Implementation and performance comparison of SSR algorithms for channel estimation in OFDM systems using SDR

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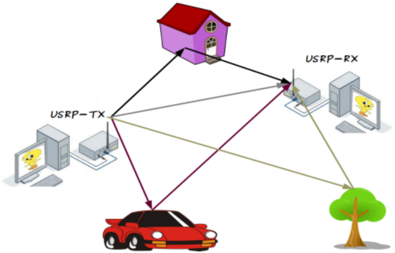
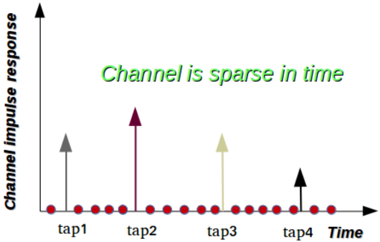
Overview

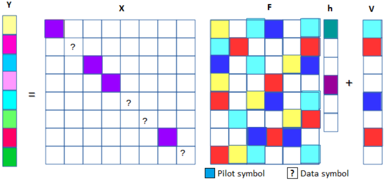
Wireless Communication Channel





Sparse in lag domain

OFDM Channel Model



$$\mathbf{y} = \mathbf{X}\mathbf{F}\mathbf{h} + \mathbf{v}$$

$\mathbf{y} \in \mathbb{C}^{N\times 1}$ is the received vector after FFT

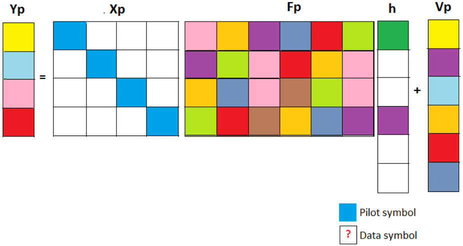
$\mathbf{X} \in \mathbb{C}^{N\times N}$ contains data symbol and pilot symbols along the diagonal

$\mathbf{F} \in \mathbb{C}^{N\times L}(N > L)$ contains the first L columns of $N \times N$ DFT matrix

$\mathbf{v} \in \mathbb{C}^{N\times 1} \sim \mathcal{CN}(0, \sigma^2\mathrm{I})$ is the AWGN noise

$\mathbf{h} \in \mathbb{C}^{L\times 1}$ is the time domain channel response

Channel Model using *Pilots* only:



$$\mathbf{y}_P = \mathbf{X}_p \mathbf{F}_p \mathbf{h} + \mathbf{v}_p, \qquad (P < L)$$
$$= \phi_p \mathbf{h} + \mathbf{v}_p$$

SBL Framework

$$\mathbf{h} \sim \mathcal{CN}(0, \mathbf{\Gamma}), \qquad \mathbf{\Gamma} = \mathrm{diag}(\gamma(1), ..., \gamma(L))$$

SBL estimation problem:

$$\begin{aligned}\hat{\mathbf{h}} &= \underset{\mathbf{h}, \gamma \in \mathbb{R}_+^{L\times 1}}{\arg\max}\ p(\mathbf{y}_p|\mathbf{h};\gamma)p(\mathbf{h};\gamma) \\ &= \underset{\mathbf{h}, \gamma \in \mathbb{R}_+^{L\times 1}}{\arg\min}\ \frac{\|\mathbf{y}_p - \mathbf{X}_p\mathbf{F}_p\mathbf{h}\|_2^2}{\sigma^2} + \log|\mathbf{\Gamma}| + \mathbf{h}^H\mathbf{\Gamma}^{-1}\mathbf{h}\end{aligned}$$

Instead of estimating $\mathbf{h}$ directly, we estimate γ first using type II ML estimate as given below

$$\hat{\gamma}_{ML} = \underset{\gamma \in \mathbb{R}_+^{L\times 1}}{\arg\max}\ p(\mathbf{y}_p;\gamma)$$

EM algorithm

$$p(\mathbf{h};\gamma) = \prod_{i=1}^{L} (\pi\gamma(i))^{-1} \exp\left(-\frac{|h(i)|^2}{\gamma(i)}\right)$$

$$\textbf{E-step:} \;\; Q\left(\gamma|\gamma^{(r)}\right) = \mathbb{E}_{\mathbf{h}|\mathbf{y}_p;\gamma^{(r)}}\left[\log\, p(\mathbf{y}_p, \mathbf{h};\gamma)\right]$$

$$\textbf{M-step:} \;\; \gamma^{(r+1)} = \underset{\gamma\in\mathbb{R}_+^{L\times 1}}{\arg\max}\; Q\left(\gamma|\gamma^{(r)}\right)$$

$$\gamma^{(r+1)}(i) = \Sigma(i,i) + |\mu(i)|^2$$

Probability densities:

$$p\left(\mathbf{h}|\mathbf{y}_p;\gamma^{(r)}\right) = \mathcal{CN}(\mu, \Sigma)$$

$$\Sigma = \Gamma^{(r)} - \Gamma^{(r)}\phi_p^H(\sigma^2 \mathrm{I}_{P_b} + \phi_p\Gamma^{(r)}\phi_p^H)^{-1}\phi_p\Gamma^{(r)}, \;\; \mu = \sigma^{-2}\Sigma\phi_p^H\mathbf{y}_p$$

Algorithm 1 SBL for estimating time domain channel taps

Input: $\mathbf{y}_p, \phi_p, r_{max}$ and ϵ.

Initialize $\Gamma^{(0)} = \mathrm{I}_L$, Set difference $= 1$, $r = 0$

while (difference$> \epsilon$ and $r < r_{max}$)

E-step: $\mu = \sigma^{-2}\Sigma\phi_p^H\mathbf{y}_p$

$\Sigma = \Gamma^{(r)} - \Gamma^{(r)}\phi_p^H \left(\sigma^2\mathrm{I}_{P_b} + \phi_p\Gamma^{(r)}\phi_p^H\right)^{-1}\phi_p\Gamma^{(r)}$

M-step: $\gamma^{(r+1)}(i) = \Sigma(i,i) + |\mu|^2$ for $i = 1, 2, \ldots, L$

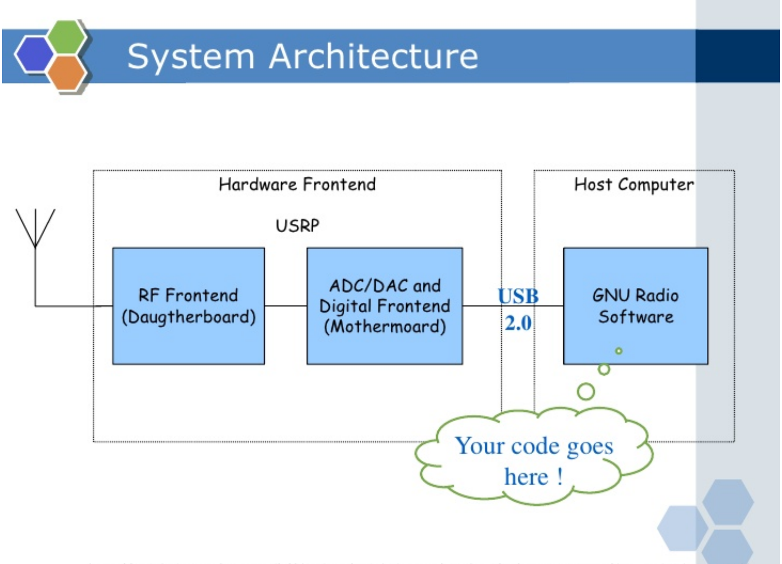
difference $\triangleq \|\gamma^{(r+1)} - \gamma^{(r)}\|_2^2$, $r \leftarrow r + 1$ **end**

output: $\mu, \gamma^{(r)}$

Introduction to GNU Radio

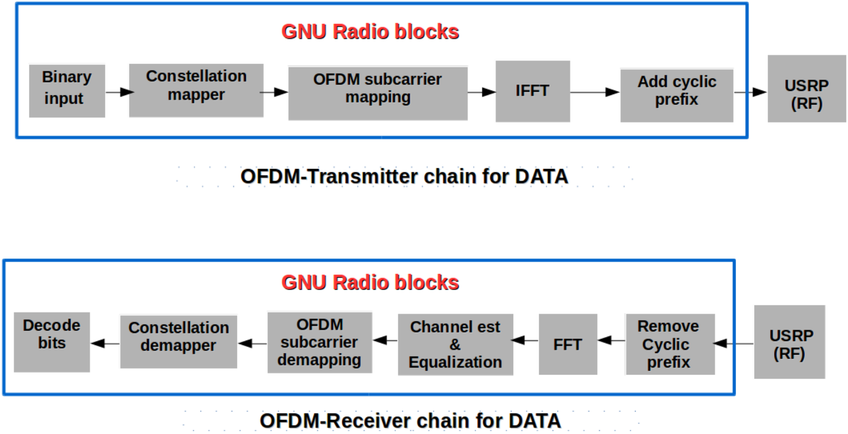
- A software development tool kit with **signal processing** blocks written in C++/Python.
- GRC (GNU Radio Companion) is the user interface for GNU Radio.
- Can be used with external RF hardware (such as USRP N210) to create Software Defined Radio(SDR).

System architecture

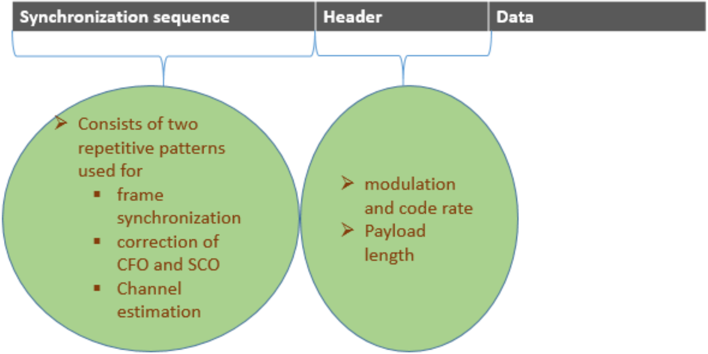


http://mobiledevices.kom.aau.dk/fileadmin/mobiledevices/teaching/software_testing/Gnu_radio_lecture.pdf

OFDM Chain in GNU Radio



Frame structure

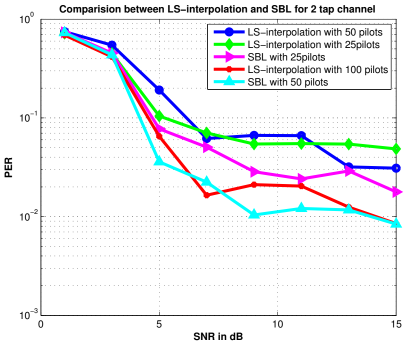


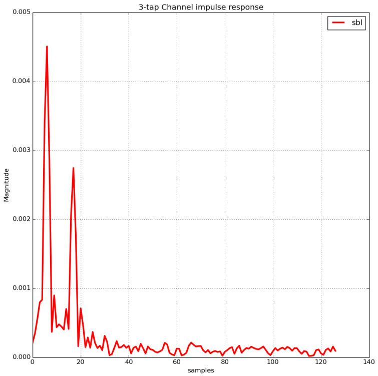
Implementation in GNU Radio

Implemented following channel estimation algorithms:

- SBL
- LS
- Linear interpolation

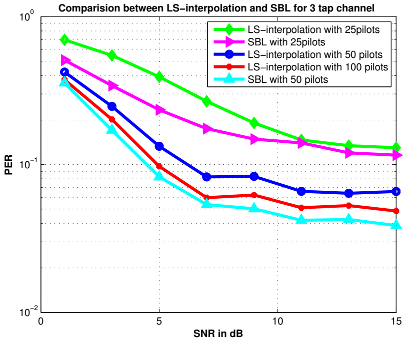
Results for 2 tap channel

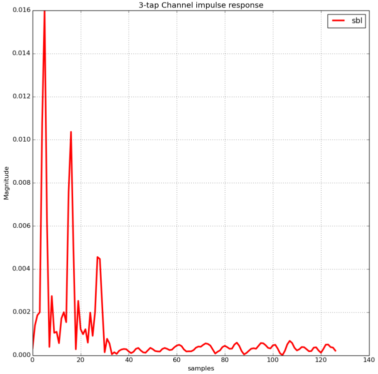




- system configuration:FFT-size=512, occupied subcarriers=200, CP length=128

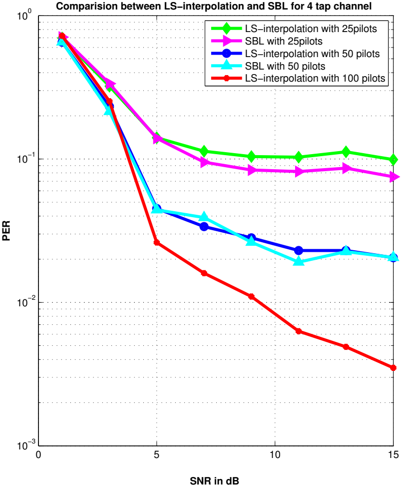
Results for 3 tap channel

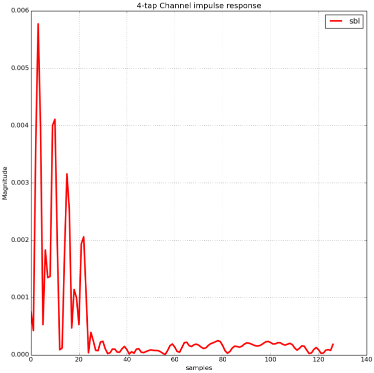




- system configuration:FFT-size=512, occupied subcarriers=200, CP length=128

Results for 4 tap channel





- system configuration:FFT-size=512,

Summary

- Implemented SBL, LS-interpolation based channel estimation algorithm in GNU radio and created the automated PER setup.
- Results confirm that SSP algorithms can work in real time and indeed can perform better with fewer observations.

Future Work

- channel estimation and data detection using SBL and other SSR based algorithms using pilots in data symbol alone, and performance comparision with traditional OFDM channel estimation techniques.