On Outage Optimal Transmission Policies for Energy Harvesting Multi-hop Links

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Outline

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- System Model
- Problem Statement
- Geometric Programming
- Proposed Solution
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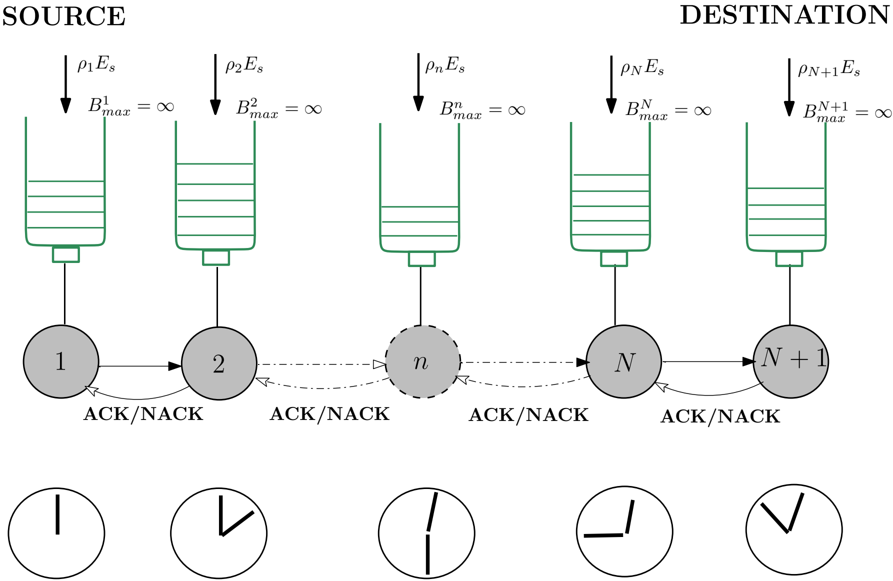
Introduction

- *Sporadically* energy is harvested from environment for eg. solar, wind etc
- *Energy neutrality constraint* (ENC): cumulative energy used cannot exceed the total harvested energy
- Energy neutrality constraint: infinite number of constraints
- Central issue: design of energy management policies to optimize a utility function
- Policy: prescription of the transmit power on the basis of available system-state information

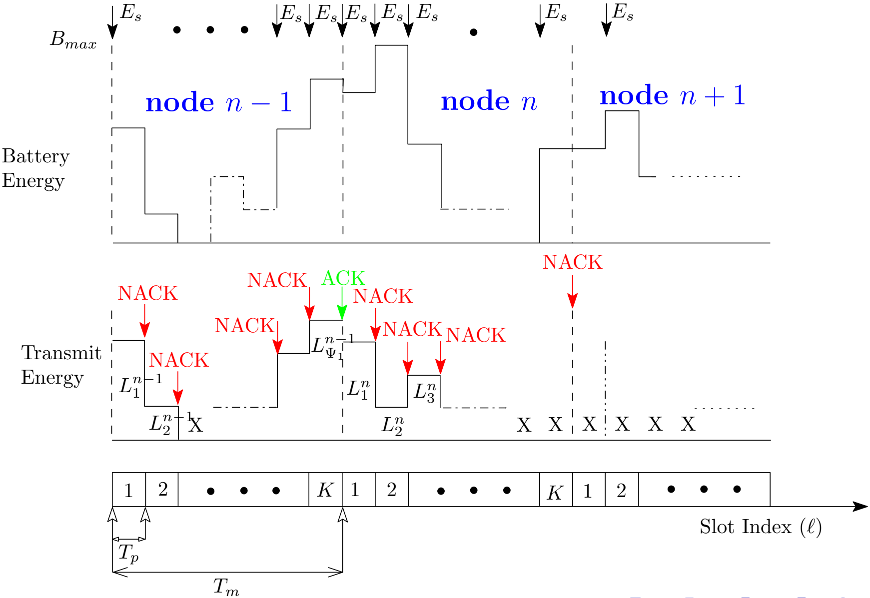
System Model



System Model

- N-hop EH link with block fading channel
- All nodes are EH nodes (EHN)
- Periodically gets a packet, to be delivered by a deadline (multi-hop frame duration T_f)
- NK slots of duration T_p, $NK \triangleq \lfloor T_f / T_p \rfloor$
- Harvesting rate ρE_s per slot, $\forall n$
- Retransmission protocol: ARQ, HARQ-CC
- Rx sends ACK/NACK, for decoding success/failure
- Tx does not have access to CSI
- A packet remains in outage if not decoded correctly
- Decoding cost for each attempt = RE_s
- Packet is dropped if doesn't reach $N + 1^{\text{th}}$ node by the end of the frame

System Dynamics



Problem Statement

- The *drop probability*

$$P_{\mathrm{D}} = 1 - \Pr[N+1]$$

$$\Pr[N+1] : \Pr\left[N+1^{\text{th}} \text{ node receive the packet correctly}\right]$$

- Design goal:

$$\min_{\{E_1^n, E_2^n, \ldots, E_K^n \geq 0\}_{n=1}^{N}} \quad P_{\mathrm{D}}$$

$$\text{s.t.,} \quad \sum_{t=1}^{t_0} \rho_n E_s \geq \sum_{t=1}^{t_0} E_t^n \qquad \forall n, t_0$$

Modified Energy Neutrality Constraint

- Average Power Constraint (APC): on average, EHN consumes energy lower than the harvesting rate
- APC with large battery capacity:
  - Battery evolution has a net positive drift
  - Battery has sufficient energy to make all K attempts
  - It is throughput optimal
- For large battery system operating under APC, ENC is *equivalent* to APC
- Infinite battery approximation
  - Maximum transmit power is limited by the RF front-end hardware
  - Finite number of attempts per packet

Modified Problem Statement: ARQ Slow Fading

$$\min_{\left\{\left\{E_k^n\right\}_{k=1}^{K_n}\right\}_{n=1}^{N}} \quad P_{\text{out}} = 1 - \Pr\left[N+1\right]$$

subject to

$$\Pr\left[n\right].\sum_{k=1}^{K_n} E_k^{n^*}\mathbb{E}_\gamma\left\{\prod_{i=1}^{k-1} P_e(E_i^{n^*},\gamma)\right\} + \Pr\left[n-1\right].RE_s.\sum_{k=1}^{K_{n-1}} \mathbb{1}_{\{E_k \neq 0\}}$$
$$.\mathbb{E}_\gamma\left\{\prod_{i=1}^{k-1} P_e(E_i^{(n-1)^*},\gamma)\right\} \leq NK\rho_n E_s \qquad \forall \quad n = 1,\ldots,N+1$$

where,

$$\Pr\left[n\right] = \prod_{n'=1}^{n-1}\mathbb{E}_\gamma\left\{\sum_{i=1}^{K_{n'}}\Big(1 - P_e(E_i^{n^*},\gamma)\Big)\prod_{\ell=1}^{i-1} P_e(E_\ell^{n^*},\gamma)\right\}$$

Alternative Formulation

- Subproblem 1:

$$\min_{\{(Tx_1,Rx_2),(Tx_2,Rx_3),\ldots,(Tx_N,Rx_{N+1})\}} P_{\text{out}}$$

s.t.

$$\begin{aligned} Tx_1 &\leq NK\rho_1 E_s \\ Rx_2 + Tx_2 &\leq NK\rho_2 E_s \\ &\vdots \\ Rx_N + Tx_N &\leq NK\rho_N E_s \\ Rx_{N+1} &\leq NK\rho_{N+1} E_s \end{aligned}$$

where, Rx_n, and Tx_n are average energy spent for transmission and reception by nth node

Subproblem 2 (SP2)

For all $n = 1, \ldots, N$

$$\min_{\{E_1^n, \ldots, E_{K_n}^n\}} P_{\text{out}}^n = \mathbb{E}_\gamma \left\{ \prod_{\ell=1}^{K_n} P_e(E_\ell^n, \gamma) \right\}$$

s.t.

$$\sum_{k=1}^{K_n} E_k^n \; \mathbb{E}_\gamma \left\{ \prod_{i=1}^{k-1} P_e(E_i^n, \gamma) \right\} \quad \leq \quad \frac{Tx_n}{\Pr[n]} = T_n$$

$$RE_s. \sum_{k=1}^{K_n} \mathbb{1}_{\{E_k \neq 0\}} \mathbb{E}_\gamma \left\{ \prod_{i=1}^{k-1} P_e(E_i^n, \gamma) \right\} \quad \leq \quad \frac{Rx_{n+1}}{\Pr[n]} = R_n$$

$$0 \leq E_k^n \leq E_{\max} \qquad \forall \quad k = 1, \ldots, K_n$$

Equality of Optimal Solutions

Theorem

Let $\bar{E}^* = \{\bar{E}_1^*, \ldots, \bar{E}_N^*\}$*, and* $\bar{E}^{**} = \{\bar{E}_1^{**}, \ldots, \bar{E}_N^{**}\}$ *be the solutions to the original, and alternative problems, respectively, then*

$$P_{out}(\bar{E}^*) = P_{out}(\bar{E}^{**})$$

Proof.

□

SP2 for Rayleigh fading

$$\min_{\{E_1,\ldots,E_K\}} P_{\text{out}} = \left(1 + \sum_{\ell=1}^{K} \frac{E_\ell}{\mathcal{N}_0}\right)^{-1}$$

s.t.

$$\sum_{k=1}^{K} E_k \left(1 + \sum_{\ell=1}^{k-1} \frac{E_\ell}{\mathcal{N}_0}\right)^{-1} \leq T_n \leftrightarrow$$

$$RE_s.\left[1 + \sum_{k=2}^{K} \mathbb{1}_{\{E_k \neq 0\}} \left(1 + \sum_{\ell=1}^{k-1} \frac{E_\ell}{\mathcal{N}_0}\right)^{-1}\right] \leq R_n$$

$$0 \leq E_k \leq E_{\max} \qquad \forall \quad k = 1, \ldots, K$$

Geometric Programming: Terminology

- Monomial: $f : \mathbf{R}^{\mathbf{n}}_{++} \to \mathbf{R}$:

$$f(\mathbf{x}) = d x_1^{a^{(1)}} x_2^{a^{(2)}} \dots x_n^{a^{(n)}}$$

  where, $d \geq 0$, and $a^{(j)} \in \mathbf{R}, j = 1, \dots, n$

- Posynomial: Sum of monomials

$$f(\mathbf{x}) = \sum_{k=1}^{K} d_k x_1^{a_k^{(1)}} x_2^{a_k^{(2)}} \dots x_n^{a_k^{(n)}}$$

  where, $d_k \geq 0$, $a_k^{(j)} \in \mathbf{R}$, $k = 1, 2, \dots, K, j = 1, 2, \dots, n$

- Examples:

$$\text{Posynomial :} \qquad 2x_1^{-\pi} x_2^{0.5} + 3x_1 x_3^{100}, \frac{x_1}{x_2}$$

$$\text{Not a Posynomial :} \qquad x_1 - x_2, \frac{x_1 + x_2}{x_3 + x_1}$$

Geometric Programming: Standard form

Standard form: (non convex problem)

$$
\begin{aligned}
\min \quad & f_0(\mathbf{x}) \\
\text{subject to} \quad & f_i(\mathbf{x}) \leq 1 \quad \forall \quad i = 1, 2, \ldots, m \\
& h_\ell(\mathbf{x}) = 1 \quad \forall \quad i = 1, 2, \ldots, M
\end{aligned}
$$

where,

$$f(\mathbf{x}) = \sum_{k=1}^{K_i} d_{ik} x_1^{a_{ik}^{(1)}} x_2^{a_{ik}^{(2)}} \ldots x_n^{a_{ik}^{(n)}}$$

$$\text{and,} \quad h_\ell(\mathbf{x}) = d_\ell x_1^{a_\ell^{(1)}} x_2^{a_\ell^{(2)}} \ldots x_n^{a_\ell^{(n)}}$$

Geometric Programming: Convex form

- Let $y_i = \log x_i$, $b_{ik} = \log d_{ik}$, and $b_\ell = \log d_\ell$

$$
\begin{aligned}
\min \quad P_0(\mathbf{y}) &= \log \sum_{k=1}^{K_0} \exp(\mathbf{a}_{0k}^T \mathbf{y} + b_{0k}) \\
\text{s. t.} \quad P_i(\mathbf{y}) &= \log \sum_{k=1}^{K_i} \exp(\mathbf{a}_{ik}^T \mathbf{y} + b_{ik}) \le 0 \quad \forall \quad i = 1, 2, \ldots, m \\
q_\ell(\mathbf{y}) &= \mathbf{a}_\ell^T \mathbf{y} + b_\ell = 0 \quad \forall \quad \ell = 1, 2, \ldots, M
\end{aligned}
$$

- References:

  - M. Chiang, "Geometric Programming for Communication Systems," *Foundations and Trends of Communication and Information Theory*, vol. 2, no 1-2, pp 1-156, Aug. 2005
  - S. Boyd, S. J. Kim, L. Vandenberghe, and A. Hassibi, "A Tutorial on Geometric Programming,"*Optimization and Engineering*, pp 67-127, April 2007.

SP2 as a Geometric Program

Let $z_i = 1 + \frac{1}{\mathcal{N}_0} \sum_{\ell=1}^{i} E_\ell \implies \quad E_i = (z_i - z_{i-1})\mathcal{N}_0$, and $Z_0 = 1$

$$\max_{\{z_1,\ldots,z_K\}} (z_K - 1)$$

s.t.

$$\sum_{\ell=1}^{K} z_\ell z_{\ell-1}^{-1} \leq \frac{T_n}{\mathcal{N}_0} + K$$

$$\sum_{k=1}^{K-1} \mathbb{1}_{\{z_{i+1} \neq z_i\}} z_i^{-1} \leq \frac{R_n}{RE_s} - 1$$

$$0 \leq (z_\ell - z_{\ell-1})\mathcal{N}_0 \leq E_{\max} \qquad \forall \quad \ell = 1, \ldots, K$$

Approximation for posynomial ratio

- Let $g(\mathbf{x}) = \sum_i u_i(\mathbf{x})$. Approximate a ratio of polynomials $\frac{f(\mathbf{x})}{g(\mathbf{x})}$ with $\frac{f(\mathbf{x})}{\tilde{g}(\mathbf{x})}$ where

$$\tilde{g}(\mathbf{x}) = \prod_i \left(\frac{u_i(\mathbf{x})}{\alpha_i} \right)^{\alpha_i} \leq g(\mathbf{x})$$

- Directly follows from AM-GM inequality $\sum_i \alpha_i v_i \geq \prod_i v_i^{\alpha_i}$
- If, $\alpha_i = \frac{u_i(\mathbf{x_0})}{g(\mathbf{x_0})} \quad \forall \quad i$, for any fixed $\mathbf{x}_0 > 0$, then

$$\tilde{g}(\mathbf{x}_0) = g(\mathbf{x}_0)$$

- $\tilde{g}(\mathbf{x}_0)$, it is the best local monomial approximation $g(\mathbf{x}_0)$ near x_0, in the sense of first order Taylor approximation.

Algorithm: finds locally optimal power allocation

Initialize with feasible $\mathbf{z} = \{z_1, \ldots, z_K\}$

1. Evaluate the denominator posynomial with the given z
2. Compute for each term i in this posynomial

$$\alpha_i = \frac{\text{value of } i\text{th term in posynomial}}{\text{value of posynomial}}$$

3. Approximate the denominator of the posynomial ration by $\tilde{g}(\mathbf{z})$ using weights α_i
4. Solve the resulting GP
5. Go to step 1, using $\mathbf{z}$ of step 4
6. Terminate the k^{th} loop if

$$||\mathbf{z}^{(k)} - \mathbf{z}^{(k-1)}|| \leq \epsilon$$

Outputs: A locally optimal $\mathbf{z}$

Subproblem 2 for HARQ-CC with slow fading

$$P_e\left(\sum_{\ell=1}^{i} E_\ell, \gamma\right) = \exp\left(\frac{-\sum_{\ell=1}^{i} E_\ell.\gamma}{\mathcal{N}_0}\right)$$

$$P_{\text{out}} = \mathbb{E}_\gamma\left\{\prod_{\ell=1}^{K} P_e\left(\sum_{\ell=1}^{i} E_\ell, \gamma\right)\right\}$$

for Rayleigh fading

$$P_{\text{out}} = \frac{1}{\sum_{\ell=1}^{i}(K-\ell+1)E_\ell}$$

Subproblem 2 for HARQ-CC with slow fading

$$
\begin{aligned}
\max_{\{E_1,\ldots,E_K\}} \quad & \sum_{\ell=1}^{K}(K-\ell+1)E_\ell \\
\text{s.t.} \quad & \sum_{\ell=1}^{K} E_\ell \left(\sum_{i=1}^{\ell-1}(\ell-i+1)E_i+1\right)^{-1} \leq E_{av}^{t} \\
& \sum_{\ell=1}^{K} \mathbb{1}_{\{E_{\ell \neq 0}\}} E_\ell \left(\sum_{i=1}^{\ell-1}(\ell-i+1)E_i+1\right)^{-1} \leq \frac{E_{av}^{r}}{RE_s} \\
& 0 \leq E_i \leq E_{\max}
\end{aligned}
$$

Converting to GP

Let $z_1 = 1 + E_1$, $z_2 = 1 + 2E_1 + E_2$, $z_3 = 1 + 3E_1 + 2E_2 + E_3$

$$E_i = z_i - 2z_{i-1} + z_{i-2}$$

$$\begin{aligned}
\max_{\{z_1,\ldots,z_K\}} \quad & z_K \\
\text{s.t.} \quad & \sum_{\ell=1}^{K} z_{\ell-1}^{-1}(z_\ell + z_{\ell-2}) \leq E_{av}^t - 2K + 1 \\
& \sum_{\ell=1}^{K} z_{\ell-1}^{-1} \leq \frac{E_{av}^r}{RE_s} \\
& 0 \leq z_i - 2z_{i-1} + z_{i-2} \leq E_{\max}
\end{aligned}$$

Subproblem 1

- Can be solved using merit-based sequential quadratic programming (MSQP)
- MSQP guarantees global convergence to local optimum under some weak conditions.