Resource Allocation in OFDMA Systems: Relay-Enhanced Cellular Networks

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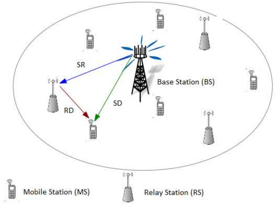
Overview

Motivation

- Ubiquitous high data-rate coverage is the theme of next-generation wireless networks.
- Ubiquitous coverage demands that service has to reach users in the most unfavorable channel conditions.
- Shrink the size of cells, Cost-wise ineffective solution.
- Multihop relaying scheme is one of the attractive solution.
- Resource allocation (RA) becomes more complicated and challenging!

System Model

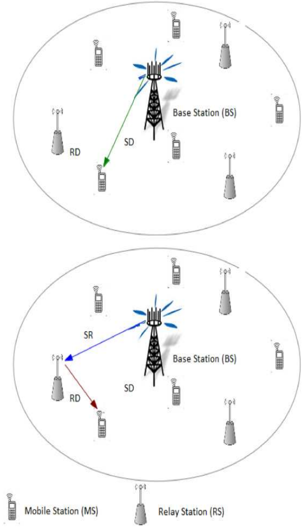
Consider an OFDMA relay-enhanced cellular network as shown below



Notations:

- $\mathcal{K} = \{0,1,2,\ldots,K\}$ denotes the index of BS or a RS.
- $\mathcal{M} = \{1,2,\ldots,M\}$ denotes the index of a user.
- $\mathcal{N} = \{1,2,\ldots,N\}$ denotes the subchannel index.
- p_*^n denotes the power allocated to subchannel n on link $'*'$
- $R_{k,m}^n$ denotes the achievable data rate of m^{th} user.
- γ_*^n denotes the CNRs of the link $'*'$ on the n^{th} subchannel.

Problem Formulation



Achievable rates

Direct path:

$$R^n_{0,m} = R^n_{SD_m} = \log_2(1 + p^n_{SD_m}\gamma^n_{SD_m})$$

Relaying path:

$$R^n_{k,m} = \frac{1}{2}\min\{R^n_{SR_k}, R^n_{R_kD_m}\}$$

Where,

$$R^n_{SR_k} = \log_2(1 + p^n_{SR_k}\gamma^n_{SR_k})$$
$$R^n_{R_kD_m} = \log_2(1 + p^n_{R_kD_m}\gamma^n_{R_kD_m})$$

Problem Formulation

Mathematically, Resource Allocation (RA) problem is formulated as following

$$
\begin{aligned}
\max_{p_*^n} \quad & \sum_{m\in\mathcal{M}} \sum_{n\in\mathcal{N}} \sum_{k\in\mathcal{K}} R_{k,m}^n \\
\text{s.t} \quad \text{c1}: \quad & \sum_{m\in\mathcal{M}} \sum_{n\in\mathcal{N}} \{p_{SD_m}^n + \sum_{k\in\mathcal{K},k\neq 0} \frac{1}{2}(p_{SR_k}^n + p_{R_kD_m}^n)\} \leq P_T \\
& \qquad\qquad or \\
\text{c1}: \quad & \sum_{m\in\mathcal{M}} \sum_{n\in\mathcal{N}} p_{R_kD_m}^n \leq P_R \quad ,\forall k \\
\text{c2}: \quad & p_{SD_m}^n, p_{SR_k}^n, p_{R_kM_m}^n \geq 0 \quad ,\forall k, m, n \\
\text{c3}: \quad & p_{SD_m}^n p_{SR_k}^n = 0, p_{SD_m}^n p_{R_kM_m}^n = 0 \quad ,\forall n
\end{aligned} \tag{1}
$$

Problem Re-Formulation

- Let us introduce $(K+1)MN$ new variables $\{p^n_{k,m} \forall k, m, n\}$

$$p^n_{k,m} = \begin{cases} p^n_{SD_m}, & \text{if } k{=}0 \\ \frac{1}{2}(p^n_{SR_k} + p^n_{R_k D_m}), & \text{otherwise} \end{cases} \quad (2)$$

- The optimal power allocated to the first hop-link of a relaying path and that allocated to its second-hop link should satisfy

$$\frac{p^n_{SR_k}}{p^n_{R_k D_m}} = \frac{\gamma^n_{R_k D_m}}{\gamma^n_{SR_k}} \quad (3)$$

Problem Re-Formulation

With (2) and (3), we get the achievable rate of the k^{th} path of user m on the n^{th} subchannel as

$$R^n_{k,m} = C1_k \log_2(1 + C2^n_{k,m} p^n_{k,m}) \tag{4}$$

Where

$$C1_k = \begin{cases} 1 & \text{if } k = 0 \\ \frac{1}{2} & \text{otherwise} \end{cases}$$

$$C2^n_{k,m} = \begin{cases} \gamma^n_{SD_m} & \text{if } k = 0 \\ \frac{2\gamma^n_{SR_k}\gamma_{R_k D_m}}{\gamma^n_{SR_k}+\gamma^n_{R_k D_m}} & \text{otherwise} \end{cases}$$

Problem Re-Formulation

So the optimization problem (1) becomes

$$
\begin{aligned}
\max_{\{p_{k,m}^n\}} \quad & \sum_{m\in\mathcal{M}} \sum_{n\in\mathcal{N}} \sum_{k\in\mathcal{K}} C1_k \log_2(1 + C2_{k,m}^n p_{k,m}^n) \\
\text{s.t} \quad \text{c1}: \quad & \sum_{m\in\mathcal{M}} \sum_{n\in\mathcal{N}} \sum_{k\in\mathcal{K}} p_{k,m}^n \le P_T \\
\text{c2}: \quad & 0 \le p_{k,m}^n \le P^{\max} \quad ,\forall k, m, n \\
\text{c3}: \quad & p_{k,m}^n p_{r,s}^n = 0 \quad ,\forall\ k \ne r \text{ and/or } m \ne s; \\
& \qquad k, r \in \mathcal{K};\ m, s \in \mathcal{M};\ \text{n} \in \mathcal{N}
\end{aligned} \tag{5}
$$

Problem Re-Formulation

By introducing a group of binary variables $y_{k,m}^n$, problem (5) can be reformulated as

$$\begin{aligned}
\max_{\{p_{k,m}^n, y_{k,m}^n\}} \quad & \sum_{m\in\mathcal{M}}\sum_{k\in\mathcal{K}}\sum_{n\in\mathcal{N}} C1_k \log_2(1 + C2_{k,m}^n p_{k,m}^n) \\
\text{s.t} \quad \text{c1}: \quad & \sum_{m\in\mathcal{M}}\sum_{k\in\mathcal{K}}\sum_{n\in\mathcal{N}} p_{k,m}^n \le P_T \\
\text{c2}: \quad & 0 \le p_{k,m}^n \le y_{k,m}^n P^{\max} \quad , \forall k, m, n \\
\text{c3}: \quad & \sum_{m\in\mathcal{M}}\sum_{k\in\mathcal{K}} y_{k,m}^n \le 1 \quad , \; n \in \mathcal{N} \\
\text{c4}: \quad & y_{k,m}^n \in \{0, 1\} \quad , \; m \in \mathcal{M}, \; k \in \mathcal{K}, \; n \in \mathcal{N}
\end{aligned} \tag{6}$$

State-of-the-art Algorithms

- Method A : Relaxation, Divide and Conquer !
- Method B : Allocate channels to users with maximum gain and then solve for power.
- Method C : Relax, Dual optimization problem, solve for power and then solve for paths and assign subchannels.
- **Room for Improvement!**

Goal and Approach

- **Approach 1** : SBL for RA
- **Approach 2** : IRM Framework for RA

IRM Framework for RA

Goal : Solve the following optimization problem using IRM framework

$$
\begin{aligned}
\max_{\{p_{k,m}^n, y_{k,m}^n\}} \quad & \sum_{m\in\mathcal{M}} \sum_{k\in\mathcal{K}} \sum_{n\in\mathcal{N}} C1_k \log_2(1 + C2_{k,m}^n p_{k,m}^n) \\
\text{s.t} \quad \text{c1}: \quad & \sum_{m\in\mathcal{M}} \sum_{k\in\mathcal{K}} \sum_{n\in\mathcal{N}} p_{k,m}^n \le P_T \\
\text{c2}: \quad & 0 \le p_{k,m}^n \le y_{k,m}^n P^{\max} \quad , \forall k, m, n \\
\text{c3}: \quad & \sum_{m\in\mathcal{M}} \sum_{k\in\mathcal{K}} y_{k,m}^n \le 1 \quad , \; n \in \mathcal{N} \\
\text{c4}: \quad & y_{k,m}^n \in \{0, 1\} \quad , \; m \in \mathcal{M}, \; k \in \mathcal{K}, \; n \in \mathcal{N}
\end{aligned}
\tag{7}
$$

Step 1 : Relaxation

Let $x_{k,m}^{n} = \frac{p_{k,m}^{n}}{P^{max}}$, the above problem can be equivalently rewritten as following:

$$
\begin{aligned}
\max_{\{x_{k,m}^{n}, y_{k,m}^{n}\}} \quad & \sum_{m\in\mathcal{M}}\sum_{n\in\mathcal{N}}\sum_{k\in\mathcal{K}} C1_k \log_2(1 + C2_{k,m}^{n} P^{max} x_{k,m}^{n}) \\
\text{s.t} \quad c1: \quad & \sum_{n\in\mathcal{N}}\sum_{k\in\mathcal{K}}\sum_{n\in\mathcal{N}} P^{\max} x_{k,m}^{n} \leq P_T \quad , m \in \mathcal{M} \\
c2: \quad & 0 \leq x_{k,m}^{n} \leq y_{k,m}^{n} \quad , \forall k, m, n \\
c3: \quad & \sum_{m\in\mathcal{M}}\sum_{k\in\mathcal{K}} y_{k,m}^{n} = 1 \quad , \; n \in \mathcal{N} \\
c4: \quad & y_{k,m}^{n} \in \{0, 1\} \quad , \; m \in \mathcal{M}, \; k \in \mathcal{K}, \; n \in \mathcal{N}
\end{aligned}
\tag{8}
$$

Step 1 : Relaxation Contd.

In particular, $y_{k,m}^n$ satisfies the last two constraints of the above problem if and only if $y_{k,m}^n$ solves the following minimization problem.

$$\begin{aligned}
\min_{\{y_{k,m}^n\}} \quad & \sum_{m\in\mathcal{M}}\sum_{k\in\mathcal{K}}\sum_{n\in\mathcal{N}}(y_{k,m}^n+\epsilon)^q \quad q\in(0,1) \\
\text{subject to} \quad & \sum_{m\in\mathcal{M}}\sum_{k\in\mathcal{K}} y_{k,m}^n = 1 \quad , n\in\mathcal{N} \\
& y_{k,m}^n \geq 0 \quad , k\in\mathcal{K}, m\in\mathcal{M}, n\in\mathcal{N}
\end{aligned} \tag{9}$$

Step 1 : Relaxation Contd.

Based on (9), we can relax (8) to

$$
\begin{aligned}
-\min_{\{x_{k,m}^{n}, y_{k,m}^{n}\}} \quad & -\sum_{m\in\mathcal{M}}\sum_{n\in\mathcal{N}}\sum_{k\in\mathcal{K}} C1_k \log_2(1 + C2_{k,m}^{n} P^{max} x_{k,m}^{n}) + \\
& \lambda \sum_{m\in\mathcal{M}}\sum_{k\in\mathcal{K}}\sum_{n\in\mathcal{N}} (y_{k,m}^{n} + \epsilon)^{q} \\
\text{s.t} \quad \text{c1}: \quad & \sum_{m\in\mathcal{M}}\sum_{k\in\mathcal{K}}\sum_{n\in\mathcal{N}} P^{\max} x_{k,m}^{n} \leq P_T \quad , m \in \mathcal{M} \\
\text{c2}: \quad & 0 \leq x_{k,m}^{n} \leq y_{k,m}^{n} \quad , \forall k, m, n \\
\text{c3}: \quad & \sum_{m\in\mathcal{M}}\sum_{k\in\mathcal{K}} y_{k,m}^{n} = 1 \quad , \text{ n} \in \text{N}
\end{aligned}
\tag{10}
$$

Here $\lambda < 0$.

Step 2: Iterative Re-weighted Minimization (IRM) Framework

Iterate : Until Convergence / Maximum iteration

$$
\begin{aligned}
-\min_{\{x_{k,m}^n, y_{k,m}^n\}} \quad & -\sum_{m\in\mathcal{M}}\sum_{n\in\mathcal{N}}\sum_{k\in\mathcal{K}} C1_k \log_2(1 + C2_{k,m}^n P^{max} x_{k,m}^n) + \\
& \lambda q \sum_{m\in\mathcal{M}}\sum_{k\in\mathcal{K}}\sum_{n\in\mathcal{N}} w_{k,m}^n y_{k,m}^n \\
\text{s.t} \quad \text{c1} : \quad & \sum_{k\in K}\sum_{n\in\mathcal{N}} P^{\max} x_{k,m}^n \leq P_T \\
\text{c2} : \quad & 0 \leq x_{k,m}^n \leq y_{k,m}^n \quad , \forall k, m, n \\
\text{c3} : \quad & \sum_{m\in\mathcal{M}}\sum_{k\in\mathcal{K}} y_{k,m}^n = 1 \quad , \ n \in \mathcal{N} \\
\text{Update:} \quad & W_{k,m}^n(t+1) = [y_{k,m}^n(t+1) + \epsilon(t+1)]^{q-1} \\
& \epsilon(t+1) = \text{Fix it} \quad \textit{or} \quad \text{update it adaptively}
\end{aligned}
$$

Algorithm : IRM Framework for RA

Input : CSI and maximum power allowed for per frame.

Iterate : Until Convergence / Maximum iteration

$$-\min_{\{x^n_{k,m}, y^n_{k,m}\}} \quad -\sum_{m\in\mathcal{M}}\sum_{n\in\mathcal{N}}\sum_{k\in\mathcal{K}} C1_k \log_2(1 + C2^n_{k,m} P^{max} x^n_{k,m}) + \lambda q \sum_{m\in\mathcal{M}}\sum_{k\in\mathcal{K}}\sum_{n\in\mathcal{N}} w^n_{k,m} y^n_{k,m}$$

$$\text{s.t} \quad c1: \quad \sum_{k\in\mathcal{K}}\sum_{n\in\mathcal{N}} P^{\text{max}} x^n_{k,m} \leq P_T$$

$$c2: \quad 0 \leq x^n_{k,m} \leq y^n_{k,m} \quad , \forall k, m, n$$

$$c3: \quad \sum_{m\in\mathcal{M}}\sum_{k\in\mathcal{K}} y^n_{k,m} = 1 \quad , \; n \in \mathcal{N}$$

$$\text{Update:} \quad W^n_{k,m}(t+1) = [y^n_{k,m}(t+1) + \epsilon(t+1)]^{q-1}$$

$$\epsilon(t+1) = \text{Fix it} \quad \textit{or} \quad \text{update it adaptively}$$

Output : Power and channel allocation matrix, $x^n_{k,m}$ and $y^n_{k,m}$, respectively

Variants of the problem

- Different Relay Co-operative schemes (AF and CF).
- Partial/imperfect CSI Knowledge.
- Variable time length.
- Time-sharing of sub-channels, $y_{k,m}^{n} \in [0, 1]$

Summary

- The proposed algorithm uses IRM framework based on an effective relaxation of RA problem.
- The proposed algorithm jointly allocates the channel and power resources. Hence, different from state-of-the-art algorithms.