An Iterative Re-Weighted Minimization Framework for Joint Channel and Power Allocation in the OFDMA System

Peng Liu,Ya-Feng Liu and Jiandong Li
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Presented by
Chethan Kumar A
SPC Lab

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Overview

Abstract

- Joint channel and rate allocation with power minimization in OFDMA has attracted extensive attention.
- The problem is to find a joint channel and power allocation strategy to minimize the total transmission power subject to QoS and the OFDMA constraints.
- In this paper, authors propose a *novel iterative reweighted minimization* framework based on an effective relaxation which jointly allocates the channel and power resources to users.
- Outperforms state-of-the-art algorithms!

System Model

- Consider a multi-user single cell OFDMA system with **K** users sharing **N** channels(**N** $\geq$**K**).

Notations:

- $\mathcal{K} = \{1,2.....\mathbf{K}\}$ denote the set of users.
- $\mathcal{N} = \{1,2.....\mathbf{N}\}$ denote the set of channels.
- $\mathfrak{p}_k^n$ denote the transmit power to user k through channel n.
- α_k^n denote the channel gain between the BS and user k of channel n.
- $SINR_k^n$ denote the SINR of user k on subcarrier n.
- $\mathcal{R}_k$ denote the achievable data rate of k^{th} user.

$$SINR_k^n = \frac{\alpha_k^n \mathfrak{p}_k^n}{\eta_k^n}; \quad \mathcal{R}_k = \sum_{n \in \mathcal{N}} \log_2(1 + SINR_k^n)$$

Problem Formulation

- Mathematically, joint subcarrier and power allocation problem for multi-user OFDMA system can be formulated as

$$\begin{aligned} \min_{\{\mathfrak{p}_k^n\}} \quad & \sum_{k\in\mathcal{K}} \sum_{n\in\mathcal{N}} \mathfrak{p}_k^n \\ \text{subject to} \quad & \sum_{n\in\mathcal{N}} \log_2(1 + SINR_k^n) \geq \gamma_k \quad , k \in \mathcal{K} \\ & 0 \leq \mathfrak{p}_k^n \leq \mathcal{P}_k^n \quad , k \in \mathcal{K}, n \in \mathcal{N} \\ & \mathfrak{p}_k^n \mathfrak{p}_j^n = 0 \quad \forall j \neq k; k, j \in \mathcal{K}, n \in \mathcal{N} \end{aligned} \tag{1}$$

Problem Re-Formulation

- By introducing a group of binary variables $\mathcal{Y}_k^n$ problem(1) can be reformulated as

$$
\begin{aligned}
\min_{\{\mathfrak{p}_k^n, \mathcal{Y}_k^n\}} \quad & \sum_{k \in \mathcal{K}} \sum_{n \in \mathcal{N}} \mathfrak{p}_k^n \\
\text{subject to} \quad & \sum_{n \in \mathcal{N}} \log_2(1 + SINR_k^n) \geq \gamma_k \quad , k \in \mathcal{K} \\
& 0 \leq \mathfrak{p}_k^n \leq \mathcal{Y}_k^n \mathcal{P}_k^n \quad , k \in \mathcal{K}, n \in \mathcal{N} \\
& \sum_{k \in \mathcal{K}} \mathcal{Y}_k^n \leq 1 \quad , n \in \mathcal{N} \\
& \mathcal{Y}_k^n = \{0, 1\} \quad , k \in \mathcal{K}, n \in \mathcal{N}
\end{aligned} \tag{2}
$$

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& \sum_{k\in\mathcal{K}} \mathcal{Y}_k^n \leq 1 \quad , n \in \mathcal{N} \\
& \mathcal{Y}_k^n = \{0, 1\} \quad , k \in \mathcal{K}, n \in \mathcal{N}
\end{aligned}
\tag{3}
$$

- The above problem is strongly-NP hard!

MA Problem Relaxation

- Let $\mathcal{X}_k^n = \frac{\mathfrak{p}_k^n}{\mathcal{P}^{Max}}$ and $\beta_k^n = \frac{\alpha_k^n \mathfrak{p}_k^n}{\eta_k^n}$,the MA problem(2) can be re-written as

$$
\begin{aligned}
\min_{\{\mathcal{X}_k^n, \mathcal{Y}_k^n\}} \quad & \sum_{k \in \mathcal{K}} \sum_{n \in \mathcal{N}} P^{Max} \mathcal{X}_k^n \\
\text{subject to} \quad & \sum_{n \in \mathcal{N}} \log_2(1 + \beta_k^n \mathcal{X}_k^n) \geq \gamma_k \quad , k \in \mathcal{K} \\
& 0 \leq \mathcal{X}_k^n \leq \mathcal{Y}_k^n \quad , k \in \mathcal{K}, n \in \mathcal{N} \\
& \sum_{k \in \mathcal{K}} \mathcal{Y}_k^n = 1 \quad , n \in \mathcal{N} \\
& \mathcal{Y}_k^n = \{0, 1\} \quad , k \in \mathcal{K}, n \in \mathcal{N}
\end{aligned} \tag{4}
$$

MA Problem Relaxation(Contd..)

- In particular $\{\mathcal{Y}_k^n\}$ satisifes the last 2 constraints of problem(3) if it solves the following optimization problem

$$
\begin{aligned}
\min_{\{\mathcal{Y}_k^n\}} \quad & \sum_{k\in\mathcal{K}}\sum_{n\in\mathcal{N}} (\mathcal{Y}_k^n+\epsilon)^q \quad q\in(0,1) \\
\text{subject to} \quad & \sum_{k\in\mathcal{K}} \mathcal{Y}_k^n = 1 \quad , n\in\mathcal{N} \\
& \mathcal{Y}_k^n \geq 0 \quad , k\in\mathcal{K}, n\in\mathcal{N}
\end{aligned}
\tag{5}
$$

MA Problem Relaxation(Contd..)

- Based on the previous reformulation ,we can relax problem(3) to

$$
\begin{aligned}
\min_{\{\mathcal{X}_k^n, \mathcal{Y}_k^n\}} \quad & \sum_{k \in \mathcal{K}} \sum_{n \in \mathcal{N}} P^{Max} \mathcal{X}_k^n + \lambda \sum_{k \in \mathcal{K}} \sum_{n \in \mathcal{N}} (\mathcal{Y}_k^n + \epsilon)^q \\
\text{subject to} \quad & \sum_{n \in \mathcal{N}} \log_2(1 + \beta_k^n \mathcal{X}_k^n) \geq \gamma_k \quad , k \in \mathcal{K} \\
& 0 \leq \mathcal{X}_k^n \leq \mathcal{Y}_k^n \quad , k \in \mathcal{K}, n \in \mathcal{N} \\
& \sum_{k \in \mathcal{K}} \mathcal{Y}_k^n = 1 \quad , n \in \mathcal{N}
\end{aligned}
\tag{6}
$$

Iterative Re-Weighted Framework

- Problem(6) is solved using iterative re-weighted minimization framework.

Relaxed MA problem using IRM framework

- Iterate : Until Convergance

$$\begin{aligned} \min_{\{\mathcal{X}_k^n, \mathcal{Y}_k^n\}} \quad & \sum_{k \in \mathcal{K}} \sum_{n \in \mathcal{N}} P^{Max} \mathcal{X}_k^n + \lambda q \sum_{k \in \mathcal{K}} \sum_{n \in \mathcal{N}} \mathcal{W}_k^n(t) \mathcal{Y}_k^n \\ \text{subject to} \quad & 6(a), 6(b) and 6(c) \\ Update: \quad & \mathcal{W}_k^n(t+1) = [\mathcal{Y}_k^n(t+1) + \epsilon(t+1)]^{q-1} \\ & \epsilon(t+1) = 1e^{-4} \quad or \quad min\{\epsilon(t), \gamma.max(\mathcal{X}_k^n)_2\} \end{aligned} \tag{7}$$

Algorithm for solving MA problem

- Input : Desired rate target γ_k and perfect CSI (Unrealistic!).
- Initialization :λ=N$\mathcal{P}^{Max}$; $\mathcal{W}_k^n(0)$=1 $\forall k \in \mathcal{K}, n \in \mathcal{N}$ and $\epsilon(0)$=1.
- Iterate : Until Convergance

$$\begin{aligned} \min_{\{\mathcal{X}_k^n,\mathcal{Y}_k^n\}} \quad & \sum_{k\in\mathcal{K}}\sum_{n\in\mathcal{N}} P^{Max}\mathcal{X}_k^n + \lambda q \sum_{k\in\mathcal{K}}\sum_{n\in\mathcal{N}} \mathcal{W}_k^n(t)\mathcal{Y}_k^n \\ \text{subject to} \quad & 6(a), 6(b) and 6(c) \\ Update: \quad & \mathcal{W}_k^n(t+1) = [\mathcal{X}_k^n(t+1) + \epsilon(t+1)]^{q-1} \\ & \epsilon(t+1) = 1e^{-4} \quad or \quad min\{\epsilon(t), \gamma.max(\mathcal{X}_k^n)_2\} \end{aligned} \tag{8}$$

- Output : Channel allocation matrix $\mathcal{Y}_k^n$ and Power allocation matrix $\mathcal{X}_k^n$

Experimental setup

- Cell radius is 20 m.
- $L_k^n = 38.48 + 20log(dk), \alpha_k^n = 10^{L_k^n/10}\zeta_k^n$.
- Each subcarrier can have atmost power of 5mW , $\mathcal{P}_k^n = 5$mW.
- Desired data rate for each user is 100bps, $\gamma_k = 100bps$.
- Number of subcarriers is 3 times of number of users, N = 3*K.
- Updates:
  Method 1:

$$w(t+1) = (\mathcal{X}_k^n(t) + \epsilon(t+1))^{q-1}$$
$$\epsilon(t+1) = min\{\epsilon(t), \gamma.max(\mathcal{X}_k^n)_2\}$$

  Method 2:

$$w(t+1) = 1/(\mathcal{X}_k^n(t) + \epsilon(t+1))$$
$$\epsilon(t+1) = min\{\epsilon(t), \gamma.max(\mathcal{X}_k^n)_2\}$$

Total transmission power P_t vs. Number of users K

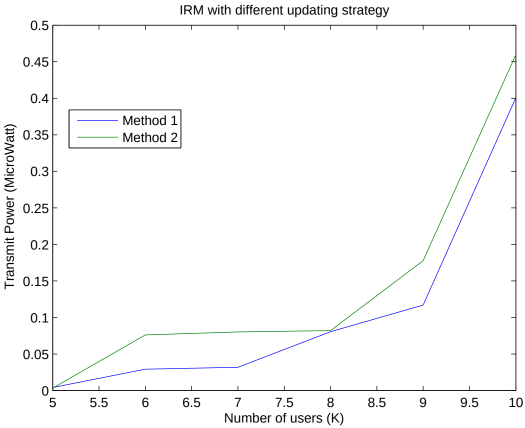

Figure: Total transmit power as a function of Number of users.The following result is obtained after averaging 25 channel realizations.

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Summary

- The proposed algorithm uses *novel iterative reweighted framework* based on an effective relaxation of MA problem.
- The proposed algorithm simultaneously allocates the channel and power resources.Hence, different from state-of-the-art algorithms.
- In fact, it outperforms state-of-the-art algorithms.