delta_{r,f}$. Rearranging the above, we obtain $\pi_r^-\delta_{r,c}-\pi_r^+\delta_{r,f} = fN_r\left(\mathrm{P}_{\mathrm{o},r}-\mathrm{P}_{\mathrm{d},r}\right) - \mathrm{P}_{\mathrm{o},r}\delta_{r,f}$. Further, from $\pi_r^+ = 1-\pi_r^-$, we have $\pi_r^+\delta_{r,c}-\pi_r^-\delta_{r,f} = \delta_{r,c}-\delta_{r,f}+\pi_r^+\delta_{r,f}-\pi_r^-\delta_{r,c}$. Now, in Lemma 2 below, we show that $\mathrm{P}_{\mathrm{d},r} = \Theta\left(\exp\left(\frac{s_*^r B_{\max}^r}{2}\right)\right)$, where $s_*^r < 0$. Similarly, it can be shown that $\mathrm{P}_{\mathrm{o},r} = \Theta\left(\exp\left(\frac{s_*^r B_{\max}^r}{2}\right)\right)$. Hence, $\pi_r^-\delta_{r,c}-\pi_r^+\delta_{r,f} = O(\mathrm{P}_{\mathrm{d},r})$. Thus, depending on whether $\delta_{r,c}-\delta_{r,f}$ is negative or positive, the second term in (14) goes to zero as $O\left(\frac{\mathrm{P}_{\mathrm{d},r}}{f}\right)$ or $O\left(\frac{\mathrm{P}_{\mathrm{d},r}}{f}\right)+O\left(\frac{1}{f}\right)$, respectively. Also, it can be shown that $\left(1+\frac{\delta_{r,f}}{fN_r}\right)\left(1-\frac{\delta_{r,c}}{fN_r}\right) = \Theta\left(1-\frac{1}{f^2}\right)$. Thus, if $\delta_{r,c}-\delta_{r,f} \le 0$ the R.H.S. in (14) converges to $\frac{\mathcal{R}(N_r\mu_1)}{N_r}$ as $O\left(\frac{1}{f^2}\right)+O\left(\frac{\mathrm{P}_{\mathrm{d},r}}{f}\right)$, otherwise it goes to zero as $O\left(\frac{1}{f}\right)+O\left(\frac{\mathrm{P}_{\mathrm{d},r}}{f}\right)$.

In the above arguments, we used the following Lemma, which asserts that for the policy $\mathcal{P}_{\text{td}}$, the battery discharge probability at the transmitter and receiver decay polynomially and exponentially, respectively, with the size of the battery.

**Lemma 2.** *Let the battery discharge probability at the transmitter and receiver be denoted by $P_{d,t}$ and $P_{d,r}$, with $P_{d,t}$ defined in a similar fashion as $P_{d,r}$ in Theorem 1. Then $P_{d,t} = \Theta\left(B_{\max}^{1\ -\beta_1}\right)$ and $P_{d,r} = \Theta\left(\exp\left(\frac{s_*^r B_{\max}^r}{2}\right)\right)$, where $\beta_1 \ge 2$ and $s_*^r$ is the negative root of the asymptotic log MGF of the drift process $D_n^r \triangleq \mathcal{E}_n^r - p_n^r(B_n^r)$. The asymptotic log MGF is defined as $\Lambda(s) \triangleq \lim_{N\to\infty}\frac{1}{N}\log\mathbb{E}\left[\exp\left(s\sum_{n=1}^{N}D_n^r\right)\right]$, where $s\in\mathbb{R}$. In the above, $\mathcal{E}_n^r$ and $p_n^r(B_n^r)$ denote the amount of energy harvested and used by the receiver in the $n^{th}$ slot.*

*Proof.* See Appendix C. ∎

*2) $\mathcal{R}^{(1)}(N_r\mu_1)$ term:* Collecting the $\mathcal{R}^{(1)}(N_r\mu_1)$ terms in (13), we obtain

$$\mathcal{R}^{(1)}(N_r\mu_1)\left[\sum_{\ell_c=0}^{f}\sum_{K_c=0}^{N_{oc}}\frac{\pi_{\ell_c K_c}(\ell_c\delta_{s_1}+(f-\ell_c)\delta_{s_2})}{\lceil fN_r\rceil}+\sum_{\ell_f=0}^{f}\sum_{K_f=0}^{N_{of}}\frac{\pi_{\ell_f K_f}(\ell_f\delta_{s_3}+(f-\ell_f)\delta_{s_4})}{\lfloor fN_r\rfloor}\right].$$

Substituting for $\delta_{s_1},\delta_{s_2},\delta_{s_3}$ and $\delta_{s_4}$, from (10) and Table II, we get

$$\begin{aligned}&=\mathcal{R}^{(1)}(N_r\mu_1)\Bigg[-\delta_1+2\delta_1\left(\sum_{K_f=0}^{N_{of}}\frac{\pi_{K_f}K_f}{\lfloor fN_r\rfloor}+\sum_{K_c=0}^{N_{oc}}\frac{\pi_{K_c}K_c}{\lceil fN_r\rceil}\right)\\ &\qquad+2\delta_1\left(\sum_{\ell_f=0}^{f}\frac{\pi_{\ell_f}\ell_f}{\lfloor fN_r\rfloor}+\sum_{\ell_c=0}^{f}\frac{\pi_{\ell_c}\ell_c}{\lceil fN_r\rceil}\right)\\ &\qquad+\left(\frac{\delta_{r,c}\mu_1\sum_{\ell_c=0}^{f}\pi_{\ell_c}}{\lceil fN_r\rceil}-\frac{\delta_{r,f}\mu_1\sum_{\ell_f=0}^{f}\pi_{\ell_f}}{\lfloor fN_r\rfloor}\right)\Bigg].\end{aligned} \tag{15}$$

In the second term of (15), $0\le K_c\le\lceil fN_r\rceil - f\le\lceil fN_r\rceil$ and $0\le K_f\le\lfloor fN_r\rfloor - f\le\lfloor fN_r\rfloor$. Hence, the second term is $O(\delta_1)$. Similarly, in the third term above $0\le\ell_c\le f\le\lceil fN_r\rceil$ and $0\le\ell_f\le f\le\lfloor fN_r\rfloor$. Hence, this term is also $O(\delta_1)$. The last term in (15) can be simplified as

$$\frac{\mu_1\left[N_r f\left(\pi_r^-\delta_{r,c}-\pi_r^+\delta_{r,f}\right)-\delta_{r,f}\delta_{r,c}\right]}{(N_r f+\delta_{r,c})(N_r f-\delta_{r,f})}. \tag{16}$$

Since $\pi_r^-\delta_{r,c}-\pi_r^+\delta_{r,f} = O\left(\mathrm{P}_{\mathrm{d},r}\right)$, the last term in (15) converges to zero as $O\left(\frac{\mathrm{P}_{\mathrm{d},r}}{f}\right)+O\left(\frac{1}{f^2}\right)$. As a result, the entire $\mathcal{R}^{(1)}(N_r\mu_1)$ term goes to zero as $O(\delta_1)+O\left(\frac{\mathrm{P}_{\mathrm{d},r}}{f}\right)+O\left(\frac{1}{f^2}\right)$. Since the second and higher terms decay faster than the first order term, this completes the proof.

## C. Proof of Lemma 2

Recall that, due to use of the super-capacitor, the battery evolution at the transmitter and receiver are independent. In [16, Lemma 2], it is shown that, for an EHN with energy

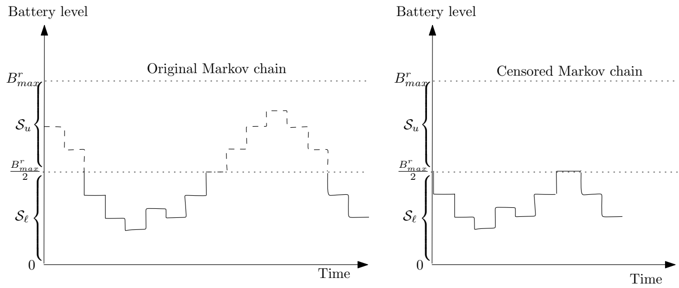



Fig. 6: Censored Markov chain: the battery state is observed only when it is less than half-full, i.e., for the set $\mathcal{S}_\ell$.

harvesting rate greater than or equal to its energy consumption rate, the battery discharge probability decays exponentially with the battery size. Note that, for our system, when the battery at the receiver is less than half full, the average energy consumed by the time-dilation based policy is indeed strictly less than the average harvesting rate. However, this is not true when the battery is more than half full, and hence, we partition the set of battery states into the lower and upper halves, and consider the *censored* Markov chain [40] associated with the set of battery states $\left\{0, 1, \ldots, \frac{B^r_{\max}}{2}\right\}$. The state of this censored Markov chain has a positive drift, and hence [16, Lemma 2] can be applied to complete the proof.

Let $\mathcal{M}_r \triangleq (B^r_0, B^r_1, B^r_2, \ldots)$ denote the Markov chain describing the evolution of the battery at the receiver with its state space and transition probability matrix (TPM) being $\mathcal{S}_r = \{0, 1, \ldots, B^r_{\max}\}$ and $\mathbf{P}$, respectively. As shown in Fig. 6, let $\mathcal{S}_\ell \triangleq \left\{0, \ldots, \frac{B^r_{\max}}{2}\right\}$ and $\mathcal{S}_u \triangleq \mathcal{S}_r \setminus \mathcal{S}_\ell$. We define a random stopping time $T^{(n)}_{\mathcal{S}_\ell}$ for $n \geq 0$ as $T^{(n+1)}_{\mathcal{S}_\ell} \triangleq \min\{m > T^{(n)}_{\mathcal{S}_\ell} | B^r_m \in \mathcal{S}_\ell\}$, where $T^{(0)}_{\mathcal{S}_\ell} \triangleq \min\{m \geq 0 | B^r_m \in \mathcal{S}_\ell\}$

We obtain a new *censored* Markov chain $\mathcal{M}^c_r$ by observing the original Markov chain $\mathcal{M}_r$ only when it is visiting the set $\mathcal{S}_\ell$, i.e., $\mathcal{M}^c_r(n) = \mathcal{M}_r(T^{(n)}_{\mathcal{S}_\ell})$. Let $\mathbf{P} \triangleq \begin{bmatrix} \mathbf{P}_{\mathcal{S}_\ell \mathcal{S}_\ell} & \mathbf{P}_{\mathcal{S}_\ell \mathcal{S}_u} \\ \mathbf{P}_{\mathcal{S}_u \mathcal{S}_\ell} & \mathbf{P}_{\mathcal{S}_u \mathcal{S}_u} \end{bmatrix}$ be the TPM of the original Markov chain $\mathcal{M}_r$ in block form. The TPM, $\mathbf{Q}$, of the Markov chain $\mathcal{M}^c_r$ is the first stochastic complement of $\mathbf{P}$. This, in turn, is given by [41] $\mathbf{Q} = \mathbf{P}_{\mathcal{S}_\ell \mathcal{S}_\ell} + \mathbf{P}_{\mathcal{S}_\ell \mathcal{S}_u} \left(\mathbf{I} - \mathbf{P}_{\mathcal{S}_u \mathcal{S}_u}\right)^{-1} \mathbf{P}_{\mathcal{S}_u \mathcal{S}_\ell}$, where $\mathbf{I}$ is the identity matrix of size $\frac{B^r_{\max}}{2} \times \frac{B^r_{\max}}{2}$. Note that, both the Markov chains $\mathcal{M}_r$ and $\mathcal{M}^c_r$ are stationary and ergodic, and hence have a unique stationary distribution. Thus, if $\boldsymbol{\pi} = \{\pi_0, \ldots, \pi_{B^r_{\max}}\}$ and $\boldsymbol{\pi}^c = \{\pi^c_0, \ldots, \pi^c_{\frac{B^r_{\max}}{2}}\}$ are the stationary distributions corresponding to $\mathbf{P}$ and $\mathbf{Q}$, respectively, then $\pi^c_i = \frac{\pi_i}{\sum_{j \in \mathcal{S}_\ell} \pi_j}, i \in \mathcal{S}_\ell$ [40]. We observe that for the Markov chain $\mathcal{M}^c_r$, the average rate of energy consumption is strictly less than the average harvesting rate, i.e., it has a positive drift towards $\frac{B_{\max}}{2}$. Equivalently, it can be considered to be a Markov chain associated with the evolution of the battery of an EHN operating under an average power constraint, with a battery of size $\frac{B^r_{\max}}{2}$. The desired result now follows from [16, Lemma 2], which shows that, for a node operating under average power constraint, the battery discharge probability decays exponentially with the size of the battery. The result on the battery discharge probability at transmitter can be shown by using arguments similar to the above. This completes the proof.

### D. Proof of Theorem 2

*Proof.* To prove the desired result, similar to the proof of Theorem 1, we compute the throughput achieved by the policy $\mathcal{P}_{\text{fu}}$ and show that it converges to the upper bound. First, we consider the energy accumulated in the supercapacitor during $N_{oc} = \lceil N_r f \rceil - f$ slots, i.e., when the receiver is *off*. If the battery at the transmitter is more than half full for $0 \leq K_c \leq N_c$ slots,

$$\begin{aligned} C_1(N_{\text{off}}) &= K_c(\mu_1 + \delta_1) + (N_{oc} - K_c)(\mu_1 - \delta_1) \\ &= N_{oc}(\mu_1 - \delta_1) + 2K_c\delta_1. \end{aligned}$$

Since the energy accumulated in the supercapacitor is divided equally over the $f$ transmission slots, the transmit energy in a slot when the transmitter's battery is more than half full is $p_{t,u} = \frac{C_1(N_{\text{off}})}{f} + \mu_1 + \delta_1 = N_r\mu_1 + \delta_{s_u}$, where $\delta_{s_u} \triangleq -\frac{\lceil N_r f \rceil \delta_1}{f} + 2\delta_1 + \frac{\delta_{r,c}\mu_1}{f} + \frac{2K_c\delta_1}{f}$ and $\delta_{r,c} = \lceil N_r f \rceil - N_r f$. Similarly, the transmit power in a slot when the battery at the transmitter is less than half full is given by $p_{t,\ell} = \frac{C_1(N_{\text{off}})}{f} + \mu_1 - \delta_1 = N_r\mu_1 + \delta_{s_\ell}$, where $\delta_{s_\ell} = \delta_{s_u} - 2\delta_1 = -\frac{\lceil N_r f \rceil \delta_1}{f} + \frac{\delta_{r,c}\mu_1}{f} + \frac{2K_c\delta_1}{f}$. In the above, $K_c$ is a random variable whose value depends on the harvesting process. Let $\pi_{u,K_c}$ (and $\pi_{\ell,K_c}$) denote the fraction of time when the transmitter's battery in current slot is more than (and less than, respectively) $\frac{B^1_{\max}}{2}$ *and* the transmitter deposits $\mu_1 + \delta_1$ energy in the supercapacitor in $K_c$ out of $N_{oc}$ slots. Then, the overall time-averaged throughput can be written as

$$\mathcal{R}_{\text{fu}} = \frac{f}{\lceil f N_r \rceil} \sum_{K_c=0}^{N_{oc}} \left( \sum_{s \in \mathcal{S}_{K_c,u}} \pi_{u,K_c} \mathcal{R}(p_{t,u}) + \sum_{s \in \mathcal{S}_{K_c,\ell}} \pi_{\ell,K_c} \mathcal{R}(p_{t,\ell}) \right). \quad (17)$$

In the above, $\mathcal{S}_{K_c,u}$ (and $\mathcal{S}_{K_c,\ell}$) denote the set of $N_{oc}+1$ length transmitter battery state sequences in which the transmitter deposits $\mu_1+\delta_1$ energy in the supercapacitor in $K_c$ slots *and* the last element is more (and less) than $\frac{B^1_{\max}}{2}$. Note that (17) does not account for the rate-loss incurred due to energy outages at the receiver. Recall that the probability that receiver does not have sufficient energy to receive the data decays as $\mathrm{P}_{\mathrm{d},r} = e^{\frac{s^r_* B^r_{\max}}{2}}$. Hence, without loss of generality, we can assume that receiver is always *on*. However, in the end, to account for the effect of energy outage at the receiver, we multiply by the probability that the receiver is *on*, i.e., $1 - \mathrm{P}_{\mathrm{d},r}$. In (17), since $\mathcal{R}$ is an analytic function, using a Taylor series expansion, we obtain

$$\begin{aligned} \mathcal{R}_{\text{fu}} = \frac{f}{\lceil f N_r \rceil} \Bigg[ & \mathcal{R}(N_r\mu_1) \\ & + \mathcal{R}^{(1)}(N_r\mu_1) \sum_{K_c=0}^{N_{oc}} \left( \sum_{s \in \mathcal{S}_{K_c,u}} \pi_{u,K_c} \delta_{s_u} + \sum_{s \in \mathcal{S}_{K_c,\ell}} \pi_{\ell,K_c} \delta_{s_\ell} \right) \\ & + \mathcal{R}^{(2)}(N_r\mu_1) \sum_{K_c=0}^{N_{oc}} \left( \sum_{s \in \mathcal{S}_{K_c,u}} \pi_{u,K_c} \delta^2_{s_u} + \sum_{s \in \mathcal{S}_{K_c,\ell}} \pi_{\ell,K_c} \delta_{s^2_\ell} \right) \end{aligned}$$

$$+ o\left(\delta_{s_u}^2\right) + o\left(\delta_{s_\ell}^2\right)\Bigg] \quad (18)$$

Since $\frac{f}{\lceil N_r f \rceil} = \frac{1}{N_r\left(1+\frac{\delta_{r,c}}{N_r f}\right)}$, and using $\frac{1}{1+x} = 1 - x + o(x^2)$, the zeroth order term goes to $\mathcal{R}(N_r \mu_1)$ as $O\left(\frac{1}{f}\right)$. Since $\delta_1 \geq 0$, $\delta_{s_u} \geq \delta_{s_\ell}$. Hence, for the first-order term, we can write

$$\frac{f}{\lceil N_r f \rceil} \sum_{K_c=0}^{N_{oc}} \left( \sum_{s \in \mathcal{S}_{K_c,u}} \pi_{u,K_c} \delta_{s_u} + \sum_{s \in \mathcal{S}_{K_c,\ell}} \pi_{\ell_c,K_c} \delta_{s_\ell} \right)$$
$$\leq \frac{f}{\lceil N_r f \rceil} \left[ 2\delta_1 + \frac{\lceil N_r f \rceil}{f} \delta_1 + \frac{\delta_{r,c} \mu_1}{f} \right].$$

The above goes to zero as $O(\delta_1) + O\left(\frac{1}{f}\right)$. Moreover, depending on whether $|\delta_{s_u}| \geq |\delta_{s_\ell}|$ the second order term in the above is $O(\delta_{s_u}^2)$ or $O(\delta_{s_\ell}^2)$, where $|x|$ is the modulus of $x$. Finally, to account for the energy outages at the receiver we multiply (18) with $1 - \mathrm{P}_{\mathrm{d},r}$, and hence the time-averaged throughput converges to the upper bound as $O(\delta_1) + O\left(\frac{\mathrm{P}_{\mathrm{d},r}}{f}\right) + O\left(\frac{1}{f}\right)$. This, along with the observation that $\delta_1 = \frac{\log B_{\max}^1}{B_{\max}^1}$, completes the proof. ∎

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