On the Sum Spectral Efficiency of Dynamic TDD Enabled Cell-Free Massive MIMO Systems

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***Abstract*—We examine the sum spectral efficiency (SE) performance of a cell-free massive multiple-input multiple-output (CF-mMIMO) system, where each access point (AP) can operate either in the uplink or downlink mode in each slot, corresponding to dynamic time-division duplexing (DTDD) across the APs. We derive the sum SE of the system under a weighted combining of the signals received at the central processing unit. We show that the sum SE is a sub-modular function of the subset of active APs. We exploit this to develop a novel, low-complexity, greedy algorithm for choosing the mode of operation of the APs which is guaranteed to achieve within $(1-1/e)$ of the sum SE attained via a full-complexity brute-force search. Our results show that DTDD with greedy AP mode selection can nearly *double* the sum SE compared to a TDD based CF-system where all APs operate in the uplink or downlink modes simultaneously. Thus, it is a promising duplexing scheme for beyond 5G communications.**

***Index Terms*—Cell-free massive MIMO, dynamic TDD, sub-modular optimization.**

I. Introduction

Cell-free massive multi-input multi-output (CF-mMIMO) is a promising technology for beyond 5G wireless communication due to its capability to enhance the spectral efficiency (SE) compared to the canonical co-located cellular mMIMO systems [1]. CF-mMIMO refers to an architecture where multiple access points (APs) jointly and coherently serve multiple user-equipment (UEs) distributed over a given area [2]. The proximity of a (different) subset of these APs with (each of) the UEs improves the macro-diversity and mitigates the detrimental effects of path-loss and shadowing.

In a parallel development, dynamic time division duplexing (DTDD) has been incorporated in the cellular standards to simultaneously cater to both uplink (UL) and downlink (DL) UEs [3]. DTDD allows each cellular base station (BS) to adaptively partition the UL-DL time frame depending on the local UL-DL traffic demands per cell, unlike TDD wherein all the BSs simultaneously operate either in UL or in DL [4]. Although DTDD improves spectrum utilization compared to TDD, the key challenges in a DTDD based system are the cross link interferences (CLIs) from DL to UL APs and UL to DL UEs. An extensive survey of CLI mitigation in DTDD for cellular mMIMO can be found in [5].

At this point, we note that the conventional TDD based CF-deployments perform poorly under heterogenous UL-DL data demands in the system. On the other hand, the current DTDD enabled cellular mMIMO system performance inherits the drawbacks of cellular architecture, such as multi-cell interference, poor performance of cell edge UEs, etc [5]. In this work, we analyze the performance of DTDD enabled CF-mMIMO, as it can potentially reap the benefits of both technologies. Now, in order to improve the SE via enabling DTDD in a CF-system, we need to appropriately activate the UL and DL modes across the APs, based on the local UL-DL traffic conditions. Thus, our goal is to analyze the sum UL-DL SE across all the UEs and find the optimal UL/DL AP configuration to maximize the SE.

Recently, the problem of AP mode selection has been addressed in [2], [6]. In [2], we lower-bounded the sum UL-DL SE by the product signal to interference plus noise ratios (SINRs) of UEs, and showed it to be a sub-modular set function of the active AP set. However, it is unclear whether sub-modularity holds for the sum SE. In this work, we prove that the UL and DL SINR and the sum UL-DL SE are modular and sub-modular set functions of the active AP set, respectively. Also, in [2], [6] the APs directly relay the combined signals to the central processing unit (CPU) for joint decoding. In contrast, in this work, each AP weighs the combined signal prior to forwarding it to the CPU so as to maximize the received SINR at the CPU. This weighting turns out to be crucial in establishing the modularity of SINRs.

The key contributions of this work are follows:

1) We analyze the UL/DL SINRs and SE under an SINR-maximizing weighted combining scheme at the CPU. We prove that, under a weighted precoding/combining scheme introduced in this work, UL and the DL SINRs are monotonically non-decreasing *modular* functions of the activated AP set, and the sum UL-DL SE is a *sub-modular* function of the activated AP set. The analysis in this paper holds for perfect, statistical, as well as trained channel state information (CSI).
2) We leverage the sub-modularity property to develop a greedy algorithm, where, at each iteration, an AP is activated in either UL or in DL if that AP offers the highest incremental gain in the sum UL-DL SE. This circumvents the exponential complexity of exhaustive search based AP mode selection, and determines the activated AP set in linear time.
3) We empirically show that DTDD enabled CF-mMIMO almost *doubles the sum UL-DL SE* compared to a canonical TDD based system. (See Fig. 2b.) Essentially, DTDD enabled CF-mMIMO exploits both the joint signal processing of a CF system and the adaptive UL-DL slot selection at the APs based on the local traffic demands.

We thus conclude that DTDD is a promising duplexing scheme that can be incorporated in a CF-mMIMO system to meet the heterogeneous UL/DL traffic demands in next generation wireless systems.

Notation: Matrices, vectors, and sets are denoted by bold uppercase, bold lowercase, and calligraphic letters. $(\cdot)^T$, $(\cdot)^H$,

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and $(\cdot)^*$ represent transposition, Hermitian, and complex conjugation operations. $|\cdot|$, $\backslash$, $'$, and $\cup$ denote the cardinality, set-minus, complement, and union of sets. $\mathbb{E}[\cdot]$ and $\mathtt{var}\{\cdot\}$ denote the mean and variance of a random variable. $\mathbf{x} \sim \mathcal{CN}(\mathbf{0}, \boldsymbol{\Sigma})$ indicates that $\mathbf{x}$ is a zero mean circularly symmetric complex Gaussian random vector with covariance matrix $\boldsymbol{\Sigma}$. Finally, $\mathbf{I}_N$ denotes the $N \times N$ identity matrix.

II. System Model

We consider a CF-mMIMO system with M half-duplex APs (HD-APs) each equipped with N antennas jointly and coherently serving K single-antenna UEs. Each AP is connected to a CPU via an ideal back-haul link. The channel from kth UE to the mth AP is modeled as $\mathbf{f}_{mk} = \sqrt{\beta_{mk}}\mathbf{h}_{mk} \in \mathbb{C}^N$, where β_{mk} is the path loss coefficient and $\mathbf{h}_{mk} \overset{\text{i.i.d.}}{\sim} \mathcal{CN}(\mathbf{0}, \mathbf{I}_N)$ is the quasi-static fast fading component.

The simultaneous UL and DL traffic results in inter-AP and inter-UE CLIs, in addition to the multi-user interference. The inter-AP channels are typically slowly varying, and additionally, the transmitted DL data vectors are known at the CPU [7]. Therefore, the entire signal transmitted by any DL AP can be used as a training signal to acquire CSI at the UL APs, and given a long enough sequence, it will be nearly orthogonal to all other APs' signals. This can be exploited to form accurate estimates of the channel and cancel the interference. Hence, for the theoretical analysis, we assume that the inter-AP interference can be eliminated at the CPU; however, the greedy AP scheduling algorithm is applicable even with imperfect inter-AP CLI cancelation. We also numerically validate the robustness of the AP-scheduling algorithm including the effect of inter-AP CLI in Sec. V. For the inter-UE CLIs, we model the channel between nth UL UE and the kth DL UE as $\mathrm{g}_{nk} \sim \mathcal{CN}(0, \epsilon_{nk})$, and is independent across all UE pairs [8], where $\epsilon_{nk} > 0$ denotes the inter-UE channel variance.

1) Problem Statement: Let $\mathcal{U}_u$ and $\mathcal{U}_d$ be the index sets of UEs demanding UL and DL access, respectively. Let $\mathcal{A}$ be the set of AP indices, with $M = |\mathcal{A}|$. The index sets $\mathcal{A}_u$ and $\mathcal{A}_d$ comprise of APs activated in UL and DL modes, respectively. The CPU needs to activate the APs so as to maximize the sum SE, $\mathcal{R}_{\mathrm{s}}(\mathcal{A}_u, \mathcal{A}_d)$, over all possible choices of $\mathcal{A}_u$ and $\mathcal{A}_d$, i.e.,

$$\begin{aligned} &\max_{\mathcal{A}_u, \mathcal{A}_d} \quad \mathcal{R}_{\mathrm{s}}(\mathcal{A}_u, \mathcal{A}_d) \\ &\text{s.t.} \quad \mathcal{A}_u, \mathcal{A}_d \subseteq \mathcal{A}, \quad \mathcal{A}_u \cap \mathcal{A}_d = \emptyset, \quad \mathcal{A}_u \cup \mathcal{A}_d = \mathcal{A}. \end{aligned} \tag{1}$$

We observe from (1) that searching over all possible UL/DL configurations requires the evaluation of the sum SE corresponding to 2^M choices, making exhaustive search computationally challenging. This motivates us to develop a low complexity AP-mode (UL/DL) selection algorithm that can solve (1) in polynomial time, along with a guarantee regarding the optimality of such a method. To this end, we leverage the sub-modular nature of the function $\mathcal{R}_{\mathrm{s}}(\mathcal{A}_u, \mathcal{A}_d)$. In the next section, we present an analytical expression for $\mathcal{R}_{\mathrm{s}}$.

III. Performance Analysis

For the ease of understanding, in this section, we first analyze the sum SE when perfect CSI (PCSI) is available at the APs and CPU. The kth stream of the received signal

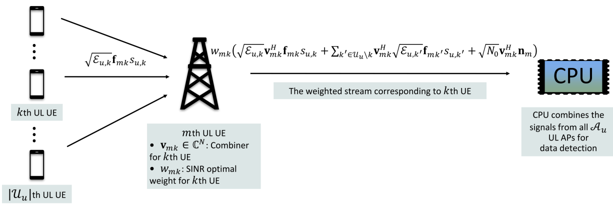


Fig. 1. Signal flow in the UL of a CF-mMIMO system. The APs are connected to the CPU via error free backhaul links.

(corresponding to the signal transmitted by the kth UL UE) at the mth UL AP ($m \in \mathcal{A}_u$) is given by

$$\begin{aligned} r_{u,mk} &= \sqrt{\mathcal{E}_{u,k}}\mathbf{v}_{mk}^H\mathbf{f}_{mk}s_{u,k} \\ &\quad + \textstyle\sum_{k' \in \mathcal{U}_u \backslash k}\mathbf{v}_{mk}^H\sqrt{\mathcal{E}_{u,k'}}\mathbf{f}_{mk'}s_{u,k'} + \sqrt{N_0}\mathbf{v}_{mk}^H\mathbf{n}_m, \end{aligned}$$

where $s_{u,k}$ is the signal transmitted by kth UL UE with power $\mathcal{E}_{u,k}$, $\mathbf{v}_{mk} \in \mathbb{C}^N$ is the combiner vector at mth UL AP for kth UL UE, and $\mathbf{n}_m \sim \mathcal{CN}(\mathbf{0}, \mathbf{I}_N)$ is the additive noise.

Now, since the SINR of the kth UE is different at the different APs, the signals forwarded by the APs to the CPU need to be appropriately scaled so as to maximize the SINR of the combined signal at the CPU. This can be accomplished by scaling $r_{u,mk}$ by a weight $w_{mk} \in \mathbb{R}^+$. Then, for the kth UE, the accumulated signal at the CPU can be expressed as

$$\begin{aligned} r_{u,k} &= \textstyle\sum_{m \in \mathcal{A}_u} w_{mk}(\sqrt{\mathcal{E}_{u,k}}\mathbf{v}_{mk}^H\mathbf{f}_{mk}s_{u,k} \\ &\quad + \textstyle\sum_{k' \in \mathcal{U}_u \backslash k}\mathbf{v}_{mk}^H\sqrt{\mathcal{E}_{u,k'}}\mathbf{f}_{mk'}s_{u,k'} + \sqrt{N_0}\mathbf{v}_{mk}^H\mathbf{n}_m), \end{aligned} \tag{2}$$

where w_{mk} is computed as [9][1]

$$w_{mk} \triangleq \frac{\sqrt{\mathcal{E}_{u,k}}\mathbb{E}\big[\mathbf{v}_{mk}^H\mathbf{f}_{mk}\big]}{\mathbb{E}[|\sum_{k' \in \mathcal{U}_u \backslash k}\mathtt{I}_{u,mkk'} + \sqrt{N_0}\mathbf{v}_{mk}^H\mathbf{n}_m|^2]}, \tag{3}$$

with $\mathtt{I}_{u,mkk'} \triangleq \mathbf{v}_{mk}^H\sqrt{\mathcal{E}_{u,k'}}\mathbf{f}_{mk'}s_{u,k'}$. We illustrate the UL signal flow in Fig. 1.

Next, considering maximal ratio combining (MRC) [1] in the UL, i.e., $\mathbf{v}_{mk} = \mathbf{f}_{mk}$, we get $w_{mk} = \frac{\sqrt{\mathcal{E}_{u,k}}N\beta_{mk}}{N\sum_{k' \in \mathcal{U}_u \backslash k}\mathcal{E}_{u,k'}\beta_{mk}\beta_{mk'} + NN_0\beta_{mk}} = \frac{\sqrt{\mathcal{E}_{u,k}}N\beta_{mk}}{\bar{\mathtt{I}}_{u,mk}}$, with $\bar{\mathtt{I}}_{u,mk} = N\sum_{k' \in \mathcal{U}_u \backslash k}\mathcal{E}_{u,k'}\beta_{mk}\beta_{mk'} + NN_0\beta_{mk}$. Then, the kth stream of the processed signal at the CPU becomes

$$\begin{aligned} \bar{r}_{u,k} &= \textstyle\sum_{m \in \mathcal{A}_u} w_{mk}r_{u,mk} = \sum_{m \in \mathcal{A}_u} w_{mk}\sqrt{\mathcal{E}_{u,k}}\mathbf{f}_{mk}^H\mathbf{f}_{mk}s_{u,k} \\ &+ \sum_{m \in \mathcal{A}_u} w_{mk}\Big(\sum_{k' \in \mathcal{U}_u \backslash k}\mathbf{f}_{mk}^H\sqrt{\mathcal{E}_{u,k'}}\mathbf{f}_{mk'}s_{u,k'} + \sqrt{N_0}\mathbf{f}_{mk}^H\mathbf{n}_m\Big). \end{aligned} \tag{4}$$

We present the UL sum SE in the following Theorem.

Theorem 1. *The UL sum SE, denoted by $\mathcal{R}_u(\mathcal{A}_u)$, can be expressed as $\mathcal{R}_u(\mathcal{A}_u) = \sum_{k \in \mathcal{U}_u}\log\left[1 + \eta_{u,k}(\mathcal{A}_u)\right]$, with kth UL UE's SINR being*

$$\eta_{u,k}(\mathcal{A}_u) = \textstyle\sum_{m \in \mathcal{A}_u}\frac{N\mathcal{E}_{u,k}\beta_{mk}^2}{\sum_{k' \in \mathcal{U}_u \backslash k}\mathcal{E}_{u,k'}\beta_{mk}\beta_{mk'} + N_0\beta_{mk}}. \tag{5}$$

Proof. We omit the proof for brevity. ∎

Remark 1. *Typically, the APs design the combiners/precoders based on the locally available channel information and statistics [1], [2], and relay the combined signals to the CPU for the joint data decoding. However, in our work, the APs relay a weighted version of the post-combined received signals to*

[1] We note that $\mathcal{R}_{\mathrm{s}}$ in (1) is the sum of each UE's achievable SE, and hence the SINR maximizing weights also maximize $\mathcal{R}_{\mathrm{s}}$.

the CPU. The weights are chosen to maximize the SINR of the combined signal at the CPU. This weighted combination is key to establishing the modularity of the UL and DL SINRs. For example, the sum of the combined signals across the APs does not satisfy the modularity property.

Next, we present the DL SE analysis. Assuming channel reciprocity, since the jth DL AP has knowledge of the downlink channel $\mathbf{f}_{jn}$ to the nth DL UE, the precoded signal transmitted by the jth DL AP, $j \in \mathcal{A}_d$, can be written as

$$\mathbf{r}_{d,j} = \kappa_{jn}\sqrt{\mathcal{E}_{d,j}}\mathbf{f}_{jn}^{*}s_{d,n} + \textstyle\sum_{q\in\mathcal{U}_d\backslash n}\kappa_{jq}\sqrt{\mathcal{E}_{d,j}}\mathbf{f}_{jq}^{*}s_{d,q}, \quad (6)$$

where $\mathcal{E}_{d,j}$ is the total DL power budget of the jth AP, κ_{jn} is the power control coefficient for the nth DL UE, and $s_{d,n}$ is the DL transmitted symbol intended for nth DL UE with $\mathbb{E}[|s_{d,n}|^2] = 1$, and $\mathbb{E}[s_{d,n}s_{d,q}^{*}] = 0, \forall q \neq n$. Then, the signal received at the nth DL UE prior to adding noise is given by

$$\tilde{r}_{d,jn} = \kappa_{jn}\sqrt{\mathcal{E}_{d,j}}\mathbf{f}_{jn}^{T}\mathbf{f}_{jn}^{*}s_{d,n} + \textstyle\sum_{q\in\mathcal{U}_d\backslash n}\kappa_{jq}\sqrt{\mathcal{E}_{d,j}}\mathbf{f}_{jn}^{T}\mathbf{f}_{jq}^{*}s_{d,q}.$$

Similar to the UL case, let w_{jn} be a weighting coefficient designed by jth DL AP for the nth DL UE. Let $\mathtt{I}_{d,jnq} \triangleq \kappa_{jq}\sqrt{\mathcal{E}_{d,j}}\mathbf{f}_{jn}^{T}\mathbf{f}_{jq}^{*}s_{d,q}$. The SINR maximizing $w_{jn} = \frac{\kappa_{jn}\sqrt{\mathcal{E}_{d,j}}\mathbb{E}[\mathbf{f}_{jn}^{T}\mathbf{f}_{jn}^{*}]}{\mathbb{E}[|\sum_{q\in\mathcal{U}_d\backslash n}\mathtt{I}_{d,jnq}|^2]}$ [9], which reduces to

$$w_{jn} = \kappa_{jn}\sqrt{\mathcal{E}_{d,j}}\beta_{jn}/(\textstyle\sum_{q\in\mathcal{U}_d\backslash n}\kappa_{jq}^{2}\mathcal{E}_{d,j}\beta_{jn}\beta_{jq}). \quad (7)$$

Including the weighting, the signal received at the nth UE is

$$r_{d,n} = \textstyle\sum_{j\in\mathcal{A}_d}w_{jn}\kappa_{jn}\sqrt{\mathcal{E}_{d,j}}\mathbf{f}_{jn}^{T}\mathbf{f}_{jn}^{*}s_{d,n} + \sum_{k\in\mathcal{U}_u}\mathcal{E}_{u,k}\mathbb{E}|\mathrm{g}_{nk}|^2$$
$$+ \textstyle\sum_{j\in\mathcal{A}_d}w_{jn}\mathbf{f}_{jn}^{T}\sum_{q\in\mathcal{U}_d\backslash n}\kappa_{jq}\sqrt{\mathcal{E}_{d,j}}\mathbf{f}_{jq}^{*}s_{d,q} + \sqrt{N_0}n_n, \quad (8)$$

with $n_n \sim \mathcal{CN}(0,1)$ is the receiver noise at nth DL UE. We have the following theorem regarding the DL sum SE.

Theorem 2. *The DL sum SE, denoted by $\mathcal{R}_d(\mathcal{A}_d)$, can be expressed as $\mathcal{R}_d(\mathcal{A}_d) = \sum_{n\in\mathcal{U}_d}\log[1+\eta_{d,n}(\mathcal{A}_d)]$, with the DL SINR of the nth DL UE being*

$$\eta_{d,n}(\mathcal{A}_d) = N^2\big(\textstyle\sum_{j\in\mathcal{A}_d}\frac{\mathcal{E}_{d,j}\kappa_{jn}^{2}\beta_{jn}^{2}}{\sum_{q\in\mathcal{U}_d\backslash n}\mathcal{E}_{d,q}\kappa_{jq}\beta_{jn}\beta_{jq}}\big)^2$$
$$\times\big(\sum_{j\in\mathcal{A}_d}\frac{N\mathcal{E}_{d,j}\kappa_{jn}^{2}\beta_{jn}^{2}}{\sum_{q\in\mathcal{U}_d\backslash n}\mathcal{E}_{d,j}\kappa_{jq}\beta_{jn}\beta_{jq}} + \sum_{k\in\mathcal{U}_u}\mathcal{E}_{u,k}\epsilon_{nk} + N_0\big)^{-1}$$
$$\approx \textstyle\sum_{j\in\mathcal{A}_d}\frac{N\mathcal{E}_{d,j}\kappa_{jn}^{2}\beta_{jn}^{2}}{\sum_{q\in\mathcal{U}_d\backslash n}\mathcal{E}_{d,j}\kappa_{jq}\beta_{jn}\beta_{jq}}. \quad (9)$$

Proof. We omit the proof for brevity. ■

Remark 2. *We observe from (8) that the inter-UE interference power and the DL noise component do not scale with N, while the desired signal strength and multi-DL UE interference power scale with N^2 and N, respectively. Therefore, we approximate the DL SINR considering only the effect of multi-DL UE interference. However, we later numerically validate the robustness of our AP-mode selection algorithm considering both inter-UE CLI and noise, and provide experimental justification for the approximation presented in (9).*

We can now write the sum UL-DL SE as

$$\mathcal{R}_{\mathrm{s}}(\mathcal{A}_s) = \mathcal{R}_u(\mathcal{A}_u) + \mathcal{R}_d(\mathcal{A}_d), \quad (10)$$

where $\mathcal{A}_s \triangleq (\mathcal{A}_u, \mathcal{A}_d)$ is a generic set which constitutes of both the UL and DL AP-indices. Note that, as the APs are HD, $\mathcal{A}_d$ and $\mathcal{A}_d$ are mutually exclusive sets of AP-indices.

1) Statistical CSI: In deriving the UL SINR in the Theorem 1, we used the fact that $\mathbf{f}_{mk}^{H}\mathbf{f}_{mk} \approx N\beta_{mk}$. In fact, $\mathbb{E}\left[\mathbf{f}_{mk}^{H}\mathbf{f}_{mk}\right] = N\beta_{mk}$, and thus, the error due to this approximation, i.e., $\mathtt{var}\left(\mathbf{f}_{mk}^{H}\mathbf{f}_{mk} - \mathbb{E}\left[\mathbf{f}_{mk}^{H}\mathbf{f}_{mk}\right]\right)$, known as beamforming uncertainty [1], can also be incorporated in the analysis. The UL received signal becomes

$$r_{u,mk} = \sqrt{\mathcal{E}_{u,k}}(\mathbb{E}\left[\mathbf{f}_{mk}^{H}\mathbf{f}_{mk}\right] + (\mathbf{f}_{mk}^{H}\mathbf{f}_{mk} - \mathbb{E}\left[\mathbf{f}_{mk}^{H}\mathbf{f}_{mk}\right]))s_{u,k}$$
$$+ \textstyle\sum_{k'\in\mathcal{U}_u\backslash k}\mathbf{f}_{mk}^{H}\sqrt{\mathcal{E}_{u,k'}}\mathbf{f}_{mk'}s_{u,k'} + \sqrt{N_0}\mathbf{f}_{mk}^{H}\mathbf{n}_m. \quad (11)$$

It is easy to show that the SINR-optimal combining coefficient w_{mk} is $N\sqrt{\mathcal{E}_{u,k}}\beta_{mk}/\bar{\mathtt{I}}_{u,mk}$, with $\bar{\mathtt{I}}_{u,mk} = N\sum_{k'\in\mathcal{U}_u}\mathcal{E}_{u,k'}\beta_{mk}\beta_{mk'} + NN_0\beta_{mk}$, which now includes the error due to kth UE's beamforming uncertainty. Similar analysis also follows in the case of the DL SINR. We present the modified UL and DL SINRs in the following corollary.

Corollary 1. *The UL and DL SINR of the kth UL UE and the nth DL UE can be expressed as*

$$\eta_{u,k}(\mathcal{A}_u) = \textstyle\sum_{m\in\mathcal{A}_u}\frac{N\mathcal{E}_{u,k}\beta_{mk}^{2}}{\sum_{k'\in\mathcal{U}_u}\mathcal{E}_{u,k'}\beta_{mk}\beta_{mk'} + N_0\beta_{mk}}, \quad (12a)$$

$$\eta_{d,n}(\mathcal{A}_d) \approx \textstyle\sum_{j\in\mathcal{A}_d}\frac{N\mathcal{E}_{d,n}\kappa_{jn}^{2}\beta_{jn}^{2}}{\sum_{q\in\mathcal{U}_d}\mathcal{E}_{d,q}\kappa_{jq}\beta_{jn}\beta_{jq}}, \quad (12b)$$

respectively, with the sum UL-DL SE being evaluated as (10).

2) Trained CSI (TCSI): Until now, we have considered the availability of accurate CSI at the APs. Although this is a good simplifying assumption to analyze the system behavior, it is impractical in practice. Therefore, we next consider the system performance under trained CSI.

We consider that out of the total τ channel uses per coherence interval, the first $\tau_p \geq K$ are reserved for UL channel estimation. During these τ_p channel uses, all the UEs synchronously transmit τ_p-length orthonormal pilots to the APs, which are then used by the APs to obtain local estimates of the UE-AP channels. Let $\mathcal{E}_{p,k}$ be the pilot power of kth UE's transmitted pilot sequence. It is easy to show that the minimum mean squared error (MMSE) estimate of $\mathbf{f}_{mk}$, denoted by $\hat{\mathbf{f}}_{mk}$, is distributed as $\mathcal{CN}(\mathbf{0}, \sigma_{mk}^{2}\mathbf{I}_N)$, with $\sigma_{mk}^{2} = \frac{\tau_p\mathcal{E}_{p,k}\beta_{mk}^{2}}{\tau_p\mathcal{E}_{p,k}\beta_{mk} + N_0}$. Let the estimation error, orthogonal to $\hat{\mathbf{f}}_{mk}$, be denoted by $\tilde{\mathbf{f}}_{mk}$, such that $\tilde{\mathbf{f}}_{mk} \sim \mathcal{CN}(\mathbf{0}, \bar{\sigma}_{mk}^{2}\mathbf{I}_N)$, with $\bar{\sigma}_{mk} = \sqrt{\beta_{mk} - \sigma_{mk}^{2}}$.

In this case, the signal received at the mth UL AP becomes

$$r_{u,mk} = \textstyle\sum_{k'\in\mathcal{U}_u}\hat{\mathbf{f}}_{mk}^{H}(\sqrt{\mathcal{E}_{u,k'}}\mathbf{f}_{mk'}s_{u,k'} + \sqrt{N_0}\mathbf{n}_{mk})$$
$$= \sqrt{\mathcal{E}_{u,k}}\hat{\mathbf{f}}_{mk}^{H}\hat{\mathbf{f}}_{mk}s_{u,k} + \sqrt{\mathcal{E}_{u,k}}\hat{\mathbf{f}}_{mk}^{H}\tilde{\mathbf{f}}_{mk}s_{u,k}$$
$$+ \textstyle\sum_{k'\in\mathcal{U}_u\backslash k}\sqrt{\mathcal{E}_{u,k'}}\hat{\mathbf{f}}_{mk}^{H}\mathbf{f}_{mk'}s_{u,k'} + \sqrt{N_0}\hat{\mathbf{f}}_{mk}^{H}\mathbf{n}_m,$$

Now, as derived in (3), under trained CSI, $w_{mk} = N\sqrt{\mathcal{E}_{u,k}}\sigma_{mk}^{2}/\bar{\mathtt{I}}_{u,mk}$, with $\bar{\mathtt{I}}_{u,mk} = N\mathcal{E}_{u,mk}\sigma_{mk}^{2}\bar{\sigma}_{mk}^{2} + N\sum_{k'\in\mathcal{U}_u\backslash k}\mathcal{E}_{u,k'}\sigma_{mk}^{2}\beta_{mk'} + NN_0\sigma_{mk}^{2}$. Thus, the kth stream of the accumulated signal received at the CPU becomes $\bar{r}_{u,k} = \sum_{m\in\mathcal{A}_u}w_{mk}r_{u,mk}$, which can be expanded as

$$\bar{r}_{u,k} = \textstyle\sum_{m\in\mathcal{A}_u}\sqrt{\mathcal{E}_{u,k}}w_{mk}\hat{\mathbf{f}}_{mk}^{H}(\hat{\mathbf{f}}_{mk} + \tilde{\mathbf{f}}_{mk})s_{u,k}$$
$$+ \textstyle\sum_{m\in\mathcal{A}_u}w_{mk}\hat{\mathbf{f}}_{mk}^{H}(\sum_{k'\in\mathcal{U}_u\backslash k}\sqrt{\mathcal{E}_{u,k'}}\mathbf{f}_{mk'}s_{u,k'} + \sqrt{N_0}\mathbf{n}_m).$$

The UL SINR under trained CSI can be derived following similar arguments as discussed in Theorem 1, as follows.

Lemma 1. *Under trained CSI, the UL sum SE can be expressed as* $\mathcal{R}_u(\mathcal{A}_u) = \frac{\tau-\tau_p}{\tau}\sum_{k\in\mathcal{U}_u}\log[1+\eta_{u,k}(\mathcal{A}_u)]$*, with the UL SINR of the kth UL UE, denoted as* $\eta_{u,k}(\mathcal{A}_u)$*, being*

$$= \sum_{m\in\mathcal{A}_u}\frac{N\mathcal{E}_{u,k}\sigma_{mk}^4}{\mathcal{E}_{u,k}\sigma_{mk}^2\bar{\sigma}_{mk}^2+\sum\limits_{k'\in\mathcal{U}_u\setminus k}\mathcal{E}_{u,k'}\sigma_{mk}^2\beta_{mk'}+N_0\sigma_{mk}^2}. \quad (13)$$

Similarly, considering matched filter precoding, the DL received signal at the nth UE can be written as

$$\begin{aligned} r_{d,n} =& \sum_{j\in\mathcal{A}_d} w_{jn}\kappa_{jn}\sqrt{\mathcal{E}_{d,j}}(\hat{\mathbf{f}}_{jn}+\tilde{\mathbf{f}}_{jn})^T\hat{\mathbf{f}}_{jn}^* s_{d,n} \\ &+\sum_{j\in\mathcal{A}_d} w_{jn}\mathbf{f}_{jn}^T\sum_{q\in\mathcal{U}_d\setminus n}\kappa_{jq}\sqrt{\mathcal{E}_{d,j}}\hat{\mathbf{f}}_{jq}^* s_{d,q} \\ &+\sum_{k\in\mathcal{U}_u}\mathcal{E}_{u,k}\mathbb{E}\left|g_{nk}\right|^2+\sqrt{N_0}n_n, \end{aligned} \quad (14)$$

where $w_{jn} = \kappa_{jn}\sqrt{\mathcal{E}_{d,j}}\sigma_{jn}^2/(\sum_{q\in\mathcal{U}_d\setminus n}\kappa_{jq}^2\mathcal{E}_{d,q}\sigma_{jn}^2\beta_{jq} + \kappa_{jn}^2\mathcal{E}_{d,j}\bar{\sigma}_{jn}^2)$, evaluated similarly as (7).

Lemma 2. *Under trained CSI, the DL sum SE can be expressed as* $\mathcal{R}_d(\mathcal{A}_d) = \frac{\tau-\tau_p}{\tau}\sum_{n\in\mathcal{U}_d}\log\left[1+\eta_{d,n}(\mathcal{A}_d)\right]$*, with the DL SINR of the nth DL UE being* $\eta_{d,n}(\mathcal{A}_d) \approx \sum\limits_{j\in\mathcal{A}_d}\frac{N\mathcal{E}_{d,j}\kappa_{jn}^2\sigma_{jn}^4}{\sum\limits_{q\in\mathcal{U}_d\setminus n}\kappa_{jq}^2\mathcal{E}_{d,j}\sigma_{jn}^2\beta_{jq}+\kappa_{jn}^2\mathcal{E}_{d,j}\bar{\sigma}_{jn}^2}$.

We next discuss the greedy AP scheduling technique leveraging the sub-modularity of the sum UL-DL SE.

IV. Greedy AP Mode (UL/DL) Selection

In this section, we establish the modularity of the UL and DL SINRs and the sub-modularity [10] of the sum UL-DL SE.

Theorem 3. *The UL SINR of the kth UE,* $\forall k \in \mathcal{U}_u$*, is a monotonically non-decreasing modular function of the activated AP set, i.e., given* $\mathcal{A}_s$ *and* $\mathcal{A}_t$*, where,* $\mathcal{A}_s\subseteq\mathcal{A}_t\subseteq\mathcal{A}$*, and for any* $\{j\}\notin\mathcal{A}_t$*, we have* $\eta_{u,k}(\mathcal{A}_s)\leq\eta_{u,k}(\mathcal{A}_t)$*, and* $\eta_{u,k}(\mathcal{A}_s\cup\{j\})-\eta_{u,k}(\mathcal{A}_s)=\eta_{u,k}(\mathcal{A}_t\cup\{j\})-\eta_{u,k}(\mathcal{A}_t)$*, where* $\eta_{u,k}$ *is evaluated as* (5), (12a), *and* (13) *for perfect CSI, statistical CSI, and trained CSI, respectively.*

Proof. See Appendix A. ■

Similarly, we can show that DL SINR is a monotonic non-decreasing modular function of the activated AP set.

Theorem 4. *The sum UL-DL SE, under perfect and trained CSI, is a monotonically non-decreasing sub-modular function of the activated AP set, i.e., given* $\mathcal{A}_s$ *and* $\mathcal{A}_t$*, with,* $\mathcal{A}_s\subseteq\mathcal{A}_t\subseteq\mathcal{A}$*, and for any* $\{j\}\notin\mathcal{A}_t$*,* $\mathcal{R}_s(\mathcal{A}_s)\leq\mathcal{R}_s(\mathcal{A}_t)$*, and*

$$\mathcal{R}_s(\mathcal{A}_s\cup\{j\})-\mathcal{R}_s(\mathcal{A}_s)\geq\mathcal{R}_s(\mathcal{A}_t\cup\{j\})-\mathcal{R}_s(\mathcal{A}_t), \quad (15)$$

where $\mathcal{R}_s(.)$ *is evaluated according to* (10) *with the UL and DL SEs obtained via Theorem 1 & Theorem 2 under perfect CSI; and Lemma 1 & Lemma 2 under trained CSI.*

Proof. See Appendix B. ■

We can exploit the sub-modular nature of sum UL-DL SE to schedule the APs in via the greedy algorithm presented in [2]. In each iteration of the algorithm, we activate an AP and its corresponding mode of operation such that the incremental gain in $\mathcal{R}_s$ as evaluated by (10) is maximized, and repeat the procedure until the last AP is activated. Due to the sub-modular nature of $\mathcal{R}_s$, the sum UL-DL SE achieved by the solution obtained via the greedy algorithm is guaranteed to be within a $(1-\frac{1}{e})$-fraction of its global optimal value [10] obtained via exhaustive search. We note that the complexity of exhaustive search is $O(2^M)$. However, complexity of greedy is $\mathcal{O}(M)$. Hence, whenever there is a change in the data demand, we only need to perform M iterations of the algorithm, which substantially reduces the complexity.

V. Numerical Results

Our setup is as follows. The UE locations are generated uniformly at random over a 1 km^2 area, and Monte Carlo simulations are performed over 10^4 UE locations and channel instantiations. For the CF-DTDD system, M HD-APs with N-antennas each are deployed in a uniform grid. The path-loss exponent and the reference distance from each AP are assumed to be -3.76 and 10 m, respectively [1]. The coherence interval (τ) is taken as 600 symbols, and we set $\tau_p = K$. The carrier frequency is 1.9 GHz and the signal bandwidth is 20 MHz. The UL SNR is set by fixing the noise variance N_0 to unity and varying the UL powers $\mathcal{E}_{u,k}$ such that $\mathcal{E}_{u,k}/N_0$ equals the desired value. In the DL, κ_{jn} is designed as in [1]. We consider 50% of the UEs demand UL data per time slot.[2] The acronyms used in the plots are as follows: (i) PCSI (TCSI): perfect (trained) CSI (ii) PCSI+Intf. (TCSI+Intf.): perfect (trained) CSI including inter-AP interference as well as inter-UE interference in the sum UL-DL SE evaluation.

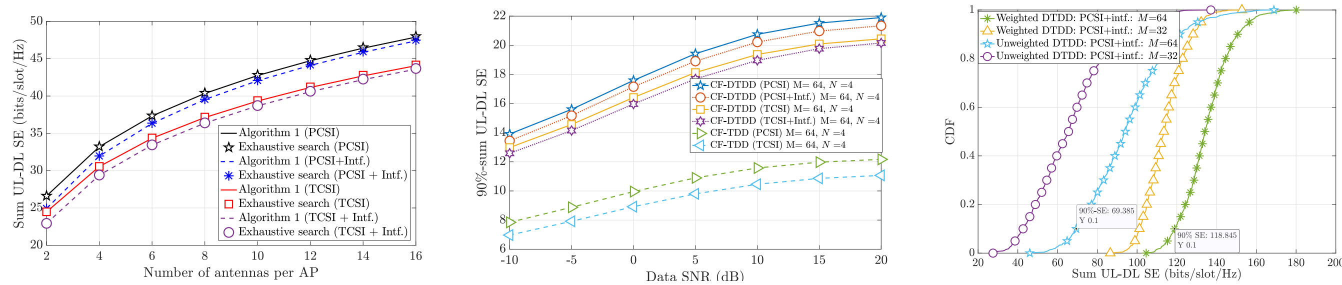
Figure 2a illustrates the near-optimality of greedy AP scheduling by comparing it with exhaustive search based AP-scheduling. The sum UL-DL SE attained via exhaustive search matches with the greedy algorithm under both perfect and trained CSI. This holds true even in the presence of inter-UE and inter-AP CLIs.[3] Also, the difference in the sum SEs with and without the CLIs is marginal, which justifies the approximations in Theorem 2.

Next, in Fig. 2b, we plot the average 90%-sum UL-DL SE versus the data SNR. Although the APs are HD in both TDD and DTDD CF-mMIMO schemes, DTDD allows simultaneous UL/DL transmission, which greatly enhances the sum UL-DL SE compared to the TDD case.

In Fig. 2c, we compare weighted combining/precoding with the approach in [2], where the APs are activated based on the sub-modularity of the product SINRs and the CPU only obtains the sum of the combined signals from the APs. To ensure that weighting does not alter the radiated power at each AP, we consider a scaled version of w_{jn}, denoted by $\dot{w}_{jn} = \sqrt{\mu_j}w_{jn}$, which ensures equal radiated power for both weighted and unweighted scheme. It is easy to show that $\mu_j = \dfrac{\sum_{q\in\mathcal{U}_d}\beta_{jq}}{\sum_{n\in\mathcal{U}_d}w_{jn}^2\beta_{jn}}$ normalizes the weights correctly. The 90%-likely SE achieved via the weighted scheme with ($M=64$) is more than double that can be attained via the

[2]Since the UE locations are random, the UL/DL traffic load at each AP is different, and for each instantiation, the APs are activated using Algo. 1 [2].

[3]For the plots corresponding to (PCSI+Intf.) and (TCSI+Intf.), we include inter-AP CLI in UL SINR to illustrate the robustness of the greedy algorithm. Specifically, we have considered imperfect inter-AP interference cancelation and modeled residual DL AP to UL AP interference as in [7].



(a) Validation of the greedy algorithm with $(M = 8, K = 16, N = 8)$.

(b) 90%-sum UL-DL SE with $(M = 64, N = 4, K = 40)$.

(c) Comparison of the weighted combiner/precoder with [2], with $K = \tau_p = 40$.

Fig. 2. Verification of greedy algorithm and performance comparisons.

unweighted scheme, which underlines the utility of weighted combining over the conventional unweighted scheme.

VI. Conclusions

In this paper, we analyzed the performance of DTDD in a CF-mMIMO system. We formulated a sum UL-DL SE maximization problem for scheduling the UL/DL mode of the APs based on the local UL/DL traffic demands of the UEs. We proved that sum UL-DL SE is a sub-modular function of the underlying AP set, and then employed a greedy algorithm to activate the APs in polynomial time. Our numerical experiments revealed that DTDD enabled CF-mMIMO substantially improves the sum SE compared to conventional TDD based CF-systems. The extension to MMSE-type precoding/combining and consideration of limited backhaul capacity are potential directions for future work.

Appendix A: Proof of Theorem 3

We first focus on the UL SE, considering perfect CSI. If $\{j\} \notin \mathcal{A}_s$ is activated in the DL, then, from (9), the UL SINR remains unchanged, i.e., $\eta_{u,k}(\mathcal{A}_s \cup \{j\}) = \eta_{u,k}(\mathcal{A}_s)$, for $\{j\} \in \mathcal{A}_d$. If jth AP is activated in the UL, then from (5)

$$\begin{aligned}\eta_{u,k}(\mathcal{A}_s \cup \{j\}) &= \textstyle\sum_{m \in \mathcal{A}_s} \frac{N\mathcal{E}_{u,k}\beta_{mk}^2}{\sum_{k' \in \mathcal{U}_u \setminus k} \mathcal{E}_{u,k'}\beta_{mk}\beta_{mk'} + N_0\beta_{mk}} \\ &\quad + \frac{N\mathcal{E}_{u,k}\beta_{jk}^2}{\sum_{k' \in \mathcal{U}_u \setminus k} \mathcal{E}_{u,k'}\beta_{jk}\beta_{jk'} + N_0\beta_{jk}} \\ &= \eta_{u,k}(\mathcal{A}_s) + \eta_{u,k}(\{j\}) > \eta_{u,k}(\mathcal{A}_s), \qquad (16)\end{aligned}$$

establishing the monotonicity. Also,

$$\begin{aligned}&\eta_{u,k}(\mathcal{A}_s \cup \{j\}) - \eta_{u,k}(\mathcal{A}_s) \\ &\quad = \begin{cases} 0, \text{ if } \{j\} \text{ operates in DL} \\ \eta_{u,k}(\{j\}), \text{ if } \{j\} \text{ operates in UL} \end{cases}, \qquad (17)\end{aligned}$$

and therefore, it is easy to see that $\eta_{u,k}(\mathcal{A}_s \cup \{j\}) - \eta_{u,k}(\mathcal{A}_s) = \eta_{u,k}(\mathcal{A}_t \cup \{j\}) - \eta_{u,k}(\mathcal{A}_t)$. Thus, the UL SINR is a modular function of the underlying activated AP set. Finally, we can easily extend the above steps for the trained CSI case.

Appendix B: Proof of Theorem 4

Recall that $\mathcal{R}_s(\mathcal{A}_s) = \mathcal{R}_s(\mathcal{A}_u) + \mathcal{R}_s(\mathcal{A}_d)$, where $\mathcal{A}_u$ and $\mathcal{A}_d$ are mutually exclusive index sets. Hence, if the UL and DL sum SEs are sub-modular functions of the index set of activated APs, then the sum UL-DL SE is also sub-modular.

From Theorem 3, the UL SINR is a monotonically non-decreasing function of the activated AP set, and since $\log(1 + x)$ is monotonically increasing for $x \geq 0$, the UL SE is also a monotonically non-decreasing function of the activated AP set. We now prove the sub-modular nature of the UL-SE. As the UL-SINR is modular, we can write $(1 + \eta_{u,k}(\mathcal{A}_s \cup \{j\})) - (1 + \eta_{u,k}(\mathcal{A}_s)) = (1 + \eta_{u,k}(\mathcal{A}_t \cup \{j\})) - (1 + \eta_{u,k}(\mathcal{A}_t))$, which implies $\frac{1+\eta_{u,k}(\mathcal{A}_s \cup \{j\})}{1+\eta_{u,k}(\mathcal{A}_s)} \geq \frac{1+\eta_{u,k}(\mathcal{A}_t \cup \{j\})}{1+\eta_{u,k}(\mathcal{A}_t)}$. Here, we use the fact that $1/(1 + \eta_{u,k}(\mathcal{A}_t)) \leq 1/(1 + \eta_{u,k}(\mathcal{A}_t))$ due to the monotonic non-decreasing nature of the SINR. Also, as $\frac{1+\eta_{u,k}(\mathcal{A}_s \cup \{j\})}{1+\eta_{u,k}(\mathcal{A}_s)}$ and $\frac{1+\eta_{u,k}(\mathcal{A}_t \cup \{j\})}{1+\eta_{u,k}(\mathcal{A}_t)}$ are both ≥ 1, using the monotonicity of $\log(\cdot)$, we have

$$\begin{aligned}&\log(1 + \eta_{u,k}(\mathcal{A}_s \cup \{j\})) - \log(1 + \eta_{u,k}(\mathcal{A}_s)) \\ &\geq \log(1 + \eta_{u,k}(\mathcal{A}_t \cup \{j\})) - \log(1 + \eta_{u,k}(\mathcal{A}_t)), \qquad (18)\end{aligned}$$

which establishes the sub-modularity of UL SE of kth UE, $\forall k \in \mathcal{U}_u$. We can similarly prove the sub-modularity of the DL SINR, and as the linear sum of sub-modular functions is sub-modular [10], Theorem 4 holds true.

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