Power Controlled Training in a Reciprocal MIMO Multiuser system

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Outline

k^{th} UT is being scheduled

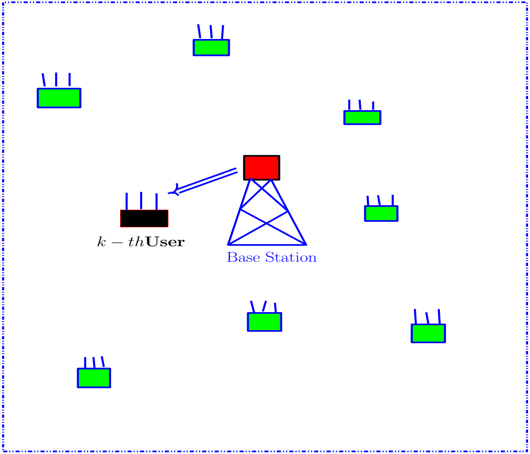
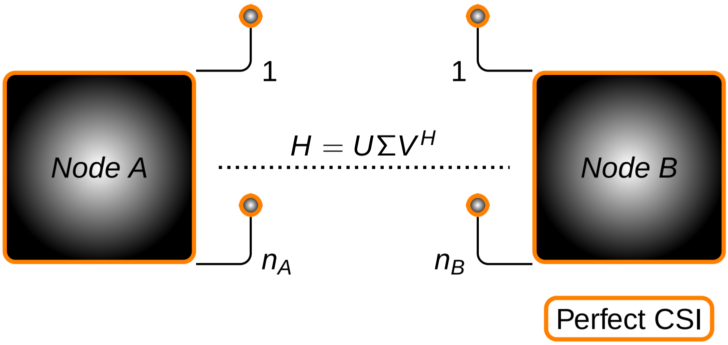


Figure: Multi-user system model and scheduling of the k^{th} **UT**. There are M users in the network.

System model

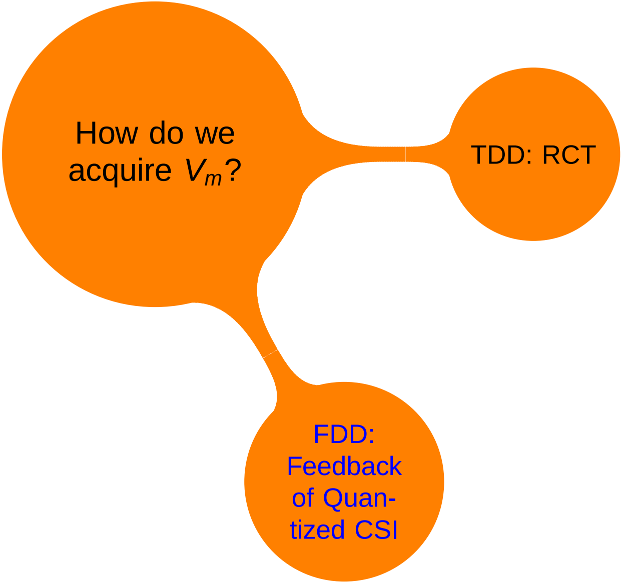


Data Transmission Scheme

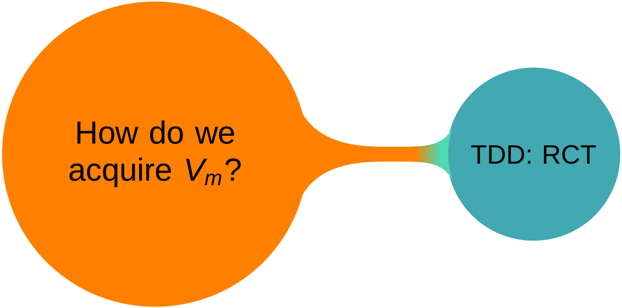
- Spatial Multiplexing (SM) with equal power allocation
  - Transmission over m dominant modes of the channel ($H = U\Sigma V^H$)
  - Use V_m, the first m columns of V as a pre-coding matrix
  - Input output equation (data transmission):

$$\mathbf{y}_{B,d} = HV_m\mathbf{x}_{A,d} + \mathbf{w}_{B,d}$$

- Need to acquire the matrix V_m at *Node A*!


How do we acquire V_m?
TDD: RCT
FDD: Feedback of Quantized CSI

1. Assumptions
    - SM with equal power allocation during data transmission
    - Perfect reciprocity of the channel (TDD System)
    - Perfect CSI at *Node B*
2. Question: What should the RCT signal be?



Training Sequence Design

- Reverse-link training:

$$Y_{A,\tau} = H^H X_{B,\tau} + W_{A,\tau}$$

- Problem: Find $X_{B,\tau}$ that optimizes a metric

- Metric:
  1. Mean Square Error (MSE)
  2. Capacity Lower Bound

- The proposed training sequence:

$$X_{B,\tau} = \sqrt{P_{B,\tau} L_{B,\tau} \phi_c} UD$$

where $D = \mathrm{diag}\{d_1, \ldots, d_m\}$ such that $\|D\|_F^2 = 1$ and $\mathbb{E}\phi_c = 1$

- Received training signal at *Node A*:

$$\bar{Y}_{A,\tau} \triangleq \frac{Y_{A,\tau}}{\sqrt{P_{B,\tau} L_{B,\tau}}} = \sqrt{\phi_c} V \Sigma^H D + \frac{W_{A,\tau}}{\sqrt{P_{B,\tau} L_{B,\tau}}}$$

- Estimate of the k^{th} BF vector:

$$\hat{\mathbf{v}}_k = \frac{\bar{\mathbf{y}}_{k,A,\tau}}{\|\bar{\mathbf{y}}_{k,A,\tau}\|_2}, \quad 1 \le k \le m$$

where $\bar{\mathbf{y}}_{k,A,\tau}$ is the k^{th} column of $\bar{Y}_{A,\tau}$

Problem Statement

- Problem:

$$\min_{D,\phi_c : \|D\|_F^2 = 1, \text{and}, \mathbb{E}\phi_c = 1} \mathbb{E}\|V_m - \hat{V}_m\|_F^2$$

MSE as a metric

Theorem

$$\left| \mathbb{E}\|V_m - \hat{V}_m\|_F^2 - MSE_{approx} \right| = \mathcal{O}\left(\frac{1}{(P_{B,\tau} L_{B,\tau})^2} \right),$$

where

$$MSE_{approx} = \left(\frac{2n_A - 1}{2} \right) \mathbb{E} \sum_{k=1}^{m} \frac{1}{\sigma_k^2 d_k^2 \phi_c}$$

Optimization problem:

1.

$$\min_{d_k : \sum_{i=1}^{m} d_i^2 = 1} \mathbb{E} \sum_{k=1}^{m} \frac{1}{\sigma_k^2 d_k^2}$$

Solution: MSE as a metric

Lemma

The optimal D and ϕ_c are given by

$$d_k^2 = \frac{\sigma_k^{-1}}{\sum_{i=1}^m \sigma_i^{-1}}, \quad (1)$$

and

$$\phi_c^* = \frac{\sum_{i=1}^m \sigma_i^{-1}}{\mathbb{E}\sum_{i=1}^m \sigma_i^{-1}}. \quad (2)$$

The corresponding approximate expression for MSE is

$$\mathbb{E}\|V_m - \hat{V}_{m,approx}\|_F^2 = \frac{2n_A - 1}{2P_{B,\tau}L_{B,\tau}} \left(\mathbb{E}\sum_{i=1}^m \sigma_i^{-1}\right)^2. \quad (3)$$

Performance gains: MSE as a metric

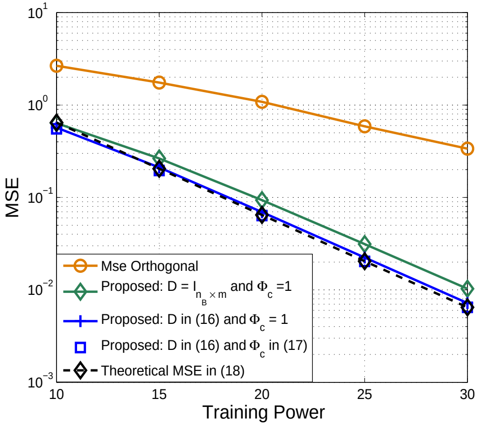


Figure: MSE versus training power for a 3×4 MIMO system with $m = 3$ with $P_{A,d} = 0.5 P_{B,\tau}$

Capacity lower bound

A capacity lower bound for the proposed data transmission scheme:

$$C_{exact} \triangleq \frac{L_c - L_{B,\tau}}{L_c} \mathbb{E} \log_2 \left| I_{m\times m} + \frac{P_{A,d}}{m} \frac{GG^H}{\frac{1}{m}\mathbb{E}_H \|\tilde{\mathbf{w}}_{eff}\|_F^2} \right|, \quad (4)$$

where $G \triangleq \Sigma_{m,m} - \Sigma_m V^H \mathbb{E}\{V_e|H\}$, $V_e \triangleq V_m - \hat{V}_m$, and

$$\tilde{\mathbf{w}}_{eff} \triangleq \sqrt{\frac{P_{A,d}}{m}} \Sigma_m V^H \mathbb{E}\{V_e|H\} \mathbf{x}_{A,d} - \sqrt{\frac{P_{A,d}}{m}} \Sigma_m V^H V_e \mathbf{x}_{A,d} + \tilde{\mathbf{w}}_{B,d}. \quad (5)$$

Approximate capacity lower bound

- Approximate capacity lower bound:

$$C_a = \frac{L_c - L_{B,\tau}}{L_c} \mathbb{E} \log \left| I_m + \frac{P_{A,d}}{m} \frac{\Sigma_m \Sigma_m^H}{\sigma_{eff}^2 + 1} \right|,$$

with $\sigma_{eff}^2 = \frac{P_{A,d}}{P_{B,\tau} L_{B,\tau} m^2} \sum_{i=1}^{m} \frac{\beta_i}{\phi_c d_i^2}$ and $\beta_i \triangleq \frac{1}{2} + \frac{\sum_{j=1, j\neq i}^{m} \sigma_j^2}{\sigma_i^2}$

- More precisely,
  1. $\left| C_{exact} - C_a^{(1)} \right| \to 0$ as $P_{A,d}, P_{B,\tau} \to \infty$ s.t. $\frac{P_{A,d}}{P_{B,\tau}} = \mu > 0$
  2. obtained by ignoring the terms of the order $1/(P_{B,\tau} L_{B,\tau})^{3/2}$ and higher

- Note: D enters the expression only through σ_{eff}^2

Simulation Results:Tightness

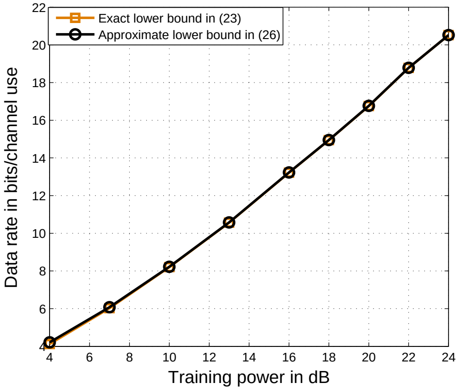


Figure: Illustration of the tightness for a 3×4 MIMO system with data power of $P_{A,d} = 0.5 P_{B,\tau}$ and training duration of 3 symbols. Here, $m = 3$

Problem Statement and Solution

- Problem Statement:

$$\max_{D,\phi_c : \|D\|_F^2=1, \mathbb{E}\phi_c=1} C_a$$

- Equivalently,

$$\max_{\phi_c, \mathbb{E}\phi_c=1} \max_{D:\|D\|_F^2=1} C_a$$

- Optimal D is obtained by solving:

$$\min_{D:\|D\|_F^2=1} \frac{P_{A,d}}{P_{B,\tau}L_{B,\tau}m^2} \sum_{i=1}^{m} \frac{\beta_i}{d_i^2}, \quad \beta_i \triangleq \frac{1}{2} + \frac{\sum_{j=1,j\neq i}^{m} \sigma_j^2}{\sigma_i^2}$$

Optimal D and ϕ_c

Theorem

The optimal D with diagonal terms $\{d_1, d_2, \ldots, d_m\}$ that maximizes C_a for a given $L_{B,\tau}$ is

$$d_i^2 = \frac{\sqrt{\beta_i}}{\sum_{j=1}^{m} \sqrt{\beta_j}}, \quad 1 \le i \le m$$

Theorem

The ϕ_c^* satisfies the following necessary and sufficient condition

$$\lambda = \mathcal{H}(\phi_c^*) \triangleq \sum_{k=1}^{m} \frac{P_{A,d}\sigma_k^2 \tau}{(P_{A,d}\sigma_k^2 + m)\phi_c^* + m\tau} \left(\frac{1}{\tau + \phi_c^*} \right), \qquad (6)$$

where λ is the Lagrange multiplier chosen s.t. $\mathbb{E}\phi_c^* = 1$

Simulation Results:Capacity Lower Bound as a Metric

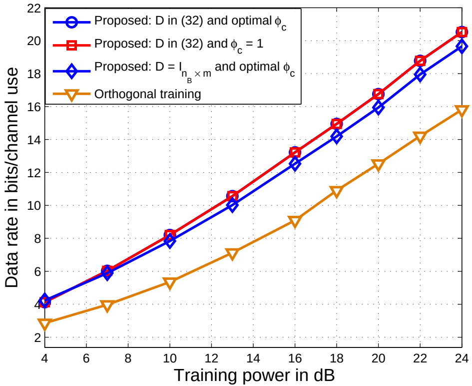


Figure: Capacity lower bound versus reverse training power for a 3×4 MIMO system with data power of $P_{A,d} = 0.5P_{B,\tau}$ and training duration of 3 symbols. Here, $m = 3$

Multi-user Scenario

Data rate optimal

- Scheduling criterion: $k^* = \arg\max_{1,\ldots,M}\{R_1, \ldots, R_M\}$, where

$$R_k \triangleq \frac{L_c - L_{B,\tau}^{(k)}}{L_c} \log_2 \left| I_m + \frac{P_{A,d}^{(k)}}{m} \frac{\Sigma_{k,m,m}\Sigma_{k,m,m}^H}{\sigma_{k,eff}^2 + 1} \right|,$$

with

$$\sigma_{k,eff}^2 \triangleq \frac{P_{A,d}^{(k)}}{P_{B,\tau}^{(k)} L_{B,\tau}^{(k)} m^2} \sum_{i=1}^{m} (\sqrt{\beta_i^{(k)}})^2, \; k = 1, \ldots, M,$$

and $\Sigma_{k,m,m}$ is the first m rows and m columns of the singular value matrix of H_k.

Multi-user data rate comparison

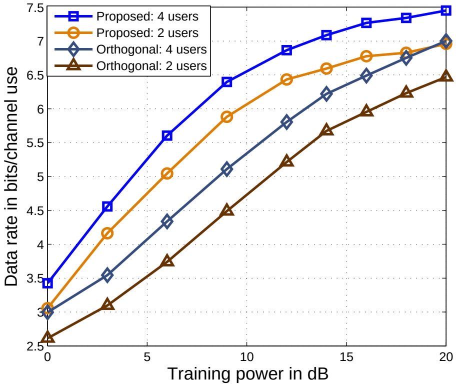


Figure: Capacity lower bound for a multi-user system with max-scheduling versus reverse training power for a 3×4 MIMO system with data power of $P_{A,d} = 0.5P_{B,\tau}$ and training duration of 3 symbols. Here, $m = 3$

Multi-user Scenario: MSE optimal scheme for BF system

Scheduling rule

- Scheduling criterion:
  $k^* = \arg\min_{1,\ldots,M}\{MSE_1, \ldots, MSE_M\}$, where

$$MSE_k \triangleq \frac{2n_A - 1}{2P_{B,\tau}^{(k)} L_{B,\tau}^{(k)}} \mathbb{E}\left(\frac{1}{\sigma_{1,k}}\right) \frac{1}{\sigma_{1,k}}$$

Average MSE achieved

- Approximate MSE

$$\mathbb{E} \min\{MSE_1, \ldots, MSE_M\}$$

- Is there a closed form expression for the above?

Closed form for the special case of $n_A = n_B$

- Approximate MSE

$$\mathbb{E}\min\{MSE_1, \ldots, MSE_M\} = \int_0^\infty (1 - \exp(-A/t^2))^M dt$$

where $A \triangleq \frac{2n_A - 1}{2P_{B,\tau}^{(k)} L_{B,\tau}^{(k)}} \mathbb{E}\left(\frac{1}{\sigma_{1,k}}\right)$

Summary

- Proposed a novel channel dependent training for an SM based TDD-MIMO system that was optimized using
  - MSE as a metric
  - a capacity lower bound as a metric
- MSE as metric
  - i-th singular value of the RCT sequence is $\propto$ the square root of the inverse of the i-th singular value of the channel
- Capacity lower bound as a metric
  - i-th singular value of the RCT sequence $\uparrow$ with the power of all $j \neq i$ modes relative to its singular value

Take home lesson

- RCT that adapts to the current CSI can significantly improve the performance of a TDD-MIMO OFDMA system compared to channel agnostic RCT scheme

References

- B. N. Bharath and C. R. Murthy, “Reverse channel training for reciprocal MIMO systems with spatial multiplexing," in Proc. IEEE-ICASSP, 2009
- R. Prasad, B. N. Bharath, and C. Murthy, “Joint Data Detection and Dominant Singular Mode Estimation in Time Varying Reciprocal MIMO Systems," Proc. IEEE-ICASSP, May 2011
- B. N. Bharath and C. R. Murthy, “On the Improvement of Diversity-Multiplexing Gain Tradeoff in a Training Based TDD-SIMO System, IEEE-ICASSP, Mar. 2010
- B. N. Bharath and C. R. Murthy, “Channel Training Signal Design for Reciprocal Multiple Antenna Systems with Beamforming," under preparation
- B. N. Bharath and C. R. Murthy, “Power controlled training in reciprocal MIMO-OFDMA system", to be submitted to ...