# Supplemental Material for "On the Performance of Distributed Antenna Array Systems with Quasi-Orthogonal Pilots"

Anubhab Chowdhury*, Pradip Sasmal*, Chandra R. Murthy*, and Ribhu Chopra†

## I. Introduction

In this document, we present an unified minimum mean square error (MMSE) estimator for uplink training for distributed antenna array (DAA) massive multiple-input-multiple-output (DAA-mMIMO) systems [1], which can be generalized to orthogonal pilot reusing (OPR) as well mutually unbiased orthonormal basis (MUOB)-based pilot codebooks [2]. We then provide the detailed steps involved in the derivation of the uplink and downlink spectral efficiencies (SEs).

## II. Chanel estimation

We consider a TDD DAA-mMIMO MIMO system consisting of $M$ APs equipped with $N$ antennas each. The APs jointly serve $K$ single antenna UEs. The channel vector between the $m$th AP and $k$th the UE is modeled as $\mathbf{h}_{mk} = \sqrt{\beta_{mk}}\mathbf{f}_{mk} \in \mathbb{C}^N$, where the pathloss component $\beta_{mk}$ is assumed to be constant for several coherence blocks, and the fast fading channel, $\mathbf{f}_{mk} \sim \mathcal{CN}(\mathbf{0}, \mathbf{I}_N)$, is to be estimated at the start of each coherence interval. Let $\mathcal{U} = \{1, 2, \ldots, K\}$ be the index set of all UEs, and the corresponding pilot sequences be $\mathbf{\Phi} \triangleq \{\boldsymbol{\varphi}_1, \boldsymbol{\varphi}_2, \ldots, \boldsymbol{\varphi}_K\}$. We consider the use of pilots of length $\tau_p$.

Let, the $k$th UE transmits a pilot signal $\boldsymbol{\varphi}_k$ with an energy $\mathcal{E}_{p,k}$, then the received signal at the $m$th AP can be expressed as

$$\mathbf{Y}_{p,m} = \sqrt{\mathcal{E}_{p,k}\tau_p}\mathbf{h}_{mk}\boldsymbol{\varphi}_k^T + \sum_{i\in\mathcal{U}\setminus\{k\}} \sqrt{\mathcal{E}_{p,i}\tau_p}\mathbf{h}_{mi}\boldsymbol{\varphi}_i^T + \mathbf{W}_{p,m} \in \mathbb{C}^{N\times\tau_p}, \quad (1)$$

where, each columns of $\mathbf{W}_{p,m}$ is distributed as $\mathcal{CN}(\mathbf{0}, N_0\mathbf{I}_N)$. Now, to estimate the $k$th UE's channel, the $m$th AP post-multiply (1) with $\boldsymbol{\varphi}_k^*$, and the processed becomes

$$\mathbf{y}_{p,m} = \mathbf{Y}_{p,m}\boldsymbol{\varphi}_k^* = \sqrt{\mathcal{E}_{p,k}\tau_p}\mathbf{h}_{mk} + \sum_{i\in\mathcal{U}\setminus\{k\}} \sqrt{\mathcal{E}_{p,i}\tau_p}\langle\boldsymbol{\varphi}_i, \boldsymbol{\varphi}_k\rangle\mathbf{h}_{mi} + \mathbf{W}_{p,m}\boldsymbol{\varphi}_k^* \in \mathbb{C}^{N\times 1}, \quad (2)$$

with $\mathbf{W}_{p,m}\boldsymbol{\varphi}_k^* \sim \mathcal{CN}(\mathbf{0}, N_0\mathbf{I}_N)$. The MMSE estimate of the $k$th UE's channel at the $m$th AP, denoted by $\hat{\mathbf{h}}_{mk}$, can be evaluated as [3]

$$\hat{\mathbf{h}}_{mk} = \frac{\mathbb{E}\left[\mathbf{h}_{mk}^H\mathbf{y}_{p,m}\right]}{\mathbb{E}\left[\mathbf{y}_{p,m}^H\mathbf{y}_{p,m}\right]}\mathbf{y}_{p,m}$$

$$= \frac{\sqrt{\mathcal{E}_{p,k}\tau_p}\beta_{mk}}{N_0 + \mathcal{E}_{p,k}\beta_{mk}\tau_p + \sum_{i\in\mathcal{U}\setminus\{k\}} \mathcal{E}_{p,i}\tau_p\beta_{mi}\left|\langle\boldsymbol{\varphi}_i, \boldsymbol{\varphi}_k\rangle\right|^2}\mathbf{y}_{p,m} \quad (3)$$

Also, we can write, $\hat{\mathbf{h}}_{mk} = \mathbf{h}_{mk} - \tilde{\mathbf{h}}_{mk}$, where, $\tilde{\mathbf{h}}_{mk} \sim \mathcal{CN}(\mathbf{0}, (\beta_{mk} - \sigma_{mk}^2)\mathbf{I}_N)$, with $\sigma_{mk}^2 = \dfrac{\mathcal{E}_{p,k}\beta_{mk}^2\tau_p}{N_0 + \mathcal{E}_{p,k}\beta_{mk}\tau_p + \sum_{i\in\mathcal{U}\setminus\{k\}} \mathcal{E}_{p,i}\beta_{mi}\tau_p\left|\langle\boldsymbol{\varphi}_i, \boldsymbol{\varphi}_k\rangle\right|^2}$.

Letting, $\texttt{Cont}_{mk} \triangleq \sum_{i\in\mathcal{U}\setminus\{k\}} \mathcal{E}_{p,i}\beta_{mi}\tau_p\left|\langle\boldsymbol{\varphi}_i, \boldsymbol{\varphi}_k\rangle\right|^2$, we can show that that [2]

$$\texttt{Cont}_{\cdot k} = \begin{cases} \sum_{j\in\mathcal{U}\setminus\{\mathcal{O}_k\cup k\}} \mathcal{E}_{p,j}\beta_{mj}, & \mathbf{\Phi} \in \text{MUOB} \\ \sum_{j \text{ s.t.} \langle\boldsymbol{\varphi}_k, \boldsymbol{\varphi}_j\rangle = 1} \tau_p\mathcal{E}_{p,j}\beta_{mj}, & \mathbf{\Phi} \in \text{OPR} \end{cases} \quad (4)$$

which is the pilot contamination experienced by the $k$th UE.

## III. Uplink and Downlink Data Processing

We now analyze the effect of pilot contamination on the system throughput. Our analysis applies for any random pilot-codebook.

### A. Uplink

Let the $k$th UE transmit the symbol $s_{u,k}$ ($\mathbb{E}[|s_{u,k}|^2] = 1$) in the uplink with an energy of $\mathcal{E}_{u,k}$. Let $\mathcal{A}_k$ be the set of AP indices that jointly and coherently processes the $k$th UE's signal. After maximal ratio combining at those APs, the $k$th stream of the accumulated received signal at the CPU becomes

$$r_{u,k} = \sqrt{\mathcal{E}_{u,k}} \sum_{m\in\mathcal{A}_k} \mathbb{E}\left[\hat{\mathbf{h}}_{mk}^H\mathbf{h}_{mk}\right] s_{u,k} + \sqrt{\mathcal{E}_{u,k}} \sum_{m\in\mathcal{A}_k} \left\{\hat{\mathbf{h}}_{mk}^H\mathbf{h}_{mk} - \mathbb{E}[\hat{\mathbf{h}}_{mk}^H\mathbf{h}_{mk}]\right\} s_{u,k} + \sum_{i\in\mathcal{U}\setminus\{k\}} \sqrt{\mathcal{E}_{u,i}} \sum_{m\in\mathcal{A}_k} \hat{\mathbf{h}}_{mk}^H\mathbf{h}_{mi}s_{u,i} + \sum_{m\in\mathcal{A}_k} \hat{\mathbf{h}}_{mk}^H\mathbf{w}_m, \quad (5)$$

where, $\mathbf{w}_m \sim \mathcal{CN}(\mathbf{0}, N_0\mathbf{I}_N)$ is the receiver noise added at the $m$th AP. The first and second term of (5) are commonly termed as array gain and beamforming uncertainty [4], respectively. Now, applying the use-and-then-forget technique [Chapter. 3, [3]], the uplink SE of $k$th UE can be expressed[1] as $\lambda(1-$

*The authors are with the Dept. of ECE, Indian Institute of Science, Bangalore, India. (e-mails:{anubhabc, pradipsasmal, cmurthy}@iisc.ac.in.)

†Ribhu Chopra is with the Dept. of EEE, Indian Institute of Technology Guwahati, India (e-mail: ribhufec@iitg.ac.in.)

[1]For a coherence interval of $\tau$, we equally partition duration of $(\tau - \tau_p)$ channel uses for uplink and downlink link data transmission. Thus, the pre-log factor $\lambda(1 - \frac{\tau_p}{\tau})$ for both uplink implies a fraction $\lambda$ ($\lambda \in [0, 1]$) of the data transmission duration is alloted for uplink.

$\frac{\tau_p}{\tau})\log_2(1+\gamma_k^u)$, where, $\gamma_k^u$ is given by (6), and the closed form expression is evaluated in the following lemma.

**Lemma 1.** *In the uplink, the signal-to-interference-plus-noise ratio (SINR) of the kth UE can written as*

$$\gamma_k^u = \frac{\mathcal{E}_{u,k}\mathtt{Gain}_{u,k}}{\mathcal{E}_{u,k}\mathtt{var}\left(\sum_{m\in\mathcal{A}_k}\hat{\mathbf{h}}_{mk}^H\mathbf{h}_{mk}\right)+\sum_{\substack{i\in\\ \mathcal{U}\setminus\{k\}}}\mathcal{E}_{u,i}\mathtt{I}_{ik}+N_0\sum_{m\in\mathcal{A}_k}\sigma_{mk}^2}, \tag{7}$$

*where,*

$$\mathtt{Gain}_{u,k} = N^2\left(\sum_{m\in\mathcal{A}_k}\sigma_{mk}^2\right)^2, \tag{8a}$$

$$\mathtt{var}\left(\sum_{m\in\mathcal{A}_k}\hat{\mathbf{h}}_{mk}^H\mathbf{h}_{mk}\right) = \sum_{m\in\mathcal{A}_k}N\sigma_{mk}^2\beta_{mk}, \tag{8b}$$

$$\mathtt{I}_{ik} = N^2\left(\sum_{m\in\mathcal{A}_k}\sigma_{mk}^2\sqrt{\frac{\mathcal{E}_{p,i}}{\mathcal{E}_{p,k}}\frac{\beta_{mi}}{\beta_{mk}}}\right)^2|\langle\boldsymbol{\varphi}_i,\boldsymbol{\varphi}_k\rangle|^2 + N\sum_{m\in\mathcal{A}_k}\sigma_{mk}^2\beta_{mi}. \tag{8c}$$

*Proof.* The array gain in (6), can be written as

$$\mathbb{E}\left[\sum_{m\in\mathcal{A}_k}\hat{\mathbf{h}}_{mk}^H\mathbf{h}_{mk}\right] = \mathbb{E}\left[\sum_{m\in\mathcal{A}_k}\hat{\mathbf{h}}_{mk}^H\left[\hat{\mathbf{h}}_{mk}^H+\tilde{\mathbf{h}}_{mk}\right]\right] = \sum_{m\in\mathcal{A}_k}\mathbb{E}\|\hat{\mathbf{h}}_{mk}\|^2 = N\sum_{m\in\mathcal{A}_k}\sigma_{mk}^2$$

Thus, the numerator of (6), becomes

$$\mathcal{E}_{u,k}\left|\mathbb{E}[\sum_{m\in\mathcal{A}_k}\hat{\mathbf{h}}_{mk}^H\mathbf{h}_{mk}]\right|^2 = \mathcal{E}_{u,k}N^2\left(\sum_{m\in\mathcal{A}_k}\sigma_{mk}^2\right)^2, \tag{9}$$

which corroborates with (8a). Next,

$$\begin{aligned}
&\mathtt{var}\left(\sum_{m\in\mathcal{A}_k}\hat{\mathbf{h}}_{mk}^H\mathbf{h}_{mk}\right)\\
&= \mathbb{E}\left[\left|\sum_{m\in\mathcal{A}_k}\hat{\mathbf{h}}_{mk}^H\mathbf{h}_{mk} - \sum_{m\in\mathcal{A}_k}\mathbb{E}\left[\hat{\mathbf{h}}_{mk}^H\mathbf{h}_{mk}\right]\right|^2\right]\\
&\overset{(a)}{=} \sum_{m\in\mathcal{A}_k}\mathbb{E}\left[\left|\hat{\mathbf{h}}_{mk}^H\mathbf{h}_{mk} - \mathbb{E}\left[\hat{\mathbf{h}}_{mk}^H\mathbf{h}_{mk}\right]\right|^2\right]\\
&= \sum_{m\in\mathcal{A}_k}\left\{\mathbb{E}\left[\left|\hat{\mathbf{h}}_{mk}^H\mathbf{h}_{mk}\right|^2\right] - \left|\mathbb{E}\left[\hat{\mathbf{h}}_{mk}^H\mathbf{h}_{mk}\right]\right|^2\right\}\\
&= \sum_{m\in\mathcal{A}_k}\left\{\mathbb{E}\left[\left|\hat{\mathbf{h}}_{mk}^H\tilde{\mathbf{h}}_{mk} + \|\hat{\mathbf{h}}_{mk}\|^2\right|^2\right] - \left|\mathbb{E}\|\hat{\mathbf{h}}_{mk}\|^2\right|^2\right\}\\
&\overset{(b)}{=} \sum_{m\in\mathcal{A}_k}\left\{\mathbb{E}\left[\left|\hat{\mathbf{h}}_{mk}^H\tilde{\mathbf{h}}_{mk}\right|^2\right] + \mathbb{E}\left[\|\hat{\mathbf{h}}_{mk}\|^4\right] - N^2\sigma_{mk}^4\right\}\\
&\overset{(c)}{=} \sum_{m\in\mathcal{A}_k}\left\{N\sigma_{mk}^2(\beta_{mk}-\sigma_{mk}^2) + N(N+1)\sigma_{mk}^4 - N^2\sigma_{mk}^4\right\}\\
&= \sum_{m\in\mathcal{A}_k}N\sigma_{mk}^2\beta_{mk},
\end{aligned} \tag{10}$$

wherein, $(a)$ follows as the variance of sum of independent random variables are sum of the respective variances. In $(b)$, we note that $\mathbb{E}\left[\tilde{\mathbf{h}}_{mk}\right] = \mathbf{0}$ and is independent of $\hat{\mathbf{h}}_{mk}$, and therefore, apply Lemma. 5. Finally, $(c)$ is obtained using (38c). Thus, (8b) follows directly.

Now, we derive the multi-user interference term. Prior to that, let us define the denominator of (3) as

$$d_{mk}^{-1} = N_0 + \mathcal{E}_{p,k}\beta_{mk}\tau_p + \sum_{i\in\mathcal{U}\setminus\{k\}}\mathcal{E}_{p,i}\tau_p\beta_{mi}\left|\langle\boldsymbol{\varphi}_i,\boldsymbol{\varphi}_k\rangle\right|^2, \tag{11}$$

and thus, $\hat{\mathbf{h}}_{mk} = \sqrt{\mathcal{E}_{p,k}\tau_p}\beta_{mk}d_{mk}\mathbf{y}_{p,m}$. For $i\neq k$, we can write,

$$\begin{aligned}
\mathtt{I}_{ik} &= \mathbb{E}\left[\left|\sum_{m\in\mathcal{A}_k}\hat{\mathbf{h}}_{mk}^H\mathbf{h}_{mi}\right|^2\right]\\
&= \mathbb{E}\left[\left|\sum_{m\in\mathcal{A}_k}\sqrt{\mathcal{E}_{p,k}\tau_p}\beta_{mk}d_{mk}\mathbf{y}_{p,m}^H\mathbf{h}_{mi}\right|^2\right]\\
&= \mathbb{E}\Bigg[\Bigg|\sum_{m\in\mathcal{A}_k}\sqrt{\mathcal{E}_{p,k}\tau_p}\beta_{mk}d_{mk}\times\\
&\qquad\left(\sum_{i'\in\mathcal{U}}\sqrt{\mathcal{E}_{p,i'}\tau_p}\langle\boldsymbol{\varphi}_{i'},\boldsymbol{\varphi}_k\rangle\mathbf{h}_{mi'} + \mathbf{W}_{p,m}\boldsymbol{\varphi}_k^*\right)^H\mathbf{h}_{mi}\Bigg|^2\Bigg]\\
&= \mathbb{E}\left[\left|\sum_{m\in\mathcal{A}_k}\sqrt{\mathcal{E}_{p,k}\tau_p}\beta_{mk}d_{mk}\boldsymbol{\varphi}_k^T\mathbf{W}_{p,m}^H\mathbf{h}_{mi}\right|^2\right]\\
&+ \mathbb{E}\Bigg[\Bigg|\sum_{m\in\mathcal{A}_k}\sqrt{\mathcal{E}_{p,k}\tau_p}\beta_{mk}d_{mk}\times\\
&\qquad\left(\sum_{i'\in\mathcal{U}}\sqrt{\mathcal{E}_{p,i'}\tau_p}\langle\boldsymbol{\varphi}_{i'},\boldsymbol{\varphi}_k\rangle\mathbf{h}_{mi'}\right)^H\mathbf{h}_{mi}\Bigg|^2\Bigg]\\
&= NN_0\sum_{m\in\mathcal{A}_k}\mathcal{E}_{p,k}\tau_p d_{mk}^2\beta_{mk}^2\beta_{mi} + \mathtt{I}_1,
\end{aligned} \tag{12}$$

where $\mathtt{I}_1$ being the second expectation term involved in (12), and can be further manipulated as shown in (14), with

$$\mathtt{I}_2 \triangleq \mathbb{E}\left[\left|\sum_{m\in\mathcal{A}_k}\sqrt{\mathcal{E}_{p,k}\tau_p}\beta_{mk}d_{mk}\sqrt{\mathcal{E}_{p,i}\tau_p}\langle\boldsymbol{\varphi}_i,\boldsymbol{\varphi}_k\rangle\|\mathbf{h}_{mi}\|^2\right|^2\right]. \tag{13}$$

Next, we expand $\mathtt{I}_2$ as shown in (15). Now, the first term of (15) can be re-written as

$$\begin{aligned}
&N^2\left(\sum_{m\in\mathcal{A}_k}\sqrt{\mathcal{E}_{p,k}\tau_p}d_{mk}\beta_{mk}\sqrt{\mathcal{E}_{p,i}\tau_p}\beta_{mi}\right)^2|\langle\boldsymbol{\varphi}_i,\boldsymbol{\varphi}_k\rangle|^2\\
&= N^2\left(\sum_{m\in\mathcal{A}_k}\mathcal{E}_{p,k}\tau_p d_{mk}\beta_{mk}^2\sqrt{\frac{\mathcal{E}_{p,i}\tau_p}{\mathcal{E}_{p,k}\tau_p}\frac{\beta_{mi}}{\beta_{mk}}}\right)^2|\langle\boldsymbol{\varphi}_i,\boldsymbol{\varphi}_k\rangle|^2\\
&= N^2\left(\sum_{m\in\mathcal{A}_k}\sigma_{mk}^2\sqrt{\frac{\mathcal{E}_{p,i}}{\mathcal{E}_{p,k}}\frac{\beta_{mi}}{\beta_{mk}}}\right)^2|\langle\boldsymbol{\varphi}_i,\boldsymbol{\varphi}_k\rangle|^2,
\end{aligned} \tag{16}$$

$$\gamma_k^u = \frac{\mathcal{E}_{u,k}\left|\sum_{m\in\mathcal{A}_k}\mathbb{E}[\hat{\mathbf{h}}_{mk}^H\mathbf{h}_{mk}]\right|^2}{\mathcal{E}_{u,k}\texttt{var}\left(\sum_{m\in\mathcal{A}_k}\hat{\mathbf{h}}_{mk}^H\mathbf{h}_{mk}\right)+\sum\limits_{i\in\mathcal{U}\backslash\{k\}}\mathcal{E}_{u,i}\mathbb{E}\left[\left|\sum_{m\in\mathcal{A}_k}\hat{\mathbf{h}}_{mk}^H\mathbf{h}_{mi}\right|^2\right]+N_0\sum_{m\in\mathcal{A}_k}\mathbb{E}\|\hat{\mathbf{h}}_{mk}\|^2}. \tag{6}$$

$$\begin{aligned}
\mathtt{I}_1 &= \mathbb{E}\left[\Big|\sum_{m\in\mathcal{A}_k}\sqrt{\mathcal{E}_{p,k}\tau_p}\beta_{mk}d_{mk}\left(\sum_{i'\in\mathcal{U}\backslash\{i\}}\sqrt{\mathcal{E}_{p,i'}\tau_p}\langle\boldsymbol{\varphi}_{i'},\boldsymbol{\varphi}_k\rangle\mathbf{h}_{mi'}+\sqrt{\mathcal{E}_{p,i}\tau_p}\langle\boldsymbol{\varphi}_i,\boldsymbol{\varphi}_k\rangle\mathbf{h}_{mi}\right)^H\mathbf{h}_{mi}\Big|^2\right]\\
&\overset{(b)}{=}\mathbb{E}\left[\Big|\sum_{m\in\mathcal{A}_k}\sqrt{\mathcal{E}_{p,k}\tau_p}\beta_{mk}d_{mk}\sqrt{\mathcal{E}_{p,i}\tau_p}\langle\boldsymbol{\varphi}_i,\boldsymbol{\varphi}_k\rangle\|\mathbf{h}_{mi}\|^2\Big|^2\right]+\mathbb{E}\left[\Big|\sum_{m\in\mathcal{A}_k}\sqrt{\mathcal{E}_{p,k}\tau_p}\beta_{mk}d_{mk}\left(\sum_{i'\in\mathcal{U}\backslash\{i\}}\sqrt{\mathcal{E}_{p,i'}\tau_p}\langle\boldsymbol{\varphi}_{i'},\boldsymbol{\varphi}_k\rangle\mathbf{h}_{mi'}\right)^H\mathbf{h}_{mi}\Big|^2\right]\\
&=\mathtt{I}_2+\underbrace{N\sum_{m\in\mathcal{A}_k}\sum_{i'\in\mathcal{U}\backslash\{i\}}\left\{\mathcal{E}_{p,k}\tau_p\beta_{mk}^2d_{mk}^2\right\}\left\{\mathcal{E}_{p,i'}\tau_p\beta_{mi}\beta_{mi'}\right\}|\langle\boldsymbol{\varphi}_{i'},\boldsymbol{\varphi}_k\rangle|^2}_{\triangleq\mathtt{I}_3}
\end{aligned}\tag{14}$$

$(b)$: Using (39).

$$\begin{aligned}
\mathtt{I}_2 &= \sum_{m\in\mathcal{A}_k}\mathcal{E}_{p,k}\mathcal{E}_{p,i}\tau_p^2d_{mk}^2\beta_{mk}^2|\langle\boldsymbol{\varphi}_i,\boldsymbol{\varphi}_k\rangle|^2\mathbb{E}\left[\|\mathbf{h}_{mi}\|^4\right]+\mathcal{E}_{p,k}\mathcal{E}_{p,i}\tau_p^2|\langle\boldsymbol{\varphi}_i,\boldsymbol{\varphi}_k\rangle|^2\mathbb{E}\left[\sum_{m\in\mathcal{A}_k}\sum_{\substack{n\in\mathcal{A}_k,\\ m\neq n}}d_{mk}d_{nk}\beta_{mk}\beta_{nk}\|\mathbf{h}_{mi}\|^2\|\mathbf{h}_{ni}\|^2\right]\\
&=N(N+1)\sum_{m\in\mathcal{A}_k}\mathcal{E}_{p,k}\mathcal{E}_{p,i}\tau_p^2d_{mk}^2\beta_{mk}^2|\langle\boldsymbol{\varphi}_i,\boldsymbol{\varphi}_k\rangle|^2\beta_{mi}^2+N^2\mathcal{E}_{p,k}\mathcal{E}_{p,i}\tau_p^2|\langle\boldsymbol{\varphi}_i,\boldsymbol{\varphi}_k\rangle|^2\sum_{m\in\mathcal{A}_k}\sum_{\substack{n\in\mathcal{A}_k,\\ m\neq n}}d_{mk}d_{nk}\beta_{mk}\beta_{nk}\beta_{mi}\beta_{ni}\\
&=N^2\left(\sum_{m\in\mathcal{A}_k}\mathcal{E}_{p,k}\mathcal{E}_{p,i}\tau_p^2d_{mk}^2\beta_{mk}^2|\langle\boldsymbol{\varphi}_i,\boldsymbol{\varphi}_k\rangle|^2\beta_{mi}^2+\mathcal{E}_{p,k}\mathcal{E}_{p,i}\tau_p^2|\langle\boldsymbol{\varphi}_i,\boldsymbol{\varphi}_k\rangle|^2\sum_{m\in\mathcal{A}_k}\sum_{\substack{n\in\mathcal{A}_k,\\ m\neq n}}d_{mk}d_{nk}\beta_{mk}\beta_{nk}\beta_{mi}\beta_{ni}\right)\\
&\qquad+\underbrace{N\sum_{m\in\mathcal{A}_k}\mathcal{E}_{p,k}\mathcal{E}_{p,i}\tau_p^2d_{mk}^2\beta_{mk}^2|\langle\boldsymbol{\varphi}_i,\boldsymbol{\varphi}_k\rangle|^2\beta_{mi}^2}_{\triangleq\mathtt{I}_4}\\
&=N^2\left(\sum_{m\in\mathcal{A}_k}\sqrt{\mathcal{E}_{p,k}\tau_p}d_{mk}\beta_{mk}\sqrt{\mathcal{E}_{p,i}\tau_p}\beta_{mi}\right)^2|\langle\boldsymbol{\varphi}_i,\boldsymbol{\varphi}_k\rangle|^2+\mathtt{I}_4.
\end{aligned}\tag{15}$$

which contributes to coherent interference. Thus, now using (14), (15), and (16), we have

$$\mathtt{I}_1 = N^2\left(\sum_{m\in\mathcal{A}_k}\sigma_{mk}^2\sqrt{\frac{\mathcal{E}_{p,i}}{\mathcal{E}_{p,k}}}\frac{\beta_{mi}}{\beta_{mk}}\right)^2|\langle\boldsymbol{\varphi}_i,\boldsymbol{\varphi}_k\rangle|^2 + \mathtt{I}_4+\mathtt{I}_3. \tag{17}$$

Now, we will simplify the second term of (14) as

$$\begin{aligned}
\mathtt{I}_3 &= N\sum_{m\in\mathcal{A}_k}\sum_{\substack{i'\in\\ \mathcal{U}\backslash\{i\}}}\{\mathcal{E}_{p,k}\tau_p\beta_{mk}^2d_{mk}^2\}\{\mathcal{E}_{p,i'}\tau_p\beta_{mi}\beta_{mi'}\}|\langle\boldsymbol{\varphi}_{i'},\boldsymbol{\varphi}_k\rangle|^2\\
&= N\sum_{m\in\mathcal{A}_k}\mathcal{E}_{p,k}\tau_p\beta_{mk}^2d_{mk}^2\beta_{mi}\times\left(\sum_{i'\in\mathcal{U}}\mathcal{E}_{p,i'}\tau_p\beta_{mi'}|\langle\boldsymbol{\varphi}_{i'},\boldsymbol{\varphi}_k\rangle|^2\right.\\
&\qquad\left.-\mathcal{E}_{p,i}\tau_p\beta_{mi}|\langle\boldsymbol{\varphi}_i,\boldsymbol{\varphi}_k\rangle|^2\right)\\
&=\underbrace{N\sum_{m\in\mathcal{A}_k}\mathcal{E}_{p,k}\tau_p\beta_{mk}^2d_{mk}^2\beta_{mi}\sum_{i'\in\mathcal{U}}\mathcal{E}_{p,i'}\tau_p\beta_{mi'}|\langle\boldsymbol{\varphi}_{i'},\boldsymbol{\varphi}_k\rangle|^2}_{\triangleq\mathtt{I}_5}\\
&\quad-N\sum_{m\in\mathcal{A}_k}\mathcal{E}_{p,k}\mathcal{E}_{p,i}\tau_p^2d_{mk}^2\beta_{mk}^2\beta_{mi}^2|\langle\boldsymbol{\varphi}_{i'},\boldsymbol{\varphi}_k\rangle|^2.
\end{aligned}\tag{18}$$

Then,

$$\mathtt{I}_5 = N\sum_{m\in\mathcal{A}_k}\mathcal{E}_{p,k}\tau_p\beta_{mk}^2d_{mk}^2\beta_{mi}\times\left(\sum_{i'\in\mathcal{U}}\mathcal{E}_{p,i'}\tau_p\beta_{mi'}|\langle\boldsymbol{\varphi}_{i'},\boldsymbol{\varphi}_k\rangle|^2\right), \tag{19}$$

and we also observe here from (11) that

$$\sum_{i'\in\mathcal{U}} \mathcal{E}_{p,i'}\tau_p\beta_{mi'}|\langle\boldsymbol{\varphi}_{i'},\boldsymbol{\varphi}_k\rangle|^2 = \left(\frac{1}{d_{mk}} - N_0\right), \tag{20}$$

which when substituted back in (19) results in

$$\begin{aligned}\mathtt{I}_5 =& N\sum_{m\in\mathcal{A}_k}\mathcal{E}_{p,k}\tau_p\beta_{mk}^2 d_{mk}\beta_{mi}\\ &- NN_0\sum_{m\in\mathcal{A}_k}\mathcal{E}_{p,k}\tau_p\beta_{mk}^2 d_{mk}^2\beta_{mi}.\end{aligned} \tag{21}$$

Therefore, inserting (21) into (18),

$$\begin{aligned}\mathtt{I}_3 =& N\sum_{m\in\mathcal{A}_k}\mathcal{E}_{p,k}\tau_p\beta_{mk}^2 d_{mk}\beta_{mi}\\ &- NN_0\sum_{m\in\mathcal{A}_k}\mathcal{E}_{p,k}\tau_p\beta_{mk}^2 d_{mk}^2\beta_{mi}\\ &- N\sum_{m\in\mathcal{A}_k}\mathcal{E}_{p,k}\mathcal{E}_{p,i}\tau_p^2 d_{mk}^2\beta_{mk}^2\beta_{mi}^2|\langle\boldsymbol{\varphi}_{i'},\boldsymbol{\varphi}_k\rangle|^2\end{aligned} \tag{22}$$

Now, substituting (17) into (12), we get

$$\begin{aligned}\mathtt{I}_{ik} =& NN_0\sum_{m\in\mathcal{A}_k}\mathcal{E}_{p,k}\tau_p d_{mk}^2\beta_{mk}^2\beta_{mi}\\ &+ N^2\left(\sum_{m\in\mathcal{A}_k}\sigma_{mk}^2\sqrt{\frac{\mathcal{E}_{p,i}}{\mathcal{E}_{p,k}}\frac{\beta_{mi}}{\beta_{mk}}}\right)^2|\langle\boldsymbol{\varphi}_i,\boldsymbol{\varphi}_k\rangle|^2 + \mathtt{I}_4 + \mathtt{I}_3.\end{aligned} \tag{23}$$

Next, substituting for $\mathtt{I}_4$ and $\mathtt{I}_3$, we get,

$$\begin{aligned}\mathtt{I}_{ik} =& NN_0\sum_{m\in\mathcal{A}_k}\mathcal{E}_{p,k}\tau_p d_{mk}^2\beta_{mk}^2\beta_{mi}\\ &+ N^2\left(\sum_{m\in\mathcal{A}_k}\sigma_{mk}^2\sqrt{\frac{\mathcal{E}_{p,i}}{\mathcal{E}_{p,k}}\frac{\beta_{mi}}{\beta_{mk}}}\right)^2|\langle\boldsymbol{\varphi}_i,\boldsymbol{\varphi}_k\rangle|^2\\ &+ N\sum_{m\in\mathcal{A}_k}\mathcal{E}_{p,k}\mathcal{E}_{p,i}\tau_p^2 d_{mk}^2\beta_{mk}^2|\langle\boldsymbol{\varphi}_i,\boldsymbol{\varphi}_k\rangle|^2\beta_{mi}^2\\ &+ N\sum_{m\in\mathcal{A}_k}\mathcal{E}_{p,k}\tau_p\beta_{mk}^2 d_{mk}\beta_{mi}\\ &- NN_0\sum_{m\in\mathcal{A}_k}\mathcal{E}_{p,k}\tau_p\beta_{mk}^2 d_{mk}^2\beta_{mi}\\ &- N\sum_{m\in\mathcal{A}_k}\mathcal{E}_{p,k}\mathcal{E}_{p,i}\tau_p^2 d_{mk}^2\beta_{mk}^2\beta_{mi}^2|\langle\boldsymbol{\varphi}_{i'},\boldsymbol{\varphi}_k\rangle|^2,\end{aligned} \tag{24}$$

and, finally,

$$\begin{aligned}\mathtt{I}_{ik} =& N^2\left(\sum_{m\in\mathcal{A}_k}\sigma_{mk}^2\sqrt{\frac{\mathcal{E}_{p,i}}{\mathcal{E}_{p,k}}\frac{\beta_{mi}}{\beta_{mk}}}\right)^2|\langle\boldsymbol{\varphi}_i,\boldsymbol{\varphi}_k\rangle|^2\\ &+ N\sum_{m\in\mathcal{A}_k}\underbrace{\mathcal{E}_{p,k}\tau_p\beta_{mk}^2 d_{mk}}_{\sigma_{mk}^2}\beta_{mi}.\end{aligned} \tag{25}$$

Lastly, the additive noise component of (7) trivially follows as $\hat{\mathbf{h}}_{mk} \sim \mathcal{CN}(\mathbf{0}, \sigma_{mk}^2\mathbf{I}_N)$. ■

*B. Downlink*

Next, let $s_{d,k}$ be intended downlink signal for the $k$th UE. Let, $\mathcal{E}_{d,m}$ be the total power budget of $m$th AP and the corresponding power control coefficient $\zeta_{mk}$ decides what fraction of power is intended for the $k$th UE. We employ reciprocity based matched filter precoding in the downlink. Now, the $m$th AP serves only a cluster of users indicated by the set $\tilde{\mathcal{U}}_m$, and therefore, the downlink transmitted signal by the $m$th AP can be expressed as

$$\mathbf{r}_{d,m} = \sum_{i\in\tilde{\mathcal{U}}_m}\sqrt{\mathcal{E}_{d,m}\zeta_{mi}}\hat{\mathbf{h}}_{mi}^* s_{d,i}. \tag{26}$$

Thus, the received signal at the $k$th UE can be expressed as

$$\begin{aligned}r_{d,k} &= \sum_{m=1}^{M}\mathbf{h}_{mk}^T\mathbf{r}_{d,m} + w_k\\ &= \sum_{m=1}^{M}\sum_{i\in\tilde{\mathcal{U}}_m}\sqrt{\mathcal{E}_{d,m}\zeta_{mi}}\mathbf{h}_{mk}^T\hat{\mathbf{h}}_{mi}^* s_{d,k} + w_k\\ &= \sum_{m\in\mathcal{A}_k}\sqrt{\mathcal{E}_{d,m}\zeta_{mk}}\mathbf{h}_{mk}^T\hat{\mathbf{h}}_{mk}^* s_{d,k}\\ &\quad+ \sum_{i\in\mathcal{U}\setminus\{k\}}\sum_{m\in\mathcal{A}_i}\sqrt{\mathcal{E}_{d,m}\zeta_{mi}}\mathbf{h}_{mk}^T\mathbf{h}_{mi}^* s_{d,i} + w_k,\end{aligned} \tag{27}$$

where, $w_k \sim \mathcal{CN}(0, N_0)$ is the receiver noise at the $k$th user. To apply Use-and-then-Forget bound, we re-write $r_{d,k}$ as

$$\begin{aligned}r_{d,k} =& \mathbb{E}\left[\sum_{m\in\mathcal{A}_k}\sqrt{\mathcal{E}_{d,m}\zeta_{mk}}\mathbf{h}_{mk}^T\hat{\mathbf{h}}_{mk}^*\right]s_{d,k}\\ &+\left\{\sum_{m\in\mathcal{A}_k}\sqrt{\mathcal{E}_{d,m}\zeta_{mk}}\mathbf{h}_{mk}^T\hat{\mathbf{h}}_{mk}^* - \mathbb{E}\left[\sum_{m\in\mathcal{A}_k}\sqrt{\mathcal{E}_{d,m}\zeta_{mk}}\mathbf{h}_{mk}^T\hat{\mathbf{h}}_{mk}^*\right]\right\}s_{d,k}\\ &+ \sum_{i\in\mathcal{U}\setminus\{k\}}\sum_{m\in\mathcal{A}_i}\sqrt{\mathcal{E}_{d,m}\zeta_{mi}}\mathbf{h}_{mk}^T\mathbf{h}_{mi}^* s_{d,i} + w_k,\end{aligned} \tag{28}$$

and thus the downlink SE becomes $(1-\lambda)(1-\frac{\tau_p}{\tau})\log_2(1+\gamma_k^d)$, where,

$$\begin{aligned}\gamma_k^d =& \left|\mathbb{E}\left[\sum_{m\in\mathcal{A}_k}\sqrt{\mathcal{E}_{d,m}\zeta_{mk}}\mathbf{h}_{mk}^T\hat{\mathbf{h}}_{mk}^*\right]\right|^2\times\\ &\left(\mathtt{var}\left(\sum_{m\in\mathcal{A}_k}\sqrt{\mathcal{E}_{d,m}\zeta_{mk}}\mathbf{h}_{mk}^T\hat{\mathbf{h}}_{mk}^*\right)\right.\\ &\left.+\sum_{i\in\mathcal{U}\setminus\{k\}}\mathbb{E}\left[\left|\sum_{m\in\mathcal{A}_i}\sqrt{\mathcal{E}_{d,m}\zeta_{mi}}\mathbf{h}_{mk}^T\hat{\mathbf{h}}_{mi}^*\right|^2\right] + N_0\right)^{-1}.\end{aligned} \tag{29}$$

We can apply exactly same analysis to derive the closed form expressions of the downlink signal gain, the beamforming error variance, and show that

$$\mathbb{E}\left[\sum_{m\in\mathcal{A}_k}\sqrt{\mathcal{E}_{d,m}\zeta_{mk}}\mathbf{h}_{mk}^T\hat{\mathbf{h}}_{mk}^*\right] = N\sqrt{\mathcal{E}_{d,m}\zeta_{mk}}\sigma_{mk}^2 \tag{30a}$$

$$\mathtt{var}\left(\sum_{m\in\mathcal{A}_k}\sqrt{\mathcal{E}_{d,m}\zeta_{mk}}\mathbf{h}_{mk}^T\hat{\mathbf{h}}_{mk}^*\right) = N\sum_{m\in\mathcal{A}_k}\mathcal{E}_{d,m}\zeta_{mk}\sigma_{mk}^2\beta_{mk}. \tag{30b}$$

However, there is a subtle difference in the multi-user interference term as the $k$th UE receives signal from the $i$th UE ($i \neq k$) transmitted from the APs that serves $i$th UE ($m \in \mathcal{A}_i$). We derive the closed form expression in the following lemma.

**Lemma 2.** *It can be shown that the downlink multi-user interference experienced by the kth UE due to the ith UE is*

$$\mathbb{E}\left[\left|\sum_{m\in\mathcal{A}_i}\sqrt{\mathcal{E}_{d,m}\zeta_{mi}}\mathbf{h}_{mk}^T\hat{\mathbf{h}}_{mi}^*\right|^2\right] = N^2\left(\sum_{m\in\mathcal{A}_i}\sqrt{\mathcal{E}_{d,m}\zeta_{mi}}\sqrt{\frac{\mathcal{E}_{p,k}}{\mathcal{E}_{p,i}}}\frac{\beta_{mk}}{\beta_{mi}}\sigma_{mi}^2\right)^2|\boldsymbol{\varphi}_i^H\boldsymbol{\varphi}_k|^2 + N\sum_{m\in\mathcal{A}_i}\mathcal{E}_{d,m}\zeta_{mi}\beta_{mk}\sigma_{mi}^2. \tag{31}$$

*Proof.* The technique of the proof is same as adopted in the uplink case. The key difference is in the uplink we substituted for the desired UE's estimated channel (i.e. $\hat{\mathbf{h}}_{mk}$) from (2), whereas here we substitute for $\hat{\mathbf{h}}_{mi}$. We show the key steps required to arrive at the final expression of (31).

$$\begin{aligned}
&\mathbb{E}\left[\left|\sum_{m\in\mathcal{A}_i}\sqrt{\mathcal{E}_{d,m}\zeta_{mi}}\mathbf{h}_{mk}^T\hat{\mathbf{h}}_{mi}^*\right|^2\right]\\
&=\mathbb{E}\Bigg[\Bigg|\sum_{m\in\mathcal{A}_i}\sqrt{\mathcal{E}_{d,m}\zeta_{mi}}\sqrt{\mathcal{E}_{p,i}\tau_p}\beta_{mi}d_{mi}\mathbf{h}_{mk}^T\times\\
&\qquad\left(\sum_{i'\in\mathcal{U}}\sqrt{\mathcal{E}_{p,i'}\tau_p}\langle\boldsymbol{\varphi}_{i'},\boldsymbol{\varphi}_i\rangle\mathbf{h}_{mi'}+\mathbf{W}_{p,m}\boldsymbol{\varphi}_i^*\right)^*\Bigg|^2\Bigg]\\
&=\mathcal{E}_{p,k}\tau_p\mathbb{E}\left[\left|\sum_{m\in\mathcal{A}_i}\bar{d}_{mi}\|\mathbf{h}_{mk}\|^2\right|^2\right]|\langle\boldsymbol{\varphi}_k,\boldsymbol{\varphi}_i\rangle|^2\\
&+\underbrace{\mathbb{E}\left[\left|\sum_{m\in\mathcal{A}_i}\bar{d}_{mi}\mathbf{h}_{mk}^T\sum_{\substack{i'\in\\ \mathcal{U}\backslash k}}\sqrt{\mathcal{E}_{p,i'}\tau_p}\langle\boldsymbol{\varphi}_{i'},\boldsymbol{\varphi}_i\rangle\mathbf{h}_{mi'}^*\right|^2\right]}_{\triangleq \mathtt{I}_5}\\
&+\underbrace{\mathbb{E}\left[\left|\sum_{m\in\mathcal{A}_i}\bar{d}_{mi}\mathbf{h}_{mk}^T\mathbf{W}_{p,m}^*\boldsymbol{\varphi}_i\right|^2\right]}_{\triangleq \mathtt{I}_6},
\end{aligned}\tag{32}$$

where in the last equality we substitute $\bar{d}_{mi} = \sqrt{\mathcal{E}_{d,m}\zeta_{mi}}\sqrt{\mathcal{E}_{p,i}\tau_p}\beta_{mi}d_{mi}$. Next, observe that, as the channel vectors of different users are uncorrelated and zero mean, and so are the channel vector and the noise component; the sum of second and third expectation of (32) reduces to

$$\mathtt{I}_5+\mathtt{I}_6 = N\sum_{m\in\mathcal{A}_i}\sum_{\substack{i'\in\\ \mathcal{U}\backslash k}}\bar{d}_{mi}^2\mathcal{E}_{p,i'}\tau_p\beta_{mk}\beta_{mi'}|\langle\boldsymbol{\varphi}_{i'},\boldsymbol{\varphi}_i\rangle|^2 + NN_0\sum_{m\in\mathcal{A}_i}\bar{d}_{mi}^2\beta_{mk}. \tag{33}$$

Next, the first expectation (32) can be expanded as

$$\begin{aligned}
&\mathbb{E}\left[\left|\sum_{m\in\mathcal{A}_i}\bar{d}_{mi}\|\mathbf{h}_{mk}\|^2\right|^2\right] = \sum_{m\in\mathcal{A}_i}\bar{d}_{mi}^2\mathbb{E}\left[\|\mathbf{h}_{mk}\|^4\right]\\
&+\mathbb{E}\left[\sum_{m\in\mathcal{A}_i}\sum_{\substack{n\in\mathcal{A}_i\\ n\neq m}}\bar{d}_{mi}\bar{d}_{ni}\|\mathbf{h}_{mk}\|^2\|\mathbf{h}_{nk}\|^2\right]\\
&= N(N+1)\sum_{m\in\mathcal{A}_i}\bar{d}_{mi}^2\beta_{mk}^2 + N^2\sum_{m\in\mathcal{A}_i}\sum_{\substack{n\in\mathcal{A}_i\\ n\neq m}}\bar{d}_{mi}\bar{d}_{ni}\beta_{mk}\beta_{nk}\\
&= N^2\left(\sum_{m\in\mathcal{A}_i}\bar{d}_{mi}\beta_{mk}\right)^2 + N\sum_{m\in\mathcal{A}_i}\bar{d}_{mi}^2\beta_{mk}^2.
\end{aligned}\tag{34}$$

Finally, substituting (34),and (33) into (32), we get

$$\begin{aligned}
&\mathbb{E}\left[\left|\sum_{m\in\mathcal{A}_i}\sqrt{\mathcal{E}_{d,m}\zeta_{mi}}\mathbf{h}_{mk}^T\hat{\mathbf{h}}_{mi}^*\right|^2\right] =\\
&\mathcal{E}_{p,k}\tau_p\left\{N^2\left(\sum_{m\in\mathcal{A}_i}\bar{d}_{mi}\beta_{mk}\right)^2+N\sum_{m\in\mathcal{A}_i}\bar{d}_{mi}^2\beta_{mk}^2\right\}|\langle\boldsymbol{\varphi}_k,\boldsymbol{\varphi}_i\rangle|^2\\
&+N\sum_{m\in\mathcal{A}_i}\sum_{\substack{i'\in\\ \mathcal{U}\backslash k}}\bar{d}_{mi}^2\mathcal{E}_{p,i'}\tau_p\beta_{mk}\beta_{mi'}|\langle\boldsymbol{\varphi}_{i'},\boldsymbol{\varphi}_i\rangle|^2\\
&+NN_0\sum_{m\in\mathcal{A}_i}\bar{d}_{mi}^2\beta_{mk}\\
&=N^2\left(\sqrt{\mathcal{E}_{p,k}\tau_p}\sum_{m\in\mathcal{A}_i}\bar{d}_{mi}\beta_{mk}\right)^2|\langle\boldsymbol{\varphi}_k,\boldsymbol{\varphi}_i\rangle|^2\\
&+N\sum_{m\in\mathcal{A}_i}\bar{d}_{mi}^2\beta_{mk}\left\{\sum_{i'\in\mathcal{U}}\mathcal{E}_{p,i'}\tau_p\beta_{mi'}|\langle\boldsymbol{\varphi}_{i'},\boldsymbol{\varphi}_i\rangle|^2\right\}\\
&+NN_0\sum_{m\in\mathcal{A}_i}\bar{d}_{mi}^2\beta_{mk}.
\end{aligned}\tag{35}$$

Now, (31) follows by substituting

$$\sqrt{\mathcal{E}_{p,k}\tau_p}\bar{d}_{mi}\beta_{mk} = \sqrt{\mathcal{E}_{d,m}\zeta_{mi}}\left\{\mathcal{E}_{p,i}\tau_p d_{mi}\beta_{mi}^2\right\}\frac{\beta_{mk}\sqrt{\mathcal{E}_{p,k}}}{\beta_{mi}\sqrt{\mathcal{E}_{p,i}}}, \tag{36a}$$

$$\left\{\mathcal{E}_{p,i}\tau_p d_{mi}\beta_{mi}^2\right\} = \sigma_{mi}^2, \tag{36b}$$

and $$\left\{\sum_{i'\in\mathcal{U}}\mathcal{E}_{p,i'}\tau_p\beta_{mi'}|\langle\boldsymbol{\varphi}_{i'},\boldsymbol{\varphi}_i\rangle|^2\right\} = \frac{1}{d_{mi}} - N_0 \tag{36c}$$

appropriately on (35). ■

## IV. Proof of Theorem 2

**Theorem 3.** *The achievable rate of the kth UE can be expressed as*

$$R_k = \left(1-\frac{\tau_p}{\tau}\right)\left[\lambda\log_2(1+\gamma_k^u)+(1-\lambda)\log_2(1+\gamma_k^d)\right],$$

*where*

$$\gamma_k^u = \frac{N\mathcal{E}_{u,k}(\sum_{m\in\mathcal{A}_k}\sigma_{mk}^2)^2}{N\ \mathtt{CohI}_k^u + \mathtt{NCohI}_k^u + N_0\sum_{m\in\mathcal{A}_k}\sigma_{mk}^2}, \tag{37a}$$

$$\gamma_k^d = \frac{N^2 \rho_d (\sum_{m\in\mathcal{A}_k} \sqrt{\zeta_{mk}} \sigma_{mk}^2)^2}{N^2\ \mathtt{CohI}_k^d + N\ \mathtt{NCohI}_k^d + 1}, \tag{37b}$$

*with* $\mathtt{CohI}_k^u \triangleq \sum_{i\in\mathcal{U}\setminus\{k\}} \mathcal{E}_{u,i} |\boldsymbol{\varphi}_k^H \boldsymbol{\varphi}_i|^2 (\sum_{m\in\mathcal{A}_k} \sigma_{mk}^2 \sqrt{\frac{\mathcal{E}_{p,i}}{\mathcal{E}_{p,k}} \frac{\beta_{mi}}{\beta_{mk}}})^2$, $\mathtt{NCohI}_k^u \triangleq \sum_{i\in\mathcal{U}} \mathcal{E}_{u,i} \sum_{m\in\mathcal{A}_k} \sigma_{mk}^2 \beta_{mi}$, $\mathtt{CohI}_k^d \triangleq \sum_{i\in\mathcal{U}\setminus\{k\}} \rho_d |\boldsymbol{\varphi}_i^H \boldsymbol{\varphi}_k|^2 (\sum_{m\in\mathcal{A}_i} \sigma_{mi}^2 \sqrt{\zeta_{mi}} \sqrt{\frac{\mathcal{E}_{p,k}}{\mathcal{E}_{p,i}} \frac{\beta_{mk}}{\beta_{mi}}})^2$, *and* $\mathtt{NCohI}_k^d \triangleq \rho_d \sum_{i\in\mathcal{U}} \sum_{m\in\mathcal{A}_i} \sigma_{mi}^2 \zeta_{mi} \beta_{mk}$.

*Proof.* In the uplink, $\gamma_k^u$ of Lemma 1 can be re-expressed as $\gamma_k^u$ of Theorem 3. The first term of (25) corresponds to the first term on the denominator of $\gamma_k^u$ in (37a), and merging (10) and $N \sum_{m\in\mathcal{A}_k} \sigma_{mk}^2 \beta_{mi}$ from (25),we obtain the second term of $\gamma_k^u$. Rest of the terms follows directly from Lemma 1. (37b) follows similarly from (30a), (30b), and Lemma 2, and $\rho_d$ be the maximum normalized (as a multiple of the noise variance $N_0$) power transmitted by each AP. ■

# Appendix

## A. Useful Lemma

**Lemma 4.** *[Appendix. A, [3]] Let two independent random vectors* $\mathbf{x}$ *and* $\mathbf{y}$ *be distributed as* $\mathcal{CN}(\mathbf{0}, \sigma_x^2 \mathbf{I}_N)$ *and* $\mathcal{CN}(\mathbf{0}, \sigma_y^2 \mathbf{I}_N)$*, respectively. Then the followings results follow*

$$\mathbb{E}\left[\|\mathbf{x}\|^2\right] = N\sigma_x^2 \tag{38a}$$

$$\mathbb{E}\left[\|\mathbf{x}\|^4\right] = N(N+1)\sigma_x^4 \tag{38b}$$

$$\mathbb{E}\left[\left|(\mathbf{x}+\mathbf{y})^H \mathbf{x}\right|^2\right] = N(N+1)\sigma_x^4 + N\sigma_x^2\sigma_y^2. \tag{38c}$$

**Lemma 5.** *[(62), [4]] If* $\mathbf{x}$ *and* $\mathbf{y}$ *are independent random vectors and* $\mathbb{E}[\mathbf{x}] = \mathbf{0}$*, then*

$$\mathbb{E}\left[|\mathbf{x}+\mathbf{y}|^2\right] = \mathbb{E}\left[|\mathbf{x}|^2\right] + \mathbb{E}\left[|\mathbf{y}|^2\right] \tag{39}$$

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