Scheduling an Energy Harvesting Network

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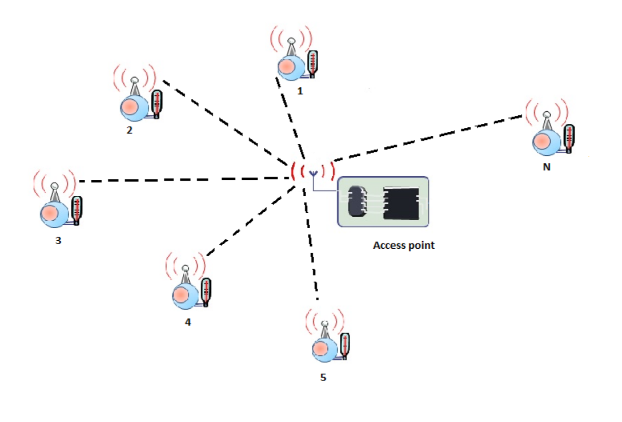
Agenda

- Motivation
- System Model
- Problem statement
- MDP Formulation
- Heuristic policy
- Simulation Results
- Multi-armed Bandit problem
- Future work

Motivation

- Cyber-physical system typically employ wireless sensors for keeping track of physical processes such as temperature and pressure.
- These measurements transmitted back to the central node (access point).
- The time between successive deliveries of packets is an important metric.
- All wireless sensor nodes are energy harvesting.

System Model



- All 'N' sensor nodes are energy harvesting sensors.
- Access point (AP) is powered by the mains.

Problem statement

- Atmost $L(< N)$ sensors can simultaneously transmit in a time slot.
- A Control message is sent at the beginning by the AP to select L sensors out of N sensors.

A scheduling policy for N wireless energy harvesting nodes is to be designed to the following constraints:

- Uniform sized packets are generated at each node when the node is scheduled by the AP.
- Each node has a given some threshold time before which it needs to be scheduled again.
- The QoS requirement of client n is specified through an integer, the packet inter-delivery time threshold τ_n.

Assumptions:

- Battery state of all the EH sensor nodes is known at the AP.
- Packet success probability of all the EH sensor nodes is known at the AP.
- The energy harvesting rate of all the nodes is known at the AP.
- The energy required to transmit a packet in a given time slot is 1 unit.
- One unit of energy is harvested at the beginning of a timeslot with harvesting rate (ρ).

MDP Formulation:

Goal: Scheduling the nodes based on the battery energy level, Success Probability and Time elapsed since the latest delivery of the client n's packet.

a) *State space:*
   - $\mathcal{B} = \{0, 1, \ldots, b_{max}\}$ is the set of battery states. $\mathcal{B}(t) := (b_1(t), \ldots, b_N(t))$, where $b_n(t)$ is the battery state of client in time slot t.
   - $X(t) := (x_1(t), \ldots, x_N(t))$, where $x_n(t)$ is the time elapsed since the latest delivery of client n's packet.

b) *Action space:* $U(t) := (u_1(t), \ldots, u_N(t))$, where $u_n(t)$ is the action taken for client n, in time slot t.

$$\sum_{n=1}^{N} u_n(t) \leq L \tag{1}$$

Contd..

$$u_n(t) = \begin{cases} 1 \text{ if client } n \text{ is selected to transmit in slot } t \\ 0 \text{ otherwise} \end{cases}$$

c) *State transition function:* Let two arbitrary states in S be $s = (B, X)$ and $s' = (B', X')$. The state transition function is the probability that the system starts in state s if it takes an action U and lands in state s'. The system state evolves as,

$$x_n(t+1) = \begin{cases} 0 \text{ if a packet of client } n \text{ is delivered in } t \\ x_n(t) + 1 \text{ otherwise} \end{cases}$$

$$b_n(t+1) = \begin{cases} \max(\min(b_n(t) + 1, b_{n(max)}) - u_n(t), 0) \text{ with probab. } \rho_\text{n} \\ \max(b_n(t) - u_n(t), 0) \text{ with probability } 1 - \rho_\text{n} \end{cases}$$

Contd..

- Consider a small network consisting of two EHS nodes and an Access point.
- Scheduling policy: The AP can schedule at most one sensor Node. $N = 2, L = \{0, 1\}$

$$B = (b_1, b_2), X = (x_1, x_2), U = (u_1, u_2)$$
$$(b_1, b_2, x_1, x_2) \xrightarrow{(u_1,u_2)} (b_1', b_2', x_1', x_2')$$
$$\text{possible action set: } (u_1, u_2) = \{(0,0), (1,0), (0,1)\}$$

- Energy harvesting rate of $client_1 = \rho_1$
- Energy harvesting rate of $client_2 = \rho_2$
- Success Probability of $client_1 = p_1$
- Success Probability of $client2 = p_2$

The State Transition Probability is given by,

$$\psi((B, X), U, (B', X')) = P((B', X')|(B, X), U)$$

$P((B', X')|(B, X)) =$

Case-1: $U = (u_1 = 0, u_2 = 0)$ None of the Nodes are Scheduled.

$$
= \begin{cases}
\rho_1\rho_2, \quad j_1 = min(i_1 + 1, b_{1max}), j_2 = min(i_2 + 1, b_{2max}), y_1 = x_1 + 1, \\
\qquad\qquad y_2 = x_2 + 1 \\
(1 - \rho_1)\rho_2, \; j_1 = i_1, j_2 = min(i_2 + 1, b_{2max}), y_1 = x_1 + 1, y_2 = x_2 + 1 \\
\\
\rho_1(1 - \rho_2), \; j_1 = min(i_1 + 1, b_{1max}), j_2 = i_2, y_1 = x_1 + 1, y_2 = x_2 + 1 \\
\\
(1 - \rho_1)(1 - \rho_2), \; j_1 = i_1, j_2 = i_2, y_1 = x_1 + 1, y_2 = x_2 + 1
\end{cases}
$$

cntd..

Case-2: $U = (u_1 = 1, u_2 = 0)$ Node1 is scheduled.

$$
= \begin{cases}
(1-p_1)\rho_1\rho_2,\ j_1 = min(i_1 + 1 - 1, b_{1max}), j_2 = min(i_2 + 1, b_{2max}), \\
\quad y_1 = x_1 + 1, y_2 = x_2 + 1 \\
(1-p_1)(1-\rho_1)\rho_2,\ j_1 = max(i_1 - 1, 0), j_2 = min(i_2 + 1, b_{2max}), \\
\quad y_1 = x_1 + 1, y_2 = x_2 + 1 \\
(1-p_1)\rho_1(1-\rho_2),\ (j_1 = min(i_1 + 1 - 1, b_{1max}), j_2 = i_2, y_1 = x_1 + 1, \\
\quad y_2 = x_2 + 1 \\
(1-p_1)(1-\rho_1)(1-\rho_2),\ j_1 = max(i_1 - 1, 0), j_2 = i_2, y_1 = x_1 + 1, \\
\quad y_2 = x_2 + 1
\end{cases}
$$

cntd...

$$
= \begin{cases}
p_1\rho_1\rho_2, \; j_1 = min(i_1 + 1 - 1, b_{1max}), j_2 = min(i_2 + 1, b_{2max}), \\
\qquad y_1 = 0, y_2 = x_2 + 1 \\
\\
p_1(1-\rho_1)\rho_2, \; j_1 = max(i_1 - 1, 0), j_2 = min(i_2 + 1, b_{2max}), \\
\qquad y_1 = 0, y_2 = x_2 + 1 \\
\\
p_1\rho_1(1-\rho_2), \; (j_1 = min(i_1 + 1 - 1, b_{1max}), j_2 = i_2, y_1 = 0, \\
\qquad y_2 = x_2 + 1 \\
\\
p_1(1-\rho_1)(1-\rho_2), \; j_1 = max(i_1 - 1, 0), j_2 = i_2, y_1 = 0, y_2 = x_2 + 1
\end{cases}
$$

Case-3: $U = (u_1 = 0, u_2 = 1)$ Node2 is scheduled.

$$
= \begin{cases}
(1-p_2)\rho_1\rho_2,\ j_1 = min(i_1+1, b_{1max}), j_2 = min(i_2+1-1, b_{2max}), \\
\quad y_1 = x_1+1, y_2 = x_2+1 \\
(1-p_2)(1-\rho_1)\rho_2,\ j_1 = i_1, j_2 = min(i_2+1-1, b_{2max}), \\
\quad y_1 = x_1+1, y_2 = x_2+1 \\
(1-p_2)\rho_1(1-\rho_2),\ j_1 = min(i_1+1, b_{1max}), j_2 = max(i_2-1, 0), \\
\quad y_1 = x_1+1, y_2 = x_2+1 \\
(1-p_2)(1-\rho_1)(1-\rho_2),\ j_1 = i_1, j_2 = max(i_2-1, 0), y_1 = x_1+1, \\
\quad y_2 = x_2+1
\end{cases}
$$

cntd...

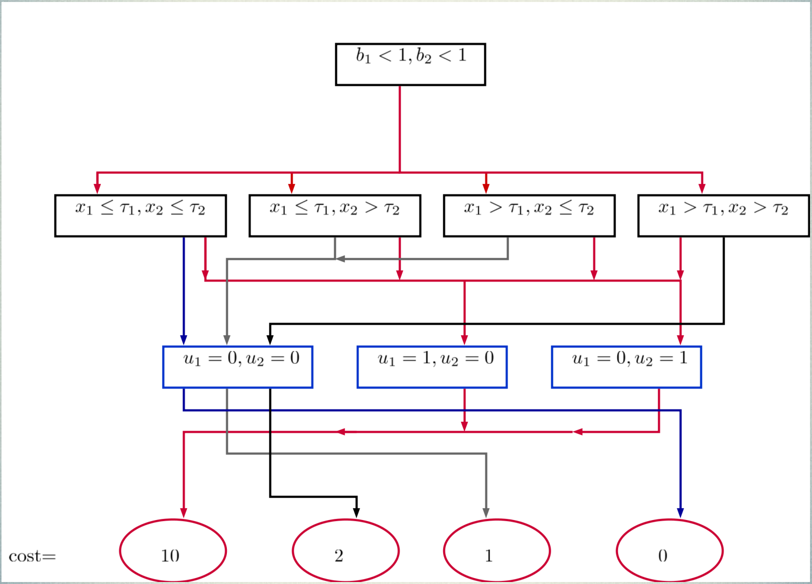
$$
= \begin{cases} p_2\rho_1\rho_2,\ j_1 = min(i_1+1, b_{1max}), j_2 = min(i_2+1-1, b_{2max}), \\ \qquad y_1 = x_1+1, y_2 = 0 \\ \\ p_2(1-\rho_1)\rho_2,\ j_1 = max(i_1-1, 0), j_2 = min(i_2+1, b_{2max}), \\ \qquad y_1 = x_1+1, y_2 = 0 \\ \\ p_2\rho_1(1-\rho_2),\ (j_1 = min(i_1+1-1, b_{1max}), j_2 = i_2, y_1 = x_1+1, \\ \qquad y_2 = 0 \\ \\ p_2(1-\rho_1)(1-\rho_2),\ j_1 = max(i_1-1, 0), j_2 = i_2, y_1 = x_1+1, y_2 = 0 \end{cases}
$$

Immediate cost:

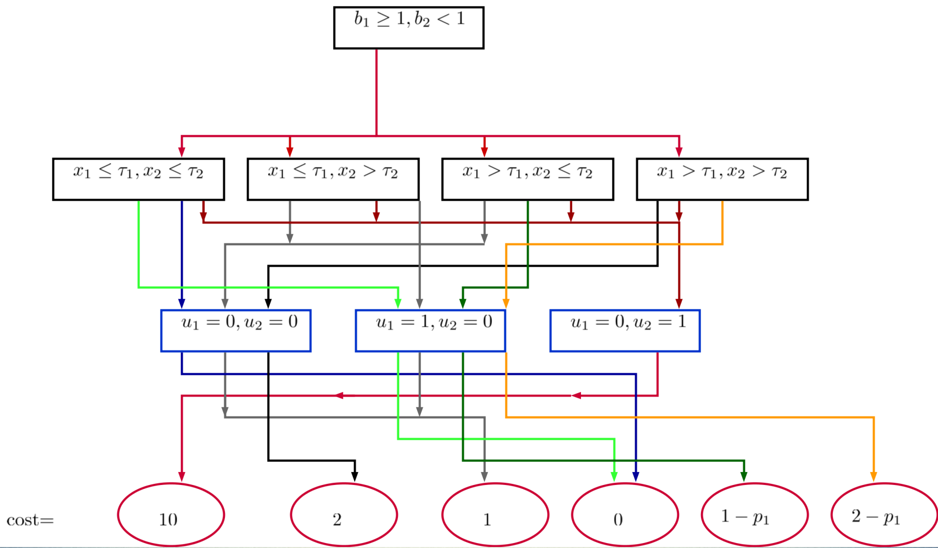
NOTE: $E = 1$ unit in all the below mentioned cases.
Cost: Let $s = (B, X)$ be the state of the system.
The expected immediate cost is defined as,
c(S,U)

$$= \begin{cases} 10 & \text{if } b_1 < E, b_2 < E,\ u_1 \neq 0, u_2 \neq 0, x_1 \leq \tau_1 \textit{ and } x_2 \leq \tau_2 \\ 10 & \text{if } b_1 < E, b_2 < E,\ u_1 \neq 0, u_2 \neq 0, x_1 > \tau_1 \textit{ and } x_2 > \tau_2 \\ & \\ 0 & \text{if } b_1 < E, b_2 < E,\ u_1 = 0, u_2 = 0, x_1 \leq \tau_1 \textit{ and } x_2 \leq \tau_2 \\ 1 & \text{if } b_1 < E, b_2 < E,\ u_1 = 0, u_2 = 0, x_1 \leq \tau_1 \textit{ and } x_2 > \tau_2 \\ 1 & \text{if } b_1 < E, b_2 < E,\ u_1 = 0, u_2 = 0, x_1 > \tau_1 \textit{ and } x_2 \leq \tau_2 \\ 2 & \text{if } b_1 < E, b_2 < E,\ u_1 = 0, u_2 = 0, x_1 > \tau_1 \textit{ and } x_2 > \tau_2 \end{cases}$$

when $b_1 < 1, b_2 < 1$:


$b_1 < 1, b_2 < 1$
$x_1 \leq \tau_1, x_2 \leq \tau_2$
$x_1 \leq \tau_1, x_2 > \tau_2$
$x_1 > \tau_1, x_2 \leq \tau_2$
$x_1 > \tau_1, x_2 > \tau_2$
$u_1 = 0, u_2 = 0$
$u_1 = 1, u_2 = 0$
$u_1 = 0, u_2 = 1$
cost=
10
2
1
0

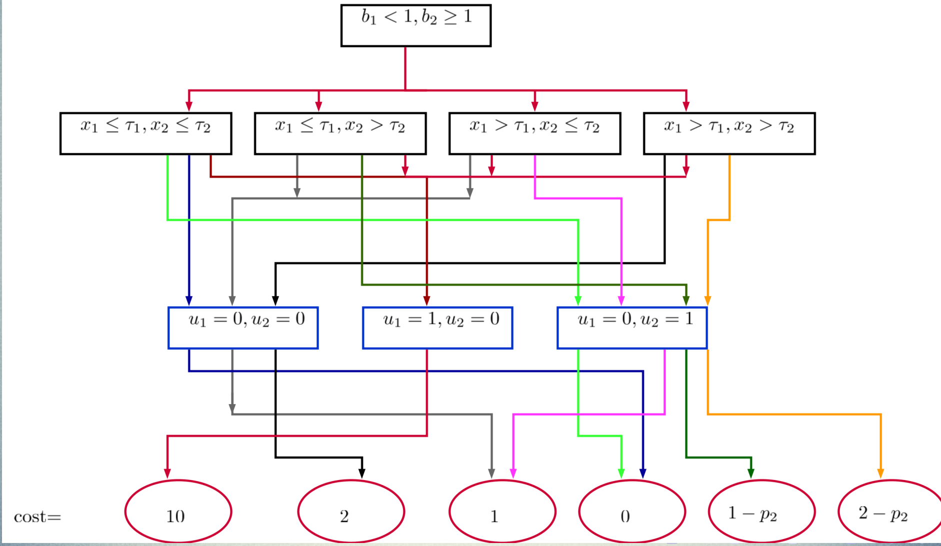
when $b_1 \geq 1, b_2 < 1$:



when $b_1 \geq 1, b_2 < 1$:

$$\begin{cases} 0 & \text{if } b_1 \geq E, b_2 < E,\ u_1 = 0, u_2 = 0, x_1 \leq \tau_1 \textit{ and } x_2 \leq \tau_2 \\ 1 & \text{if } b_1 \geq E, b_2 < E,\ u_1 = 0, u_2 = 0, x_1 \leq \tau_1 \textit{ and } x_2 > \tau_2 \\ 1 & \text{if } b_1 \geq E, b_2 < E,\ u_1 = 0, u_2 = 0, x_1 > \tau_1 \textit{ and } x_2 \leq \tau_2 \\ 2 & \text{if } b_1 \geq E, b_2 < E,\ u_1 = 0, u_2 = 0, x_1 > \tau_1 \textit{ and } x_2 > \tau_2 \\ \\ 0 & \text{if } b_1 \geq E, b_2 < E,\ u_1 = 1, u_2 = 0, x_1 \leq \tau_1 \textit{ and } x_2 \leq \tau_2 \\ 1 & \text{if } b_1 \geq E, b_2 < E,\ u_1 = 1, u_2 = 0, x_1 \leq \tau_1 \textit{ and } x_2 > \tau_2 \\ 1 - p_1 & \text{if } b_1 \geq E, b_2 < E,\ u_1 = 1, u_2 = 0, x_1 > \tau_1 \textit{ and } x_2 \leq \tau_2 \\ 2 - p_1 & \text{if } b_1 \geq E, b_2 < E,\ u_1 = 1, u_2 = 0, x_1 > \tau_1 \textit{ and } x_2 > \tau_2 \\ 10 & \text{if } b_1 \geq E, b_2 < E,\ u_1 = 0, u_2 = 1, \forall\ x_1 \text{ and } x_2 \end{cases}$$

when $b_1 < 1, b_2 \geq 1$:



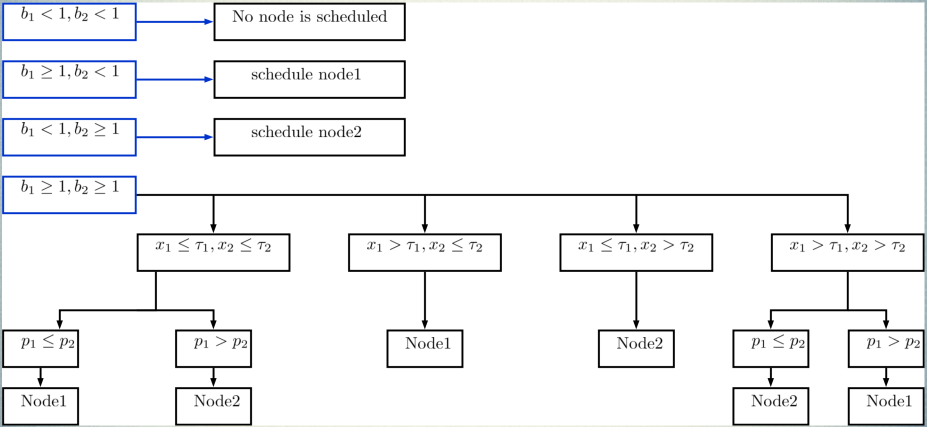
when $b_1 < 1, b_2 \geq 1$:

$$
\begin{cases}
0 & \text{if } b_1 < E, b_2 \geq E,\ u_1 = 0, u_2 = 0, x_1 \leq \tau_1 \textit{ and } x_2 \leq \tau_2 \\
1 & \text{if } b_1 < E, b_2 \geq E,\ u_1 = 0, u_2 = 0, x_1 \leq \tau_1 \textit{ and } x_2 > \tau_2 \\
1 & \text{if } b_1 < E, b_2 \geq E,\ u_1 = 0, u_2 = 0, x_1 > \tau_1 \textit{ and } x_2 \leq \tau_2 \\
2 & \text{if } b_1 < E, b_2 \geq E,\ u_1 = 0, u_2 = 0, x_1 > \tau_1 \textit{ and } x_2 > \tau_2 \\
\\
0 & \text{if } b_1 < E, b_2 \geq E,\ u_1 = 0, u_2 = 1, x_1 \leq \tau_1 \textit{ and } x_2 \leq \tau_2 \\
1 - p_2 & \text{if } b_1 < E, b_2 \geq E,\ u_1 = 0, u_2 = 1, x_1 \leq \tau_1 \textit{ and } x_2 > \tau_2 \\
1 & \text{if } b_1 < E, b_2 \geq E,\ u_1 = 0, u_2 = 1, x_1 > \tau_1 \textit{ and } x_2 \leq \tau_2 \\
2 - p_2 & \text{if } b_1 < E, b_2 \geq E,\ u_1 = 0, u_2 = 1, x_1 > \tau_1 \textit{ and } x_2 > \tau_2 \\
10 & \text{if } b_1 \geq E, b_2 < E,\ u_1 = 1, u_2 = 0, \forall\ x_1 \text{ and } x_2
\end{cases}
$$

when $b_1 \geq 1, b_2 \geq 1$:

$$\begin{cases} 0 & \text{if } b_1 \geq E, b_2 \geq E,\ u_1 = 0, u_2 = 0, x_1 \leq \tau_1 \, and \; x_2 \leq \tau_2 \\ 1 & \text{if } b_1 \geq E, b_2 \geq E,\ u_1 = 0, u_2 = 0, x_1 \leq \tau_1 \, and \; x_2 > \tau_2 \\ 1 & \text{if } b_1 \geq E, b_2 \geq E,\ u_1 = 0, u_2 = 0, x_1 > \tau_1 \, and \; x_2 \leq \tau_2 \\ 2 & \text{if } b_1 \geq E, b_2 \geq E,\ u_1 = 0, u_2 = 0, x_1 > \tau_1 \, and \; x_2 > \tau_2 \\ & \\ 0 & \text{if } b_1 \geq E, b_2 \geq E,\ u_1 = 1, u_2 = 0, x_1 \leq \tau_1 \, and \; x_2 \leq \tau_2 \\ 1 & \text{if } b_1 \geq E, b_2 \geq E,\ u_1 = 1, u_2 = 0, x_1 \leq \tau_1 \, and \; x_2 > \tau_2 \\ 1 - p_1 & \text{if } b_1 \geq E, b_2 \geq E,\ u_1 = 1, u_2 = 0, x_1 > \tau_1 \, and \; x_2 \leq \tau_2 \\ 2 - p_1 & \text{if } b_1 \geq E, b_2 \geq E,\ u_1 = 1, u_2 = 0, x_1 > \tau_1 \, and \; x_2 > \tau_2 \\ & \\ 0 & \text{if } b_1 \geq E, b_2 \geq E,\ u_1 = 0, u_2 = 1, x_1 \leq \tau_1 \, and \; x_2 \leq \tau_2 \\ 1 - p_2 & \text{if } b_1 \geq E, b_2 \geq E,\ u_1 = 0, u_2 = 1, x_1 \leq \tau_1 \, and \; x_2 > \tau_2 \\ 1 & \text{if } b_1 \geq E, b_2 \geq E,\ u_1 = 0, u_2 = 1, x_1 > \tau_1 \, and \; x_2 \leq \tau_2 \\ 2 - p_2 & \text{if } b_1 \geq E, b_2 \geq E,\ u_1 = 0, u_2 = 1, x_1 > \tau_1 \, and \; x_2 > \tau_2 \end{cases}$$

Heuristic policy



Heuristic policy

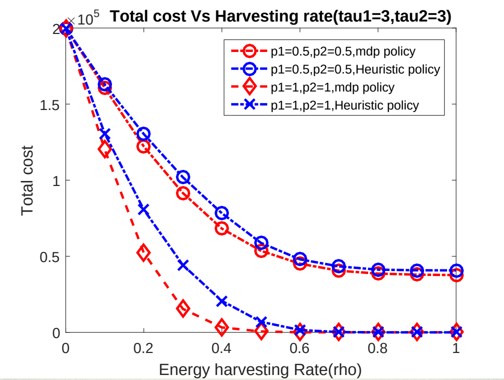
Cost Function:
The T-horizon optimal cost-to-go from initial state x, $V_T(\mathrm{x})$ is given by,

$$:= \min_{\pi:\Sigma_n U_n(t)\leq L} \mathrm{E}\{\sum_{t=0}^{T-1}\sum_{n=1}^{N}((X_n(t)+1-\tau_n)^+ 1\{X_n(t+1)=0\})|X(0)=\mathrm{x}\}$$

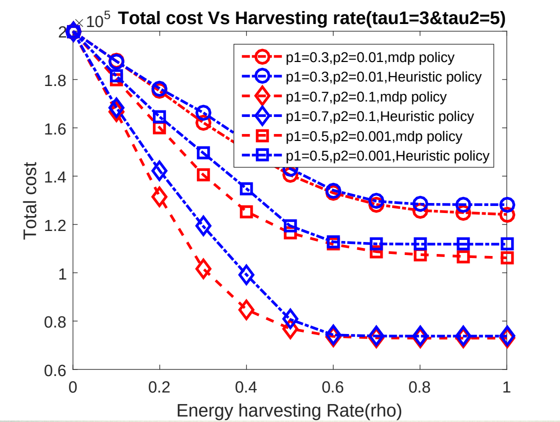
Simulation Results

Effect of Success Probability:

- For all the simulations $\rho_1 = \rho_2 = \rho$
- Total cost is decreasing as the harvesting rate increases.
- Total cost is decreasing as the success probability increases.

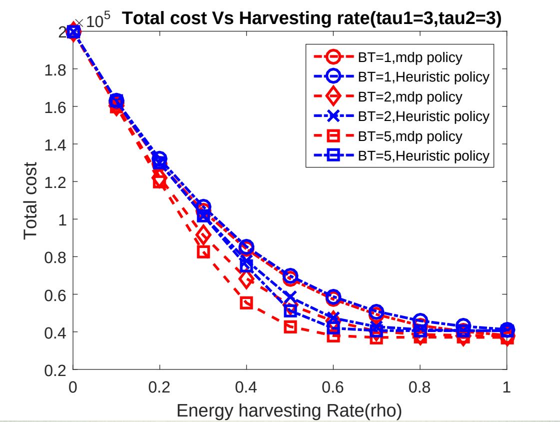


Effect of Success Probability:



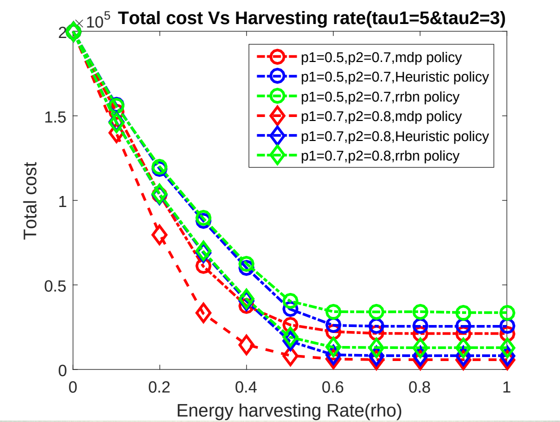
Effect of Effect of Battery Capacity:

- Total cost is independent of the battery capacity at low and high harvesting rates.
- Higher battery capacity gives good performance in the range from 0.3 to 0.8.



Comparison with Round-robin policy

- MDP policy performs better than the Round-robin policy.



Multi-armed Bandit problem

- 'L out of N' type sequential decision problems
- Simple Multi-armed bandits: only active projects/arms incur the cost and evolve
- Restless multi-armed bandits: projects/arms which are not scheduled also evolve and incur the cost, e.g., N queues served by L servers

Whittle Index based policy for RMABs

- Compute the Whittle index for each arm
- Choose arms with top L whittle index
- Such policies are near-optimal, and can be shown to be asymptotically optimal as $N \to \infty$ with $\frac{L}{N}$ fixed

ω-subsidy problem

For nth node:

The state space contains the battery and time elapsed $S_n = (B_n(t), d_n(t))$

$B_n(t) \in \{0, 1, .., b_n^{max}\}$

$d_n(t) \in \{0, 1, .., \tau_n\}$

Total no.of states $= (b_n^{max} + 1)(\tau_n + 1)$.

The transition probabilities are given by,

$$= \begin{cases} \rho \quad \text{if } B_n(t)' = \min(B_n(t) + 1, b_n^{max}), d_n(t)' = d_n(t) + 1, u = 0 \\ (1-\rho) \quad \text{if } B_n(t)' = B_n(t), d_n(t)' = d_n(t) + 1, u = 0 \\ \rho p \quad \text{if } B_n(t)' = \min(B_n(t) + 1 - 1, b_n^{max}), d_n(t)' = 0, u = 1 \\ \rho(1-p) \quad \text{if } B_n(t)' = \min(B_n(t) + 1 - 1, b_n^{max}), d_n(t)' = d_n(t) + 1, \\ \qquad u = 1 \\ (1-\rho)p \quad \text{if } B_n(t)' = \max(B_n(t) - 1, 0), d_n(t)' = 0, u = 1 \\ (1-\rho)(1-p) \quad \text{if } B_n(t)' = \max(B_n(t) - 1, 0), d_n(t)' = d_n(t) + 1, \\ \qquad u = 1 \end{cases}$$

Threshold policy:

- $B_n(t) = 0$: Only *passive* action is possible and subsidy $\omega = 0$
- $B_n(t) = 1$:(considering unit battery only, $\beta = B_n^{max} = 1$)

$$\phi(\beta, \theta, \mathrm{q}) = \begin{cases} Passive \quad \text{if} \;\; d_n(t) < \theta \\ Passive \;\; with \;\; prob. \;\; '\mathrm{q}' \;\; \text{if} \;\; d_n(t) = \theta \\ Active \quad \text{if} \; d_n(t) > \theta \end{cases}$$

The optimal average reward R is the same for all initial states and together with some vector $f = \{f(1), ..., f(n)\}$ satisfies Bellman's equation:

$$R + f(i) = \min_{u \in \{0,1\}} [c(i,u) + \sum_{j=1}^{n} p_{ij}(u) f(j)] \tag{2}$$

The Bellman's equation can be written as,

$$
\begin{aligned}
R + f(B_n(t), d_n(t)) = \max_{u \in \{0,1\}} \{ & -1\{i = \tau_n\} + \omega(1-u)\mathbb{1}\{B_n(t) \geq 1\} \\
& + \rho(1-u) f(((B_n(t)+1) \wedge B_n^{max}), ((d_n(t)+1) \wedge \tau_n)) \\
& + (1-\rho)(1-u) f(B_n(t), ((d_n(t)+1) \wedge \tau_n)) \\
& + \rho p u \mathbb{1}\{B_n(t) \geq 1\} f(((B_n(t)+1-1) \wedge B_n^{max}), 0)) \\
& + (1-\rho) p u \mathbb{1}\{B_n(t) \geq 1\} f((B_n(t)-1), 0) \\
& + \rho(1-p) u \mathbb{1}\{B_n(t) \geq 1\} f(((B_n(t)+1-1) \wedge B_n^{max}), ((d_n(t)+1) \wedge \tau_n)) \\
& + (1-\rho)(1-p) u \mathbb{1}\{B_n(t) \geq 1\} f(((B_n(t)-1) \wedge B_n^{max}), ((d_n(t)+1) \wedge \tau_n)
\end{aligned}
$$

Case1: $\phi(\beta, \theta, q) = \phi(1, 0, 0)$, $B_n^{max} = 1 and \tau_n = 1$

- $B_n = 0$: Always passive

$$R + f(0,0) = \rho f(1,1) + (1-\rho) f(0,1) \quad (3)$$

$$R + f(0,1) = -1 + \rho f(1,1) + (1-\rho) f(0,1) \quad (4)$$

- $B_n = 1$:
  $d_n = \theta \rightarrow$ Passive with prob. 'q=0'(Active)

$$R + f(1,0) = \rho p f(1,0) + (1-\rho) p f(0,0) + \rho(1-p) f(1,1) + (1-\rho)(1-p) f(0,1)$$

  $d_n > \theta \rightarrow$ Active

$$R + f(1,1) = -1 + \rho p f(1,0) + (1-\rho) p f(0,0) + \rho(1-p) f(1,1) + (1-\rho)(1-p) f(0,1)$$

Solution: $R = \rho p - 1, f(0,1) = -1, f(1,0) = p, f(1,1) = p - 1$

Case2: $\phi(\beta, \theta, q) = \phi(1, 1, 1)$, $B_n^{max} = 1 and \tau_n = 1$

- $B_n = 0$: Always passive

$$R + f(0,0) = \rho f(1,1) + (1-\rho) f(0,1) \tag{5}$$

$$R + f(0,1) = -1 + \rho f(1,1) + (1-\rho) f(0,1) \tag{6}$$

- $B_n = 1$:
  $d_n < \theta \rightarrow$ Passive

$$R + f(1,0) = \omega + \rho f(1,1) + (1-\rho) f(1,1) \tag{7}$$

$d_n = \theta \rightarrow$ Passive with prob. 'q=1'

$$R + f(1,0) = -1 + \omega + \rho f(1,1) + (1-\rho) f(1,1) \tag{8}$$

Solution: $R = \omega - 1, f(0,1) = -1, f(1,0) = \frac{\omega}{\rho}, f(1,1) = \frac{\omega - \rho}{\rho}$

Case3: $\phi(\beta, \theta, q) = \phi(1, 1, 0)$, $B_n^{max} = 1 and \tau_n = 1$

- $B_n = 0$: Always passive

$$R + f(0,0) = \rho f(1,1) + (1-\rho) f(0,1) \tag{9}$$

$$R + f(0,1) = -1 + \rho f(1,1) + (1-\rho) f(0,1) \tag{10}$$

- $B_n = 1$:
  $d_n < \theta \rightarrow$ Passive

$$R + f(1,0) = \omega + \rho f(1,1) + (1-\rho) f(1,1) \tag{11}$$

$d_n = \theta \rightarrow$ Passive with prob. 'q=0'

$$R + f(1,1) = -1 + \rho p f(1,0) + (1-\rho) p f(0,0) + \rho(1-p) f(1,1) + (1-\rho)(1-p) f(0,1)$$

Solution: $R = \frac{p\rho + p\rho^2\omega - p\rho^2 - 1}{p\rho^2 + 1}$, $f(0,1) = -1$, $f(1,0) = \frac{p + \omega - p\rho + p\rho\omega}{p\rho^2 + 1}$, $f(1,1) = \frac{-p\rho^2 + p\omega\rho + p - 1}{p\rho^2 + 1}$

Future work

- Average reward and ω-subsidy expressions are need to be computed for general θ
- Whittle index for unit battery case
- Comparing whittle index policy optimality against MDP policy
- Extending whittle index for general battery case

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Thank You