Outage Analysis for Energy Harvesting Wireless Links With i.i.d Channels

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Outline

- System Model,
- Outage Analysis,
  - ARQ with receiver connected to mains,
  - ARQ with chase combining (HARQ-CC) receiver connected to mains,
  - ARQ with energy harvesting receiver,
  - ARQ with chase combining energy harvesting receiver.
- Conclusions.

System Model[Anup (Oct. 2013)]

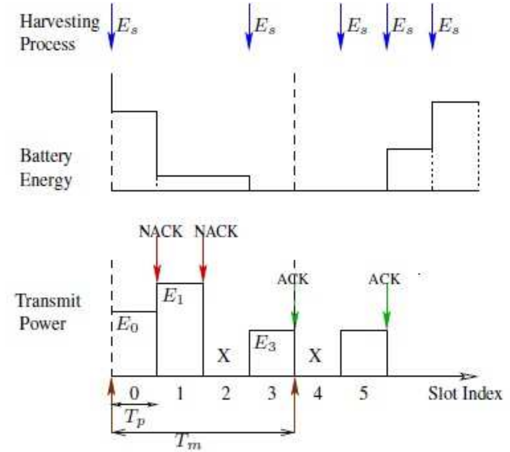

Figure : Transmission timeline of the EH node with $K = 4$, showing the random energy arrival process ($\downarrow$) and periodic data arrivals ($\uparrow$).

System Model

- Time-slotted system
- In a slot energy E_s is harvested with probability ρ.
- i.i.d Rayleigh channels,
  - slow fading $(T_c = T_p)$,
  - fast fading $(T_c = T_m)$.
- A packet is available to transmit, periodically.
- Packet is transmitted until the ACK is received or new frame starts (*Outage*).
- Received packet is decoded,
  - ARQ : independent of the previous receptions,
  - HARQ-CC : using all previously received packets.
- Packet is dropped if it doesn't get transmitted till the end of the frame
- A transmit power policy considered is,

$$\{P_1 \triangleq L_1 E_s, P_2 \triangleq L_2 E_s, \ldots, P_K \triangleq L_K E_s\},$$

where P_i is transmit power level in i^{th} slot.

Outage

- Outage : If packet transmission is not successful in K slots.
- A packet remains in outage if,

$$\gamma_r < \gamma_0.$$

where,

$$\gamma_r \quad : \quad \text{the received SNR,}$$
$$\gamma_0 \quad : \quad \text{target SNR.}$$

Outage Probability

- **ARQ :**

$$p_{\text{out}} = \Pr[\gamma_r < \gamma_0] \quad (1)$$
$$= \Pr[P_i |h_i|^2 < \gamma_0] \quad (2)$$
$$= 1 - e^{-\frac{\gamma_0 \mathcal{N}_0 T_p}{E_S \sigma_c^2}} \quad (3)$$

- **HARQ-CC:**

$$p_{\text{out}} = \Pr[\gamma_n < \gamma_0] \quad (4)$$
$$= \Pr\left[\sum_{i=1}^{n} P_i |h_i|^2 < \gamma_0\right] \quad (5)$$

Analysis

We analyze for following cases,

- Zero battery,
  - slow fading,
  - fast fading.
- With battery,
  - slow fading,
  - fast fading.

Basic ARQ: 0 Battery

Receiver is connected to mains

- Slow fading,

$$P_{\text{out}}(K) = (1-\rho)^K + \sum_{m=1}^{K} \binom{K}{m} \rho^m (1-\rho)^{K-m} p_{\text{out}}, \quad (6)$$

$$= 1 - \left(1 - (1-\rho)^K\right) e^{-\frac{\gamma_0 \mathcal{N}_0 T_p}{E_S \sigma_C^2}}. \quad (7)$$

- Fast fading,

$$P_{\text{out}}(K) = \sum_{m=0}^{K} \binom{K}{m} \rho^m (1-\rho)^{K-m} (p_{\text{out}})^m, \quad (8)$$

$$= \left(1 - \rho e^{-\frac{\gamma_0 \mathcal{N}_0 T_p}{E_S \sigma_C^2}}\right)^K. \quad (9)$$

HARQ-CC: 0 Battery

- Slow fading,

$$P_{out}(K|m) = \Pr\left[|h_i|^2 < \frac{\gamma_0 \mathcal{N}_0}{\sum_{i=1}^{m} P}\right] \quad (10)$$

$$= 1 - e^{-\frac{\gamma_0 \mathcal{N}_0 T_p}{\sigma_c^2 (mE_s)}}. \quad (11)$$

- Fast fading,

$$P_{out}(K|m) = \Pr\left[\sum_{i=1}^{m} |h_i|^2 < \frac{\gamma_0 \mathcal{N}_0 T_p}{E_s}\right]. \quad (12)$$

for i.i.d. Rayleigh channel $\sum_{i=1}^{m} |h_i|^2$ follows *Erlang* distribution, hence,

$$P_{out}(K|m) = 1 - \sum_{n=0}^{m-1} \frac{1}{n!} e^{-\frac{\gamma_0 \mathcal{N}_0 T_p}{\sigma_c^2 E_s}} \cdot \left(\frac{\gamma_0 \mathcal{N}_0 T_p}{\sigma_c^2 E_s}\right)^n \quad (13)$$

Results : Zero Battery

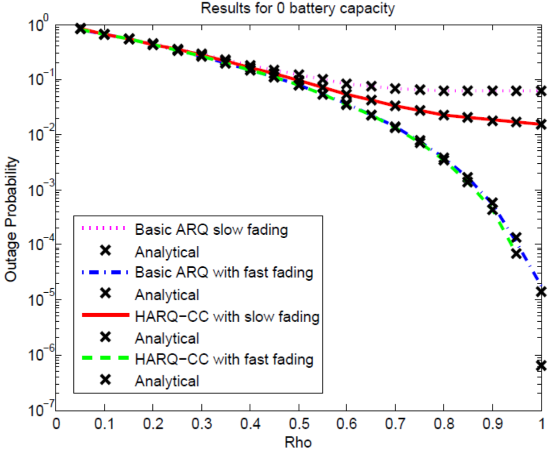

Figure : zero battery result

Outage Probability With Battery

- Process, within a frame, is a discrete time Markov chain (DTMC),
- State of this DTMC is given by tuple (B_n, U_n),
- B_n is the battery state in n^{th} slot,
- $U_n \in \{-1, 0, 1, \ldots, (K-1)\}$, represent the 'feedback state' defined as,

$$U_n = \begin{cases} -1 & \text{ACK received,} \\ 0 & \text{Start of transmission,} \\ i & i \text{ NACKs received, } i \in \{1, \ldots, K\}. \end{cases} \quad (14)$$

Outage occurs if and only if $U_K \neq -1$.

Outage Probability

The outage probability is given as

$$P_{\text{out}}(K) = \sum_{i} \pi(i) P_{\text{out}}\left(K | i, r = 0\right), \tag{15}$$

where, $\pi(i)$: the stationary probability that battery state is i.

$$\pi(j) = \sum_{i} \Pr(B_{(n+1)K} = j | B_{nK} = i) \pi(i), \tag{16}$$

and,

$$P_{\text{out}}\left(K | i, r = 0\right) = \sum_{m=0}^{K} \binom{K}{r} \rho^{m} (1-\rho)^{K-m} p_{\text{out}}(i, m), \tag{17}$$

$p_{\text{out}}(i, m)$ for ARQ

- slow fading

$$p_{\text{out}}(i, m) = \Pr\left\{\left(|h|^2 < \frac{\gamma_0 \mathcal{N}_l}{P_1}\right) \cap \ldots \cap \left(|h|^2 < \frac{\gamma_0 \mathcal{N}_l}{P_{\Psi_1}}\right)\right\},$$

$$= \Pr\left\{\left(|h|^2 < \frac{\gamma_0 \mathcal{N}_l}{P_{\Psi_1}}\right)\right\} = p_{\text{out}}(P_{\Psi_1}). \quad (18)$$

- fast fading

$$p_{\text{out}}(i, m) = \prod_{\ell=1}^{\Psi_1} p_{\text{out}}(P_\ell). \quad (19)$$

$$\Psi_1 : \text{no of attempts}, \Psi_1 = \min\{K, \kappa\}. \quad (20)$$

$$\text{Where, } \kappa = \max\{k_i | E_{\text{av}} - \sum_{k=1}^{k_i} P_k \geq 0\}, \quad (21)$$

where, E_{av}: the net energy available in the frame for transmission.

$p_{\text{out}}(i, m)$ for HARQ-CC

- slow fading,

$$p_{\text{out}}(i,m) = \Pr\left[|h_n|^2 < \frac{\gamma_0 \mathcal{N}_0}{\sum_{n=1}^{\Psi_1} P_n}\right]. \qquad (22)$$

- Fast fading,

$$p_{\text{out}}(i,m) = \Pr\left[\sum_{n=1}^{\Psi_1} L_n |h_n|^2 < \frac{\gamma_0 \mathcal{N}_0 T_p}{E_s}\right]. \qquad (23)$$

$\sum_{n=0}^{\Psi_1} L_n |h_n|^2$ is *hypoexponentially* distributed,

$$\Pr\left[\sum_{n=1}^{\Psi_1} L_n |h_n|^2 < \frac{\gamma_0 \mathcal{N}_0 T_p}{E_s}\right] = 1 - \sum_{\ell=1}^{\Psi_1} C_{\ell,\Psi_1} e^{-\frac{\gamma_0 \mathcal{N}_0 T_p}{L_\ell E_s}}, \qquad (24)$$

$$\text{where, } C_{\ell,\Psi_1} = \prod_{\ell \neq k} \frac{L_\ell}{L_k - L_\ell.} \qquad (25)$$

Transition Probability Matrix G

The probability of transition from state (i, r) to (j, s) is,

$$G_{ij}^{rs} = Pr(B_{n+1} = j, U_{n+1} = s | B_n = i, U_n = r), \qquad (26)$$

where, $i, j \in \{0, 1, \ldots, \infty\}$ and $r, s \in \{-1, 0, \ldots, K\}$.

- For $r \in \{0, \ldots, K-1\}$ and $i \geq L_r$,

$$G_{ij}^{rs} = \begin{cases} \rho Pr\left[\gamma_n < \gamma_0\right], & j = i - L_r + 1, s = r + 1, \\ \rho Pr\left[\gamma_n \geq \gamma_0\right], & j = i - L_r + 1, s = -1, \\ (1-\rho) Pr\left[\gamma_n < \gamma_0\right], & j = i - L_r, s = r + 1, \\ (1-\rho) Pr\left[\gamma_n > \gamma_0\right], & j = i - L_r, s = -1, \\ 0, & \text{else.} \end{cases} \qquad (27)$$

- For $r \in \{0, \ldots, K-1\}$ and $L_r - 1 \leq i < L_r$,

$$G_{ij}^{rs} = \begin{cases} \rho Pr\left[\gamma_n < \gamma_0\right], & j = i - L_r + 1, s = r + 1, \\ \rho Pr\left[\gamma_n \geq \gamma_0\right], & j = i - L_r + 1, s = -1, \\ (1-\rho), & j = i, s = r, \\ 0, & \text{else.} \end{cases} \quad (28)$$

- For $r \in \{0, \ldots, K-1\}$ and $0 \leq i \leq L-2$,

$$G_{ij}^{rs} = \begin{cases} \rho, & j = i + 1, s = r, \\ (1-\rho), & j = i, s = r, \\ 0, & \text{else.} \end{cases} \quad (29)$$

- For $r = -1$ and $i \geq 0$,

$$G_{ij}^{rs} = \begin{cases} \rho, & j = i + 1, s = -1, \\ (1-\rho), & j = i, s = -1, \\ 0, & \text{else.} \end{cases} \quad (30)$$

Procedure: to find $P_{\text{out}}(K)$

- Find transition probability matrix G,
- Find G^K, k-step tpm,
- Using G^K, compute the probability, $\Pr(B_{(n+1)K} = j | B_{nK} = i, U_{nK} = 0)$, as

$$= \sum_{r=-1}^{K} \Pr(B_{(n+1)K} = j, U_{(n+1)K} = r | B_{nK} = i, U_{nK} = 0)$$

- Obtain stationary probabilities $\pi(j)$ by solving (16),
- Obtain $P_{\text{out}}(K | i, r = 0)$ using (17),
- Obtain P_{out} using (15).

Results for slow fading channels: With Battery

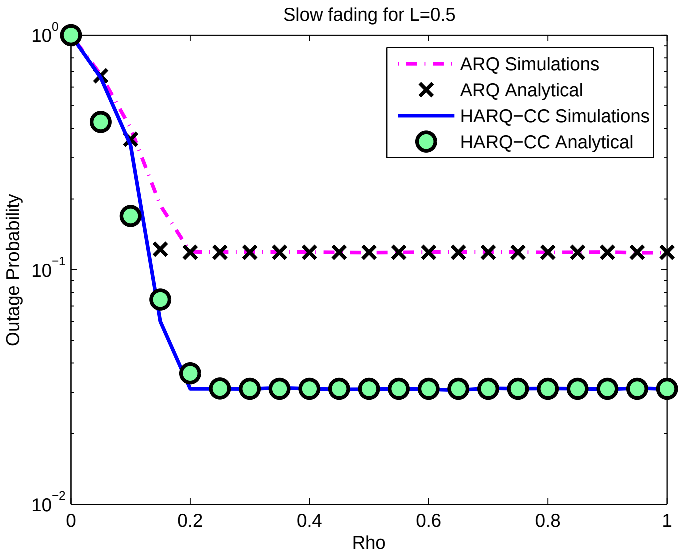

Figure : Fixed power (fractional L) transmission scheme results

Results for slow fading channels: With Battery

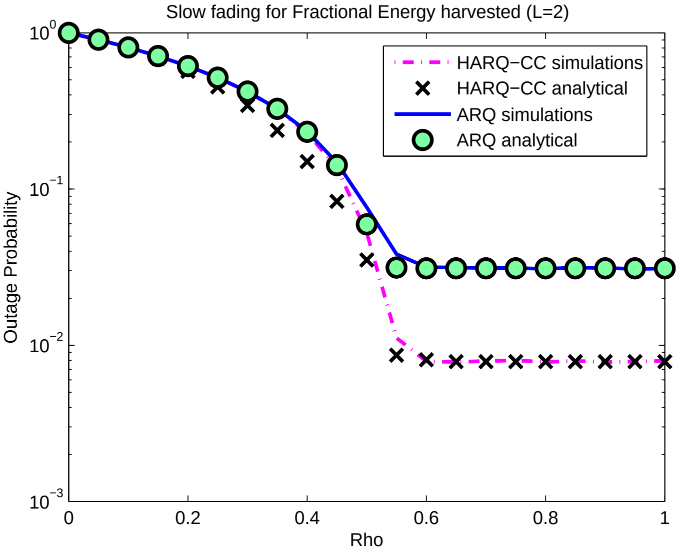

Figure : Fixed power (integer *L*) transmission scheme results

Results for fast fading channels: With Battery

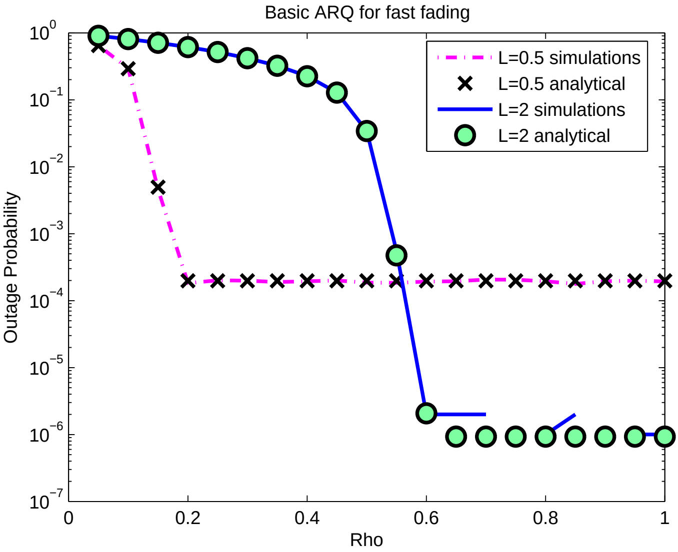

Figure : Fixed power transmission scheme results

Results for fast fading channels: With Battery

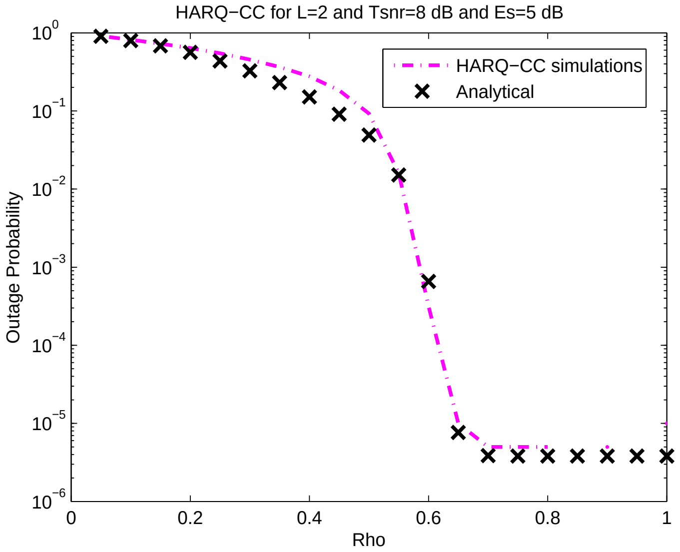

Figure : Fixed power transmission scheme results for HARQ-CC

Harvesting Unconstrained Regime (HUCR)

- HUCR is achieved if,

$$E_{\text{avg}}^{c} < \rho K E_s \tag{31}$$

where,

E_{avg}^{c} : Average energy consumed per frame,

$\rho K E_s$: Average energy harvested per frame.

- slow fading,

$$E_{\text{avg}}^{c} = \sum_{t=1}^{K} L_t p_{\text{out}}(L_{t-1}) E_s, \tag{32}$$

for HUCR,

$$\frac{1}{K} \sum_{t=1}^{K} L_t p_{\text{out}}(L_{t-1}) < \rho. \tag{33}$$

HUCR

- Fast fading,

$$E_{\text{avg}}^{c} = \sum_{t=1}^{K} L_t \prod_{p=1}^{t-1} p_{\text{out}}(L_p) E_s, \tag{34}$$

for HUCR,

$$\frac{1}{K} \sum_{t=1}^{K} L_t p_{\text{out}}(L_{t-1}) < \rho. \tag{35}$$

- HUCR for HARQ-CC: Average energy consumed is,

$$E_{avg}^{c} = \left(\sum_{k=1}^{K} L_k p_{\text{out}}(k-1) \right) E_s, \tag{36}$$

$$\text{for HUCR,} \qquad \frac{1}{K} \left(\sum_{k=1}^{K} L_k p_{\text{out}}(k-1) \right) \leq \rho, \tag{37}$$

$p_{\text{out}}(k-1)$: probability of outage in $(k-1)^{th}$ attempt.

Energy Harvesting Receiver

- Receiver harvests the fixed energy E_s with probability ρ_r.
- Protocol,
  - SOC : Start of Communication,
  - EOC : End of Communication.
- SOC signal is sent if a node has 'sufficient energy' to participate in communication,
- EOC signal is sent otherwise,
- After sending EOC, node goes into sleep mode and harvests energy,
- Constant units of power, P_r , is consumed to received and decode a packet.

EH receiver: 0 Battery

- Basic ARQ-slow fading,

$$P_{\text{out}}(K) = (1-\rho_t\rho_r)^K + \sum_{m=1}^{K}\binom{K}{m}(\rho_t\rho_r)^m(1-\rho_t\rho_r)^{K-m}p_{\text{out}},$$
$$= 1-\left(1-(1-\rho_t\rho_r)^K\right)e^{-\frac{\gamma_0\mathcal{N}_0T_p}{E_s\sigma_c^2}}, \quad (38)$$

- Basic ARQ- fast fading,

$$P_{\text{out}}(K) = \sum_{m=0}^{K}\binom{K}{m}(\rho_t\rho_r)^m(1-\rho_t\rho_r)^{K-m}(p_{\text{out}})^m, \quad (39)$$
$$= \left(1-\rho_t\rho_r e^{-\frac{\gamma_0\mathcal{N}_0T_p}{E_s\sigma_c^2}}\right)^K, \quad (40)$$

- HARQ-CC

$$P_{\text{out}}(K) = \sum_{m=0}^{K}\binom{K}{m}(\rho_t\rho_r)^m(1-\rho_t\rho_r)^{K-m}p_{\text{out}}^m \quad (41)$$

EH Receiver : With Battery

- The process in this case is also a DTMC for the intra-frame interval.
- State of this DTMC is represented by a 3-tuple (B_n^t, B_n^r, U_n),
- B_n^t and B_n^r represent the battery state at transmitter and receiver, respectively,
- U_n represent the feedback state.

the outage probability is given as,

$$P_{\text{out}}(K) = \sum_{(i,j)} \pi(i,j) P_{\text{out}}(K|i,j,r=0), \tag{42}$$

$\pi(i,j)$: stationary probability that both nodes have (iE_s, jE_s) energy

$$\pi(i_2,j_2) = \sum_{(i_1,j_1)} \Pr\left[(B^t_{n+1} = i_2, B^r_{n+1} = j_2)|(B^t_n = i_1, B^r_n = j_1)\right] \pi(i_1,j_1) \tag{43}$$

The outage probability $P_{\text{out}}(K|i,j,r=0)$,

$$P_{\text{out}}(K|i,j,r=0) = \sum_{m_t=0}^{K} \sum_{m_r=0}^{K} \binom{K}{m_t} \binom{K}{m_r} \rho_t^{m_t} \rho_r^{m_r} (1-\rho_t)^{K-m_t} (1-\rho_r)^{K-m_r} p_{\text{out}}(i,j,m_t,m_r), \tag{44}$$

- For basic ARQ, for slow and fast fading, it is given by (18) and (20), respectively,
- For HARQ-CC, for slow and fast fading, it is given by (22)-(24), respectively. where Ψ_1 given as,

$$\Psi_1 = \min\{K, \kappa_t, \kappa_r\}, \tag{45}$$

where, κ_t and κ_r is given as,

$$\kappa_t = \max\{k_i | E_{\text{av}}^t - \sum_{k=1}^{k_i} P_k \geq 0\}, \tag{46}$$

$$\kappa_r = \max\{k_j | E_{\text{av}}^r - k_j P_r \geq 0\}. \tag{47}$$

Where E_{av}^t and E_{av}^r denote the net energy available in the frame for transmission and reception, respectively.

Transition Probability Matrix G

The probability of transition from state (i_1, j_1, r) to (i_2, j_2, s) is

$$G_{i_1,j_1,r}^{i_2,j_2,s} = Pr(B_{n+1}^t = i_2, B_{n+1}^r = j_2, U_{n+1} = s | B_n^t = i_1, B_n^r = j_1, U_n = r),$$

where, $i_1, i_2, j_1, j_2 \in \{0, 1, \ldots, \infty\}$ and $r, s \in \{-1, 0, \ldots, K\}$.

- For $r \in \{0, \ldots, K-1\}$, $i_1 \geq L_r$ and $j_1 \geq R$,

$$G_{i_1,j_1,r}^{i_2,j_2,s} = \begin{cases} \rho_t \rho_r Pr\left[\gamma_n < \gamma_0\right], & i_2 = i_1 - L_r + 1, j_2 = j_1 - R + 1, s = r + 1, \\ \rho_t \rho_r Pr\left[\gamma_n \geq \gamma_0\right], & i_2 = i_1 - L_r + 1, j_2 = j_1 - R + 1, s = -1, \\ (1-\rho_t)\rho_r Pr\left[\gamma_n < \gamma_0\right], & i_2 = i_1 - L_r, j_2 = j_1 - R + 1, s = r + 1, \\ (1-\rho_t)\rho_r Pr\left[\gamma_n \geq \gamma_0\right], & i_2 = i_1 - L_r, j_2 = j_1 - R + 1, s = -1, \\ \rho_t (1-\rho_r) Pr\left[\gamma_n < \gamma_0\right], & i_2 = i_1 - L_r + 1, j_2 = j_1 - R, s = r + 1, \\ \rho_t (1-\rho_r) Pr\left[\gamma_n \geq \gamma_0\right], & i_2 = i_1 - L_r + 1, j_2 = j_1 - R, s = -1, \\ (1-\rho_t)(1-\rho_r) Pr\left[\gamma_n < \gamma_0\right], & i_2 = i_1 - L_r, j_2 = j_1 - R, s = r + 1, \\ (1-\rho_t)(1-\rho_r) Pr\left[\gamma_n \geq \gamma_0\right], & i_2 = i_1 - L_r, j_2 = j_1 - R, s = -1, \\ 0, & \text{else.} \end{cases} \quad (48)$$

- For $r \in \{0, \ldots, K-1\}$, $L_r - 1 \leq i_1 < L$ and $j_1 \geq R$,

$$G_{i_1,j_1,r}^{i_2,j_2,s} = \begin{cases} \rho_t \rho_r Pr\left[\gamma_n < \gamma_0\right], & i_2 = i_1 - L_r + 1, j_2 = j_1 - R + 1, s = r+1, \\ \rho_t \rho_r Pr\left[\gamma_n \geq \gamma_0\right], & i_2 = i_1 - L_r + 1, j_2 = j_1 - R + 1, s = -1, \\ \rho_t(1-\rho_r) Pr\left[\gamma_n < \gamma_0\right], & i_2 = i_1 - L_r + 1, j_2 = j_1 - R, s = r+1, \\ \rho_t(1-\rho_r) Pr\left[\gamma_n \geq \gamma_0\right], & i_2 = i_1 - L_r + 1, j_2 = j_1 - R, s = -1, \\ (1-\rho_t)\rho_r, & i_2 = i_1, j_2 = j_1 + 1, s = r, \\ (1-\rho_t)(1-\rho_r), & i_2 = i_1, j_2 = j_1, s = r, \\ 0, & \text{else.} \end{cases} \quad (49)$$

- For $r \in \{0, \ldots, K-1\}$, $i_1 \geq L_r$ and $R - 1 \leq j_1 < R$,

$$G_{i_1,j_1,r}^{i_2,j_2,s} = \begin{cases} \rho_t \rho_r Pr\left[\gamma_n < \gamma_0\right], & i_2 = i_1 - L_r + 1, j_2 = j_1 - R + 1, s = r+1, \\ \rho_t \rho_r Pr\left[\gamma_n \geq \gamma_0\right], & i_2 = i_1 - L_r + 1, j_2 = j_1 - R + 1, s = -1, \\ (1-\rho_t)\rho_r Pr\left[\gamma_n < \gamma_0\right], & i_2 = i_1 - L_r, j_2 = j_1 - R + 1, , s = r+1, \\ (1-\rho_t)\rho_r Pr\left[\gamma_n \geq \gamma_0\right], & i_2 = i_1 - L_r, j_2 = j_1 - R + 1, s = -1, \\ \rho_t(1-\rho_r), & i_2 = i_1 + 1, j_2 = j_1, s = r, \\ (1-\rho_t)(1-\rho_r), & i_2 = i_1, j_2 = j_1, s = r, \\ 0, & \text{else.} \end{cases} \quad (50)$$

- For $r \in \{0, \ldots, K-1\}$, $L-1 \leq i_1 < L$ and $R-1 \leq j_1 < R$,

$$G_{i_1,j_1,r}^{i_2,j_2,s} = \begin{cases} \rho_t \rho_r Pr\left[\gamma_n < \gamma_0\right], & i_2 = i_1 - L_r + 1, j_2 = j_1 - L_r + 1, s = r+1, \\ \rho_t \rho_r Pr\left[\gamma_n \geq \gamma_0\right], & i_2 = 0, j_2 = 0, s = -1, \\ (1-\rho_t)\rho_r, & i_2 = i_1, j_2 = j_1 + 1, , s = r, \\ \rho_t(1-\rho_r), & i_2 = i_1 + 1, j_2 = j_1, s = r, \\ (1-\rho_t)(1-\rho_r), & i_2 = i_1, j_2 = j_1, s = r, \\ 0, & \text{else.} \end{cases} \quad (51)$$

- For $r \in \{0, \ldots, K-1\}$, $0 \leq i_1 \leq L-2$ or $0 \leq j_1 \leq R-2$

$$G_{i_1,j_1,r}^{i_2,j_2,s} = \begin{cases} \rho_t \rho_r, & i_2 = i_1 + 1, j_2 = j_1 + 1, s = r, \\ (1-\rho_t)\rho_r, & i_2 = i_1, j_2 = j_1 + 1, s = r, \\ (1-\rho_r)\rho_t, & i_2 = i_1 + 1, j_2 = j_1, s = r, \\ (1-\rho_t)(1-\rho_r), & i_2 = i_1, j_2 = j_1, s = r, \\ 0, & \text{else.} \end{cases} \quad (52)$$

- For $r = -1$, $i_1 \geq 0$ and $j_1 \geq 0$,

$$G_{i_1,j_1,r}^{i_2,j_2,s} = \begin{cases} \rho_t \rho_r, & i_2 = i_1 + 1, j_2 = j_1 + 1, s = -1, \\ (1-\rho_t)\rho_r, & i_2 = i_1, j_2 = j_1 + 1, s = -1, \\ (1-\rho_r)\rho_t, & i_2 = i_1 + 1, j_2 = j_1, s = -1, \\ (1-\rho_t)(1-\rho_r), & i_2 = i_1, j_2 = j_1, s = -1, \\ 0, & \text{else.} \end{cases} \quad (53)$$

EH Receiver : HUCR

for slow fading channels, a transmitter node operates in HUCR if,

$$\frac{1}{K}\sum_{t=1}^{K} L_t p_{\text{out}}(L_{t-1}) < \rho_t, \quad (54)$$

while, a receiver node operates in HUCR if,

$$\frac{R}{K}\sum_{t=1}^{K} p_{\text{out}}(L_{t-1}) < \rho_t, \quad (55)$$

where, $R = \frac{P_r T_p}{E_s}$. Further, for fast fading channels the transmitter node operates in HUCR if,

$$\frac{1}{K}\sum_{t=1}^{K} L_t \prod_{p=1}^{t-1} p_{\text{out}}(L_p) < \rho_t, \quad (56)$$

and a receiver node operates in HUCR if,

$$\frac{R}{K}\sum_{t=1}^{K} \prod_{p=1}^{t-1} p_{\text{out}}(L_p) < \rho_r. \quad (57)$$

Conclusion

- We analyzed the outage property for the following cases,
  - ARQ with receiver connected to mains,
  - ARQ with chase combining (HARQ-CC) receiver connected to mains,
  - ARQ with energy harvesting receiver,
  - ARQ with chase combining energy harvesting receiver.
- Using these closed form expressions, we can apply optimization technique.