Channel Aging in Massive MIMO systems

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The questions that we will try to answer

- What is Massive MIMO?
- What is channel aging?
- What have others done about it?
- What more can we do?

Massive MIMO

What?

- MIMO systems with very large (Massive) number of antennas

Why?

- Capacity can be increased simply by adding more antennas
- Reduced powers (scale with square root of number of antennas)
- Easier analysis and reduced receiver computational complexity

One Challenge

- FDD mode

Channel aging

The channel needs to be learned and may change between training and data transmission.

Traditional block fading model not accurate enough

Can be modelled as an AR process

$$\boldsymbol{H}[n+1] = \rho \boldsymbol{H}[n] + \sqrt{1-\rho^2}\boldsymbol{\Xi}[n]$$

with the innovation process uncorrelated to $\boldsymbol{H}[n]$

This causes problems both with CSI acquisition and data transmission

What has been done about it?

Effects of channel aging in massive MIMO systems

Kien T. Troung and Robert Heath

AR 1 channel considered.

MF precoder and MRC receiver are used.

TDD Massive MIMO is assumed along with reverse link training

Channel is assumed to be static during the training phase

Uplink training with channel reciprocity assumed

Co channel interference is also considered

The received training signal is

$$Y_{p,b}[n] = \sqrt{P_p \tau} \sum_{c=1}^{C} \boldsymbol{H}_{bc}[n]\,\boldsymbol{\Psi} + \mathbf{Z}_{\mathrm{p,b}}[\mathrm{n}]$$

Channel Model

The MMSE estimate of the channel to the uth user is

$$\widehat{\boldsymbol{h}}_{bbu}[n] = \boldsymbol{R}_{bbu}\left(\frac{\sigma_b^2}{P_p\tau}\boldsymbol{I}_{N_t} + \overline{\boldsymbol{R}}_{bu}\right)^{-1} \boldsymbol{y}_{p,bu}[n]\boldsymbol{\Psi}$$

$$\boldsymbol{R}_{bu} = \sum_{c=1}^{C} \boldsymbol{R}_{bcu}$$

The channel can be decomposed as

$$\boldsymbol{h}_{bbu}[n] = \widehat{\boldsymbol{h}}_{bbu}[n] + \widetilde{\boldsymbol{h}}_{bbu}[n]$$

$$\widehat{\boldsymbol{h}}_{bbu}[n] \sim \mathcal{CN}(0, \boldsymbol{\Phi}_{bbu}) \; \widetilde{\boldsymbol{h}}_{bbu}[n] \sim \mathcal{CN}(0, \mathbf{R}_{bbu} - \boldsymbol{\Phi}_{bbu})$$

Also

$$\boldsymbol{h}_{bbu}[n+1] = \rho\boldsymbol{h}_{bbu}[n] + \sqrt{1-\rho^2}\boldsymbol{\xi}_{bbu}[n]$$

where $\rho = J_0(2\pi f_D T_s)$

More Channel Model

Therefore

$$\boldsymbol{h}_{bbu}[n+1] = \rho\widehat{\boldsymbol{h}}_{bbu}[n] + \rho\widetilde{\boldsymbol{h}}_{bbu}[n] + \sqrt{1-\rho^2}\boldsymbol{\xi}_{bbu}[n]$$

We have an aged channel estimate.

It is also possible to predict the channel using a pth order Wiener predictor V_{bbu}

$$\boldsymbol{V}_{bbu} = \rho[\boldsymbol{\Delta}(p,\rho) \otimes \mathbf{R}_{\mathrm{bbu}}]\mathbf{T}_{\mathrm{bu}}(\mathrm{p},\rho)$$

$$\delta(p,\rho) \coloneqq [1\ \rho\ \ldots\ \rho^p]$$

$$\boldsymbol{\Delta}(p,\rho) = \begin{bmatrix} 1 & \rho & \cdots & \rho^p \\ \rho & 1 & \cdots & \rho^{p-1} \\ \vdots & \vdots & \ddots & \vdots \\ \rho^p & \rho^{p-1} & \cdots & 1 \end{bmatrix}$$

If we consider an AR-1 model then why a pth order predictor?

$$\boldsymbol{T}_{bu} = \left[\boldsymbol{\Delta}(p,\rho) \otimes \overline{\boldsymbol{R}}_{bu} + \frac{\sigma_b^2}{P_p\tau}\boldsymbol{I}_{N_t(p+1)}\right]^{-1}$$

Performance Analysis

Three cases considered

- Current CSI
- Aged CSI
- Predicted CSI

The SINR and achievable rates at $(n+1)$th instant are calculated.

CSI at the BS

$$\boldsymbol{g}_{bbu}[n+1] = \begin{cases} \widehat{\boldsymbol{h}}_{bbu}[n+1] & current\ CSI \\ \alpha \widehat{\boldsymbol{h}}_{bbu}[n] & aged\ CSI \\ \overline{\boldsymbol{h}}_{bbu}[n] & predicted\ CSI \end{cases}$$

The received signal is

$$y_{r,bu}[n+1] \\ = \boldsymbol{w}_{bu}^{H}[n+1]\boldsymbol{g}_{bbu}[n+1]x_{r,bu}[n+1] \\ + w_{bu}^{H}[n+1](\boldsymbol{h}_{bbu}[n+1] - \boldsymbol{g}_{bbu}[n+1])x_{r,bu}[n+1] \\ + \sum_{(c,k)\neq(b,u)} \boldsymbol{w}_{bu}^{H}[n+1]\boldsymbol{h}_{bcu}[n+1]x_{r,ck}[n+1] + z_{r,bu}[n+1]$$

The signal power becomes

$$S_{r,bu} = \left|\boldsymbol{w}_{bu}^{H}[n+1]\boldsymbol{g}_{bbu}[n+1]\right|^2$$

This is the channel vector for MRC

More performance analysis

Interference power is

$$I_{r,bu} = \left|\boldsymbol{w}_{bu}^{H}[n+1](\boldsymbol{h}_{bbu}[n+1] - \boldsymbol{g}_{bbu}[n+1])\right|^{2} + \sum_{(c,k)\neq(b,u)} |g_{bbu}[n+1]|^{2} + \sigma_n^2 \|\boldsymbol{w}[n+1]\|_2^2$$

The deterministic equivalent of the aged SINR after substituting the weights is given as

$$\eta_{r,bu}^{(a)}[n+1] = \frac{|\rho^2 tr\{\boldsymbol{\Phi}_{bbu}\}|^2}{|\rho^2 tr\{\boldsymbol{R}_{bbu} - \boldsymbol{\Phi}_{bbu}\}|^2 + \sigma_n^2 tr\{\boldsymbol{\Phi}_{bbu}\} + \sum_{(c,k)\neq(b,u)} tr\{\boldsymbol{R}_{bck}\boldsymbol{\Phi}_{bck}\} + \rho^2 \sum_{c\neq b} |tr\{\Phi_{bcu}\}|}$$

The deterministic equivalent of the SINR with predicted CSI is

$$\eta_{r,bu}^{(p)}[n+1] = \frac{\rho^2 |tr\{\Theta_{bbu}(p,\rho)\}|^2}{tr\{(R_{bbu} - \rho^2\Theta_{bbu}(p,\rho))\Theta_{bbu}(p,\rho)\} + \sigma_n^2 tr\{\boldsymbol{\Theta}_{bbu}(p,\rho)\} + + \sum_{(c,k)\neq(b,u)} tr\{\boldsymbol{R}_{bck}\boldsymbol{\Theta}_{bbu}(p,\rho)\} + \rho^{2p} \sum_{c\neq b} |tr\{\boldsymbol{\Theta}_{bcu}(p,\rho)\}|}$$

With

$$\boldsymbol{\Theta}_{bcu}(p,\rho) = [\boldsymbol{\delta}(p,\rho) \otimes \mathbf{R}_{\mathrm{bbu}}]\mathbf{T}_{\mathrm{bu}}(\mathrm{p},\rho)[\boldsymbol{\delta}(p,\rho) \otimes \mathbf{R}_{\mathrm{bbu}}]^{\mathrm{H}}$$

Downlink performance analysis

MF precoder is used

The user only knows $E\left[h_{bbu}^{H}[n+1]f_{bu}[n+1]\right]$

The signal at user u of cell b is

$$
\begin{aligned}
& y_{f,bu}[n+1] \\
& = \sqrt{\lambda_b} E\left[\boldsymbol{h}_{bbu}^{H}[n]\boldsymbol{f}_{bu}[n+1]\right] x_{f,bu}[n+1] \\
& + \sqrt{\lambda_b}\left(\boldsymbol{h}_{bbu}^{H}[n]\boldsymbol{f}_{bu}[n+1] - E\left[\boldsymbol{h}_{bbu}^{H}[n]\boldsymbol{f}_{bu}[n+1]\right]\right) x_{f,bu}[n+1] \\
& + z_{f,bu}[n+1] + \sum_{(c,k)\neq(\mathrm{b,u})} \sqrt{\lambda_c}\, \boldsymbol{h}_{cbu}^{H}[n]\boldsymbol{f}_{ck}[n+1] x_{f,ck}[n+1]
\end{aligned}
$$

Where λ_c is a scaling factor to satisfy the total transmit power constraint

$$
\lambda_c = \frac{1}{E\left[\sum_{u=1}^{U} \boldsymbol{f}_{cu}^{H}[n+1]\boldsymbol{f}_{cu}[n+1]\right]}
$$

Similar expressions for downlink SINR are derived

Some results

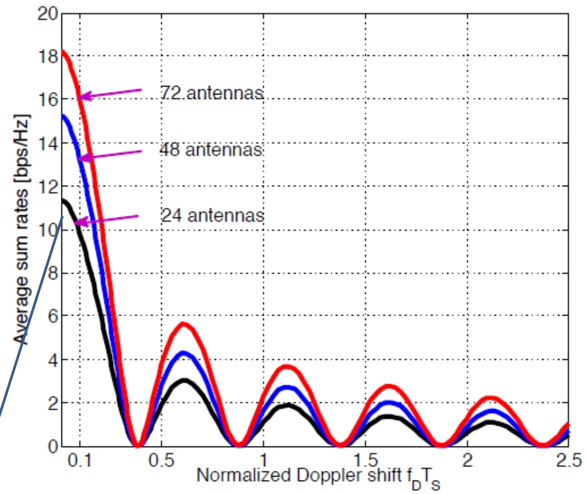


Fig. 3. The downlink average achievable sum-rates of the users in the center cell as a function of normalized Doppler shifts for different numbers of antennas at each base station.

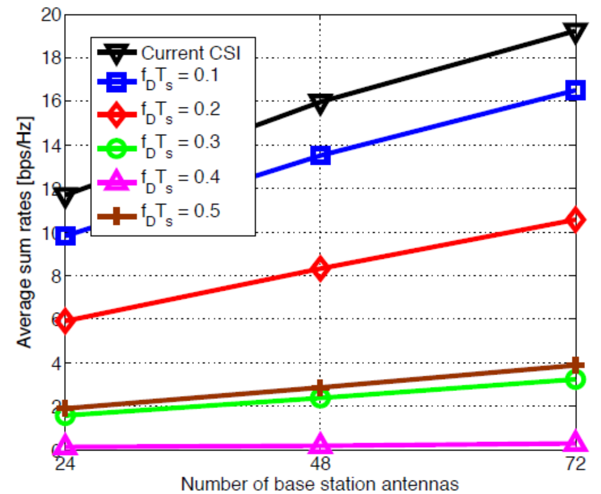


Fig. 4. The uplink average achievable sum-rates of the users in the center cell as a function of the number of antennas at a base station for different normalized Doppler shifts. Each cell has 12 active users, that are uniformly distributed in the cell area.

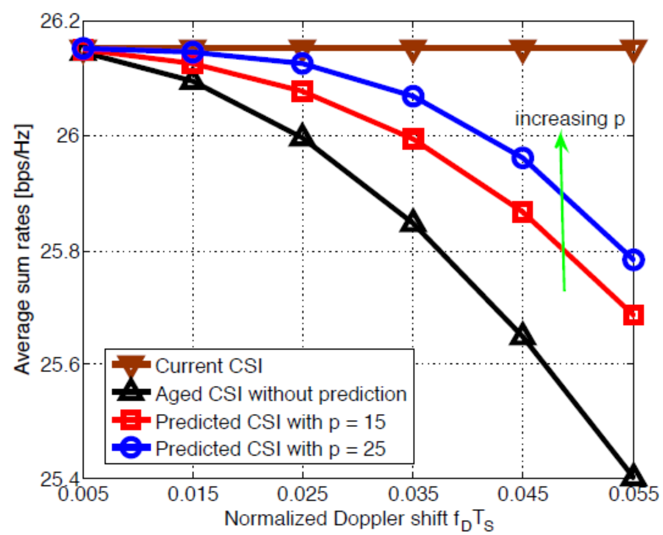


Fig. 6. The uplink achievable rate of the typical user as a function of different normalized Doppler shifts without prediction and with FIR prediction of $p = 50$.

Does not talk about training

Impact of Channel Aging in Multi-Way Relay Networks with Massive MIMO

Amarasuriya and Poor

MWRNs: Multiple spatially distributed nodes exchange signals via relays.

K single antenna user nodes, one M antenna relay.

All terminals in half duplex mode

Single slot MAC phase $K-1$ slot broadcast phase

User to relay channel F; relay to user channel $\boldsymbol{H}$ such that $\boldsymbol{F} = \boldsymbol{H}^T$

$$\boldsymbol{F}[n+1] = \rho \boldsymbol{F}[n] + \overline{\boldsymbol{E}}_F[n]$$

Channel estimate at nth instant

$$\widehat{\boldsymbol{F}}_{ll}[n] = \left(\sum_{i=1}^{L} \boldsymbol{F}_{li}[n] + \frac{\boldsymbol{N}_{F_{li}}[n]}{\sqrt{P_p}} \right) \left(\boldsymbol{D}_{F_{li}} + \frac{\boldsymbol{I}_K}{P_p} \right)^{-1} \boldsymbol{D}_{F_{ll}}$$

More System Model

L such systems are considered and they interfere with each other.

$$F_{ll}[n] = \hat{F}_{ll}[n] + \bar{F}_{ll}[n]$$
$$F_{ll}[n] = \rho\hat{\mathrm{F}}_{ll}[n-1] + E_{F_{ll}}[n]$$
$$E_{F_{ll}}[n] = \rho\bar{F}_{ll}[n] + \bar{E}_{F_{ll}}[n]$$

MAC phase: K users transmit their signals to the BS

$$y_R[n] = \sqrt{P_U}\boldsymbol{F}[n]\boldsymbol{x}_U[n] + \boldsymbol{n}_R[n]$$
$$\widetilde{\boldsymbol{y}}_R[n] = \boldsymbol{W}_R[n]\boldsymbol{y}[n]$$
$$W_R[n] = \left(\widehat{\boldsymbol{F}}^H[n-1]\hat{\mathbf{F}}[\mathrm{n}-1]\right)^{-1}\hat{\mathbf{F}}^H[n-1]$$

BC Phase : The relay uses transmit ZF precoding to transmit an amplified and permuted version of $\boldsymbol{y}_R[n]$ to the user nodes. The channel is assumed to be static during this period, therefore, this paper does not actually consider channel aging

Still More System Model

Transmitted symbol by the realy

$$\widetilde{\boldsymbol{y}}_{R_t}^{(j)}[n] = \boldsymbol{W}_T[n] G_R \boldsymbol{\Pi}_j \boldsymbol{W}_R[n] \boldsymbol{y}_R[n]$$

$$W_T[n] = \widehat{\boldsymbol{H}}^H[n-1]\left(\widehat{\boldsymbol{H}}[n-1]\widehat{\boldsymbol{H}^H}[n-1]\right)^{-1}$$

$\boldsymbol{\Pi}_j$ is the permutation matrix for the jth slot

The received signal vector at the user nodes during the jth slot is

$$\boldsymbol{y}_U^{(j)} = G_R \boldsymbol{H}[n] \boldsymbol{W}_T[n] \boldsymbol{\Pi}_j \boldsymbol{W}_R[n] \boldsymbol{y}_R[n] + \boldsymbol{n}_U[n]$$

The post processing SINR at the kth node for the jth node is given as (this is for perfect CSI)

$$\gamma_{k,j} = \rho^4 P_U P_R \left(\sum_{i=1}^{5} I_{P_i} + \sigma_U^2 \sum_{i=1}^{3} I_{G_i} \right)^{-1}$$

where

$$I_{G_1} = \rho^2 P_U tr\{\boldsymbol{V}_H^{-1}\}$$
$$I_{G_2} = \sigma_R^2 tr\{\boldsymbol{V}_F^{-1}\boldsymbol{V}_H^{-1}\}$$
$$I_{G_3} = P_U tr\left\{\boldsymbol{V}_F^{-1}\boldsymbol{V}_{E_F}^{-1}\boldsymbol{V}_{E_F}^{H}\boldsymbol{V}_F^{-1}\boldsymbol{V}_H^{-1}\right\}$$
$$I_{P_1} = \rho^2 P_R \sigma_R^2\left[\mathbf{V}_{\mathrm{F}}^{-1}\right]_{\mathrm{kk}}$$
$$I_{P_2} = \rho^2 P_U P_R\left[\boldsymbol{V}_F^{-1}\boldsymbol{V}_{E_F}\boldsymbol{V}_{E_F}^{H}\boldsymbol{V}_F^{-1}\right]_{kk}$$
$$I_{P_3} = \rho^2 P_U P_R\left[\boldsymbol{V}_{E_H}\boldsymbol{V}_H^{-1}\boldsymbol{V}_H^{-1}\boldsymbol{V}_{E_H}^{H}\right]_{kk}$$
$$I_{P_4} = P_U P_R\left[\boldsymbol{V}_{E_H}\boldsymbol{V}_H^{-1}\boldsymbol{V}_F^{-1}\boldsymbol{V}_{E_F}\boldsymbol{V}_{E_H}^{H}\boldsymbol{V}_F^{-1}\boldsymbol{V}_H^{-1}\boldsymbol{V}_{E_H}^{H}\right]_{kk}$$
$$I_{P_5} = P_R \sigma_R^2\left[\boldsymbol{V}_{E_H}\boldsymbol{V}_H^{-1}\boldsymbol{V}_F^{-1}\boldsymbol{V}_H^{-1}\boldsymbol{V}_{E_H}^{H}\right]_{kk}$$
$$\boldsymbol{V}_H = \boldsymbol{H}[n-1]\boldsymbol{H}^H[n-1]$$
$$\boldsymbol{V}_F = \boldsymbol{F}^H[n-1]\boldsymbol{F}[n-1]$$
$$\boldsymbol{V}_{E_F} = \boldsymbol{F}^H[n-1]\overline{\boldsymbol{E}}_F[n]$$
$$\boldsymbol{V}_{E_H} = \overline{\boldsymbol{E}}_H[n]\boldsymbol{H}^H[n-1]$$

Similar expressions are derived for imperfect CSI

Deterministic Equivalent Performance analysis of time varying massive MIMO systems

Papazafeiropoulos and Ratanrajah

L cells each with one BS having N antennas serving K users.

TDD system with channel reciprocity

$$\boldsymbol{h}_{jlm}[n] = \boldsymbol{R}_{jlm}^{\frac{1}{2}}[n]\boldsymbol{q}_{jlm}$$
$$\boldsymbol{q}_{jlm} \sim \mathcal{CN}(0, \boldsymbol{I}_N)$$

Training

$$\widetilde{\boldsymbol{Y}}_{p,j}[n] = \boldsymbol{Y}_{p,j}\,[n]\boldsymbol{\Psi}^H = \sqrt{p_p\tau}\left(\sum_{l=1}^{L}\boldsymbol{H}_{jl}[n]\right)\boldsymbol{\Psi}\boldsymbol{\Psi}^H + \boldsymbol{Z}_{p,j}[n]\boldsymbol{\Psi}^H$$

$$\widehat{\boldsymbol{h}}_{jjm}[n] = \boldsymbol{R}_{jjm}\left(\frac{\sigma_j^2}{P_{p\tau}}\boldsymbol{I}_N + \sum_{(l)} R_{jlm}\right)^{-1} \tilde{\mathbf{y}}_{p,jm}$$

$$h_{jjm}[n] = \hat{h}_{jjm}[n] + \tilde{h}_{jjm}[n]$$

System model

$$h_{jjm}[n+1] = \rho \hat{h}_{jjm}[n] + \tilde{e}_{jjm}[n+1]$$

pth order wiener predictor similar to the one used previously

Uplink

$$\boldsymbol{y}_{r,j}[n] = \sqrt{p_r}\boldsymbol{W}_j\boldsymbol{H}_{jj}[n]\boldsymbol{x}_{r,j}[n] + \sqrt{p_r}\sum_{l=1,l\neq j}^{L}\boldsymbol{W}_j^H\boldsymbol{H}_{jl}[n]\boldsymbol{x}_{r,l}[n] + \boldsymbol{z}_{r,j}[n]$$

Downlink

$$y_{f,jm}[n] = \sqrt{p_f\lambda_j}\boldsymbol{h}_{jjm}^H[n]\boldsymbol{f}_{jm}[n]x_{f,jm}[n] + \sum_{(l,k)\neq(j,m)}\sqrt{p_f\lambda_l}\boldsymbol{h}_{ljm}^H[n]\boldsymbol{f}_{\boldsymbol{lk}}[n]x_{f,lk}[n] + z_{f,jm}[n]$$

Asymptotic Performance analysis

Uplink Signal

$$y_{r,jm}[n+1] = \boldsymbol{w}_{jm}^{H}[n+1]\boldsymbol{g}_{jjm}[n]x_{r,jm}[n+1] + \boldsymbol{w}_{jm}^{H}[n+1](\boldsymbol{h}_{jjm}[n]-\boldsymbol{g}_{jjm}[n])x_{r,jm}[n+1] + \sum_{(l,k)\neq(j,m)} \boldsymbol{w}_{jm}^{H}[n+1]\boldsymbol{h}_{\boldsymbol{jlk}}[n+1]x_{r,lk}[n+1] + \tilde{z}_{f,jm}[n+1]$$

$$\hat{\gamma}_{r,jm}[n+1] = \frac{\rho^2\hat{\delta}_{jm}^2}{\frac{1}{N}\delta'_{r,jm} + \frac{\sigma_j^2}{p_r}\frac{1}{N}\hat{\delta}'' + \sum_{(l,k)\neq(j,m)}\frac{1}{N}\hat{\mu}_{jlkm} + \alpha^2\sum_{l\neq j}\left|\hat{v}_{jlm}\right|^2}$$

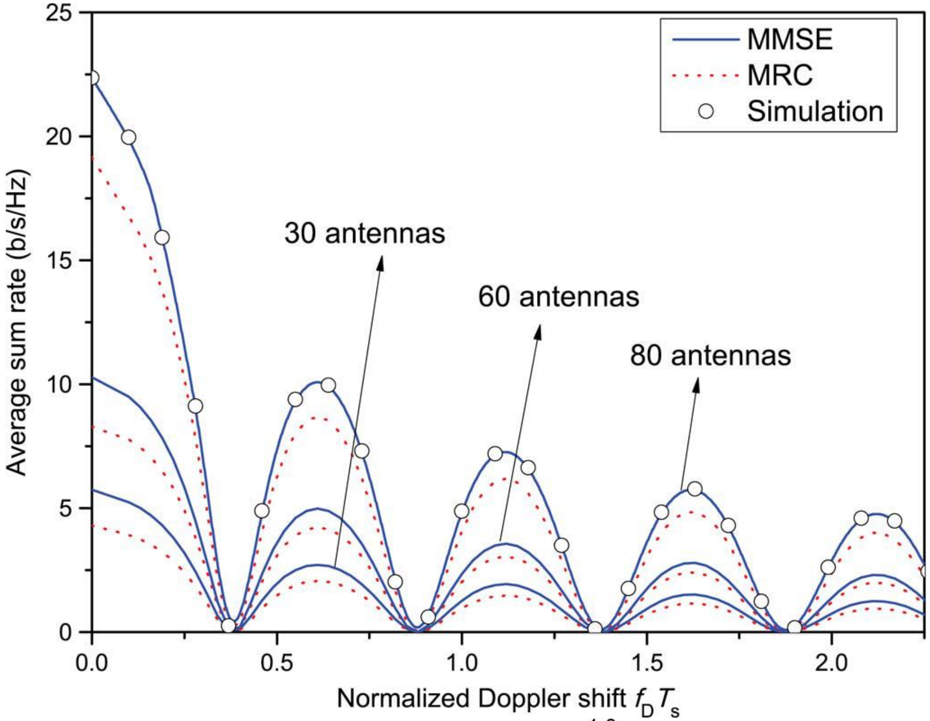
$$\bar{\gamma}_{r,jm}[n+1] = \frac{\rho^2\bar{\delta}_{jm}^2}{\frac{1}{N}\bar{\delta}'_{r,jm} + \frac{\sigma_j^2}{p_r}\frac{1}{N}\bar{\delta}'' + \sum_{(l,k)\neq(j,m)}\frac{1}{N}\bar{\mu}_{jlkm} + \alpha^2\sum_{l\neq j}\left|\bar{v}_{jlm}\right|^2}$$

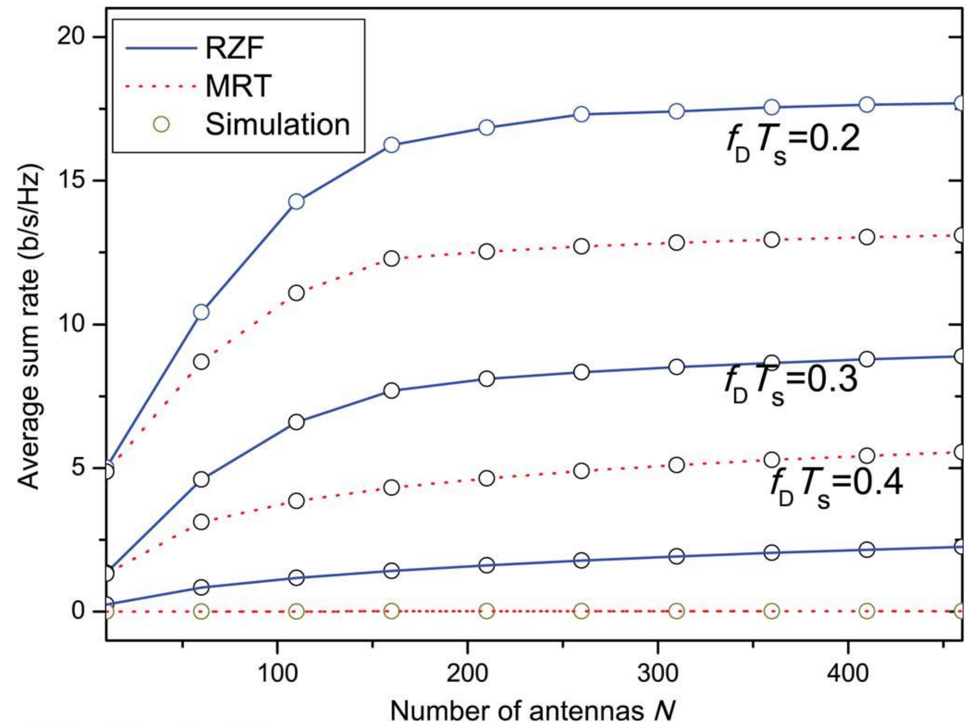
Downlink Transmission

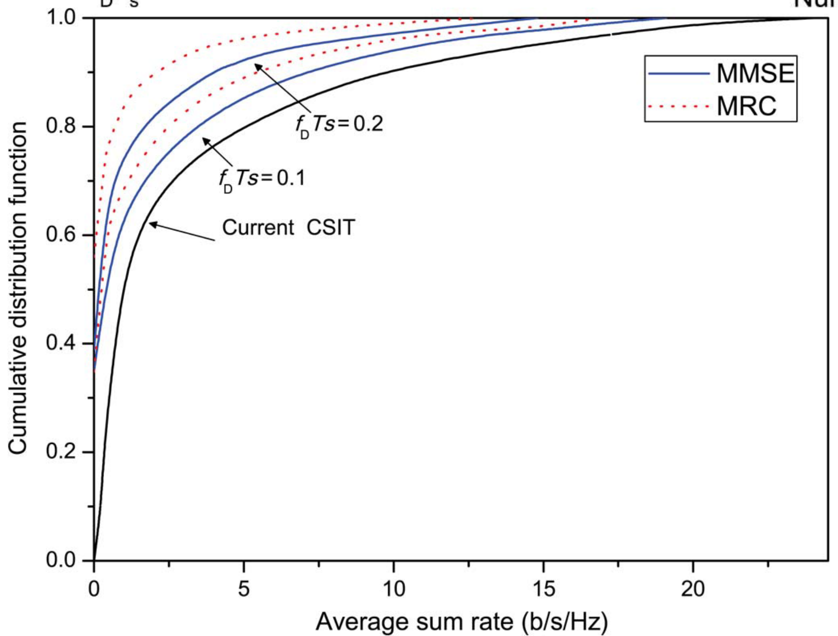
$$
\begin{aligned}
y_{f,jm}[n] &= \sqrt{\lambda_j E\left[\boldsymbol{h}_{jjm}^{H}[n]\boldsymbol{f}_{jm}[n]\right]} x_{f,jm}[n] \\
&+ \sqrt{\lambda_j}\left(\boldsymbol{h}_{jjm}^{H}[n]\boldsymbol{f}_{jm}[n] - E\left[\boldsymbol{h}_{jjm}^{H}[n]\boldsymbol{f}_{jm}[n]\right]\right) x_{f,jm}[n] \\
&+ \sum_{(l,k)\neq(j,m)} \sqrt{\lambda_l}\boldsymbol{h}_{ljm}^{H}[n]\boldsymbol{f}_{\boldsymbol{lk}}[n] x_{f,lk}[n] + z_{f,jm}[n]
\end{aligned}
$$

Similar expressions for downlink SINR are derived

Some Results







Sum Rate and Power Scaling of Massive MIMO systems with Channel Aging

Kong Zhong Papazafeiropoulos Matthaiou Zhang

Uplink model considered

$$\boldsymbol{G}[n] = \boldsymbol{H}[n]\boldsymbol{D}^{\frac{1}{2}}$$

$$\widetilde{\boldsymbol{Y}}[n] = \frac{1}{\sqrt{P_p}}\boldsymbol{J}[n]\boldsymbol{\Phi}^{\mathrm{H}}$$

Two receivers considered

$$\widehat{\boldsymbol{A}}[n+1] = \begin{cases} \overline{\boldsymbol{G}}[n+1] & MRC \\ \overline{\boldsymbol{G}}[n+1](\boldsymbol{G}^{H}[n+1]\overline{\boldsymbol{G}}[n+1])^{-1} & ZF \end{cases}$$

The kth element of the received vector is

$$\boldsymbol{r}[n+1] = \boldsymbol{A}^{H}[n+1]\boldsymbol{G}[n+1]\boldsymbol{x}[n+1] + \boldsymbol{A}^{H}[n+1]\boldsymbol{z}[n+1]$$

$$r_k[n+1] = \sqrt{p_u}\boldsymbol{a}_k^{H}[n+1]\boldsymbol{g}_k[n+1]x_k[n+1] + \sqrt{p_u}\sum_{i=1,i\neq k}^{K}\boldsymbol{a}_k^{H}[n+1]\overline{\boldsymbol{g}}_i[n+1]x_i[n+1] + \sqrt{p_u}\sum_{i=1}^{K}\boldsymbol{a}_k^{H}[n$$

Achievable uplink rate

$$R = \frac{T - \tau}{T} \sum_{k=1}^{K} R_k$$

$$R_k = E\left[\log_2\left(1\right.\right.$$

With the Wiener predictor

$$R_k^{p,mrc} - \log_2\left(1 + \frac{\alpha^2 \sum_{j=0}^{p} \alpha^{2j} \tau E_u^2 \beta_k^2}{M^{2\gamma-1}}\right) \to^{M\to\infty} 0$$

The model is extended to single cell downlink to show power scaling there as well.

Another extension is made for multi-cell cellular systems by introducing a pilot contamination term in the channel estimation part and an inter cell interference term in the SINR expression

Some Results

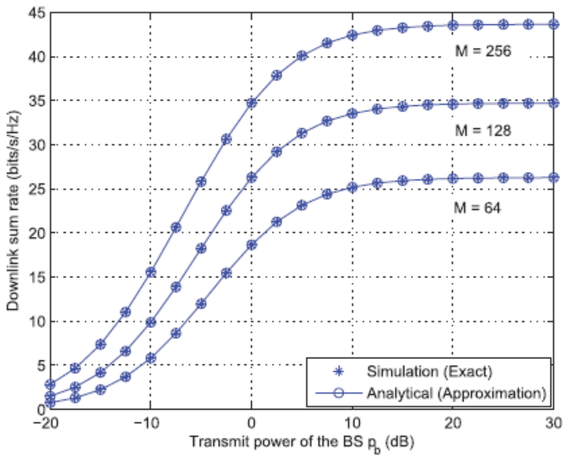


Fig. 4. Downlink sum-rate with MRT precoder versus the transmit power p_b for $K = 10$, $M = 64$, $p_p = 10$ dB, and $f_D T_s = 0.1$.

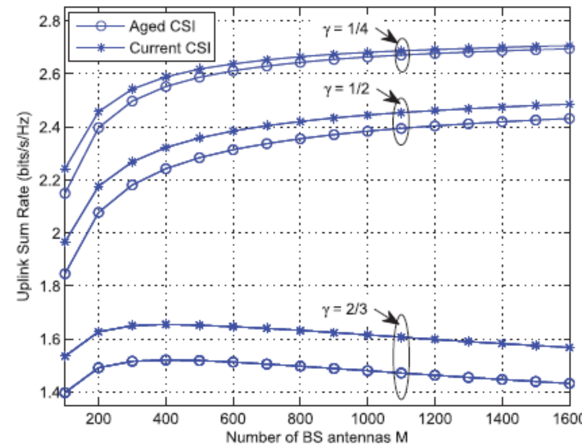


Fig. 6. Uplink sum-rate versus the number of BS antennas M for MRC receivers with aged CSI for $K = 10$, $f_D T_s = 0.1$, and $p_u = E_u/M^{\gamma}$ with $E_u = 15$ dB.

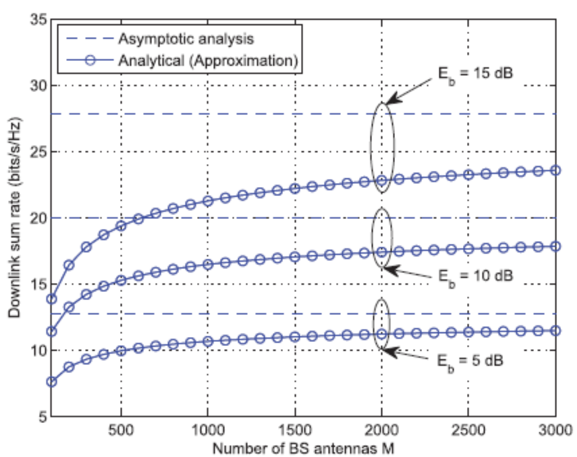


Fig. 5. Downlink sum-rate versus the number of BS antennas M with MRT precoder for $K = 5$, $f_D T_s = 0.1$, $p_p = \tau E_u/\sqrt{M}$ with $E_u = 3$ dB, and $p_b = E_b/\sqrt{M}$.

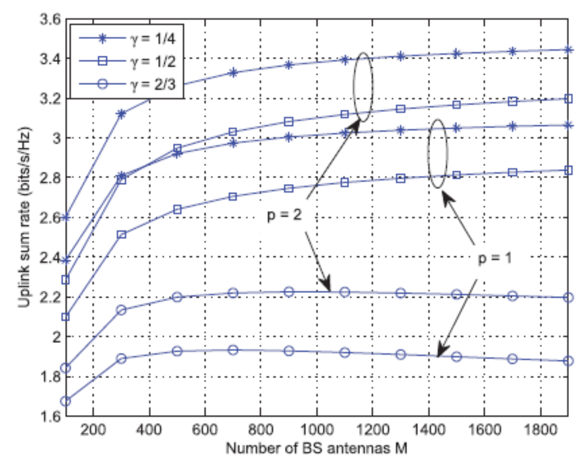


Fig. 7. Uplink sum-rate versus the number of BS antennas M for MRC receivers with channel prediction for $K = 10$, $f_D T_s = 0.1$, and $p_u = E_u/M^{\gamma}$ with $E_u = 15$ dB.

Impact of general channel aging conditions on the downlink performance of Massive MIMO

Papazafeiropoulus

User mobility is not the only cause of channel aging

Phase noise is also shown to have channel aging effects

MRT and RZF precoding based systems are considered

Deterministic equivalents for downlink SNRs are obtained

Single cell system with K single antenna non-cooperating UEs

Each BS antenna has an independent phase noise

$$\phi_m[n] = \phi_m[n-1] + \delta^{\phi_m}[n]$$
$$\varphi_m[n] = \varphi_m[n-1] + \delta^{\varphi_m}[n]$$

Both are ZMGs with variance $c_i 4\pi^2 f_c T_s$ c_i is an oscillator dependent constant.

More system model

It is explicitly stated that aging during training is neglected.

$$\boldsymbol{g}_k[n] = \boldsymbol{A}[n]\boldsymbol{g}_k[0] + \boldsymbol{e}_k[n]$$
$$= \boldsymbol{A}[n]\widehat{\boldsymbol{g}}_k[0] + \tilde{\boldsymbol{e}}_k[n]$$

$$\boldsymbol{A}[n] = J_0(2\pi f_D T_s n) e^{-\frac{\sigma_{\varphi_k}^2}{2} n} \Delta\boldsymbol{\Phi}[\mathrm{n}]$$

$$\Delta\boldsymbol{\Phi}[\mathrm{n}] = \mathrm{diag}\left\{ e^{-\frac{\sigma_{\phi_1}^2}{2} n} e^{-\frac{\sigma_{\phi_2}^2}{2} n} \ldots e^{-\frac{\sigma_{\phi_K}^2}{2} n} \right\}$$

In case when all the BS antennas are connected to the same oscillator, or all the oscillators are identical,

$$\boldsymbol{A}_n = \rho \boldsymbol{I}_M, \rho = J_0(2\pi f_D T_s n) e^{-\frac{\sigma_{\phi_k}^2 + \sigma_{\varphi_k}^2}{2} n}.$$

The received signal is

$$y_k[n] = \sqrt{\lambda_{p_d}} E\left[\boldsymbol{g}_{k,n}^H \boldsymbol{\Theta}_{k,n}^2 \boldsymbol{f}_{k,n}\right] x_{k,n} + z_{k,n} + \sum_{i \neq k} \sqrt{\lambda p_d} \boldsymbol{g}_{k,n}^H \boldsymbol{\Theta}_{k,n}^2 \boldsymbol{f}_{i,n} \boldsymbol{x}_{i,n} + \sqrt{\lambda_{p_d}} \left(E\left[\boldsymbol{g}_{k,n}^H \boldsymbol{\Theta}_{k,n}^2 \boldsymbol{f}_{k,n}\right] - \boldsymbol{g}_{k,n}^H \boldsymbol{\Theta}_{k,n}^2 \boldsymbol{f}_{k,n} \right) x_{k,n}$$

Downlink Transmission

$$R_k = \frac{1}{T_c}\sum_{n=1}^{T_c-\tau} R_{k,n} = \frac{1}{T_c}\sum_{n=1}^{T_c-\tau} \log_2(1+\gamma_k[n])$$

With MRT precoding

$$\bar{\gamma}_k[n] = \frac{e^{-2\left(\sigma_{\varphi_k}^2+\sigma_\phi^2\right)n} tr^2\{\boldsymbol{A}_n^2\boldsymbol{D}_k\}}{tr\{\boldsymbol{A}_n^2\boldsymbol{D}_k(\boldsymbol{R}_k-\boldsymbol{A}_n^2\boldsymbol{D}_k)\} + \frac{M\sigma_k^2}{p_d\bar{\bar{\lambda}}} + \sum_{i\neq \mathrm{k}} tr\{\boldsymbol{A}_n^2\boldsymbol{D}_i\boldsymbol{R}_k\}}$$

For a diagonal $\boldsymbol{D}_k$ this is shown to allow a power scaling of $\frac{1}{\sqrt{M}}$

Similar expressions for RZF transmission as well.

Some more results

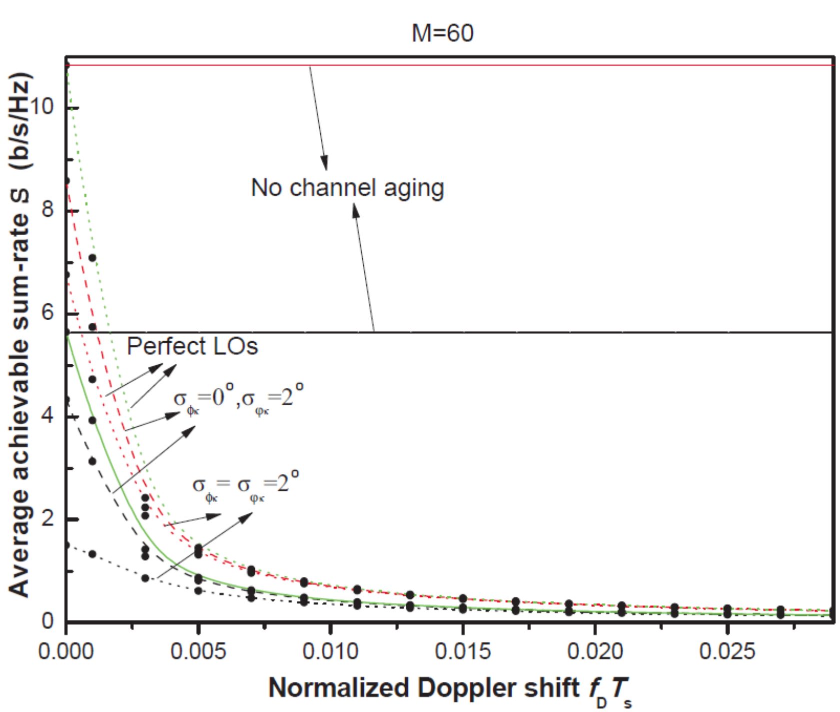


Fig. 2. Simulated and DE downlink sum-rates with MRT and RZF precoders when $M = 60$ as a function of the normalized Doppler shift for various values of phase noise. Red and black lines correspond to the theoretical sum-rates with RZF and MRT precoding, respectively, while the black bullets refer to the simulation results. The green "solid" and "dot" lines mirror a scenario with channel aging but not phase noise in the cases of MRT and RZF, respectively. Lines parallel to x-axis represent scenarios with no channel aging.

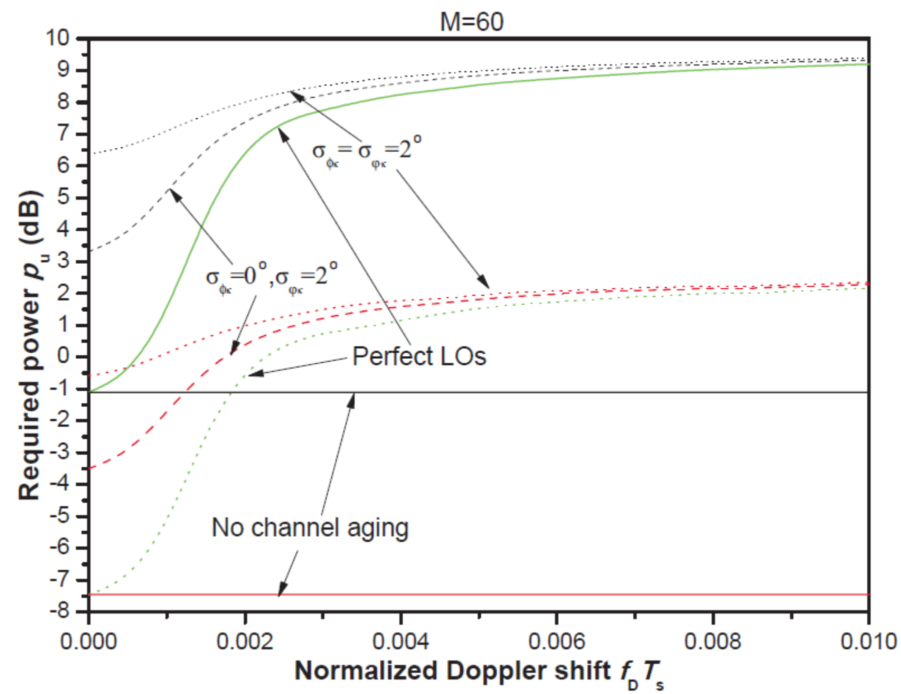


Fig. 4. Required transmit power to achieve 1 bit/s/Hz per user with MRT and RZF precoders when $M = 60$ as a function of the normalized Doppler shift for various values of phase noise. Red and black lines correspond to the theoretical sum-rates with RZF and MRT precoding, respectively. The green "solid" and "dot" lines mirror a scenario with channel aging but not phase noise in the cases of MRT and RZF, respectively. Lines parallel to x-axis represent scenarios with no channel aging.

Study of Effect of Training for Downlink Massive MIMO systems with Outdated Channel

Kim Min and Choi

The system model is similar to the one discussed previously, and the effect of training on system performance is considered

This is quantized in terms of the rate gap

$$\Delta\bar{R}_k = \bar{R}_k^{ideal} - \bar{R}_k$$

This is lower bounded as

$$\Delta\bar{\mathrm{R}}_k \geq \log_2\left(1 + \frac{p_k\beta_k\rho_{m,k}^2 B_k^2\left(\hat{\sigma}_k^2 + \hat{\mu}_k^2 - 1\right)}{I + p_k\beta_k\left(\sigma_k^2 + \rho_{m,k}^2 A_k^2\hat{\mu}_k^2\right)}\right)$$

With

$$I = \beta_k \sum_{j\neq \mathrm{k}} p_j\left(\sigma_j^2 + \mu_j^2\right) + 1$$

What have we done?

A beamforming based system considered

$$\hat{\mu} = \boldsymbol{u}^H \widehat{\boldsymbol{H}} \boldsymbol{v}$$

Dominant Eigenmode considered.

Dominant singular value $\hat{\mu} = N_t(1 + \sqrt{c})$

SINR

$$\gamma[n] = \frac{\rho^{2n}\left(1 - \frac{N_0}{\mathcal{E}_p}\right) N_t (1 + \sqrt{c})^2 \mathcal{E}_s}{\left(\rho^{2n}\frac{N_0}{\mathcal{E}_p} + (1 - \rho^{2n})\right)\mathcal{E}_s + N_0}$$

Channels is good to use until $\gamma[n]$ above a certain γ_{th}.

Can define the usable time as

$$\gamma[T_c] = \gamma_{th}$$

What have we done?

The overall throughput becomes

$$R = \left(1 - \frac{N_t}{T_c}\right)\log(1 + \gamma_{th})$$

We can select N_t, T_c and γ_{th} to optimize this.

$\mathcal{E}_s$ and $\mathcal{E}_p$ are to be chosen to satisfy

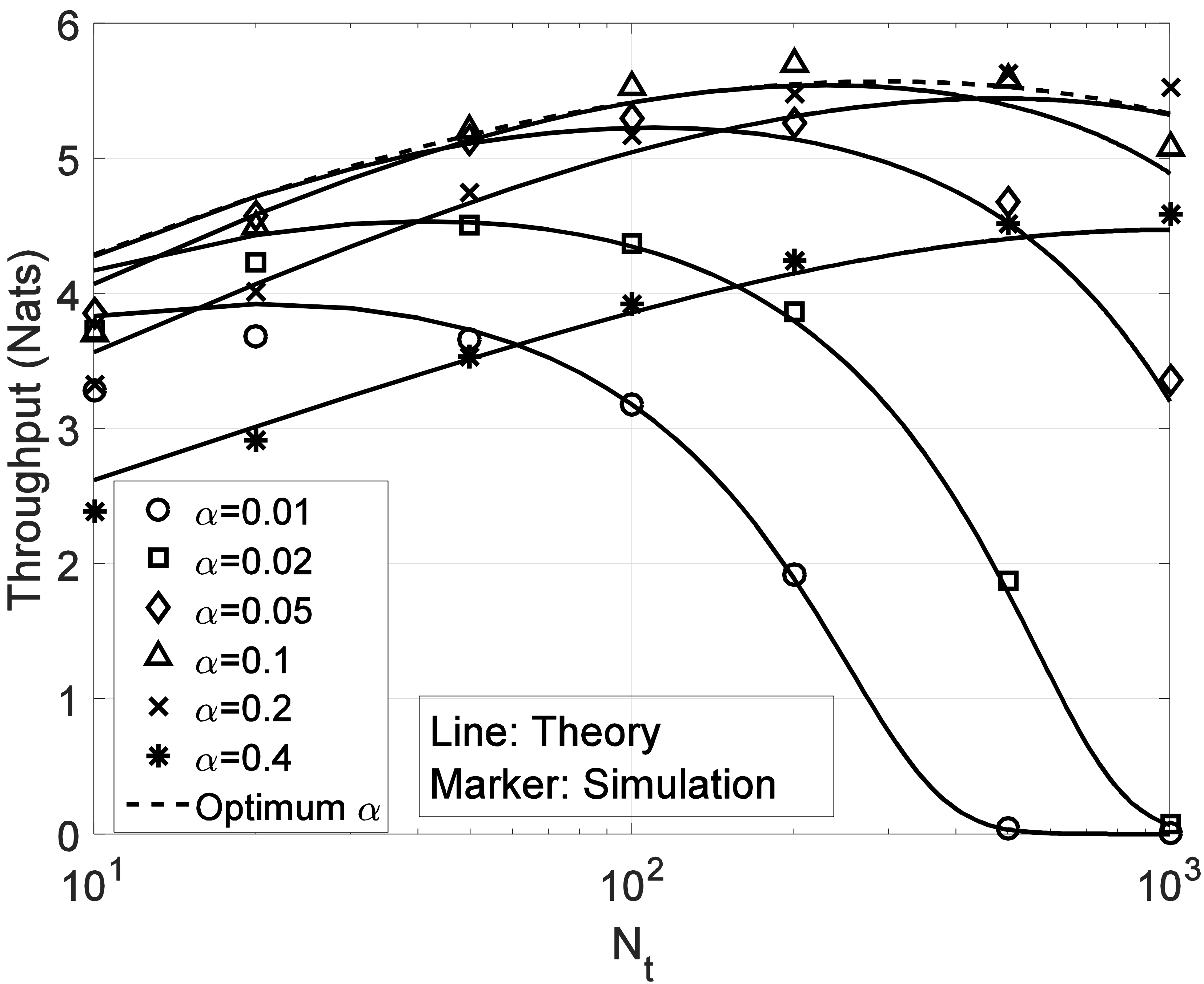
$$N_t\mathcal{E}_p + (T_c - N_t)\mathcal{E}_s = T_c\mathcal{E}_t$$

But these can also be tuned to maximize the throughput

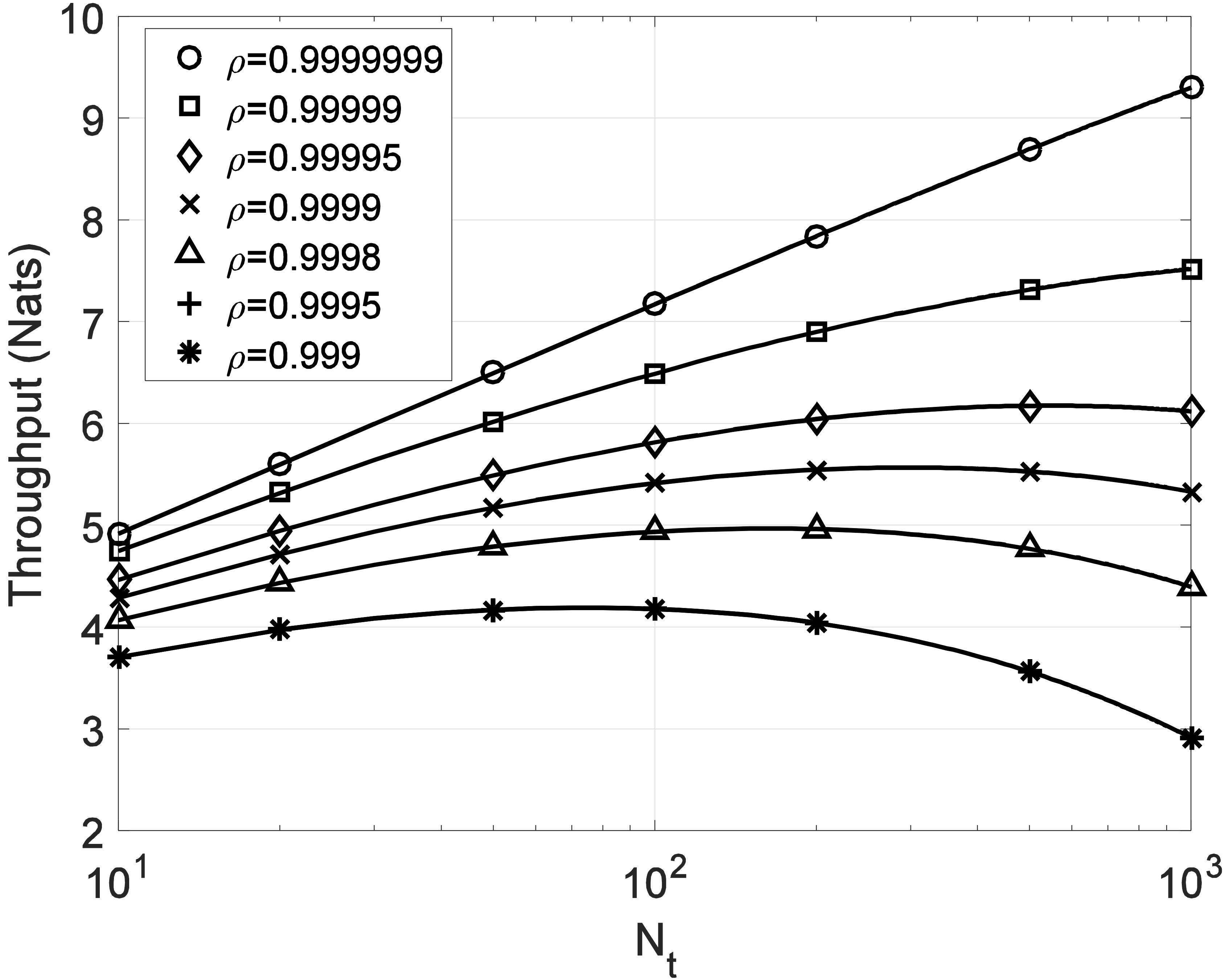
$$\alpha = \frac{N_t}{T_c}$$

The optimal $\mathcal{E}_s$ is

$$\mathcal{E}_s^* = \frac{\left(\frac{1-\alpha}{\alpha}\right)\frac{\mathcal{E}_t}{\alpha} - \sqrt{\left(\frac{1-\alpha}{\alpha}\right)\frac{\mathcal{E}_t}{\alpha}\left[\frac{1-\alpha}{\alpha}N_0 + \frac{\mathcal{E}_t}{\alpha} - N_0\right]}}{\left[\frac{1-2\alpha}{\alpha}\frac{1-\alpha}{\alpha}\right]}$$



Throughput (Nats)
N_t
0
1
2
3
4
5
6
10^1
10^2
10^3
α=0.01
α=0.02
α=0.05
α=0.1
α=0.2
α=0.4
Optimum α
Line: Theory
Marker: Simulation



ρ=0.9999999
ρ=0.99999
ρ=0.99995
ρ=0.9999
ρ=0.9998
ρ=0.9995
ρ=0.999
Throughput (Nats)
10
9
8
7
6
5
4
3
2
10^1
10^2
10^3
N_t

What more can we do?

No one has considered aging while training, in which case a Kalman filter is the optimal solution

An outage based analysis is needed for uplink and downlink systems

The CHEMP receiver

Questions ?


That's all Folks!