Diversity-Multiplexing Tradeoff in MIMO Channels with Finite Rate Feedback

Anup Aprem

Advisor : Chandra R Murthy

Outline

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DMT

Diversity-Multiplexing Tradeoff(DMT) is essentially the tradeoff between **Error Probability** and **Data Rate** of the System.

Definition

$$\lim_{SNR \to \infty} \frac{R(SNR)}{\log \text{SNR}} = g_m$$

$$\lim_{SNR \to \infty} \frac{\log P_e(\text{SNR})}{\log \text{SNR}} = -d$$

Preliminaries (cont.)

Motivation

- Zheng and Tse [1] showed that DMT(in absence of CSIT) is a linear piecewise function of the multiplexing gain.
- If perfect CSIT, then we can achieve ∞ diversity.

Notation

$$\lim_{SNR \to \infty} \frac{\mathsf{f(SNR)}}{\log \mathsf{SNR}} = b$$

is denoted as

$$\mathsf{f(SNR)} \doteq \mathsf{SNR}^b$$

Similarly $\stackrel{\circ}{\geq}$ and $\stackrel{\circ}{\leq}$ is defined

Preliminaries (cont.)

Acquiring CSIT

- Training for Reciprocal Channels (Analog Feedback)
- Finite Rate Feedback (Quantized/Digital Feedback)
- Quantized feedback shown to be superior to Analog Feedback in presence of CSIT errors[2]

Challenges

Best quantity to be fed back to the transmitter that optimizes the system performance (e.g. minimizing outage) is largely OPEN.

Preliminaries (cont.)

Side Note

- Zheng and Tse[Lemma 5] proved that the probability of error is lower bounded by the outage probability.

$$P_e(SNR) \overset{\circ}{\geq} SNR^{-d_{out}(r)}$$

- We will charachterize Outage Probability instead of Probability of error.

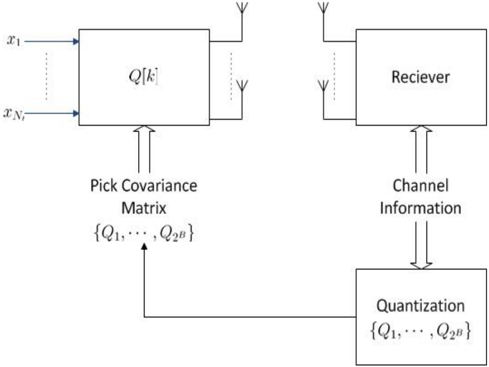


Figure: General Structure of Finite Rate Feedback

Outline

Outage Formulation

- An encoder $\mathcal{I}$ is a mapping from the channel state H to a covariance matrix Q_i, such that trace$(Q_i) \leq P_i$
- Outage Definition.

$$P_{out,K}(R) = Pr\left[\log\det\left(I_{N_r} + HQ_{\mathcal{I}(H)}H^{\dagger} \leq R\right)\right]$$

- Picking $\bar{Q}_i = P_i I_{N_t}$ gives an lower bound on the outage probability.
- Picking $\underline{Q_i} = \frac{P_i}{N_t} I_{N_t}$ gives a upper bound on the outage probability.

Outage Formulation (cont.)

- Outage Probabilty satisfies

$$Pr\left[\log\det\left(I_{N_r} + P_{\mathcal{I}} HH^{\dagger} \leq R\right)\right] \leq P_{out}(R)$$
$$\leq P\left[\log\det\left(I + \frac{P_{\mathcal{I}}}{N_t} HH^{\dagger}\right) \leq R\right]$$

- We will restrict our analysis to power codebook of form $\{P_i^*\}_{i=1}^{K}$

Two Lemmas

- Define $F(\rho, \pi) \triangleq Pr\left(I(H, \pi) \leq \rho\right)$, where

$$I(H, \pi) \triangleq \log \det \left(I_{N_t} + \frac{\pi}{N_t} HH^{\dagger} \right)$$

- Lemma 1: For a given SNR and rate R, the outage minimizing power codeboook $\{P_i^*\}_{i=1}^{K}$ solves the following optimization problem.

$$\begin{aligned} & max P_K \\ & \quad s.t \left[F(R, P_K) + 1 - F(R, P_1)\right] P_1 \\ & \qquad + \sum_{i=2}^{K} \left[F(R, P_{i-1}) - F(R, P_i)\right] P_i \leq SNR, \\ & \qquad 0 \leq P_1 \leq \cdots \leq P_K \end{aligned}$$

Two Lemmas (cont.)

The optimal deterministic mapping is given by

$$\mathcal{I}^*(H) = \begin{cases} 1 \quad if \;\; I(H, P_K^*) \leq R \\ min\{i : i \in 1, 2, \cdots, K, I(H, P_i^*) \geq R\} \end{cases}$$

Two Lemmas (cont.)

- Lemma 2: For r $\in$ (0,n), let π be a function of SNR such that $\pi \doteq SNR^p$, where p is a finite constant and $p \geq 1$. Denoting $(x)^+ = \max(x, 0)$, we have

$$F(r \log \text{SNR}, \pi) \doteq \text{SNR}^{-D(r,p)}$$

and

$$D(r,p) \triangleq \inf_{\alpha_1^n \in A} \sum_{i=1}^{n} (2i - 1 + m - n)\alpha_i$$

where,

$$A \triangleq \left\{ \alpha_1^n | \alpha_1 \geq \cdots \geq \alpha_n \geq 0, \sum_{i=1}^{n} (p - \alpha_i)^+ \leq r \right\}$$

Outline

Optimal DMT Tradeoff Theorem

Theorem

Kim[3]: The optimal D-M tradeoff of a single-rate MIMO system with K quantization regions in the feedback link is upper bounded by the outage bound

$$d^*_{out,K}(r) \;=\; D(r, 1 + d^*_{out,K-1}(r))$$

where $d_{out,0}(r) \;\triangleq\; 0, \forall r$

Optimal DMT Tradeoff Theorem (cont.)

Let $\bar{P}_i$ be the solution of the following optimization problem, which is a relaxed version of the previous problem

$$
\begin{aligned}
& max P_K \\
& \quad s.t \left[F(R, P_K) + 1 - F(R, P_1)\right] P_1 \leq SNR \\
& \qquad \left[F(R, P_{i-1}) - F(R, P_i)\right] P_i \leq SNR \quad i \geq 2 \\
& \qquad 0 \leq P_1 \leq \cdots P_K
\end{aligned}
$$

$\bar{P}_K \geq P_K^*$ due to relaxation.
Summing up the constraints, we have

$$\sum_{i=1}^{K} \frac{SNR}{\bar{P}_i} \geq 1$$

Optimal DMT Tradeoff Theorem (cont.)

We must have $\bar{P}_i \leq K\ SNR$, otherwise

$$\sum_{i=1}^{K} \frac{SNR}{\bar{P}_i} \leq K \frac{1}{K} = 1$$

Hence,

$$\bar{P}_1 \overset{\circ}{\leq} SNR$$

$$F(R, \bar{P}_1) \overset{\circ}{\geq} SNR^{-D(r,1)} = SNR^{-d^*_{out,1}(r)}$$

Second Constraint implies that

$$\frac{\text{SNR}}{\bar{P}_2} + F(R, \bar{P}_2) \overset{\circ}{\geq} SNR^{-d^*_{out,1}(r)}$$

Optimal DMT Tradeoff Theorem (cont.)

Suppose $\bar{P}_2 \doteq \mathsf{SNR}^{1+d^*_{out,1}(r)+\epsilon}$. Then

$$\frac{\mathsf{SNR}}{\bar{P}_2} + F(R, \bar{P}_2) \doteq \mathsf{SNR}^{-d^*_{out,1}(r)-\varepsilon} + \mathsf{SNR}^{-D(r,1+d^*_{out,1}(r)+\varepsilon)} \quad (1)$$

which contradicts 9 because $D(r, 1 + d^*_{out,1}(r) + \varepsilon) \geq D(r, 1) = d^*_{out,1}(r)$. Therefore, we require

$$\bar{P}_2 \overset{\circ}{\leq} SNR^{1+d^*_{out,1}(r)}$$

and thus,

$$F(R, \bar{P}_2) \overset{\circ}{\geq} SNR^{-D(r,1+d^*_{out,1}(r))} = SNR^{-d^*_{out,2}(r)}$$

Optimal DMT Tradeoff Theorem (cont.)

Following this procedure, we get for k = K,

$$F(R, \bar{P}_K) \overset{\circ}{\geq} SNR^{-D(r, 1+d^*_{out,K-1}(r))} = SNR^{-d^*_{out,K}(r)}$$

Going back to the original problem. If,

$$\underline{P_1} = \frac{SNR}{K}$$

$$\underline{P_2} = \frac{SNR}{K \; F(R, P_1)}$$

$$\vdots$$

$$\underline{P_K} = \frac{SNR}{K \; F(R, P_{K-1})}$$

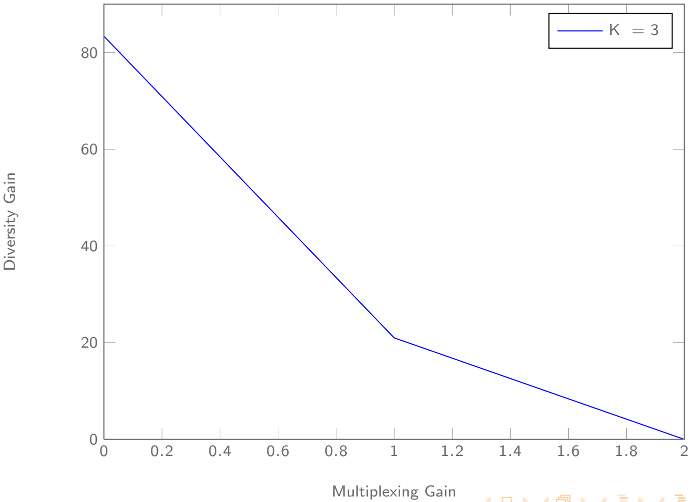
Optimal DMT Tradeoff Theorem (cont.)

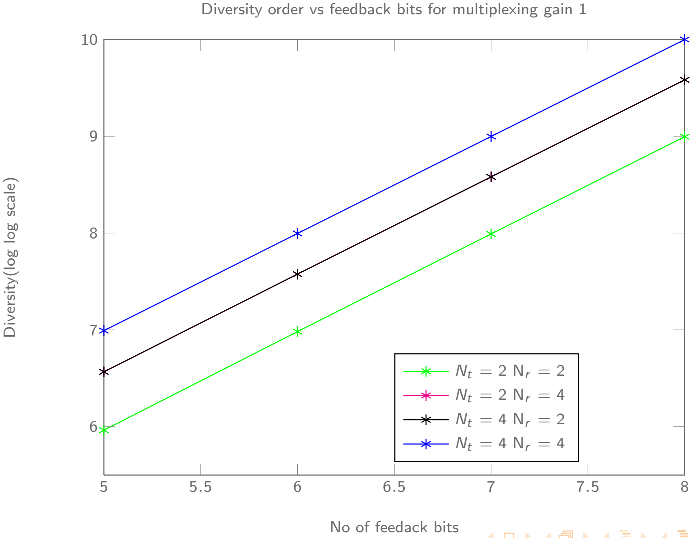
satisfy the constraints and hence we have a lower bound. Therefore by construction.

$$\begin{aligned}
\underline{P_1} &\mathrel{\dot{=}} SNR \\
\underline{P_2} &\mathrel{\dot{=}} SNR^{1+d^*_{out,1}(r)} \\
&\vdots \\
\underline{P_K} &\mathrel{\dot{=}} SNR^{1+d^*_{out,K-1}(r)}
\end{aligned}$$

□

D — M Tradeoff of single rate transmission over a 2 x 2




Diversity order vs feedback bits for multiplexing gain 1
10
9
8
7
6
5
5.5
6
6.5
7
7.5
8
Diversity(log log scale)
No of feedack bits
N_t = 2 N$_r$ = 2
N_t = 2 N$_r$ = 4
N_t = 4 N$_r$ = 2
N_t = 4 N$_r$ = 4

Outline

Problem

- DMT performance in the case of generalized covariance codebooks.
- Following the above approach, we have been able to extend the proof for the case of $\{P_i \; D\}_{i=1}^{K}$, where D is a diagonal matrix.

Reference

- L. Zheng and D. Tse, "Diversity and multiplexing: A fundamental tradeoff in multiple-antenna channels," *Information Theory, IEEE Transactions on*, vol. 49, no. 5, pp. 1073–1096, 2003.
- G. Caire, N. Jindal, M. Kobayashi, and N. Ravindran, "Quantized vs. analog feedback for the mimo broadcast channel: A comparison between zero-forcing based achievable rates," in *Information Theory, 2007. ISIT 2007. IEEE International Symposium on*, pp. 2046–2050, IEEE.
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Thank YOU